Performance Assessment of Model-Based Optimal Feedforward and Feedback Current Profile Control in NSTX-U using the TRANSP Code

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57th Annual Meeting of the APS Division of Plasma Physics

November 17, 2015

Motivation for Current Density Profile Control in NSTX-U

- **There is growing consensus that the path to an economical power-producing reactor is the "Advanced Tokamak" (AT) concept.**
- AT operational goals for the NSTX-U include **[1]**:
  - Non-inductive sustainment of the high-β spherical torus. (Fusion power scales as $P_{fus} \approx \beta^2 B^4$)
  - High performance equilibrium scenarios with neutral beam heating.
  - Longer pulse durations.
- **Active, model-based, feedback control** of the **current density profile** evolution can be useful to achieve these AT operational goals.
- Relation between ι-profile and the toroidal current density (j_ϕ) profile **[2]**:

$$\iota(\hat{\rho}, t) = \frac{R_0 \mu_0}{\hat{\rho}^2 B_\phi} \int_0^{\hat{\rho}} j_\phi(\hat{\rho}', t) \hat{\rho}' d\hat{\rho}' = \underbrace{-\frac{d\Psi}{d\Phi}}_{-(\partial\psi/\partial\hat{\rho})/B_{\phi,0}\rho_b^2\hat{\rho}}$$

- Control of the ι-profile is equivalent to control of the current density profile, $j_\phi(\hat{\rho}, t)$, and the control of the poloidal flux gradient profile, $\partial\psi/\partial\hat{\rho}$.

[1] GERHARDT, S. P., ANDRE, R., and MENARD, J. E., Nuclear Fusion **52 (2012).**
[2] J. Wesson, *Tokamaks* (Oxford University Press, 3rd edition, 2004).

First-Principles-Driven (FPD) Current Profile Modeling

Physics–based Empirical Models for: T_e , n_e , η , j_{NI} $+$ *Poloidal Flux Evolution Model (Magnetic Diffusion Equation)*

First – Principles – Driven (FPD) Current Profile Evolution Model

- The evolution of the poloidal magnetic flux is given by the **Magnetic Diffusion Equation [3]**

$$\boxed{\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_0 \rho_b^2 \hat{F}^2} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} D_\psi \frac{\partial \psi}{\partial \hat{\rho}} \right) + R_0 \hat{H} \eta(T_e) \frac{< \bar{j}_{NI} \cdot \bar{B} >}{B_{\phi,0}},} \quad (1)$$

with boundary conditions

$$\left.\frac{\partial \psi}{\partial \hat{\rho}}\right|_{\hat{\rho}=0} = 0, \qquad \left.\frac{\partial \psi}{\partial \hat{\rho}}\right|_{\hat{\rho}=1} = -\frac{\mu_0}{2\pi} \frac{R_0}{\hat{G}\big|_{\hat{\rho}=1} \hat{H}\big|_{\hat{\rho}=1}} I(t), \quad (2)$$

where $D_\psi(\hat{\rho}) = \hat{F}(\hat{\rho})\hat{G}(\hat{\rho})\hat{H}(\hat{\rho})$, and $\hat{F}$, $\hat{G}$, $\hat{H}$ are geometric factors pertaining to the magnetic configuration of a particular equilibrium.

[3] OU, Y., LUCE, T. C., SCHUSTER E. et al., Fusion Engineering and Design (2007).

First-Principles-Driven (FPD) Current Profile Modeling

- NSTX-U-tailored **[4]** empirical models **[5]** for the electron temperature, electron density, plasma resistivity, and noninductive current drives **[6]** take the form

$$n_e(\hat{\rho},t) = n_e^{prof}(\hat{\rho})u_n(t) \tag{3}$$

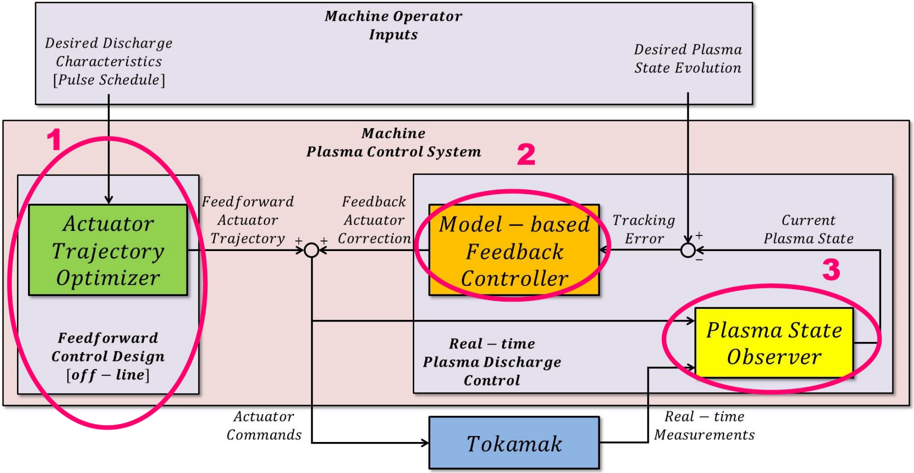
$$T_e(\hat{\rho},t) = k_{T_e}(\hat{\rho},t_r)\frac{T_e^{prof}(\hat{\rho},t_r)}{n_e(\hat{\rho},t)}I(t)\sqrt{P_{tot}(t)} \tag{4}$$

$$\eta(T_e) = \frac{k_{sp}(\hat{\rho},t_r)Z_{eff}}{T_e(\hat{\rho},t)^{3/2}} \tag{5}$$

$$\begin{aligned}\frac{\left\langle \bar{j}_{ni}\cdot\bar{B}\right\rangle}{B_{\phi,0}} &= \sum_{i=1}^{6}\frac{\left\langle \bar{j}_{nbi_i}\cdot\bar{B}\right\rangle}{B_{\phi,0}} + \frac{\left\langle \bar{j}_{bs}\cdot\bar{B}\right\rangle}{B_{\phi,0}} \\ &= \sum_{i=1}^{6} k_i^{prof}(\hat{\rho})j_i^{dep}(\hat{\rho})\frac{\sqrt{T_e(\hat{\rho},t)}}{n_e(\hat{\rho},t)}P_i(t) \\ &\quad + \frac{k_{JeV}R_0}{\hat{F}(\hat{\rho})}\left(\frac{\partial\psi}{\partial\hat{\rho}}\right)^{-1}\left[2\mathcal{L}_{31}T_e\frac{\partial n_e}{\partial\hat{\rho}} + \{2\mathcal{L}_{31}+\mathcal{L}_{32}+\alpha\mathcal{L}_{34}\}n_e\frac{\partial T_e}{\partial\hat{\rho}}\right]\end{aligned} \tag{6}$$

[4] ILHAN, Z. O. et al., 55th Annual Meeting of the APS DPP (2013)
[5] BARTON, J. E. et al., 52nd IEEE CDC (2013)
[6] SAUTER, O. et al., Physics of Plasmas (1999), (2002)

Possible Uses of the FPD Model



Schematics of the plasma profile control system

Feedforward Actuator Trajectory Optimization

- **Objective:** Design the actuator trajectories that can steer the plasma to a target state characterized by the safety factor profile $q^{tar}(\hat{\rho}, t_f)$ or rotational transform profile $\iota^{tar}(\hat{\rho}, t_f)$ at a specified time t_f during the discharge such that the achieved plasma state is as stationary in time as possible.
- **Cost functional** defined as:

$$\boxed{J(t_f) = k_q J_q(t_f) + k_{ss} J_{ss}(t_f)}$$

where k_{ss} and k_q are the weight factors representing the relative importance of the plasma state characteristics and

$$J_q(t_f) = \int_0^1 W_q(\hat{\rho}) \left[q^{tar}(\hat{\rho}) - q(\hat{\rho}, t_f)\right]^2 d\hat{\rho} \tag{7}$$

$$J_{ss}(t_f) = \int_0^1 W_{ss}(\hat{\rho}) \left[g_{ss}(\hat{\rho}, t_f)\right]^2 d\hat{\rho}, \tag{8}$$

where $W_q(\hat{\rho})$ and $W_{ss}(\hat{\rho})$ are positive weight functions and

$$g_{ss}(\hat{\rho}, t) = \frac{\partial U_p}{\partial \hat{\rho}} = -\frac{\partial \Psi}{\partial t} = -2\pi \frac{\partial \psi}{\partial t}, \tag{9}$$

where U_p is the loop-voltage profile which can be related to the temporal derivative of the poloidal magnetic flux.

Formulation of Various Constraints

- **Actuator Trajectory Parametrization:** The trajectories of the i-th control actuator (u_i) can be parametrized by a finite number n_{p_i} of *to-be-determined parameters* (x_i) at discrete points in time (t_{p_i}), i.e.,

$$t_{p_i} = [t_1,\ t_2, \ldots,\ t_k, \ldots, t_{n_{p_i}} = \ t_f] \in \mathbb{R}^{n_{p_i}}$$

$$x_i = \left[u_i^1, u_i^2, \ldots, u_i^k, \ldots, u_i^{n_{p_i}}\right] \in \mathbb{R}^{n_{p_i}}$$

- Combining all parameters to represent individual actuator trajectories into the vector $\tilde{x}$, where $\tilde{x} \in \mathbb{R}^{n_p^{tot}}$ and $n_p^{tot} = \sum_{i=1}^{n_{act}} n_{p_i}$, the control actuator trajectories can be written compactly as

$$\boxed{u(t) = \Pi(t)\tilde{x}} \qquad (10)$$

- **Actuator Constraints:** The actuator magnitude and rate constraints are given by

$$I_p^{min} \leq I_p(t) \leq I_p^{max},$$
$$P^{min} \leq P_i(t) \leq P^{max}, \quad i = 1, \ldots, n_{nbi}$$
$$-I_{p,max}^{d'} \leq dI_p/dt \leq I_{p,max}^{u'},$$

- The above actuator constraints can be written compactly as a matrix inequality as

$$\boxed{A_u^{lim}\tilde{x} \leq b_u^{lim}} \qquad (11)$$

Statement of the Optimization Problem

- The optimization problem is written mathematically as

$$\min_{\tilde{x}} \; J(t_f) = J(\dot{\theta}(t_f), \theta(t_f)), \tag{12}$$

such that

$$\dot{\theta} = g(\theta, u), \tag{13}$$

$$u(t) = \Pi(t)\tilde{x}, \tag{14}$$

$$A_u^{lim}\tilde{x} \leq b_u^{lim}, \tag{15}$$

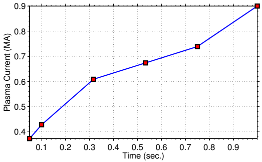
where the poloidal flux gradient, $\theta = d\psi/d\hat{\rho}$ represents the plasma state, u represents the actuators, and g is a nonlinear function representing the plasma dynamics as an additional constraint ((13) is derived from (1)-(6)).

- The optimization problem (12)-(15) can be solved iteratively in MATLAB by using the **S**equential **Q**uadratic **P**rogramming (**SQP**) method **[7]**.

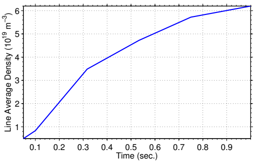
[7] J. Nocedal and S. J. Wright, *Numerical optimization,* (Springer, New York, 2006).

Feedforward Optimization: Weighting only the q-profile

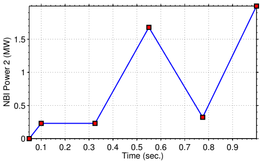
- For this application, $k_q = 1$ and $k_{ss} = 0$ in the cost functional.
- **The goal is to hit a target q-profile at $t_f = 1$ sec.**



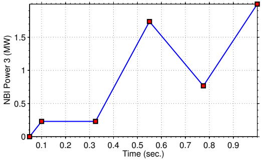
(a) Optimal plasma current $I_p(t)$



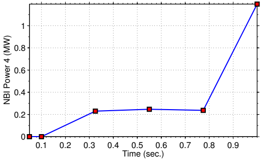
(b) Line Averaged Density $\bar{n}_e(t)$



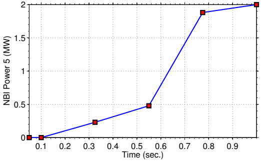
(c) Optimal NBI beam power #2



(d) Optimal NBI beam power #3



(e) Optimal NBI beam power #4

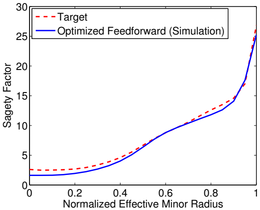


(f) Optimal NBI beam power #5

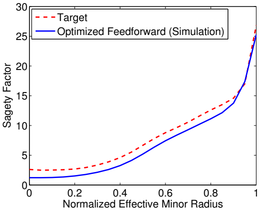
Time evolution of the optimized feedforward actuator trajectories

Feedforward Optimization: Weighting only the q-profile

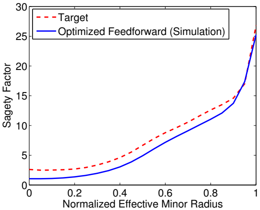
- Comparison of the target and achieved q-profiles at various times:



(a) $t = t_f = 1.0$ sec.

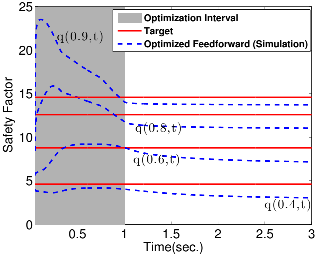


(b) $t = 2.0$ sec.



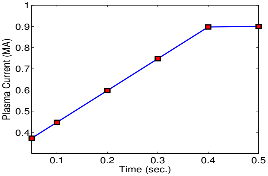
(c) $t = 3.0$ sec.

- Time evolution of the safety factor at various radial locations:

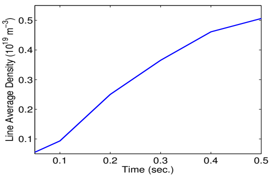


Feedforward Optimization: Weighting only Steadiness

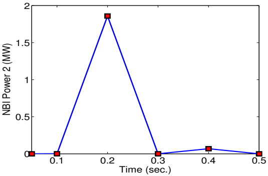
- For this application, $k_{ss} = 1$ and $k_q = 0$ in the cost functional.
- **The goal is to maintain a steady q-profile throughout the simulation.**



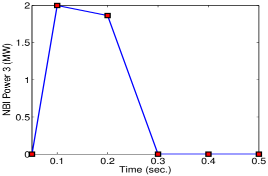
(a) Optimal plasma current $I_p(t)$



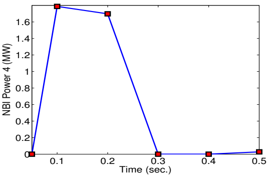
(b) Line Averaged Density



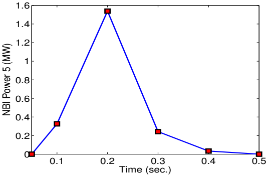
(c) Optimal NBI beam power #2



(d) Optimal NBI beam power #3



(e) Optimal NBI beam power #4

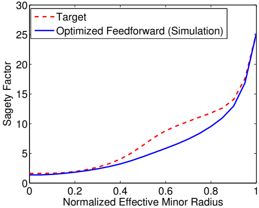


(f) Optimal NBI beam power #5

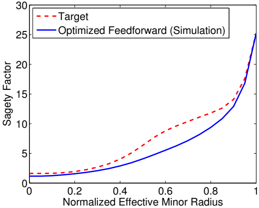
Time evolution of the optimized feedforward actuator trajectories

Feedforward Optimization: Weighting only Steadiness

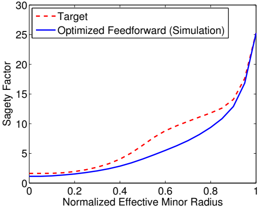
- Comparison of the target and achieved q-profiles at various times:



(a) $t = t_f = 0.5$ sec.

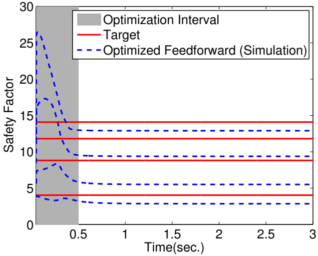


(b) $t = 1.0$ sec.



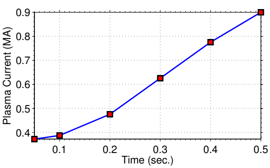
(c) $t = 2.0$ sec.

- Time evolution of the safety factor at various radial locations:

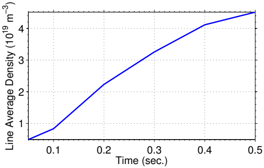


Feedforward Optimization: Weighting q + Steadiness

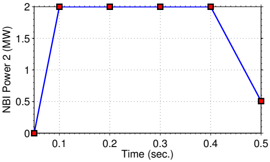
- For this application, $k_{ss} = 1$ and $k_q = 1$ in the cost functional.
- **The goal is to hit a target q-profile at $t = 0.5$ sec. and maintain it throughout a 3 sec. simulation.**



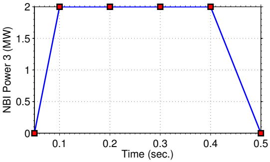
(a) Optimal plasma current $I_p(t)$



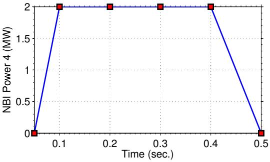
(b) Line Averaged Density



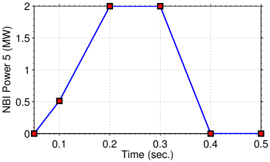
(c) Optimal NBI beam power #2



(d) Optimal NBI beam power #3



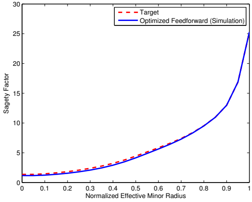
(e) Optimal NBI beam power #4



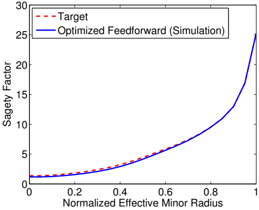
(f) Optimal NBI beam power #5

Time evolution of the optimized feedforward actuator trajectories

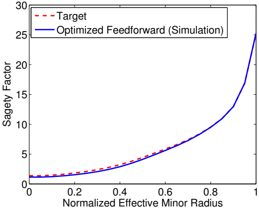
- Comparison of the target and achieved q-profiles at various times:



(a) $t = t_f = 0.5$ sec.

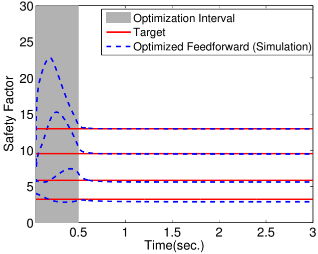


(b) $t = 1.0$ sec.



(c) $t = 2.0$ sec.

- Time evolution of the safety factor at various radial locations:



Optimal Feedback Control of the Current Density Profile

- **L**inear-**Q**uadratic-**I**ntegral **(LQI)** Optimal feedback controller has been designed in MATLAB based on the FPD, control-oriented model.
- The effectiveness of the designed controller is first tested in MATLAB by simulating the nonlinear magnetic diffusion equation (1).
- Early results on control design and numerical testing have been presented in **[8], [9].**
- The proposed feedback controller is now implemented in TRANSP for performance assessment before experimental testing in NSTX-U.
- Recently developed Expert routine **[10]** provides a framework to perform closed-loop predictive simulations within the TRANSP source code.

[8] ILHAN, Z. O. et al., 56^{th} Annual Meeting of the APS DPP (2014)

[9] ILHAN, Z. O. et al., IEEE Multi-Conference on Systems and Control (2015)

[10] BOYER, M. D., ANDRE, R., GATES, D. A., et al., Nuclear Fusion **55 (2015)**

Closed-Loop Control Simulation Study in TRANSP

- The control objective is to track a target state trajectory $\iota_r(\rho, t)$ with *minimum control effort.*
- The target state trajectory $\iota_r(\rho, t)$ is generated through an open-loop TRANSP simulation with the following constant reference inputs.

$\mathbf{n_e (m^{-3})}$	5.0×10^{19}
$\mathbf{P_1 (W)}$	0.2×10^{6}
$\mathbf{P_2 (W)}$	0.4×10^{6}
$\mathbf{P_3 (W)}$	0.6×10^{6}

$\mathbf{P_4 (W)}$	0.8×10^{6}
$\mathbf{P_5 (W)}$	1.0×10^{6}
$\mathbf{P_6 (W)}$	1.2×10^{6}
$\mathbf{I_p (A)}$	0.7×10^{6}

- Starting from the first second of the simulation, the controller is tested against perturbed initial conditions and constant input disturbances, i.e.,

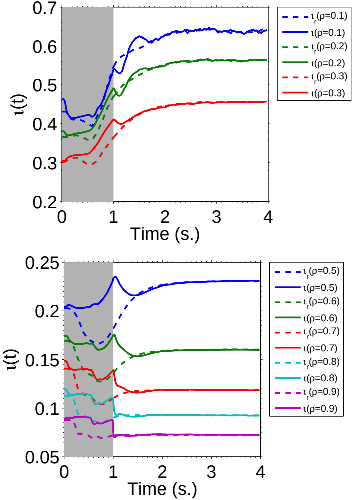
$$u(t) = \begin{cases} u_r + u_d, & t \leq 1\,\text{s.} \\ u_r + u_d + \Delta u(t), & t > 1\,\text{s.} \end{cases}$$

where u_r represents the constant reference inputs, u_d stands for the constant disturbance inputs (15% for I_p, 10% for P_1, P_3, P_5 and P_6), and $\Delta u(t)$ is the output of the feedback controller.

CASE 1: Actuation with I_p and Neutral Beams

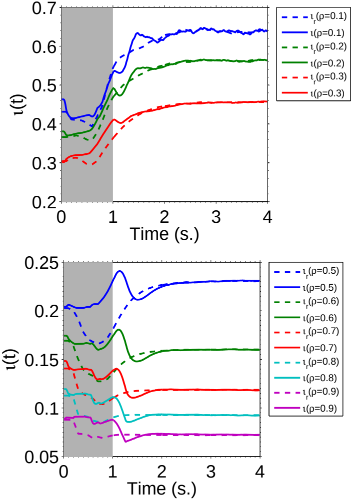
CASE 1A

(without I_p rate saturation)



CASE 1B

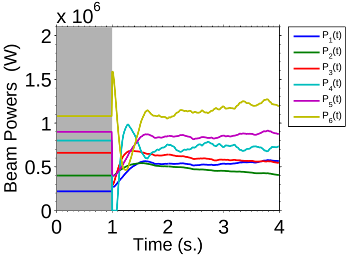
(with I_p rate saturation)

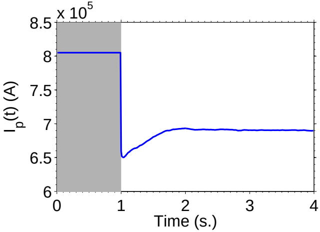


Figures (left & right): Time evolution of the optimal outputs (solid) with their respective targets (dashed). Controller is off in the grey region.

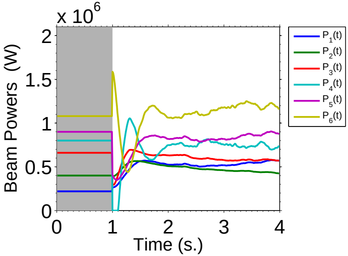
CASE 1: Actuation with I_p and Neutral Beams

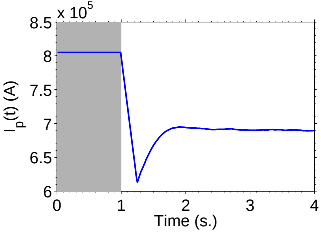
CASE 1A
(without I_p rate saturation)





CASE 1B
(with I_p rate saturation)



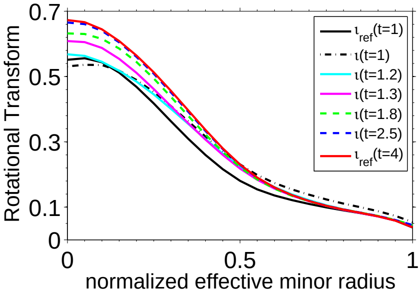


Figures (upper left & right): Time evolution of the optimal beam powers.
Figures (lower left & right): Time evolution of the optimal plasma current.

CASE 1: Actuation with I_p and Neutral Beams

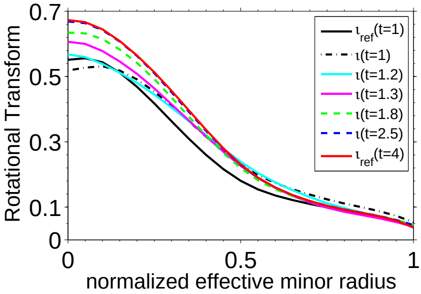
CASE 1A

(without I_p rate saturation)



CASE 1B

(with I_p rate saturation)

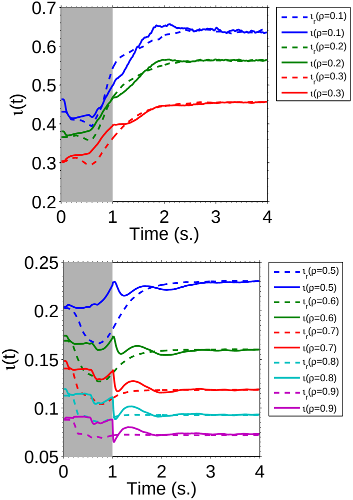


Figures (left & right): Time evolution of the rotational transform (ι-profile).

CASE 2: Actuation with n_e, I_p and Neutral Beams

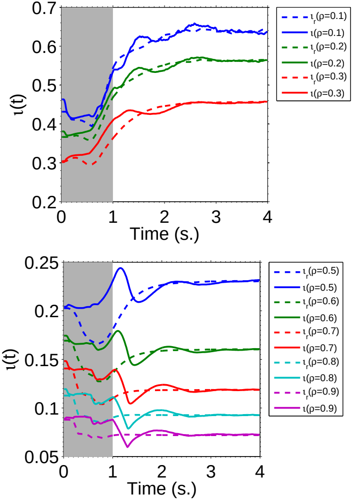
CASE 2A

(without I_p&n_e rate saturation)



CASE 2B

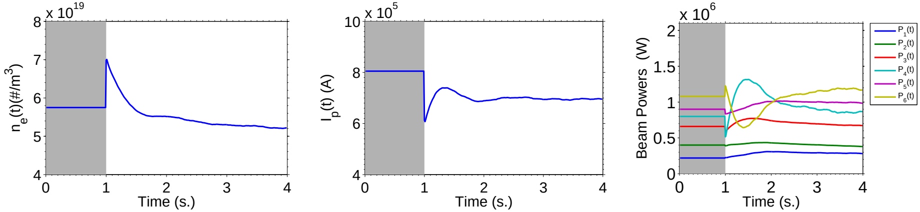
(with I_p&n_e rate saturation)



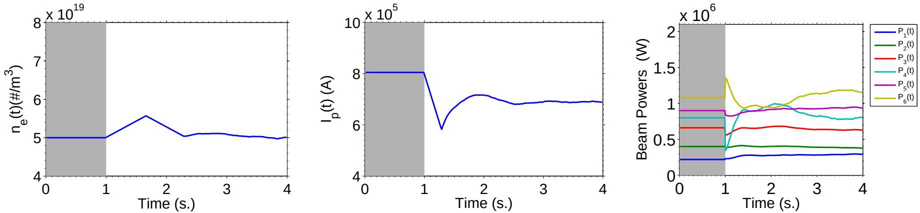
Figures (left & right): Time evolution of the optimal outputs (solid) with their respective targets (dashed). Controller is off in the grey region.

CASE 2: Actuation with n_e, I_p and Neutral Beams

- **CASE 2A** (without $I_p \& n_e$ rate saturation)



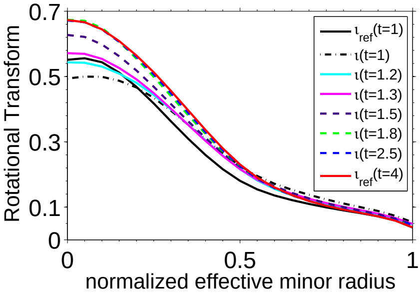
- **CASE 2B** (with $I_p \& n_e$ rate saturation)



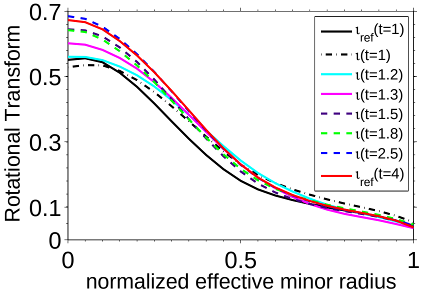
Figures: (left) Time evolution of the optimal line-averaged electron density, (center) time evolution of the optimal plasma current, and (right) time evolution of the optimal beam powers.

CASE 2: Actuation with n_e, I_p and Neutral Beams

CASE 2A
(without I_p&n_e rate saturation)



CASE 2B
(with I_p&n_e rate saturation)



Figures (left & right): Time evolution of the rotational transform (ι-profile).

CASE 3: Control Against Changing Confinement Factor

- In predictive TRANSP simulations, n_e and T_e profile evolutions are *not modeled by first-principles calculations.* **[11]**
- A reference n_e profile is specified based on an experimental profile measured on NSTX and then scaled to achieve a particular Greenwald fraction, f_{GW}.
- Similarly, T_e profile is also taken from an experiment and scaled to achieve a particular global confinement time **[12]**

$$\tau_{ST} = H_{ST}\, 0.1178\, I_p^{0.57}\, B_T^{1.08}\, n_e^{0.44}\, P_{\mathrm{Loss,th}}^{-0.73}$$

- When performing closed-loop simulations in TRANSP, the simulation must be constrained to follow a specific confinement level all the time although the actuators are varied based on the calculations of the feedback controller ⇒ This is achieved by manipulating the confinement factor H_{ST} through a user-defined waveform. **[13]**
- However, the H_{ST} factor can deviate from the user-supplied waveform in the NSTX-U experiments ⇒ ***creating additional source of disturbance.***

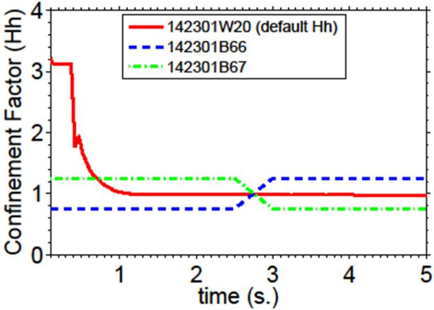
[11] GERHARDT, S. P., ANDRE, R., and MENARD, J. E., Nuclear Fusion **52 (2012).**

[12] KAYE, S. et al., Nuclear Fusion **46 (2006).**

[13] BOYER, M. D., ANDRE, R., GATES, D. A., et al., Nuclear Fusion **55 (2015).**

CASE 3: Control Against Changing Confinement Factor

- Two closed-loop TRANSP simulations are carried out to verify disturbance rejection against changing confinement factors:
  - Run 142301B66 has a step increase in the H_{ST} from 0.75 to 1.25.
  - Run 142301B67 has a step decrease in the H_{ST} from 1.25 to 0.75.
- Note that the target profile corresponds to the open-loop run 142301W20, which has $H_{ST} \approx 1$ when $t \in [1\ 5]$ s., during which the controller is on.
- Only I_p and neutral beams are used as actuators without considering rate saturations.

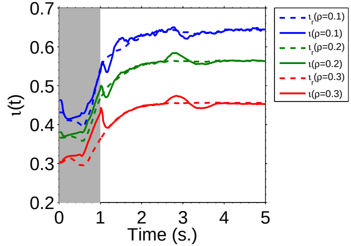


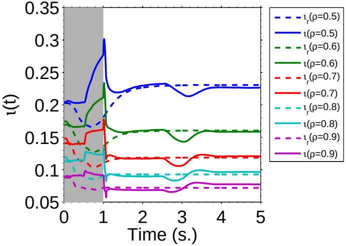
Time evolution of the confinement factors.

CASE 3: Control Against Changing Confinement Factor

CASE 3A

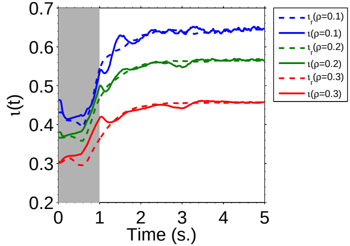
(increasing H_{ST} - 142301B66)

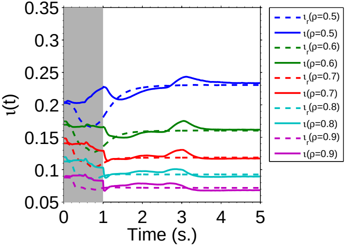




CASE 3B

(decreasing H_{ST} - 142301B67)



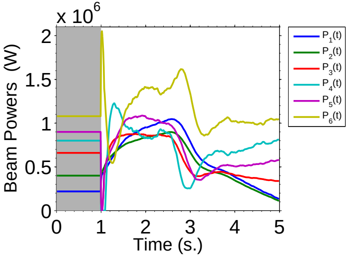


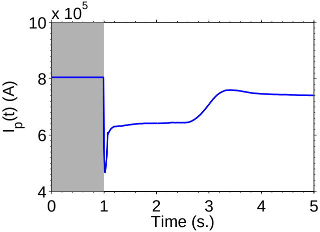
Figures (left & right): Time evolution of the optimal outputs (solid) with their respective targets (dashed). Controller is off in the grey region.

CASE 3: Control Against Changing Confinement Factor

CASE 3A

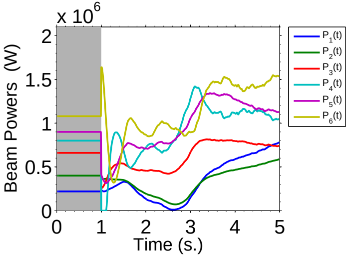
(increasing H_{ST} - 142301B66)

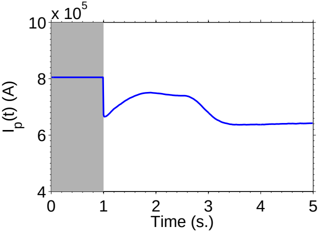




CASE 3B

(decreasing H_{ST} - 142301B67)



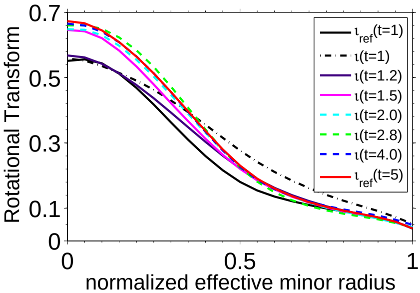


Figures (upper left & right): Time evolution of the optimal beam powers.
Figures (lower left & right): Time evolution of the optimal plasma current.

CASE 3: Control Against Changing Confinement Factor

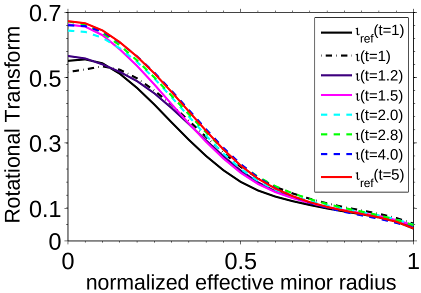
CASE 3A

(increasing H_{ST} - 142301B66)



CASE 3B

(decreasing H_{ST} - 142301B67)



Figures (left & right): Time evolution of the rotational transform (ι-profile).

Conclusion and Future Work

- A nonlinear, control-oriented, physics-based model has been proposed to describe the evolution of the poloidal magnetic flux profile, which can be related to the q-profile (ι-profile) $\Rightarrow$ the current density profile.
- Using this first-principles-driven (FPD), control-oriented model, a two-component control design approach has been proposed for the regulation of the current density profile:
  1. A feedforward trajectory optimizer (controller) to compute offline actuator requests to achieve specific plasma scenarios.
  2. A feedback control algorithm to track a desired current density profile while adding robustness against model uncertainties and disturbances to the overall current profile control scheme.
- The performance of the feedback controller has been validated in TRANSP simulations through the recently developed Expert routine, which provides a framework to perform closed-loop predictive simulations within the TRANSP source code.
- The immediate next step is to test the feedforward actuator trajectory optimizer in TRANSP and then in the actual NSTX-U machine.
- A longer-term next step is the implementation of the feedback controller in the NSTX-U PCS with the ultimate goal of experimental testing.