NSTX-U

Supported by U.S. DEPARTMENT OF ENERGY | Office of Science

Equilibrium reconstruction including kinetic effects and impact on MHD stability interpretation

Coll of Wm & Mary
Columbia U
CompX
General Atomics
FIU
INL
Johns Hopkins U
LANL
LLNL
Lodestar
MIT
Lehigh U
Nova Photonics
Old Dominion
ORNL
PPPL
Princeton U
Purdue U
SNL
Think Tank, Inc.
UC Davis
UC Irvine
UCLA
UCSD
U Colorado
U Illinois
U Maryland
U Rochester
U Tennessee
U Tulsa
U Washington
U Wisconsin
X Science LLC

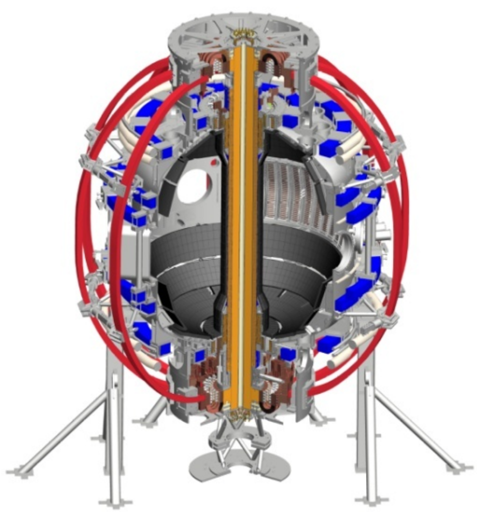
Jon Menard (PPPL)

Z. Wang, Y. Liu (CCFE), R.E. Bell, S.M. Kaye, J.-K. Park, K. Tritz

and the NSTX Research Team

56th Annual Meeting of the APS-DPP
October 27-31, 2014
New Orleans, Louisiana

*This work supported by US DoE contract DE-AC02-09CH11466



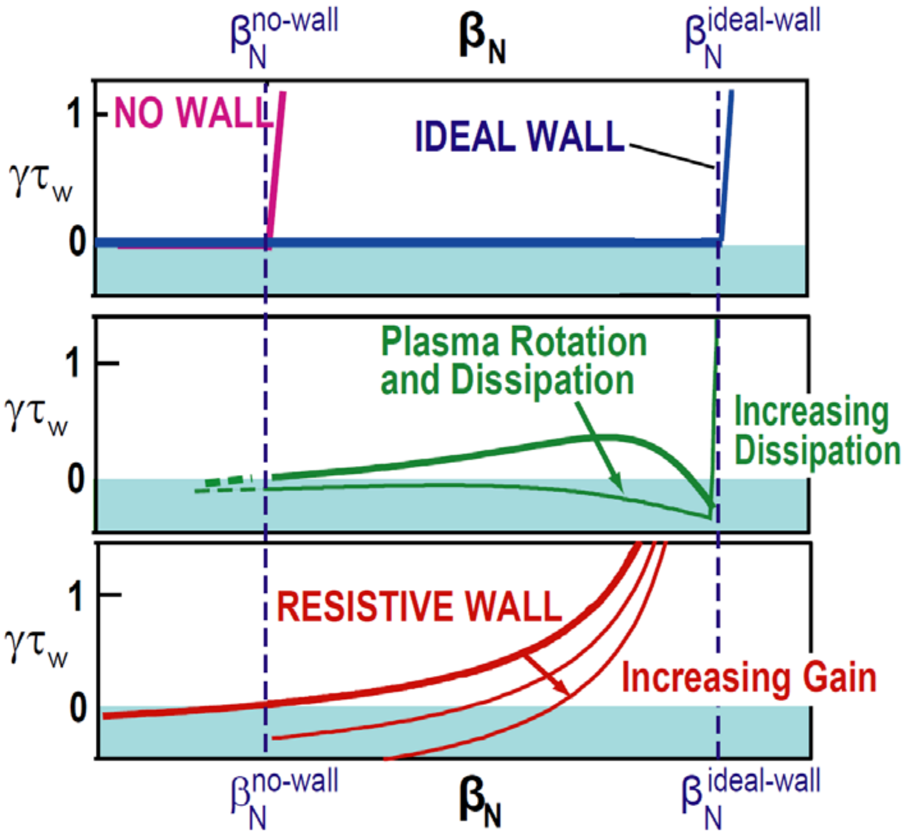
Culham Sci Ctr
York U
Chubu U
Fukui U
Hiroshima U
Hyogo U
Kyoto U
Kyushu U
Kyushu Tokai U
NIFS
Niigata U
U Tokyo
JAEA
Inst for Nucl Res, Kiev
Ioffe Inst
TRINITI
Chonbuk Natl U
NFRI
KAIST
POSTECH
Seoul Natl U
ASIPP
CIEMAT
FOM Inst DIFFER
ENEA, Frascati
CEA, Cadarache
IPP, Jülich
IPP, Garching
ASCR, Czech Rep

ABSTRACT

Non-ideal plasma equilibrium effects such as toroidal rotation and the presence of fast-ions from neutral beam heating can play an important role in MHD stability for both ideal-wall-mode and resistive-wall-mode instabilities. Systematic comparisons between measured and predicted ideal-wall-mode instability characteristics (such as marginal stability threshold and mode real frequency) have been carried out and highlight the sensitivity of the results to the rotation profile and fast-ion density and pressure profiles. A key uncertainty is the potential redistribution of fast-ions by higher frequency Alfvenic instabilities. Analysis indicates that utilizing reconstructed total pressure and rotation profiles as opposed to using modeled/predicted fast-ion pressure and angular momentum profiles from TRANSP in the limit of zero anomalous fast-ion diffusion can yield better agreement between measured and predicted stability characteristics – consistent with apparent redistribution of fast-ions. Reconstruction methodologies and associated stability implications will be discussed.

Pressure-driven kink limit is strong physics constraint on maximum fusion performance

$$P_{fusion} \propto n^2\langle\sigma v\rangle \propto p^2 \propto \beta_T{}^2 B_T{}^4 \propto \beta_N{}^4 B_T{}^4 (1+\kappa^2)^2 / A f_{BS}{}^2$$



M. Chu, et al., Plasma Phys. Control. Fusion 52 (2010) 123001

- Modes grow rapidly above kink limit:
  - γ ~ 1-10% of $\tau_A{}^{-1}$ where τ_A ~ 1μs
- Superconducting "ideal wall" can increase stable β_N up to factor of 2
- Real wall resistive → slow-growing "resistive wall mode" (RWM)
  - $\gamma\ \tau_{wall}$ ~ 1 →
  - ms instead of μs time-scales
- RWM can be stabilized with:
  - kinetic effects (rotation, dissipation)
  - active feedback control

Here we focus on <u>ideal-wall mode</u> (IWM)

Background

- Characteristic growth rates and frequencies of RWM and IWM
  - RWM: $\gamma\tau_{wall} \sim 1$ and $\omega\tau_{wall} < 1$
  - IWM: $\gamma\tau_A \sim$ 1-10% ($\gamma\tau_{wall} >> 1$) and $\omega\tau_A \sim \Omega_\phi\tau_A$ (1-30%) ($\omega\tau_{wall} >> 1$)
- Kinetic effects important for RWM (see J. Berkery poster PP8.00064)
  - Publications: Berkery, et al. PRL 104 (2010) 035003, Sabbagh, et al., NF 50 (2010) 025020
- Rotation and kinetic effects largely unexplored for IWM
  - Such effects generally higher-order than fluid terms (∇p, $J_{||}$, $|\delta B|^2$, wall)
- **Calculations for NSTX indicate both rotation and kinetic effects can modify both IWM (and RWM) stability limits**
  - High toroidal rotation generated by co-injected NBI in NSTX
    - Fast core rotation: $\Omega_\phi / \omega_{sound}$ up to ~1, $\Omega_\phi / \omega_{Alfven}$ ~ up to 0.1-0.3
  - Fluid/kinetic pressure is dominant instability drive in high-β ST plasmas

MARS-K: self-consistent linear resistive MHD including toroidal rotation and drift-kinetic effects

- **Perturbed single-fluid linear MHD:**

Y.Q. Liu, et al., Phys. Plasmas 15, 112503 2008

$$(\gamma + in\Omega)\xi = \mathbf{v} + (\xi\cdot\nabla\Omega)R^2\nabla\phi$$

$$\rho(\gamma + in\Omega)\mathbf{v} = \mathbf{j}\times\mathbf{B} + \mathbf{J}\times\mathbf{Q} - \nabla\cdot\mathbf{p}$$
$$\boxed{+\rho\left[2\Omega\hat{\mathbf{Z}}\times\mathbf{v} - (\mathbf{v}\cdot\nabla\Omega)R^2\nabla\phi\right] - \nabla\cdot(\rho\xi)\Omega\hat{\mathbf{Z}}\times\mathbf{V}_0}$$

$$(\gamma + in\Omega)\mathbf{Q} = \nabla\times(\mathbf{v}\times\mathbf{B}) + (\mathbf{Q}\cdot\nabla\Omega)R^2\nabla\phi - \nabla\times(\eta\mathbf{j})$$

$$(\gamma + in\Omega)p = -\mathbf{v}\cdot\nabla P - \Gamma P\nabla\cdot\mathbf{v} \qquad \mathbf{j} = \nabla\times\mathbf{Q}$$

- **Drift-kinetic effects in perturbed anisotropic pressure *p*:**

$$\mathbf{p} = p\mathbf{I} + p_{\parallel}\hat{\mathbf{b}}\hat{\mathbf{b}} + p_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})$$

$$p_{\parallel}e^{-i\omega t + in\phi} = \sum_{e,i}\int d\Gamma M v_{\parallel}^2 f_L^1$$

$$p_{\perp}e^{-i\omega t + in\phi} = \sum_{e,i}\int d\Gamma \frac{1}{2} M v_{\perp}^2 f_L^1$$

$$f_L^1 = -f_\epsilon^0 \epsilon_k e^{-i\omega t + in\phi}\sum_{m,l,u} X_m^u H_{ml}^u \lambda_{ml} e^{-in\tilde{\phi}(t) + im\langle\dot{\chi}\rangle t + il\omega_b t}$$

$$H_L = \frac{1}{\epsilon_k}[M v_{\parallel}^2 \vec{\kappa}\cdot\xi_{\perp} + \mu(Q_{L\parallel} + \nabla B\cdot\xi_{\perp})]$$

- Rotation and rotation shear effects:
- Mode-particle resonance operator:

Diamagnetic

$$\lambda_{ml} = \frac{n[\omega_{*N} + (\hat{\epsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n(\langle\omega_d\rangle + \omega_E) + [\alpha(m + nq) + l]\omega_b - i\nu_{\text{eff}} - \omega}$$

Precession ExB Transit and bounce Collisions

- Fast ions: analytic slowing-down f(v) model – isotropic or anisotropic

This poster

- Include toroidal flow only: $\mathbf{v}_\phi = \mathbf{R}\Omega_\phi(\psi)$ and $\omega_E = \omega_E(\psi)$

Real part of complex energy functional provides equation for growth-rate useful for understanding instability sources

Dispersion relation

$$\delta K + \delta W = 0$$

Kinetic energy

$$\delta K = \frac{1}{2}\int d^3x\rho(\gamma + in\Omega)^2 \left|\vec{\xi}_\perp\right|^2$$

Potential energy

$$\delta W = -\frac{1}{2}\int d^3x\, \mathbf{F}\cdot\xi_\perp^*$$

$$\delta K_1 = -\frac{1}{2}\int d^3x\rho\left|\vec{\xi}_\perp\right|^2$$

Growth rate equation: mode growth for $\delta W^{re} < 0$

$$\left(\gamma^{re}\right)^2 = \left(\delta W_K^{re} + \delta W_F^{re} + \delta W_{vb} + \delta W_{rot}^{re}\right) / \delta K_1$$

$$\delta W_K = -\frac{1}{2}\int d^3x\mathbf{F}^K\cdot\xi_\perp^* \qquad \mathbf{F}^K = -\nabla\cdot\mathbf{p}^{\text{kinetic}}$$

$$\delta W_{rot} = \delta W_\Omega + \delta W_{d\Omega} + \delta W_{cf} + \delta K_2$$

$$\delta W_F^p = -\frac{1}{2}\int d^3x\mathbf{F}^p\cdot\xi_\perp^*$$
$$= \frac{1}{2}\int d^3x\left[(\xi_\perp\cdot\nabla P)\nabla\cdot\xi_\perp^* + \Gamma P|\nabla\cdot\xi|^2 - \Gamma P(\nabla\cdot\xi)(\nabla\cdot\xi_\parallel^*)\right] + S_F^p$$

$$\delta W_F^j = -\frac{1}{2}\int d^3x\mathbf{F}^j\cdot\xi_\perp^* = \frac{1}{2}\int d^3x|Q|^2 + S_F^j$$

$$\delta W_F^Q = -\frac{1}{2}\int d^3x\mathbf{F}^Q\cdot\xi_\perp^* = \frac{1}{2}\int d^3x\left[J_\parallel\hat{\mathbf{b}}\cdot\xi_\perp^*\times\mathbf{Q}_\perp - \frac{Q_\parallel}{B}(\xi_\perp^*\cdot\nabla P)\right]$$

$$S_F^p = -\frac{1}{2}\int[(\xi_\perp\cdot\nabla P) + \Gamma P\nabla\cdot\xi]\,\xi_\perp^*\cdot d\mathbf{s}$$

$$S_F^j = \frac{1}{2}\int BQ_\parallel\xi_\perp^*\cdot d\mathbf{s}$$

Coriolis - Ω

$$\delta W_\Omega = \frac{1}{2}\int d^3x\left[-2\rho\Omega(\gamma + in\Omega)\mathbf{Z}\times\vec{\xi}_1\cdot\vec{\xi}_\perp^*\right]$$

Coriolis - dΩ/dρ

$$\delta W_{d\Omega} = \frac{1}{2}\int d^3xR\left(2\rho\Omega\left(\vec{\xi}_1\cdot\nabla\Omega\right)\vec{\xi}_{\perp\mathbf{R}}^*\right)$$

Centrifugal

$$\delta W_{cf} = \frac{1}{2}\int d^3xR\Omega^2\nabla\cdot\left(\rho\vec{\xi}_1\right)\vec{\xi}_{\perp\mathbf{R}}^*$$

Differential kinetic

$$\delta K_2 = -\frac{1}{2}\int d^3x\rho(\omega + n\Omega)^2\left|\vec{\xi}_\perp\right|^2$$

Note: n = -1 in MARS

Equilibrium force balance model including toroidal rotation

Force balance for species s: $$\vec{J}_s \times \vec{B} = \nabla p_s + \rho_s \vec{v}_s \cdot \nabla \vec{v}_s + Z_s e n_s \nabla \Phi$$

Assume: $T_s = T_s(\psi)$ $\quad v_{\phi s} = R\Omega_{\phi s}$ $\quad \Omega_{\phi s} = \Omega_{\phi s}(\psi)$

B• above → $$n_s(\psi, R) = N_s(\psi)\exp\left(\frac{m_s\Omega_{\phi s}^2(R^2 - R_{\text{axis}}^2)}{2k_B T_s} - \frac{Z_s e\Phi(\psi,\theta)}{k_B T_s}\right)$$

Exact multi-species solution requires iteration to enforce quasi-neutrality → simplify → intrinsically quasi-neutral if all n_s have same exponential dependence. This approximate solution assumes main ions dominate centrifugal potential.

$$\vec{J} \times \vec{B} = \sum_s \nabla(n_s T_s(\psi)) + \sum_s m_s n_s {\Omega_{\phi s}}^2 \nabla\left(\frac{R^2}{2}\right) \qquad 0 = \sum_s N_s(\psi) Z_s$$

$$n_s(\psi, R) = N_s(\psi)\exp\left(U(\psi)\left(\frac{R^2}{R_{\text{axis}}^2} - 1\right)\right) \qquad U(\psi) = \frac{P_\Omega(\psi)}{P_K(\psi)}$$

$$P_\Omega(\psi) = \frac{\sum_s N_s(\psi) m_s {\Omega_{\phi s}}^2 R_{\text{axis}}^2}{2} \qquad P_K(\psi) = \sum_s N_s(\psi) T_s(\psi)$$

Grad-Shafranov Equation (GSE) including toroidal rotation

Total force balance:

$$\rho \vec{v} \cdot \nabla \vec{v} \approx -\rho \Omega_\phi^2 \nabla R^2/2 = \left(\frac{J_\phi}{R} - \frac{FF'}{\mu_0 R^2} \right) \nabla \psi - \nabla p$$

$$\vec{B} = \nabla \psi \times \nabla \phi + F \nabla \phi$$

Rotation-modified GSE:

$$\frac{J_\phi}{R} = \frac{FF'}{\mu_0 R^2} + \left. \frac{\partial p}{\partial \psi} \right|_R \qquad \rho \Omega_\phi^2 R = \left. \frac{\partial p}{\partial R} \right|_\psi$$

$$p(\psi, R) = P_{\mathrm{K}}(\psi) \exp \left(U(\psi) \left(\frac{R^2}{R_{\mathrm{axis}}^2} - 1 \right) \right)$$

LRDFIT reconstructions with rotation determine 3 flux functions:

- **$U(\psi)$ - <u>based on fitting electron density profile asymmetry</u> (not C^{6+} rotation data)**
- **$P_K(\psi)$ and $FF'(\psi)$ – full kinetic reconstruction → fit to magnetics, iso-T_e, MSE with E_r corrections, thermal pressure between r/a = 0.6-1.**

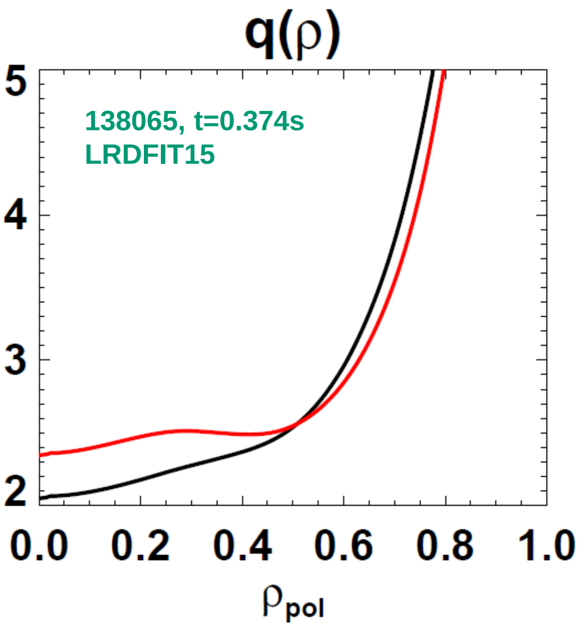
Study 2 classes of IWM-unstable plasmas spanning low to high β_N

- Low β_N limit ~3.5, often saturated/long-lived mode
  - q_{min} ~ 2-3
  - Common in early phase of current flat-top
  - Higher fraction of beam pressure, momentum (lower n_e)

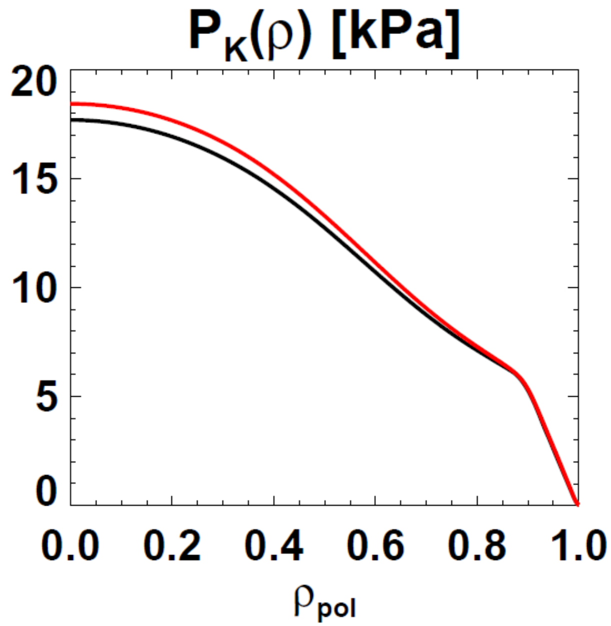
- Intermediate β_N limit ~ 5
  - q_{min} ~1.2-1.5
  - Typical good-performance H-mode, H_{98} ~ 0.8-1.2

Impact of including rotation on q, P_K, P_Ω

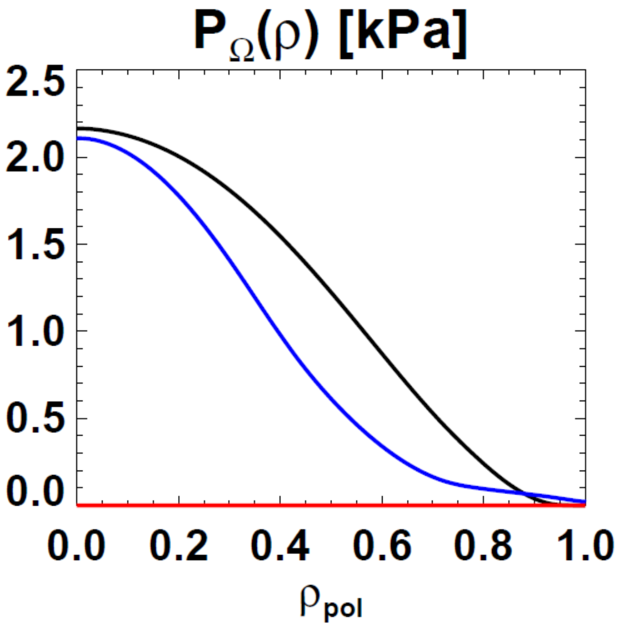
- Black – rotation included (from fit to n_e profile asymmetry)
- Red – rotation set to 0 in reconstruction



Reconstructed core q(ρ) lower with rotation included



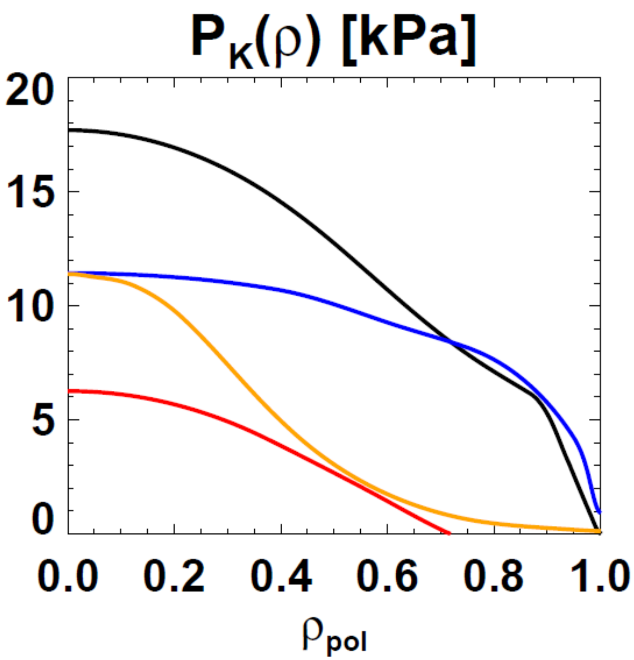
Reconstructed core $P_K(\rho)$ slightly lower w/ rotation included



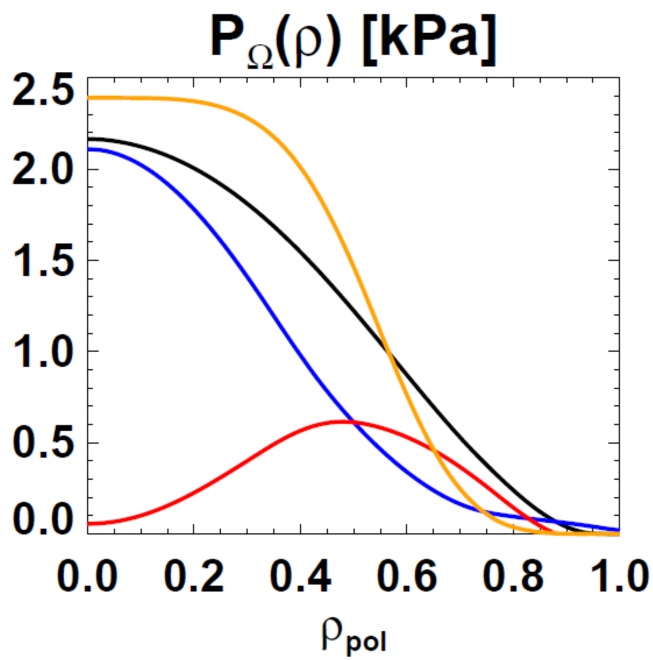
Reconstructed core $P_\Omega(\rho)$ comparable to measured thermal ion rotation pressure (they should be similar)

Reconstructions imply significant fast-ion profile broadening

- Black: reconstruction with rotation included (n_e asymmetry)
- Blue: measured thermal
- Red: recons. minus thermal, Orange: TRANSP (no FI diffusion)



Reconstructed core fast ion $P_K(\rho)$ significantly lower than TRANSP calculation



Reconstructed fast ion $P_\Omega(\rho)$ significantly broader, lower than TRANSP calculation

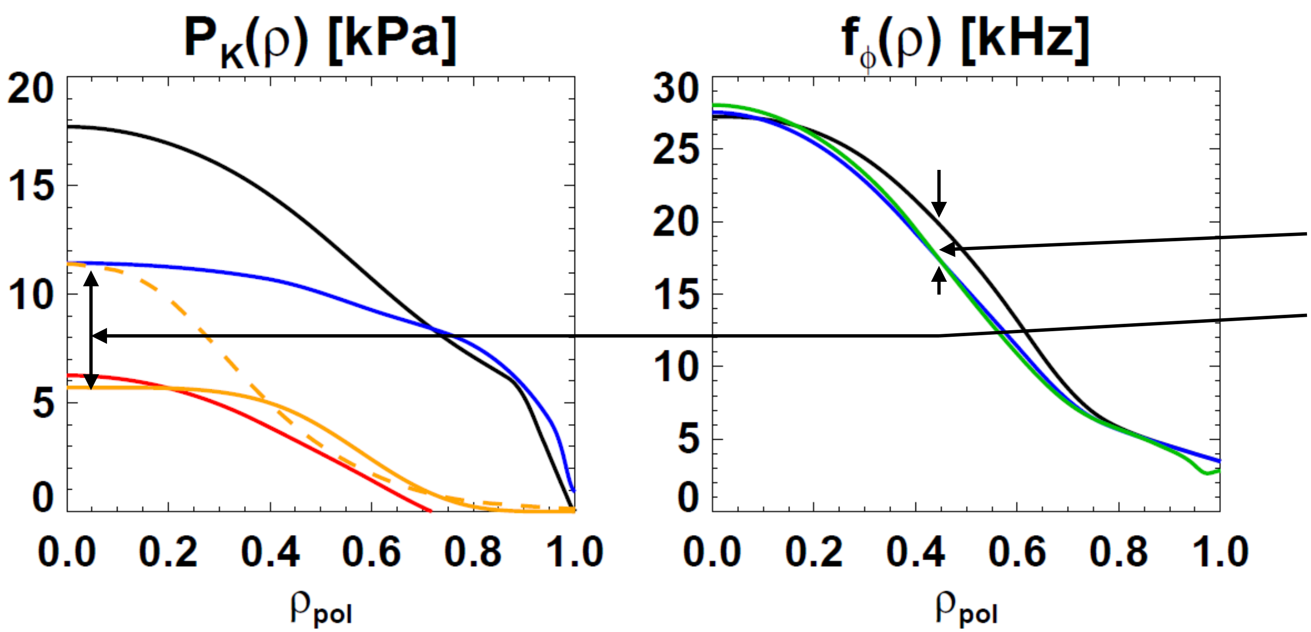
$$U(\psi) = \frac{P_\Omega(\psi)}{P_K(\psi)}$$

$$\exp\left(U(\psi)\left(\frac{R^2}{R_{\rm axis}^2} - 1\right)\right)$$

NOTE: there is substantial uncertainty in P_Ω near the magnetic axis since U is indeterminate there, i.e. U could be much larger or smaller w/o impacting the density asymmetry fit

Profiles after fast-ion density profile broadening

- Black: reconstruction with rotation included (n_e asymmetry)
- Blue: measured thermal, Red: recons. minus thermal
- Orange: TRANSP with FI density profile broadening (post-facto)



Because fast ion density is low, the impact of fast-ions on total toroidal rotation f_ϕ is weaker than impact on P_Ω

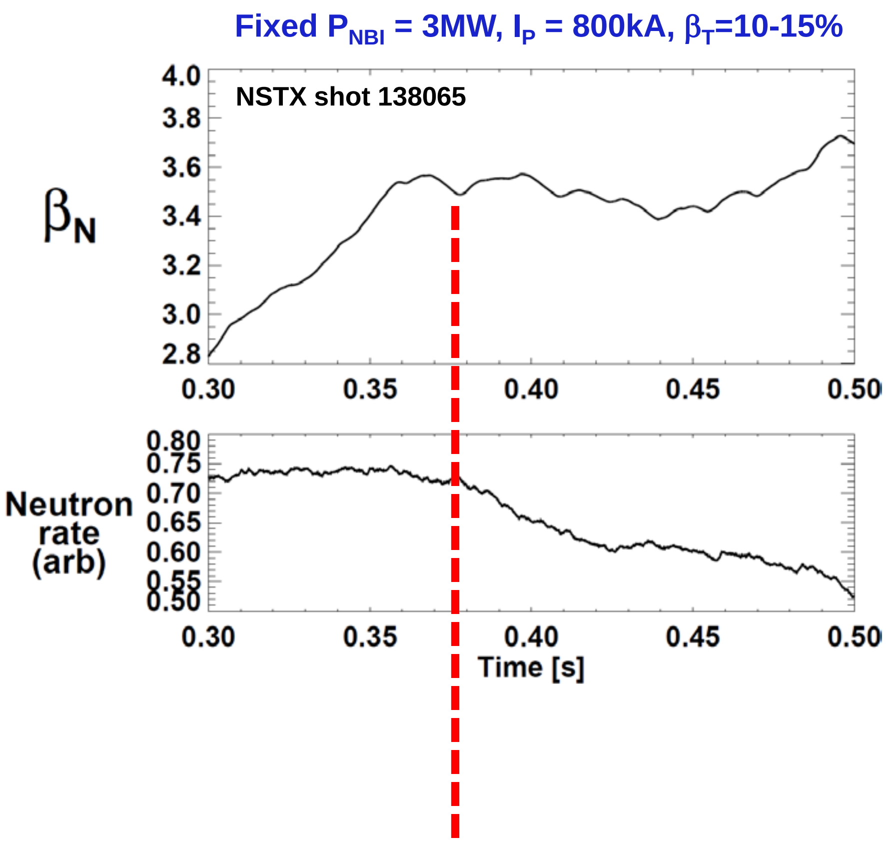
Implication: fast-ion redistribution or loss likely more important for pressure than rotation

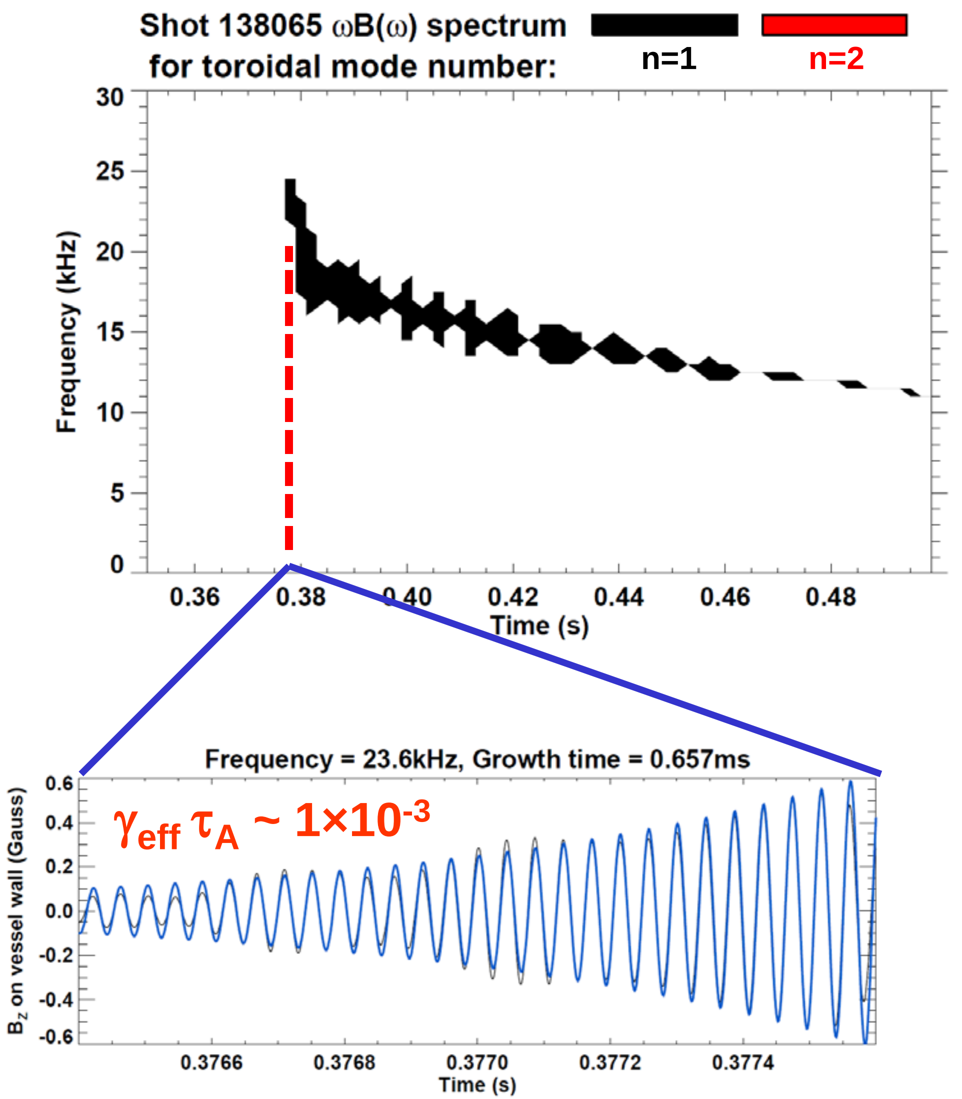
Dash → Solid = fast-ion density broadening: conserve FI stored energy, number, also T_{fast} (ρ)

f_ϕ(r) including fast ions is broader than $f_{\phi\text{–Carbon}}(\rho)$ & f_{ExB}

MARS-K uses P_K, f_ϕ , f_{ExB}

Low β_N limit ~ 3.5: Saturated f=15-30kHz n=1 mode common during early I_P flat-top phase

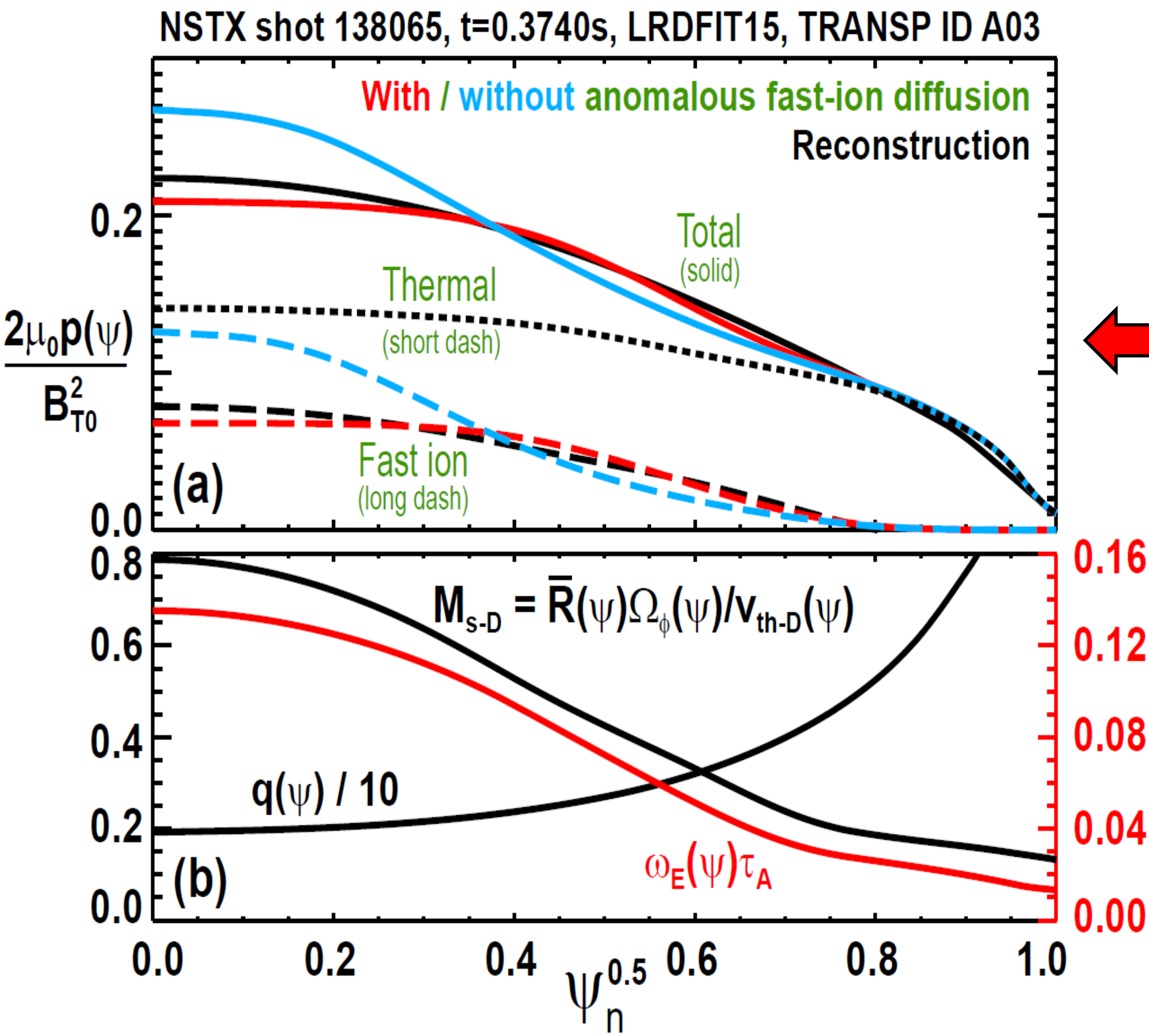




Mode clamps β_N to ~3.5, reduces neutron rate ~20%

sometimes slows → locks → disrupts

Kinetic profiles used in analysis



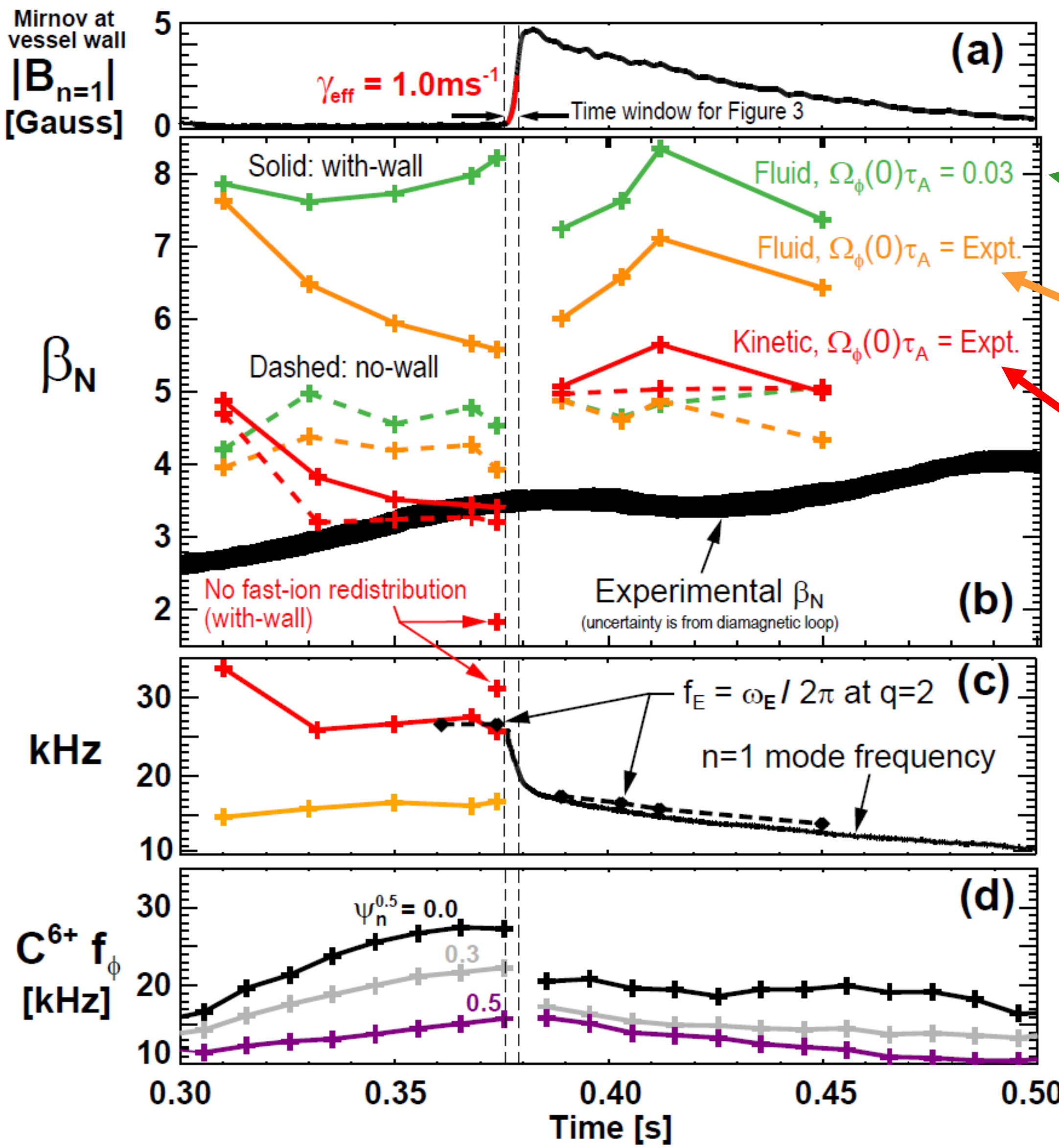
Use broadened fast-ion *n, p* profiles (red curves)

(consistent w/ reconstruction)

- q ≈ 2 in core
- D sound Mach number $M_{s\text{-}D}$ → 0.8 on-axis → significant drive for rotational instability

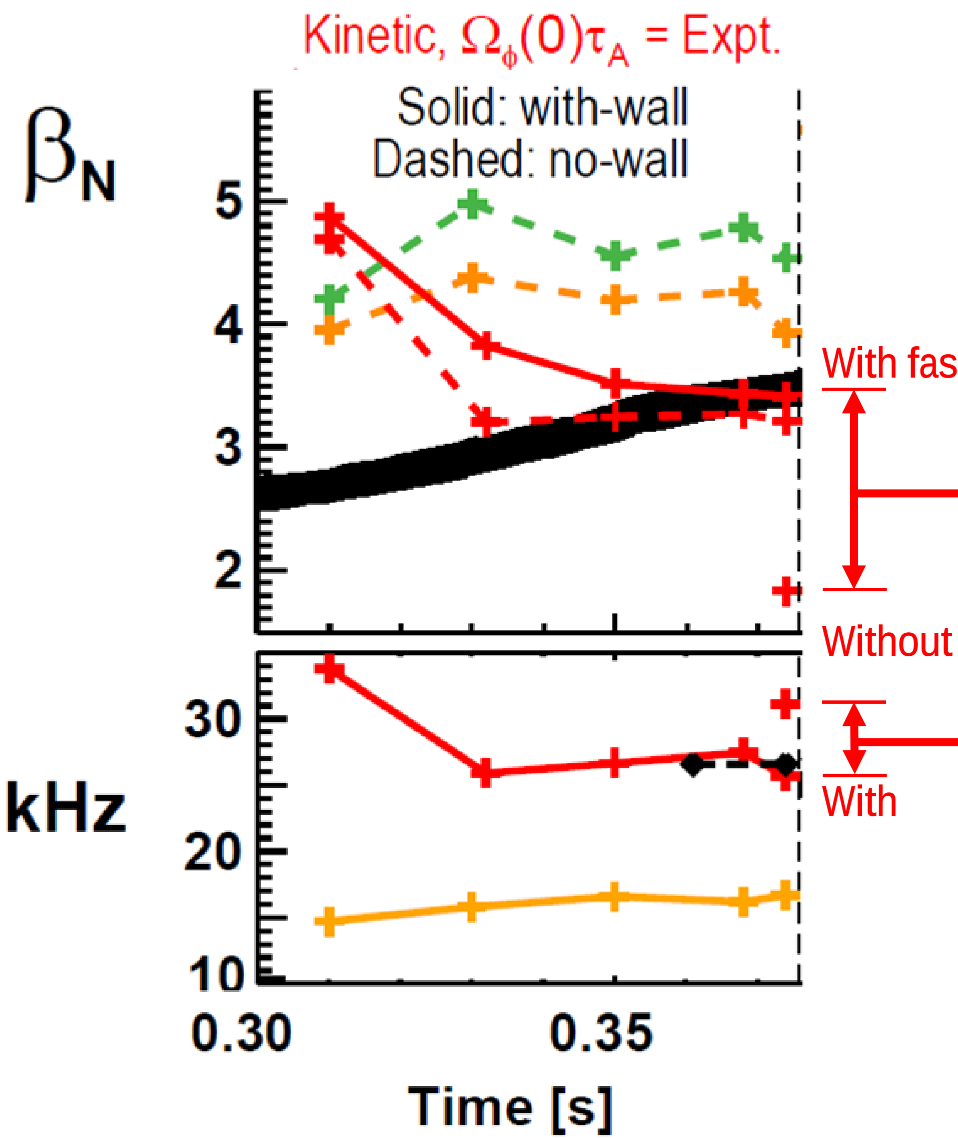
$$\delta \hat{W}_{rot} \sim \delta W_{\nabla p} \Rightarrow v_\phi \sim v_{th-ion}/\sqrt{q}$$
$$\Rightarrow \Omega_\phi \tau_A \sim \sqrt{\beta_{thermal}/2q}$$

Predicted stability evolution using MARS-K compared to experiment



- Mode grows quickly, but decays slowly – kink or tearing mode?
- Low-rotation fluid with-wall limit is very high → marginal β_N ~ 7-8
- Increasing rotation lowers max β_N to ~5.5 at mode onset time
- Full kinetic treatment including fast-ions → marginal β_N ~3.5 → most consistent with experiment
- Full kinetic: predicted mode *f* matches measured *f* ~ 26kHz
- Fluid model under-predicts *f*
- Rotation increasing until mode onset, then drops, then remains lower while mode is present.

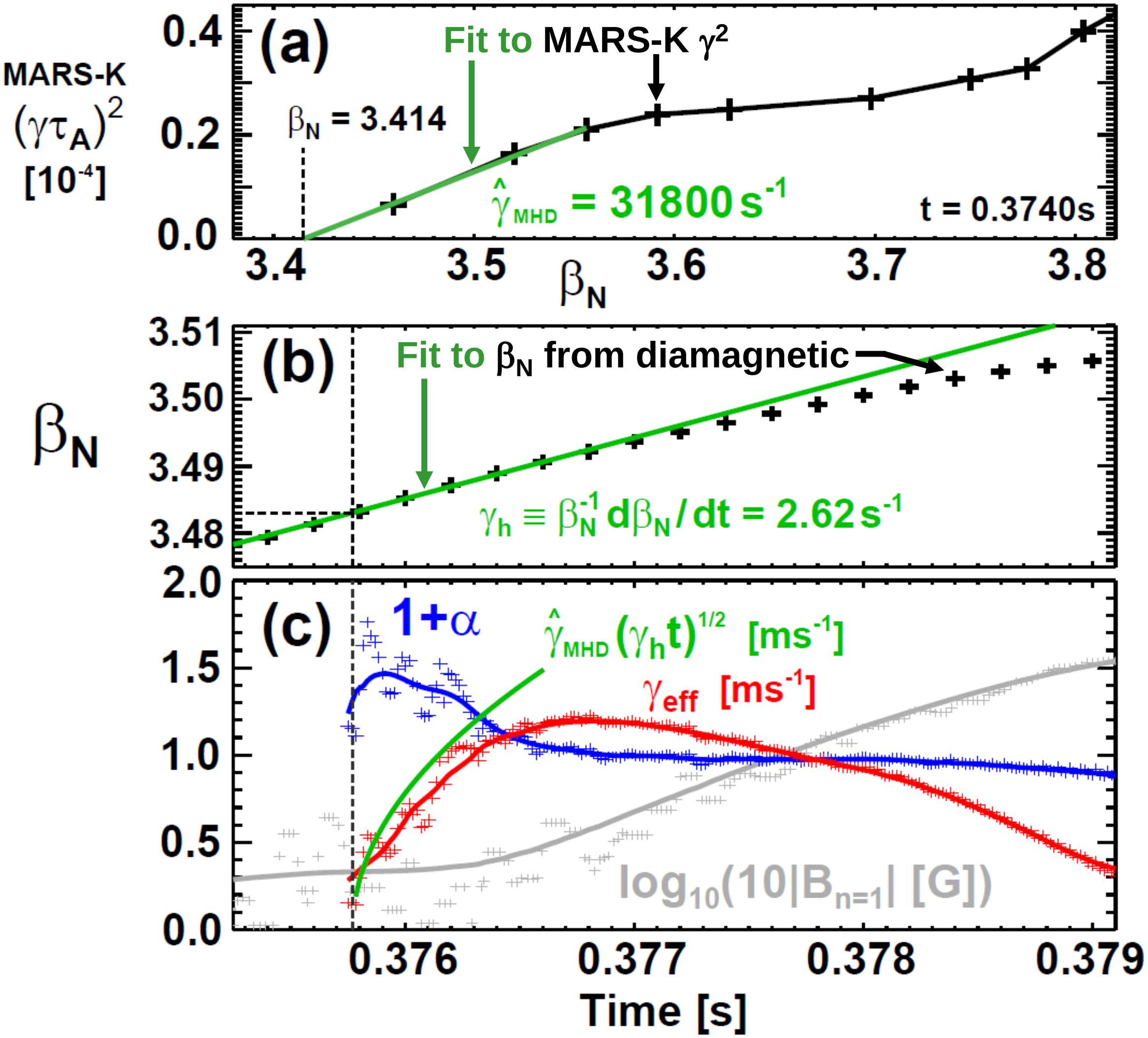
Fast ion broadening has significant impact on predicted stability



Without broadening, predicted marginally stable β_N would be much lower than experiment

And predicted frequency would be higher than observed

Observed mode initial γ consistent with kink/MARS-K



- Use Callen method

  J. D. Callen *et al.*, Physics of Plasmas **6**, 2963 (1999)

- MARS-K γ^2 linear in β_N near marg. stability

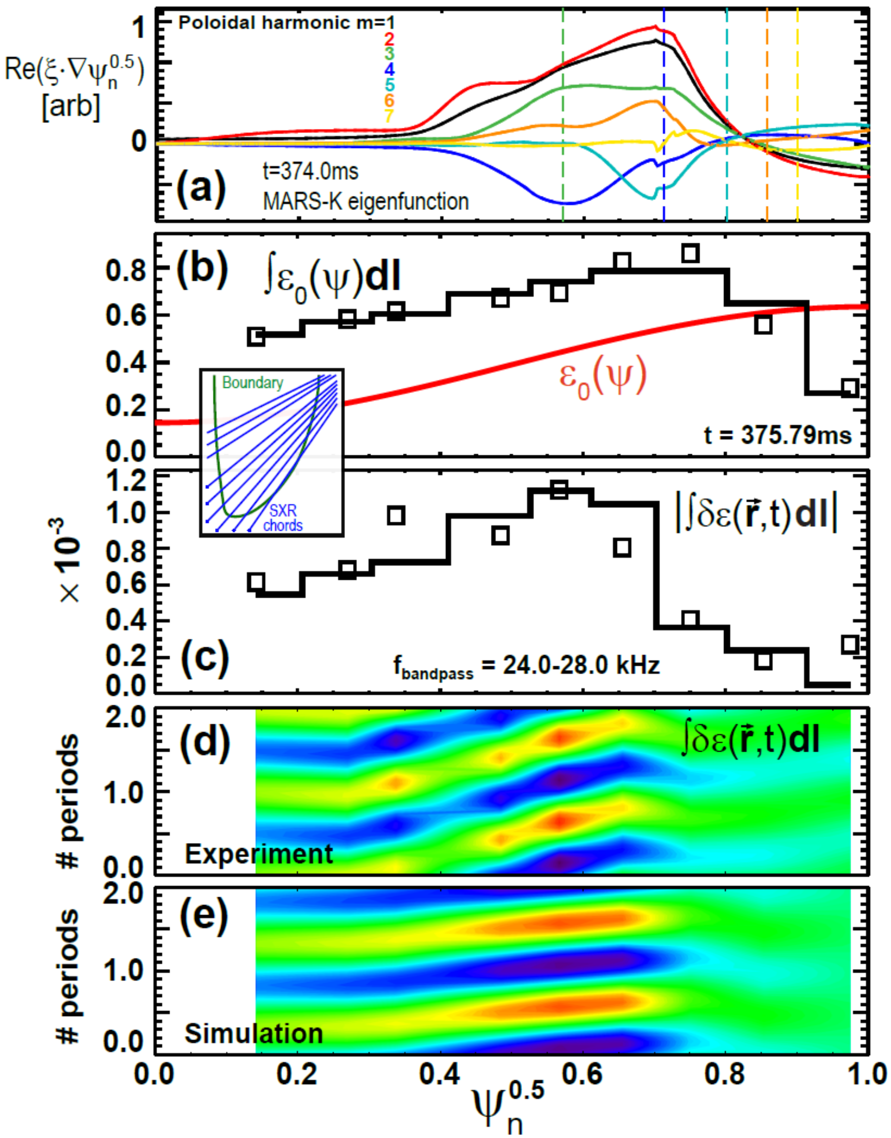
- Rate of rise of β_N tracked using diamagnetic loop

- For first 0.5ms, growth is consistent with Callen hybrid γ model for ideal instability:

$$\xi = \xi_0 \exp[(\gamma_{eff} t)^{(1+\alpha)}]$$

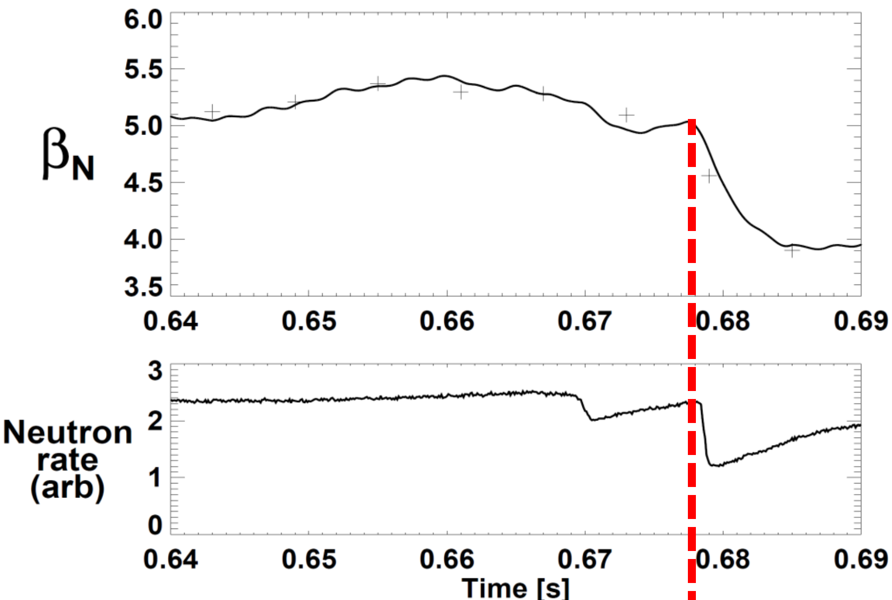
α = 0.5 for ideal mode

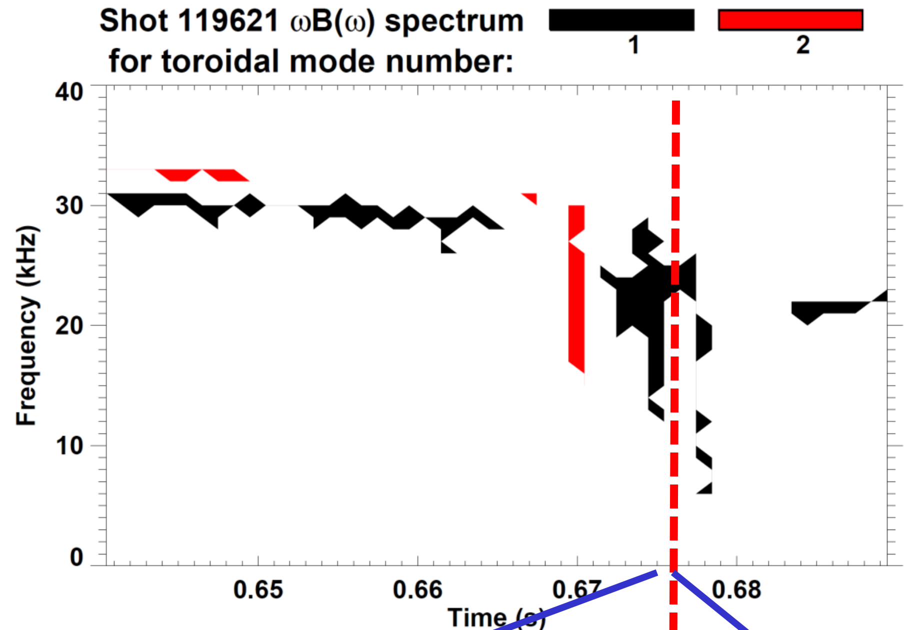
SXR data also consistent with kink/MARS-K at onset

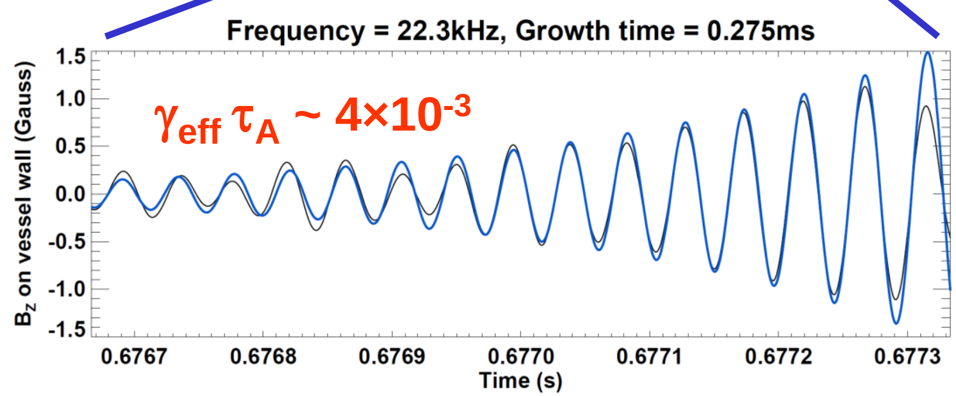


- MARS-K IWM kink eigenfunction largest amplitude for r/a = 0.5-0.8
- Simple/smooth emission profile $\varepsilon_0(\psi)$ can reproduce line-integrated SXR
- …and can reproduce line-integrated SXR fluctuation amplitude profile
- …and has same kink-like structure vs. time and SXR chord position
  - Although the "slope" of the simulated eigenfunction is shallower than measured… rotation or fast-ion effect?

Intermediate β_N limit ~ 5: Small f=30kHz continuous n=1 mode precedes larger 20-25kHz n=1 bursts





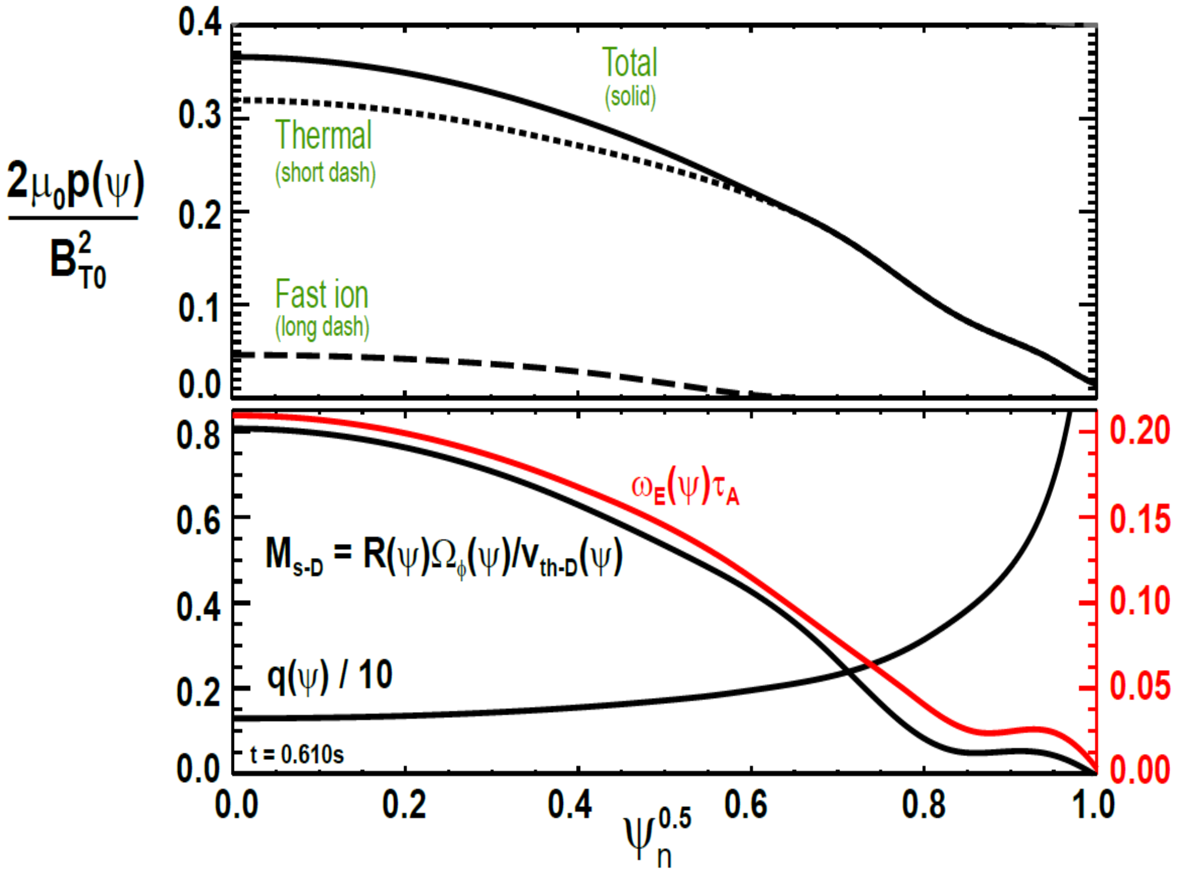


First large n=1 burst → 20% drop in β_N

50% neutron rate drop

Later n=1 modes → full disruption

Kinetic profiles used in analysis



Fast ion pressure lower in this shot due to higher n_e

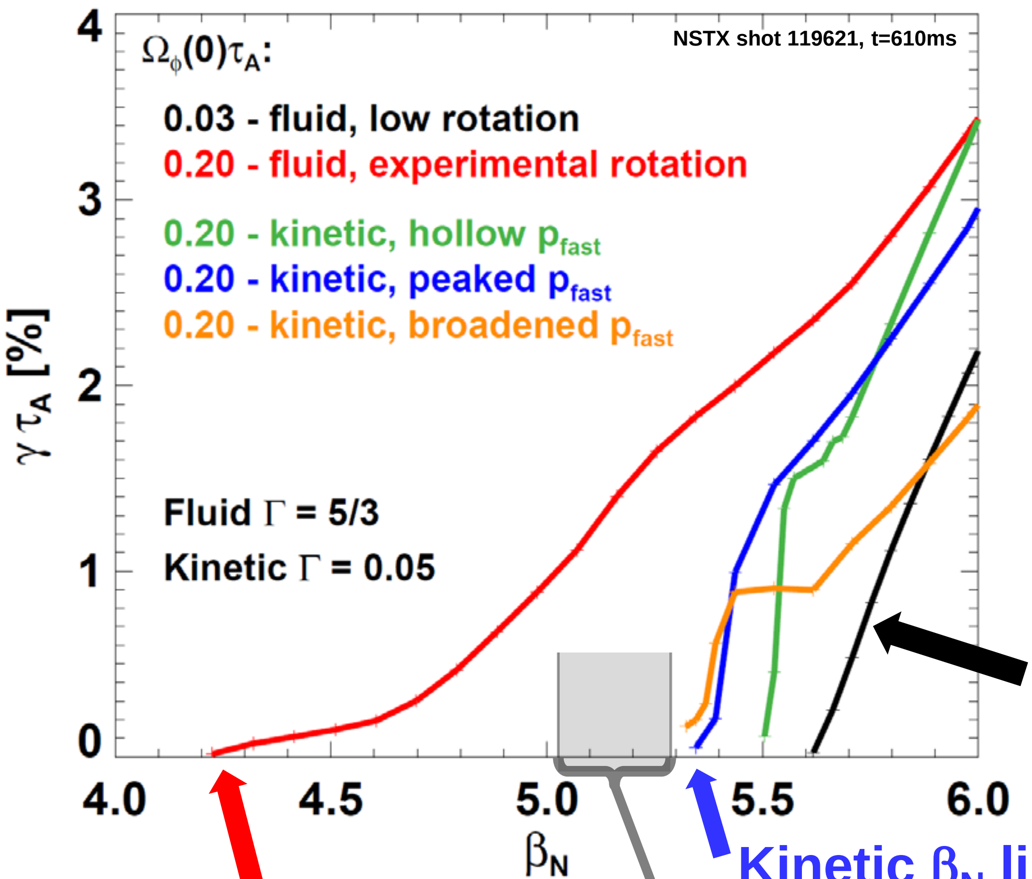
Compute fast-ion from reconstructed total - thermal

- $q \approx 1.3$ in core
- D sound Mach number $M_{s\text{-}D} \rightarrow 0.8$ on-axis $\rightarrow$ significant drive for rotational instability

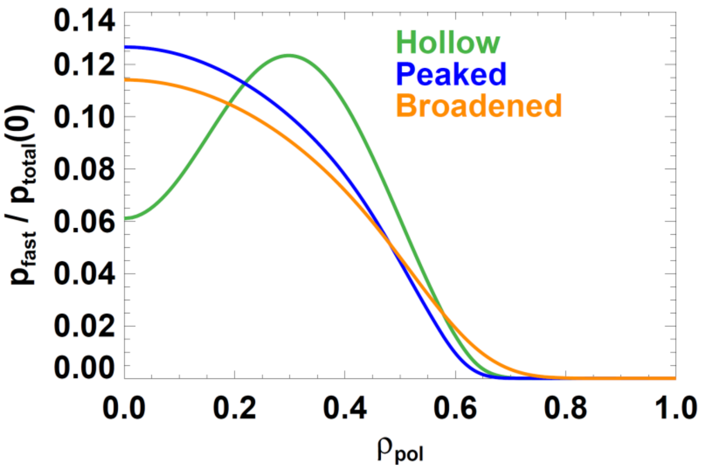
- But, expect weaker rotational destabilization since $M_{s\text{-}D}$ similar, q lower

$$\delta \hat{W}_{rot} \sim \delta W_{\nabla p} \Rightarrow v_\phi \sim v_{th-ion}/\sqrt{q}$$

Kinetic IWM β_N limit consistent with experiment, fluid calculation under-predicts experimental limit



Fast-ion p profiles used in kinetic calculations



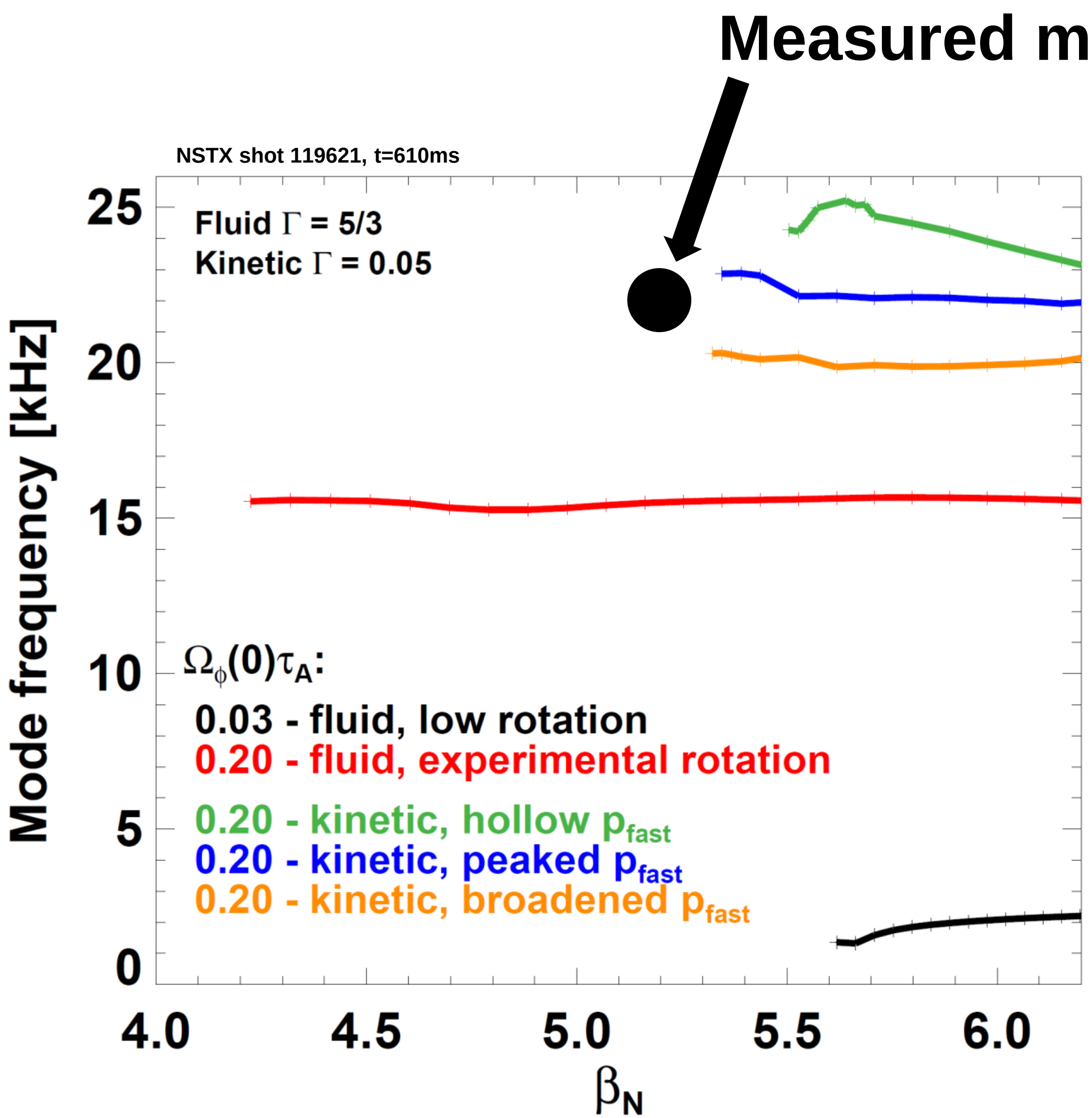
Fluid β_N limit for low Ω_ϕ

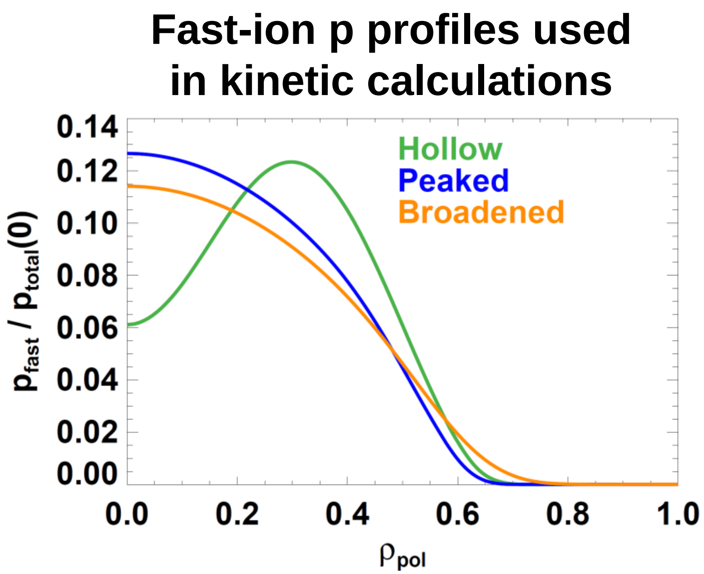
Kinetic β_N limit for experimental Ω_ϕ (weakly dependent on fast-ion peaking factor)

Experimental β_N for n=1 mode onset

Fluid β_N limit for experimental Ω_ϕ ($\Delta\beta_N$ = -1.4 vs. low rotation)

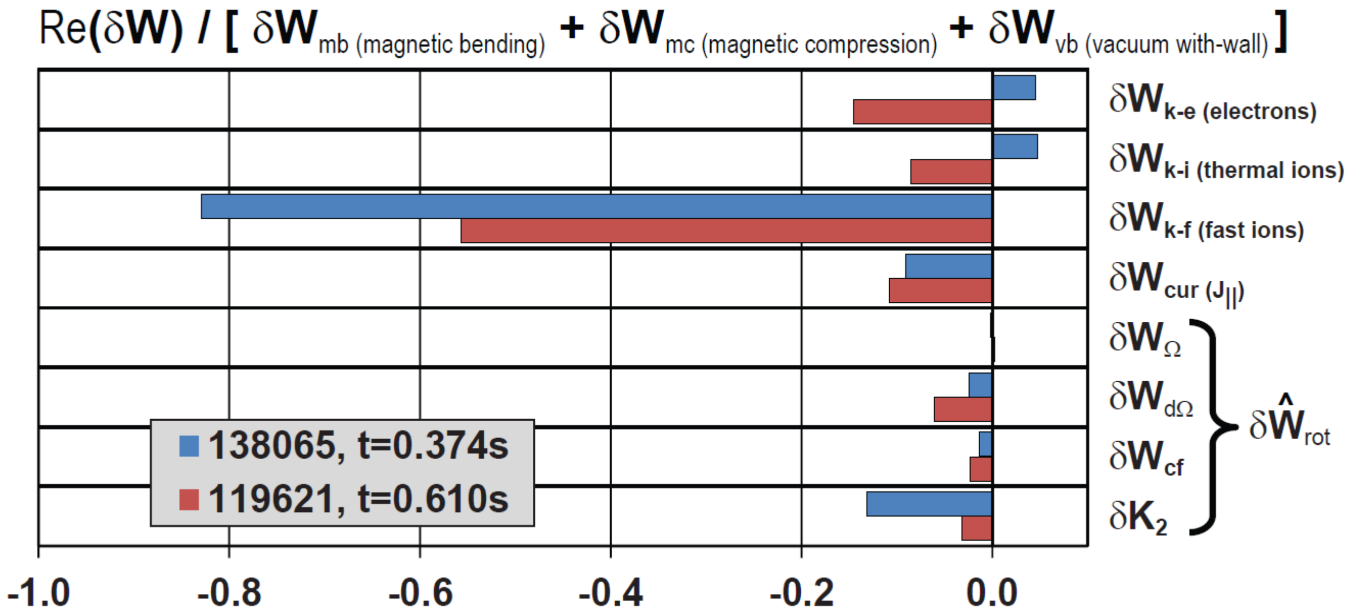
Measured IWM real frequency more consistent with kinetic model than fluid model





IWM energy analysis near marginal stability elucidates trends from growth-rate scans

- Fast-ions in $\mathrm{Re}(\delta W_k)$ = dominant destabilization in both shots
  - Balanced against field-line bending+compression + vacuum stabilization
- Shot 138065 has larger destabilization from fast-ions & rotation
  - Consistent w/ larger 55% reduction in β_N = 7-8 → 3.5 (vs. 5.5→ 4.2 or 25%)
- Kelvin-Helmholtz-like $\delta W_{d\Omega}$ and δK_2 are dominant δW_{rot} terms
  - Rotational Coriolis and centrifugal effects weaker



Summary

- Accurate q profile requires inclusion of rotation in reconstruction
- Significant fast-ion redistribution apparent in many shots in reconstructed kinetic and rotational pressure profiles (P_K, P_Ω)
- Rotation, fast-ion/kinetic effects can strongly modify IWM
  - Rotation near sonic → potentially large reduction in with-wall marginal β_N
  - High fast-ion pressure fraction further reduces marginal β_N
- Initial calculations show good agreement between MARS-K predicted and measured mode characteristics: $\beta_{N\text{-crit}}$, ω, γ, ξ
  - Kinetic values/limits closer to experiment than fluid treatment
- Inclusion of wall stabilization, rotation, fast-ions (w/ broadening or loss) in MARS-K necessary to achieve good agreement between measured & predicted stability characteristics