

# The Tokamak Density limit: a Thermoresistive Disruption Mechanism

Presented by D. A. Gates

In collaboration with

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# Outline

- Review density limit
- Radiation driven island theory and scaling
- Issues with simple (slab) theory
- New physics in the MRE
- Results from the new model
  - Have reproduced the Greenwald limit quantitatively from a first principles model with reasonable assumptions for impurity behavior
- Summary

# Where does the Greenwald limit come from?

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- The empirical tokamak operational limit (aka the Greenwald limit) relates the maximum achievable density to the circular-equivalent current density

$$\bar{n}_e (10^{20} m^{-3}) < \frac{I_p}{\pi a^2} (MA / m^2)$$

- A radiative limit should scale as  $P^{1/2}$
- The Greenwald limit is a fairly robust result

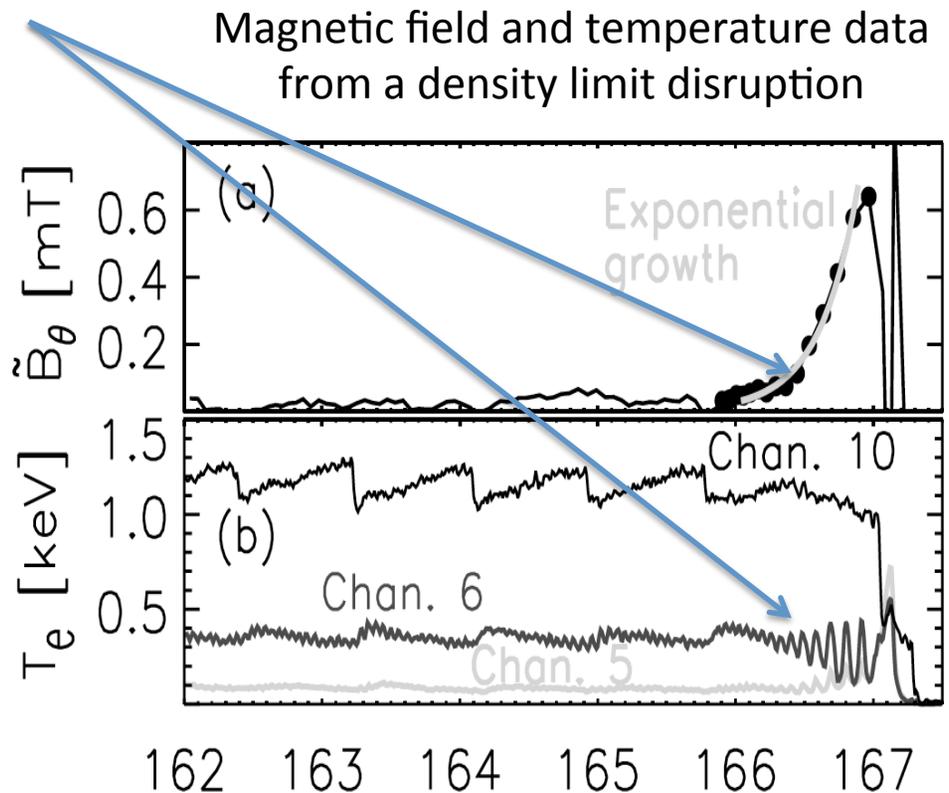
# Puzzles associated with the Greenwald limit

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- 1) The scaling is universal, but the phenomenon appears to be associated with radiative collapse and tearing modes, which can be complicated
- 2) If the physics is associated with radiative collapse, why is the density limit so weakly dependent on heating power?
- 3) Why is the limit only weakly dependent on  $Z_{eff}$ ?
- 4) Why is the density limit power scaling different in stellarators?
- 5) Why are tearing modes associated with a radiative collapse?

# Tearing modes precede the density limit collapse

- MHD mode preceding collapse is ubiquitous
- Explained by Wesson as a classical  $\Delta'$  change caused by the  $I_i$  increase
  - This explanation has not been robustly linked to the Greenwald limit



# Radiation increases at the Greenwald limit

- Radiation physics matters!
  - Why doesn't the Greenwald limit depend on heating power?
- Collapse is not associated with fixed  $P_{\text{rad}}/P_{\text{tot}}$

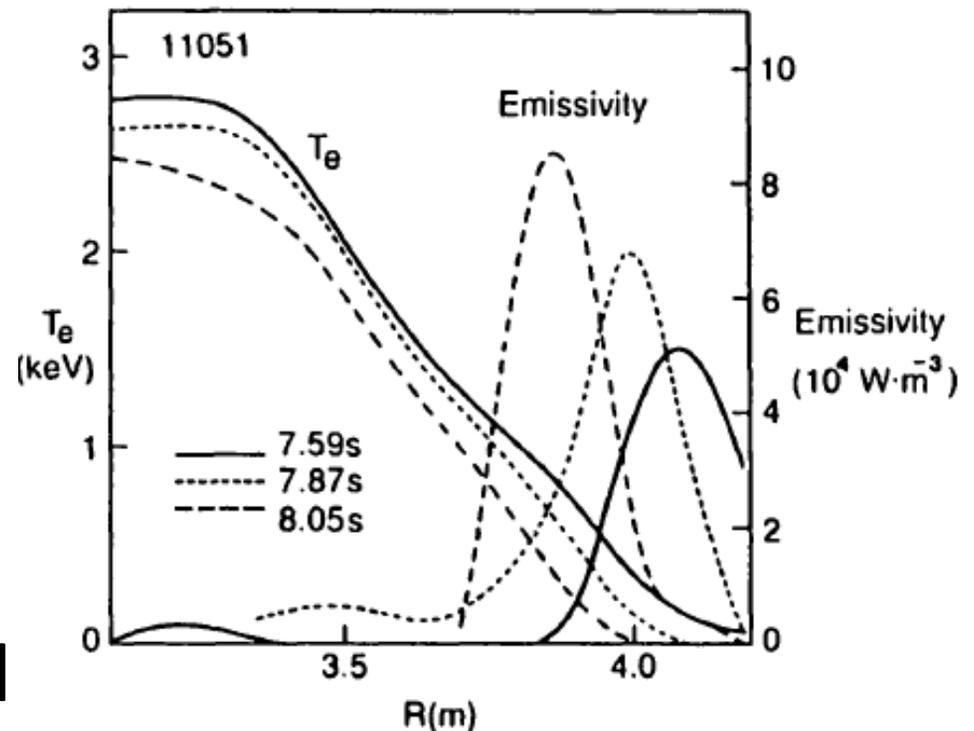
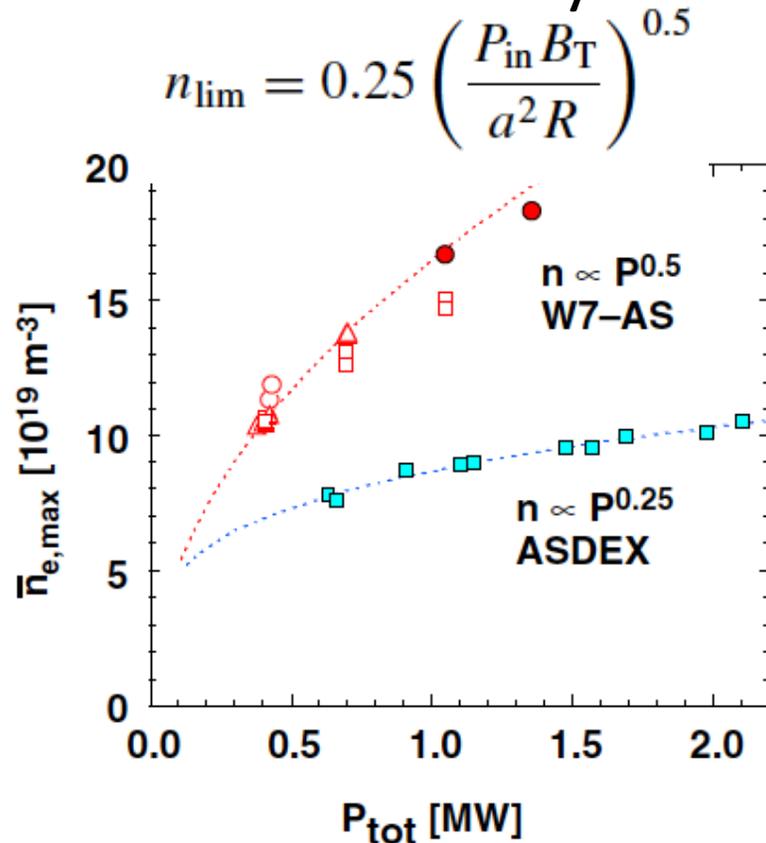


FIG. 12. Temperature and radiated power profiles during the plasma contraction.

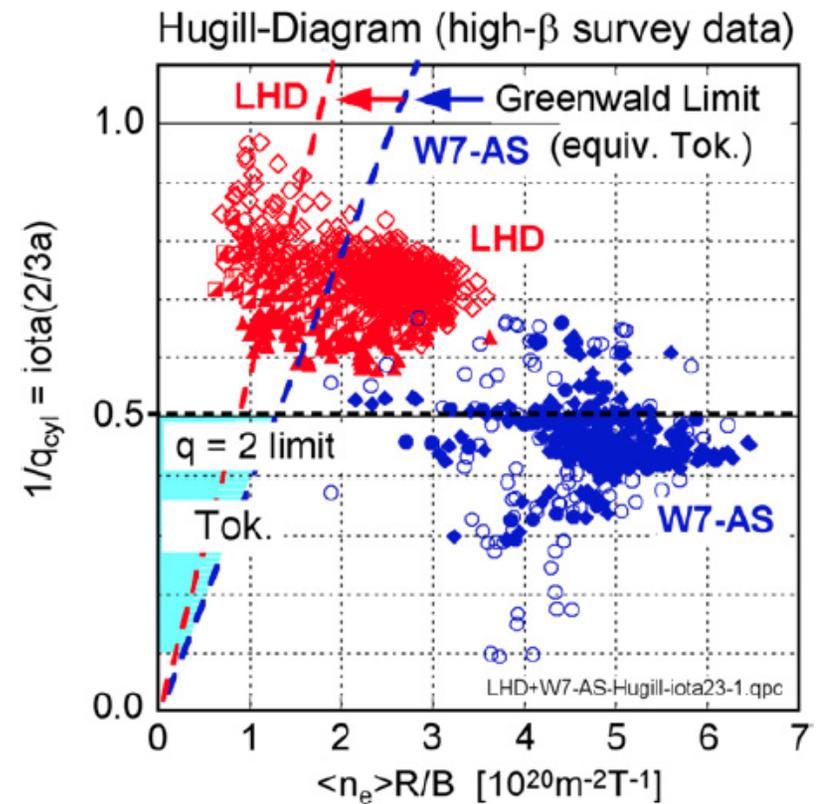
\*J. A. Wesson, R. D. Gill, M. Hugon, F. C. Schuller, J. A. Snipes, et al., Nucl. Fusion **29** (1989) 641

# Stellarators are different than tokamaks

- Density limit clearly does not obey tokamak scalings
- Stellarator density limit is given by the Sudo limit



M. Greenwald, et al., Plasma Phys. Control. Fusion **44** (2002) R27–R80



A. Weller, et al., Nucl. Fusion **49** (2009) 065016

But density limit does not vary with  $Z_{eff}$

- Density limit almost independent of  $Z_{eff}$  until  $Z_{eff} \sim 3$
- $Z_{eff}$  is a good proxy for Brehmstrahlung

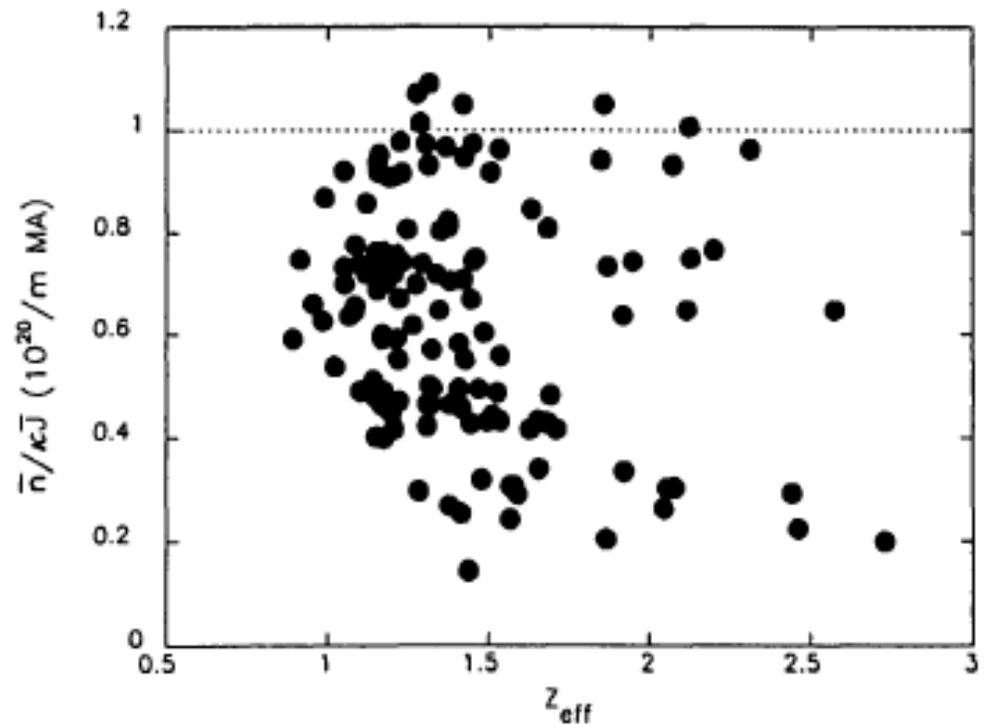


FIG. 9. Density (normalized to  $\kappa J$ ) versus  $Z_{eff}$ . The dashed curve, which represents the scaled limit,  $\bar{n} = \kappa J$ , can be reached for plasmas with  $Z_{eff}$  substantially above 1 (Alcator C).

# The islands at the density limit have been tentatively identified as radiation driven

- Suttrop et al. did extensive study on ASDEX-U (1997)
  - Did not draw a causal connection between islands and the density limit
  - Did identify at least some of the islands as potentially radiative

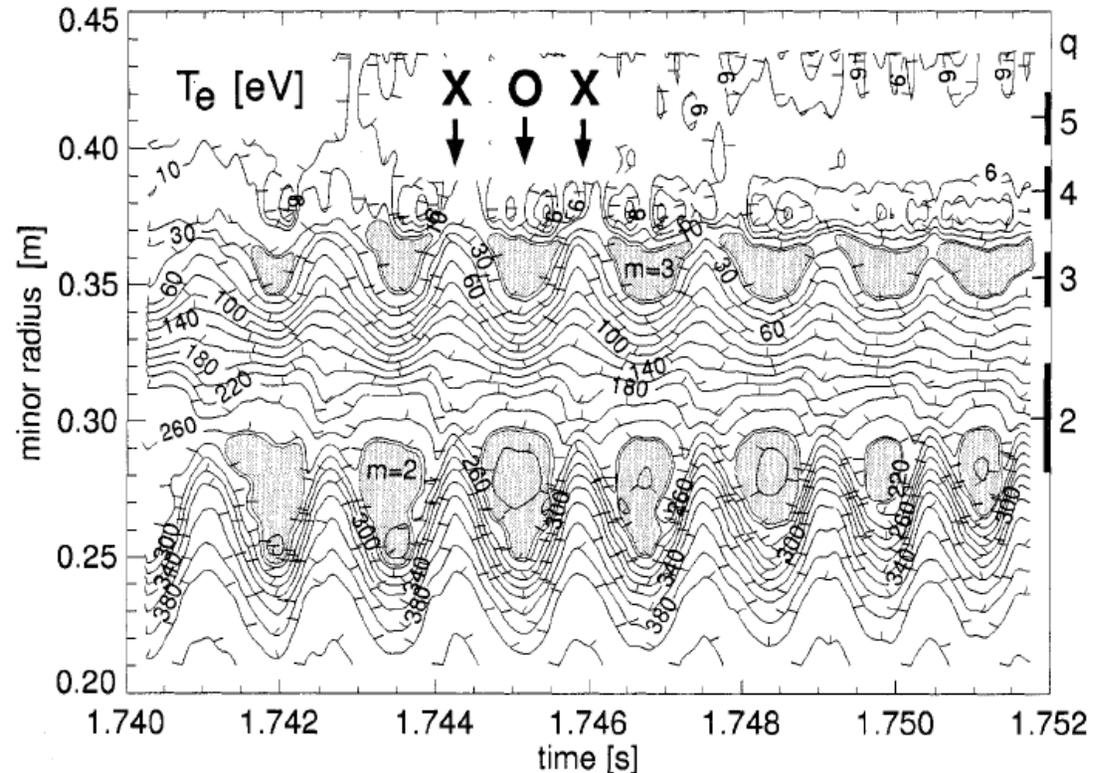
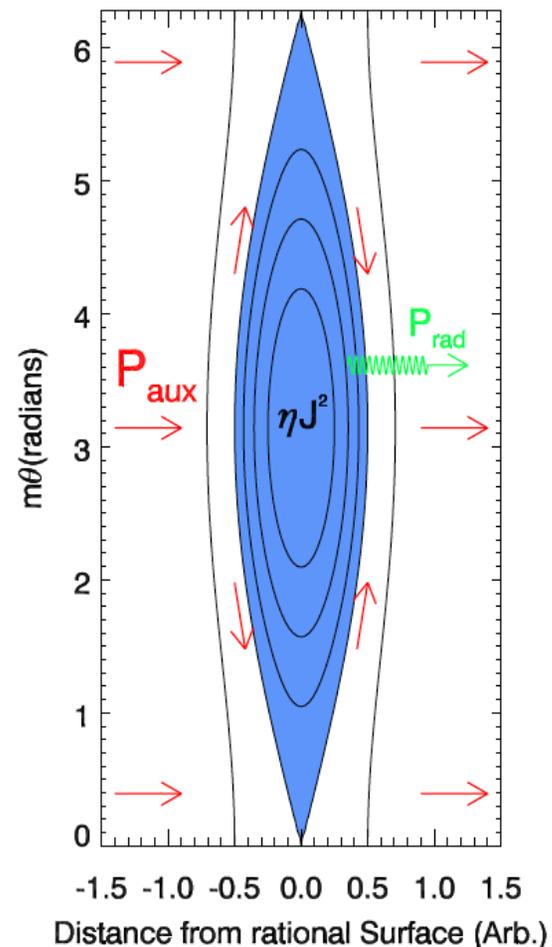


FIG. 2. Reconstruction of coupled (2,1) and (3,1) islands from  $T_e$  measurements in a time interval during current profile contraction between two minor disruptions. Islands recognized by regions of flat  $T_e$  are marked by shaded areas. While the (3,1) island grows, the (2,1) island shrinks.  $q(r)$  is derived from equilibrium reconstruction at  $t = 1.75$  s with radial uncertainties indicated.

# Radiation driven islands

- The island is magnetically insulated from its surroundings
- So radiation can cool the island,
- Lower temperature leads to increased resistivity
- This enhances the helical current perturbation
- The island then grows, increasing the magnetic insulation, causing the process to exponentiate.

P. H. Rebut and M. Hugon, Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984), Vol. 2, IAEA, Vienna, 197, (1985).



# Radiation drive in the MRE

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- Power balance in the island

$$n_e \chi_{\perp} \nabla T_e A_{island} = \delta P * V_{island}$$

- where  $A_{island}$  is the surface area the island and  $V_{island}$  is the volume
- $\delta P = P_{heat} - P_{loss}$ , is the net power in the island

- Relate the current to the temperature using resistivity and use Rutherford  $\Delta'$  formula

$$\frac{\delta J}{J} = -\frac{3\delta T}{2T} \quad \Delta' = 16k_1 \frac{\delta J}{swJ}$$

- Find the radiation drive term

$$\Delta' = 3 \frac{r_s s_I}{s} \frac{\delta P}{n_e \chi_{\perp} T_e} w$$

# Modified Rutherford equation with radiation

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- For now, ignore the bootstrap and polarization terms (consider low to moderate  $\beta_p$ )
- The MRE then becomes:

$$\frac{k_0}{\eta} \frac{dw}{dt} = \Delta' r_s + C_3 w$$

Rutherford/White term
Radiation term

Exponential growth

Where:  $C_3 = 3(r_s s_I / s) (\delta P / [n_e \chi_{\perp} T_e])$

- The radiation term changes sign when  $\delta P = 0$  or

$$P_{rad} = P_{heat} \longrightarrow P_{rad} \sim \eta J^2$$

# Radiation drive term changes sign when island cools

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$$P_{rad} < \eta J^2 \quad \text{or} \quad n_e E_{ave} v_{eZ} < \frac{m_e v_{ei}}{e^2 n_e} J^2$$

$$n_e < \sqrt{\frac{m_e v_{ei}}{e^2 E_{eff} v_{eZ}} J}$$

- Assume ohmic heating dominates inside of the island
- Auxiliary power is shunted around the island by parallel conduction, consistent with density limit being independent of heating power
  - Constant temperature island boundary
- Quantity in square root is nearly independent of temperature\*
- Reminiscent of the Greenwald limit

\*F. W. Perkins and R. A. Hulse, Phys. Fluids **28** (1985) 1837.

# Simple cylindrical model relates local density and current to global values

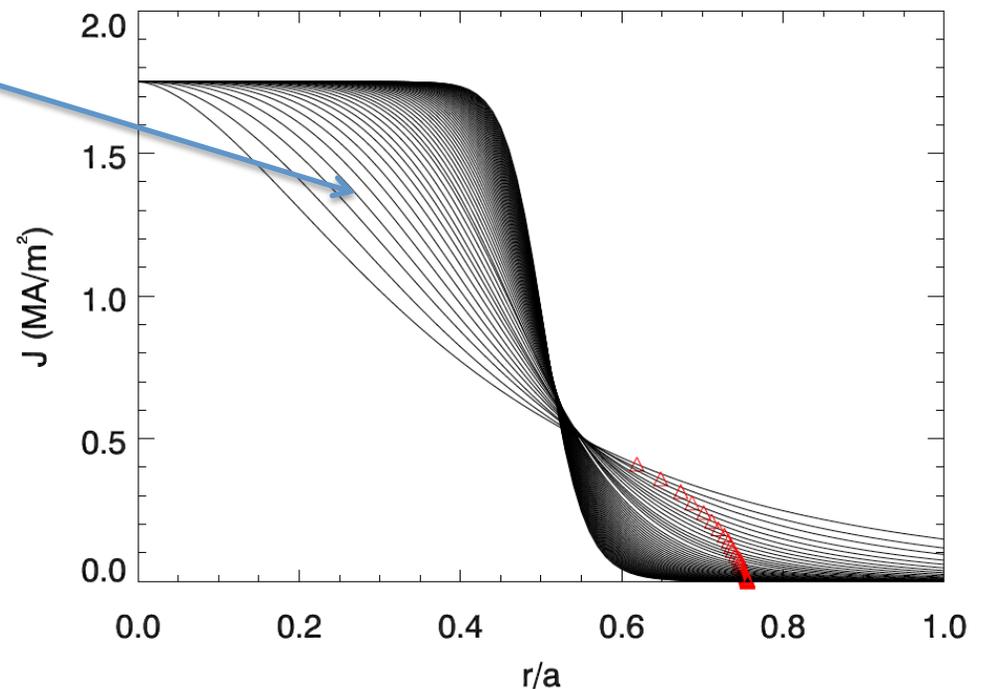
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- Use a simple profile model

$$J = \frac{J_0}{\left(1 + \left(\frac{r}{r_0}\right)^{2\nu}\right)^{1 + \frac{1}{\nu}}}$$

- Assume parabolic density profile
- Still to many variables
  - Need additional information to determine  $J(r)$  at the density limit

Current profiles used in simple density limit model at constant-q



# Current profile peaking at the density limit

- Corresponding to the density limit there is a corresponding (simultaneous)  $l_i$ -limit
- Fit this curve with a line

$$l_i = 0.12q_{edge} + 0.6$$

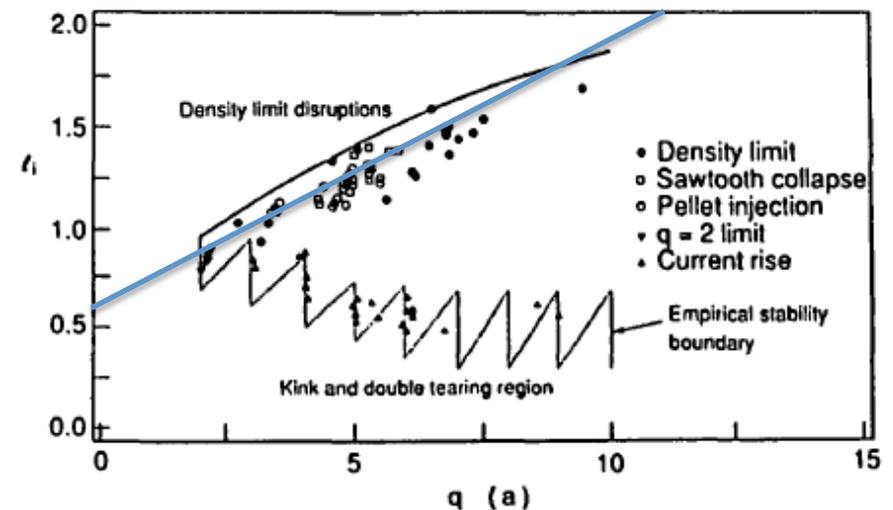


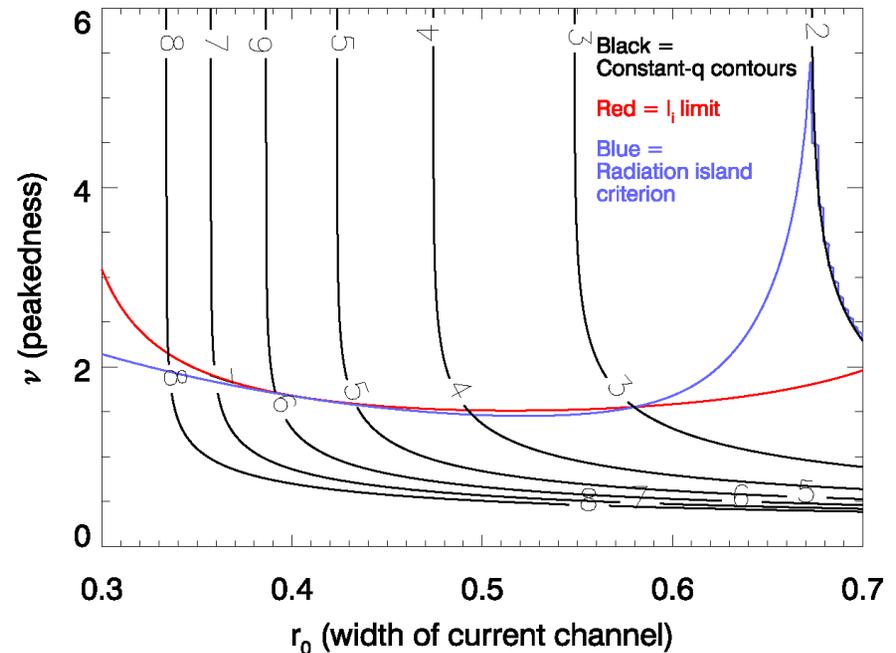
FIG. 6. Empirical stability diagram for JET, showing the  $l_i$ - $q$  plane. The lower boundary (dotted) indicates the stability boundary for rotating MHD modes during the current rise. The upper boundary (solid) indicates the region where major disruptions occur. The symbols indicate the onset of quasi-stationary modes in various situations.

\*J. A. Wesson, R. D. Gill, M. Hugon, F. C. Schuller, J. A. Snipes, et al., Nucl. Fusion **29** (1989) 641

# A contour of constant local power balance corresponds with the contour of maximum $I_i$

- Indicates that the local and the global scaling laws are co-linear if the current profile corresponds with the  $I_i$  observed at the density limit

➤ Island net power threshold corresponds to the Greenwald limit



Contour plot of the 1)  $q_{edge}$  (black) as a function of the profile parameters  $\nu$  and  $r_0$ . Also shown in the plot are 2)  $I_i$  (red) and the 3) island net power threshold (blue)

D. A Gates, L. Delgado-Aparicio, Phys. Rev. Letters **108** 165004 (2012)

# Slab model doesn't reproduce observations

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$$\frac{k_0}{\eta} \frac{dw}{dt} = \Delta'(w)r_s + C_3 w$$

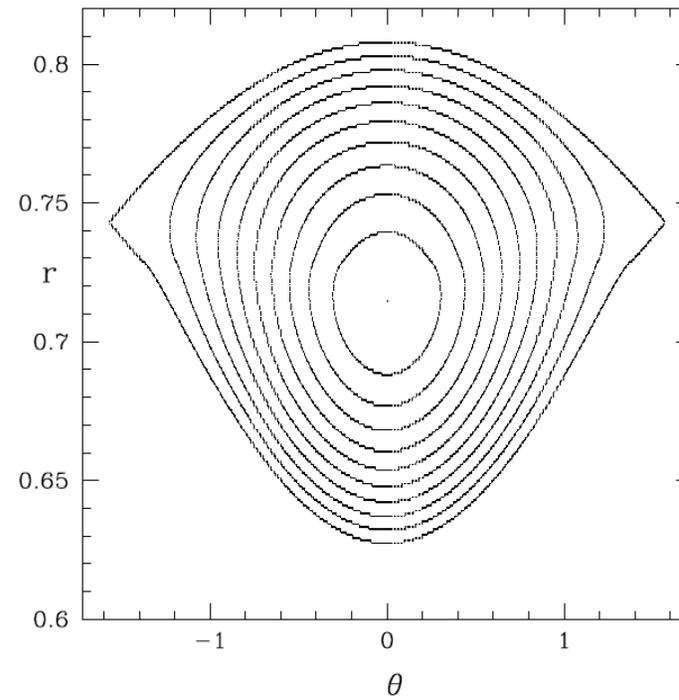
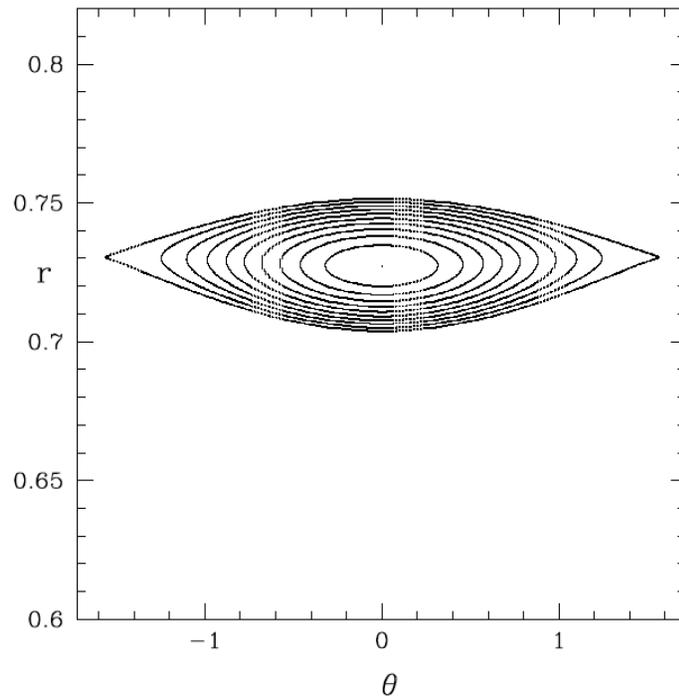
$$\Delta'(w)r_s = (\Delta' - \alpha w)r_s$$

White saturation term

- White saturation term becomes stabilizing at large width
- $\delta T$  required to drive a large island is too big
- Large temperature depression in the center of islands is not observed in experiments
- Something else must be going on...

# Island from linear theory is asymmetric at large width

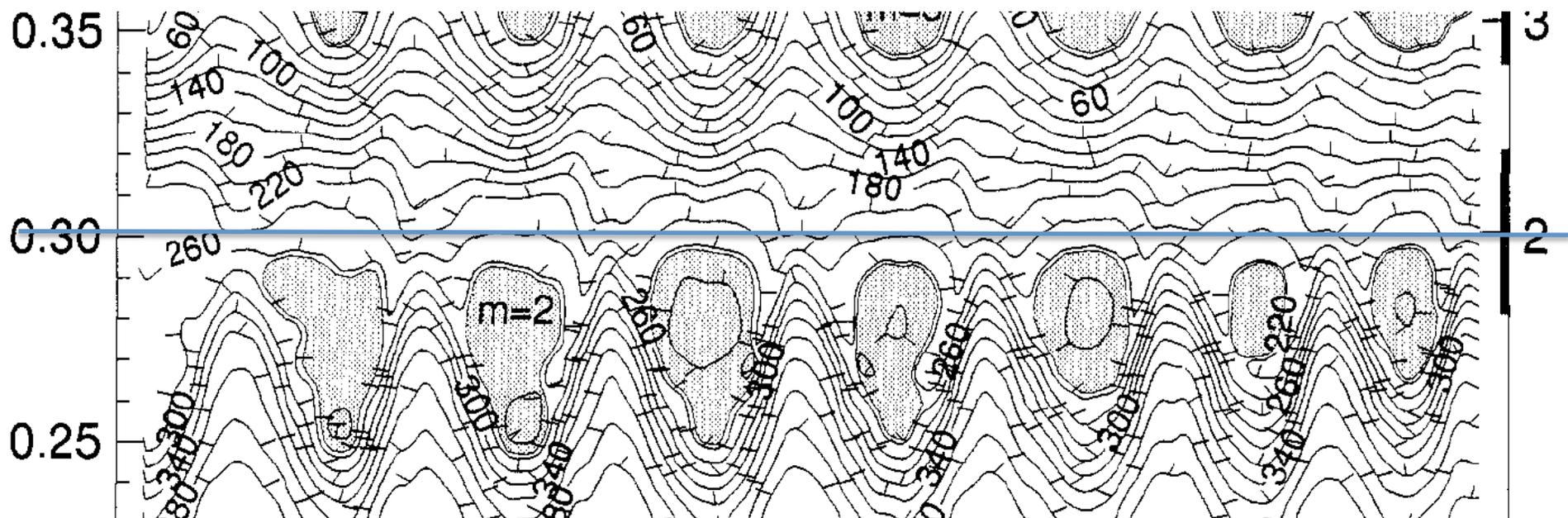
- Asymmetry at large width is not a new non-linearity— well established



# Observed islands are also asymmetric

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- O-point of the 2/1 island is shifted 3cm inside the nominal  $q=2$  surface
- X-point appears shifted outside of  $q=2$  surface
- Inward shift of the island is destabilizing for flattened islands

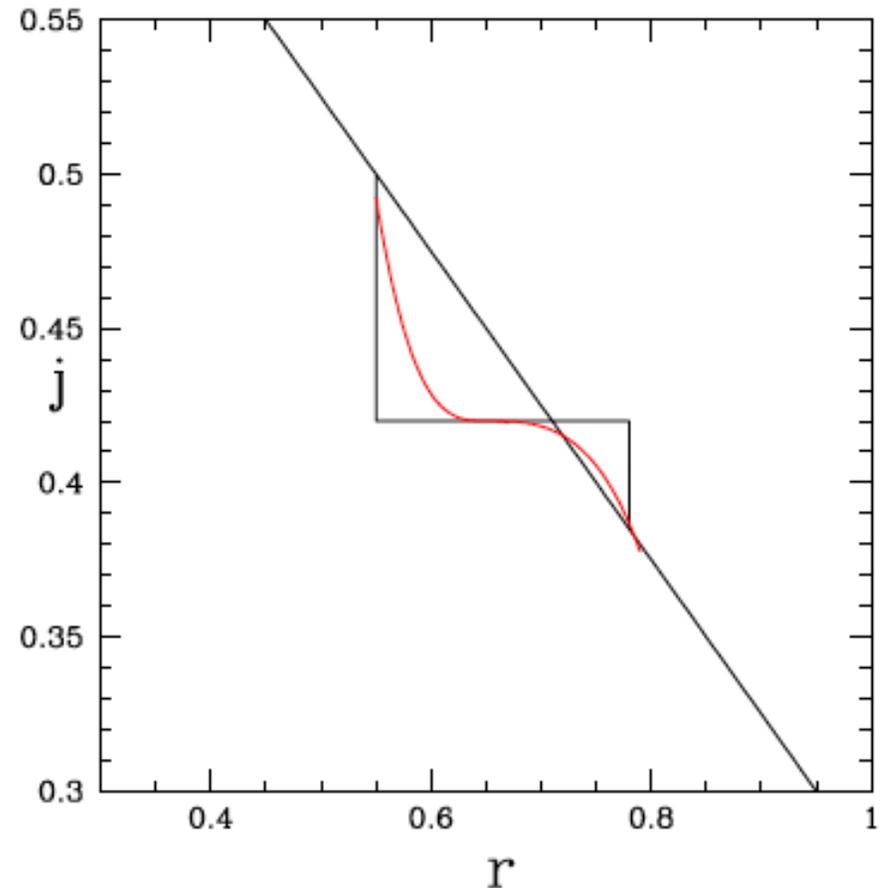


# 3D resistivity with asymmetry adds new destabilizing term

- Asymmetric resistivity perturbation with flattening leads to net negative current perturbation, so island grows

$$\Delta'_A(w) = f_F \frac{\int [j(r_x) - j(r)] \cos(m\theta) dr d\theta}{\psi_1(r_s)}$$

- Must also include “Fitzpatrick effect” to avoid small island singularities



# New model with 3D resistivity

- New MRE including all terms with 3D resistivity

$$\frac{dw}{dt} = r_s^2 [\Delta'(w) + \Delta'_{\delta j}(w) + \Delta'_A(w)]$$

Rutherford/White term

Rebut term

Asymmetry (Westerhof/White) term

$$\Delta'_{\delta j}(w) = -\frac{32 \delta j_1}{\pi \psi_0''} \frac{w}{w^2 + w_F^2}, \quad \Delta'_A(w) = \frac{8 j'(r_s)}{\pi \psi_0''} \frac{w^2}{w^2 + w_F^2} f_A,$$

Fitzpatrick term

- Coupled to simple thermal equation

$$0 = \kappa \psi_0''(r_s) \frac{dT}{d\psi} + H(T) - R(T)$$

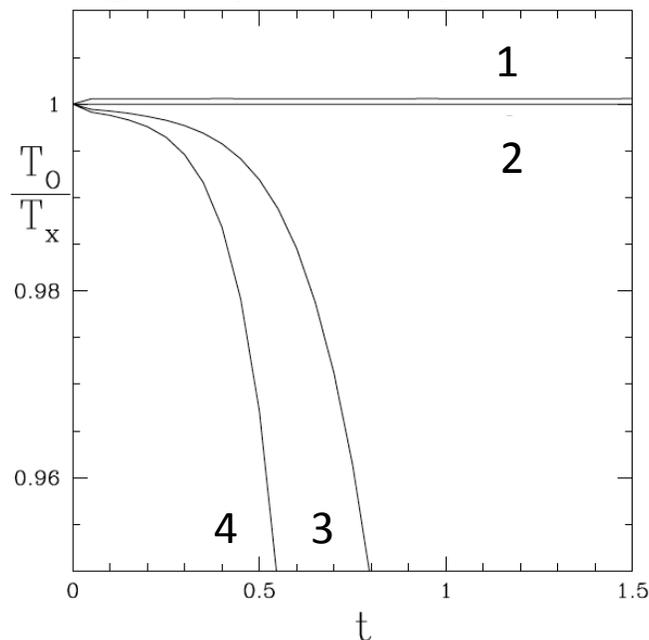
Heating

Radiation

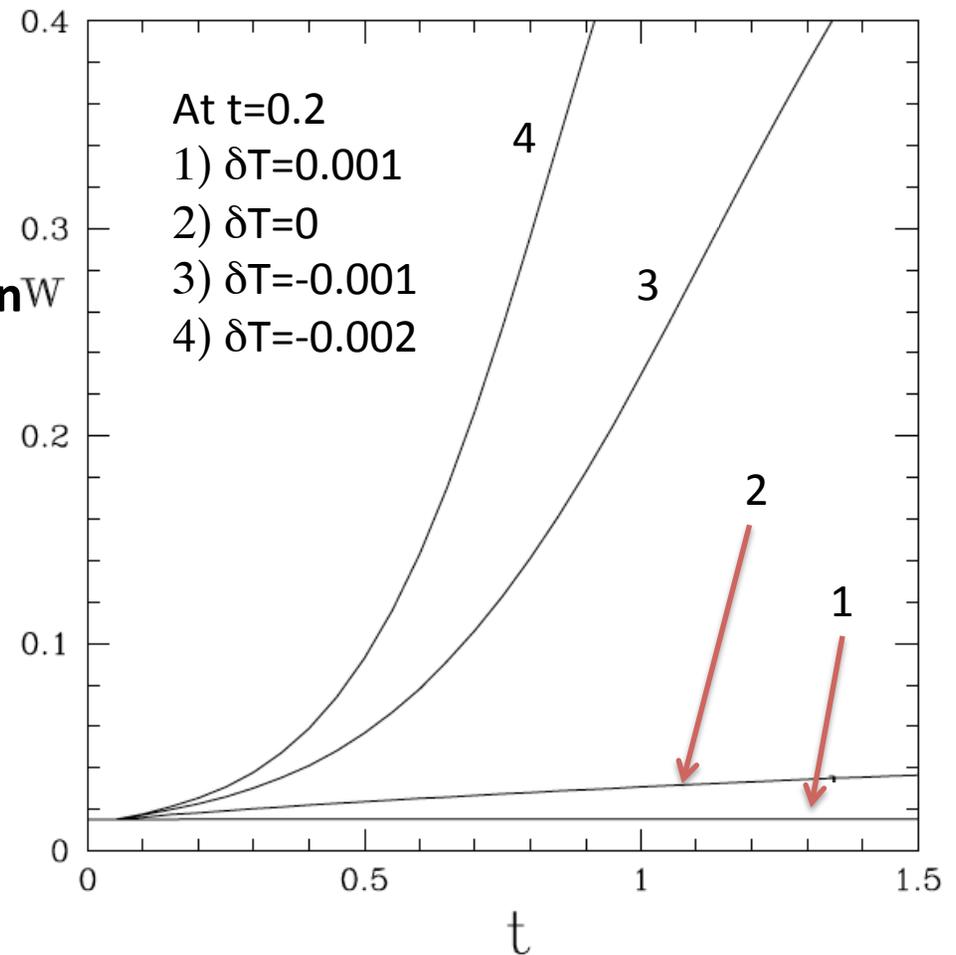
# New model shows robust threshold for mode onset at identified density limit criteria

- Small change in internal temperature leads to rapid change in growth rate
- Resultant island has small temperature perturbation at the O-point

Corresponding temperature perturbation  $\frac{T_0}{T_x}$

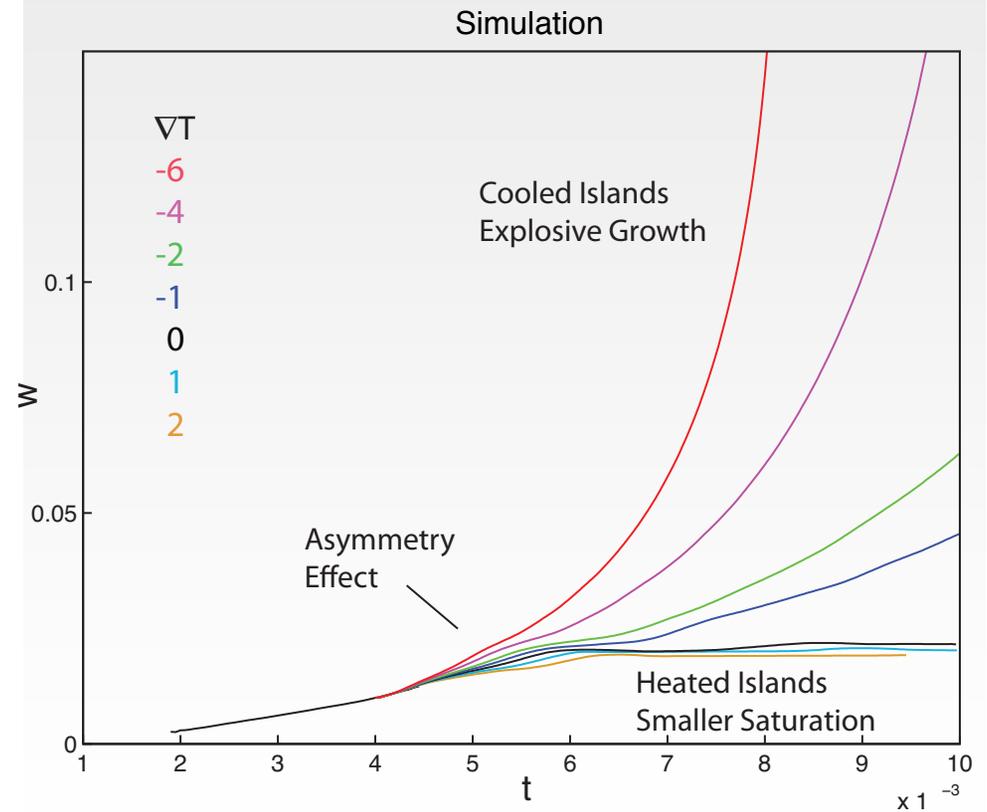
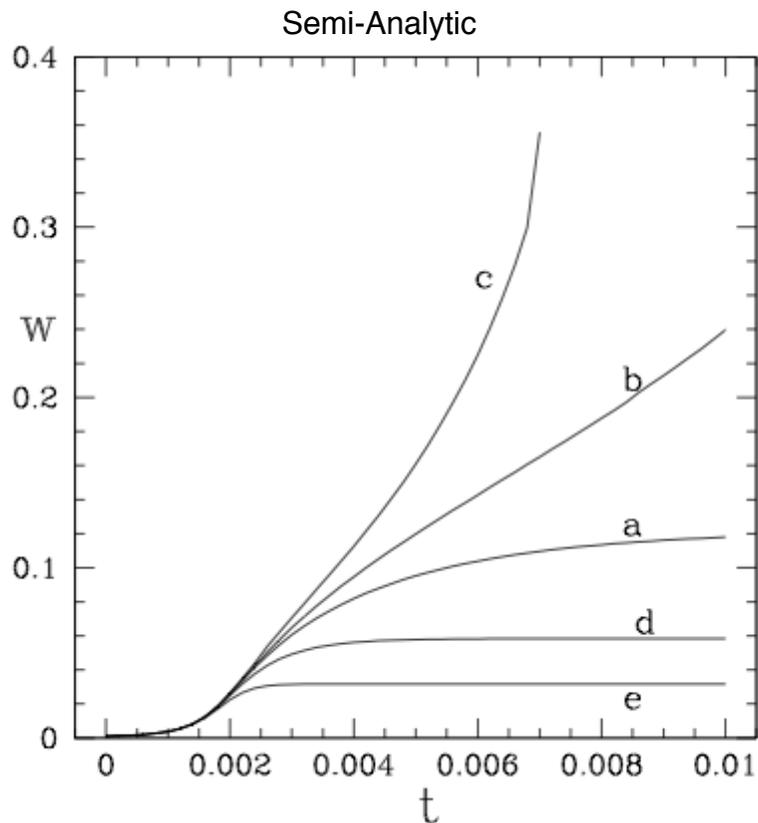


Time history of island width for different fixed internal temperature gradients



# Simulations agree with reduced analysis, cooling causes exponential growth, heating causes saturation

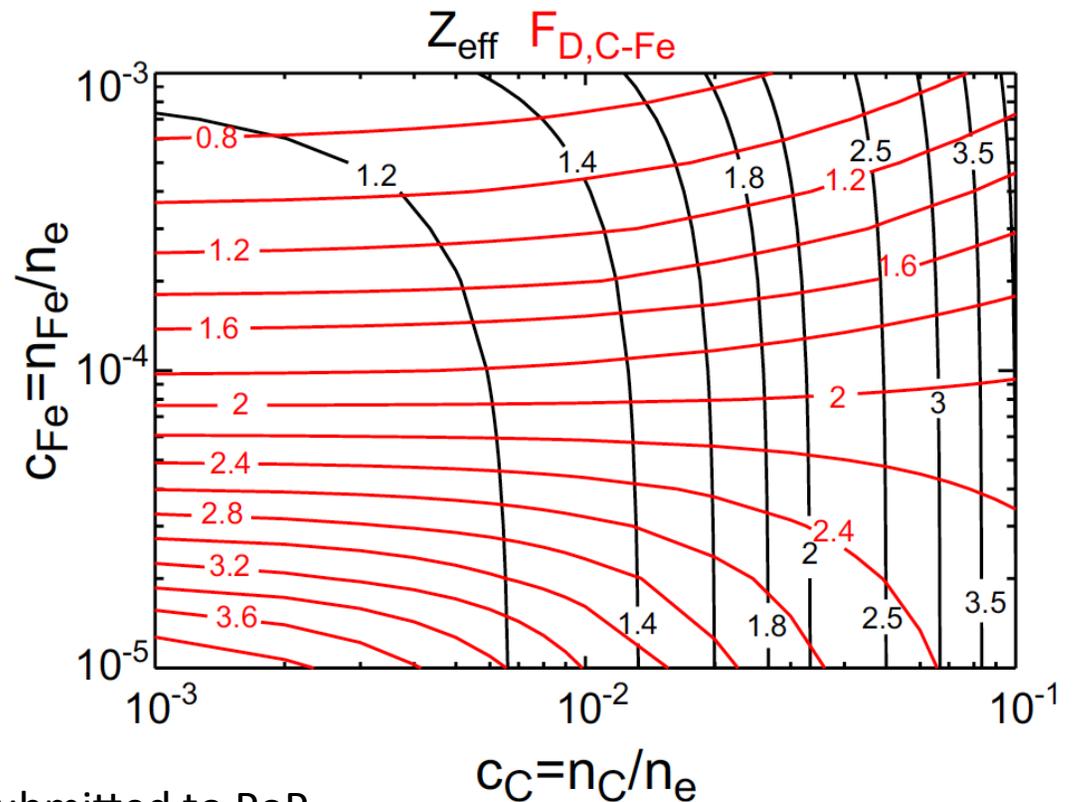
Islands with small amount of cooling eventually exponentially grow in  $w$   
Temperature perturbations are small, mostly below experimental observation  
Despite tiny  $\Delta T$ , heated islands saturate at small size ( $\eta(T)$  has strong effect)



# Density limit coefficient is nearly independent of $Z_{eff}$

- Local coefficient between current and density depends predominantly on line radiation from high-Z impurities
  - $Z_{eff}$  depends predominantly on low-Z

Contours of coefficient between density and current along with contours of constant  $Z_{eff}$  for a three species plasma.



L. Delgado-Aparicio, D. A. Gates, to be submitted to PoP  
D.E. Post, et al., At. Data Nucl. Data Tables **20**, 397 (1977).

# Simple low-beta tokamak model

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- Assume 1% C and  $1.1 \times 10^{-4}$  Fe
- Assume steady state (constant)  $E = j/\eta$
- Ad hoc model for relationship between  $l_i$  and  $n_e$  that is consistent with published data at the density limit


$$l_i = \begin{cases} 0.12 \times q_{edge} \times \frac{n_e}{n_G} + 0.6 & \text{if } n_e/n_G > 0.7 \\ 0.084 \times q_{edge} + 0.6 & \text{if } n_e/n_G \leq 0.7 \end{cases}$$

- Use definitions:

$$P_{heat} = \eta j^2$$

$$\eta = 1.03 \times 10^{-4} Z \ln \Lambda T [eV]^{-3/2} \Omega m$$

$$P_{rad} = n_e n_D L_D(T_e) + \sum_Z n_e n_Z L_Z(T_e)$$

$L_Z$  defined in D.E. Post, et al., At. Data Nucl. Data Tables **20**, 397 (1977).

# Model regenerates the Greenwald limit quantitatively

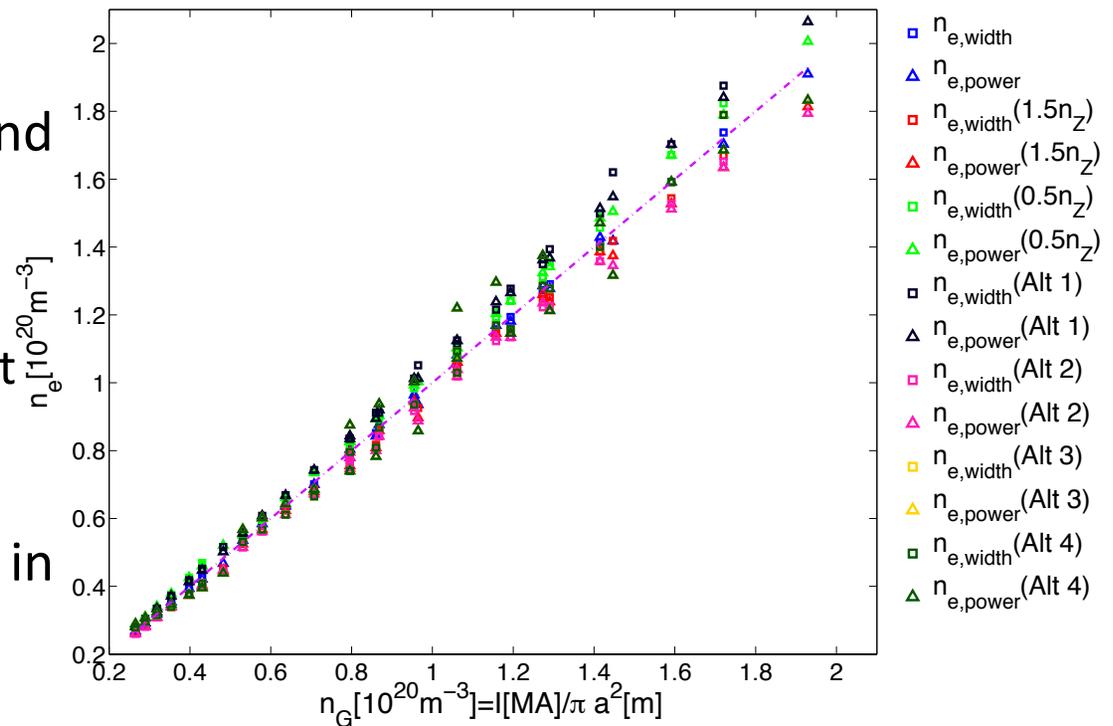
- Use model to solve coupled equations

$$\frac{dw}{dt} = r_s^2 [\Delta'(w) + \Delta'_{\delta j}(w) + \Delta'_A(w)]$$

$$0 = \kappa T'' \langle (\nabla \psi)^2 \rangle + \kappa T' \langle (\nabla^2 \psi) \rangle + H(T) - R(T)$$

Mode onset and power balance threshold from simple model

- as a function  $n_e$  and  $I_p$
- Reasonable assumptions and simple model give quantitative agreement with:
  - Scaling of the density limit
  - Sudden mode onset
  - $Z_{eff}$  independence
- Model robust to variations in assumptions



**Q. Teng, Poster NP12.00012, Wednesday 9:30 - Submitted to PRL**

# Implications and future plans

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- This theory provides a testable quantitative prediction of the density limit based on local measurements and points to methods for exceeding the limit and controlling/avoiding disruptions
  - Key issue for ITER
- Theory predicts exponentially growing islands with a sudden robust onset condition
  - Consistent with a robust density limit and observed rapidly growing 2/1 tearing mode
  - Quantitatively predicts Greenwald limit with reasonable assumptions and simple model of a tokamak
- Need to directly verify local power balance criteria
  - Data analysis proceeding on NSTX
  - Experiments proposed on DIII-D, EAST, KSTAR
- Theory is robust – may be more widely applicable