BOUT++ simulations for pedestal plasmas with testable models

X. Q. Xu

Lawrence Livermore National Laboratory, USA

C-Mod/NSTX pedestal planning workshop

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Anomalous Electron Viscosity for Pedestal Plasmas

- **With addition of the anomalous electron viscosity μ_e under the assumption that $\mu_e \sim \chi_e$, it is found from simulations using a realistic high Lundquist number S, BOUT++ simulations show that**
  - **the pedestal collapse is limited to the edge region**
  - **the ELM size is about 5-10% of the pedestal stored energy.**

  These are consistent with observations of large ELMs.

- **CDBM transport model: Itoh et al**
  - Thermal diffusivities of the CDBM model is based on the theory of self-sustained turbulence due to the ballooning mode driven by the turbulent current diffusivity.
    - Itoh K. *et al 1993 Plasma Phys. Control. Fusion* ***35 543***
    - Itoh K. *et al 1994 Plasma Phys. Control. Fusion* ***36 279***

- **J Drake's latest 3D PIC simulations for earth's magnetosphere show that**
  - Breaking field lines during reconnection: it's anomalous viscosity not anomalous resistivity
    - J. Drake, Cambridge Summer Workshop on Gyrokinetics, July 26,2010

The basic set of equations for the MHD peeling-ballooning modes

$$\frac{\partial \varpi}{\partial t} + v_E \cdot \nabla \varpi = B_0^2 \nabla_{||}\left(\frac{j_{||}}{B_0}\right) + 2b_0 \times \kappa \cdot \nabla P,$$

$$\frac{\partial P}{\partial t} + v_E \cdot \nabla P = 0,$$

$$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}(\phi + \Phi_0) + \frac{\eta}{\mu_0}\nabla_\perp^2 A_{||} - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_{||},$$

$$\varpi = \frac{n_0 M_i}{B_0}\left(\nabla_\perp^2 \phi + \nabla_\perp^2 P\right),$$

$$j_{||} = J_{||0} - \frac{1}{\mu_0}\nabla_\perp^2 A_{||}, v_E = \frac{1}{B_0} b_0 \times \nabla(\phi + \Phi_0)$$

Non-ideal physics

✓Using resistive MHD term, resistivity can renormalized as Lundquist Number $S=\mu_0 R v_A/\eta$

✓Using hyper-resistivity η_H

$S_H=\mu_0 R^3 v_A/\eta_H=S/\alpha_H$

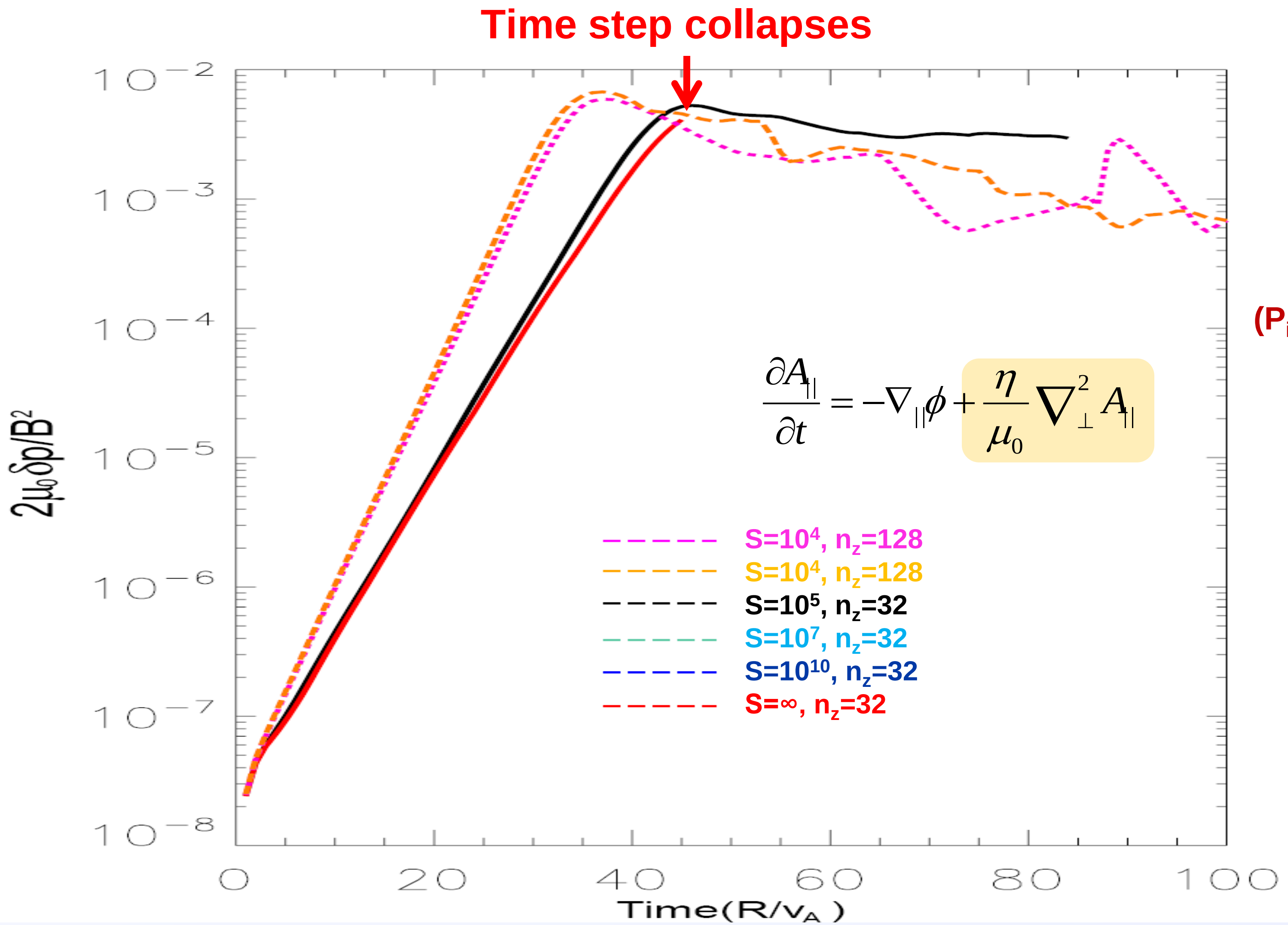
✓After gyroviscous cancellation, the diamagnetic drift modifies the vorticity and additional nonlinear terms

✓Using force balance and assuming no net rotation,

$E_{r0}=(1/N_i Z_i e)\nabla_\perp P_{i0}$

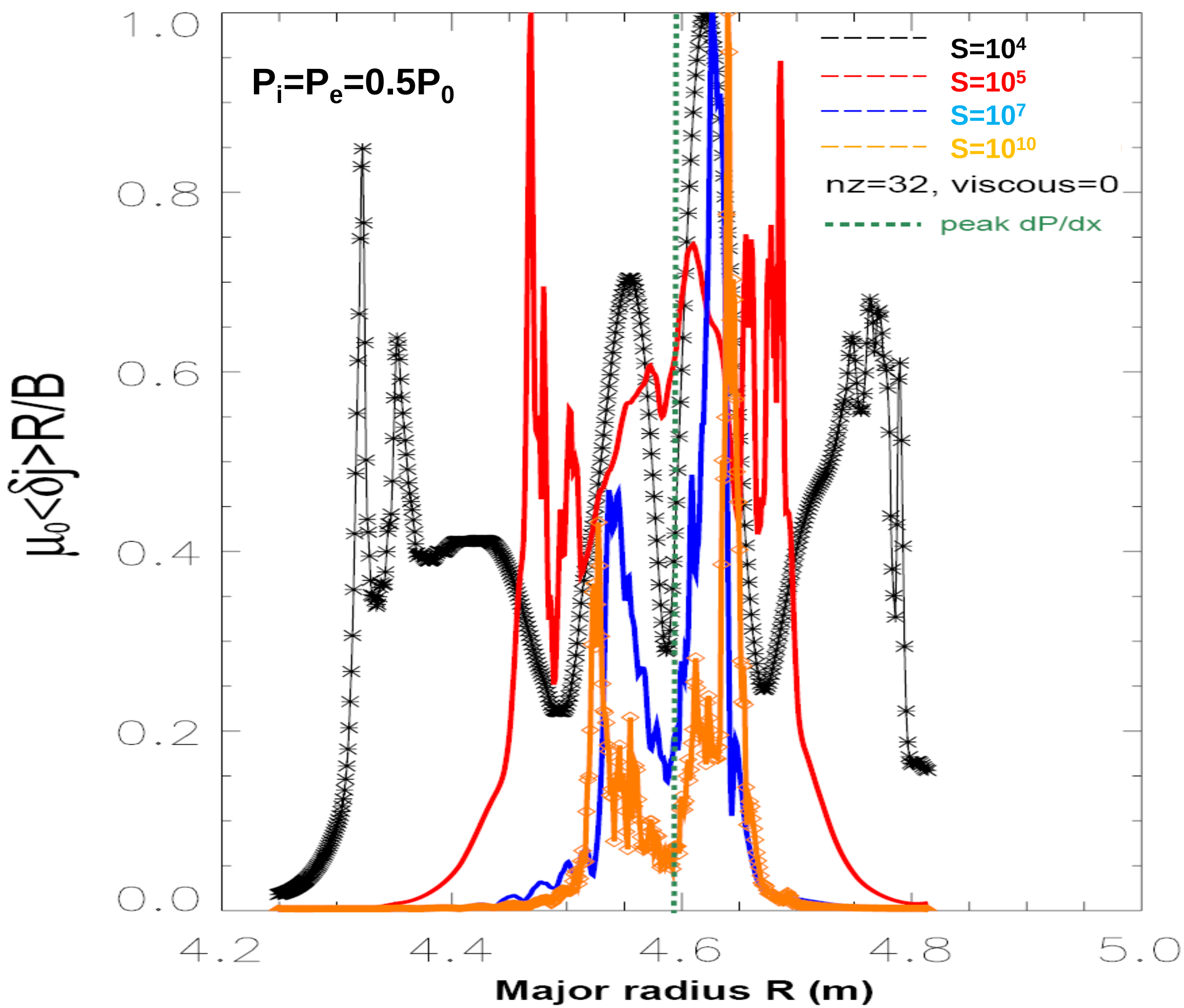
Lundquist number S plays a critical role on nonlinear ideal ballooning mode

Time step collapses at high Lundquist number S w/o η_H



($P_i = P_e = 0.5P_0$)

Radial profile of averaged perturbed parallel current $\delta j_{||}$ at outside midplane: resolving the classical current sheet is a formidable task



$$\frac{\partial \delta A_{||}}{\partial t} = \cdots + \frac{1}{S}\nabla_\perp^2 \delta A_{||}$$

$$\hat{\Delta}_\perp = \left(\frac{1}{\hat{\omega} S}\right)^{1/2}$$

$$S \approx 1\times10^8 - 1\times10^{10},$$

$$\hat{\omega} \approx 0.1,$$

$$\hat{\Delta}_\perp \equiv \frac{\Delta_\perp}{R} \approx 1\times10^{-4}$$

$$\frac{R}{\Delta r} \geq 1\times10^4, \Delta r \approx \rho_e \leq 100 microns$$

Other physics should be included to prevent the tiny current layer

Scaling for the generalized Ohm's law

Hyper-resistivity prevents $J_{||}$ layer from collapsing to the resistive scale

- **The magnetic flux surfaces must reconnect before the pedestal plasma collapses**

- **The relevant radial scale lengths for the generalized Ohm's law:**

  - ✓ **q spacing Δ_q**
  - ✓ **J layer Δ_J**
  - ✓ **Hall-MHD ($\sim\delta_i$)**
  - ✓ **Skin depth($\sim\delta_e$)**

$$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}\phi + \frac{\eta}{\mu_0}\nabla_\perp^2 A_{||} + \frac{1}{ne} b\cdot\nabla\cdot\vec{\vec{p}} - \frac{m_e}{ne^2}\frac{\partial j_{||}}{\partial t},$$

$$\frac{\partial \hat{A}_{||}}{\partial \hat{t}} = -\hat{\nabla}_{||}\hat{\phi} + \left(\frac{1}{S}\right)\hat{\nabla}_\perp^2 \hat{A}_{||} + \left(\frac{\delta_i}{L_{||}}\right) b\cdot\hat{\nabla}\cdot\hat{\vec{\vec{p}}}_e - \left(\frac{\delta_e}{L_{||}}\right)^2 \frac{\partial j_{||}}{\partial t},$$

Enhanced resistivity due to turbulence: a common practice

- **leads to significantly different growth rates and instability thresholds**
- **In nonlinear RMHD, the pedestal pressure collapses deep inside the plasma core**

Scaling for the generalized Ohm's law

Hyper-resistivity prevents $J_{||}$ layer from collapsing to the resistive scale in low collision regime

➢The magnetic flux surfaces must reconnect before the pedestal plasma collapses

➢The relevant radial scale lengths for the generalized Ohm's law :

- ✓**q spacing Δ_q**
- ✓**J layer Δ_J**
- ✓**Hall-MHD (~δ_i)**
- ✓**Skin depth(~δ_e)**

$$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}\phi + \frac{\eta}{\mu_0}\nabla_\perp^2 A_{||} - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_{||} + \frac{1}{ne} b\cdot\nabla\cdot\vec{\vec{p}} - \frac{m_e}{ne^2}\frac{\partial j_{||}}{\partial t}, \eta_H \propto \mu_e$$

$$\frac{\partial \hat{A}_{||}}{\partial \hat{t}} = -\hat{\nabla}_{||}\hat{\phi} + \left(\frac{1}{S}\right)\hat{\nabla}_\perp^2 \hat{A}_{||} - \left(\frac{1}{S_H}\right)\hat{\nabla}_\perp^4 \hat{A}_{||} + \left(\frac{\delta_i}{L_{||}}\right) b\cdot\hat{\nabla}\cdot\hat{\vec{\vec{p}}}_e - \left(\frac{\delta_e}{L_{||}}\right)^2 \frac{\partial j_{||}}{\partial t},$$

Enhanced resistivity due to turbulence: a common practice

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Viscous MHD(Δ_H): (?)

The mechanisms for hyper-resistivity η_H in pedestal plasmas

- The mechanism for hyper-resistivity η_H is often attributed to the presence of chaos in the magnetic field structure of a plasma
    - η_H leads to a flattening of the current density profile
- In edge plasmas, η_H may be generated by a broad spectrum of kinetic scale electron turbulence with closely spaced mode rational surfaces:
    - Dissipative drift-wave turbulence
    - Dissipative trapped electron modes (DTEM)
    - Electron temperature gradient driven modes (ETG)
    - Rechester-Rosenbluth-type electron diffusion in stochastic B-field
- Hyper-resistivity has often been used in MHD computations as a form of subgrid modeling, such as
    - in RFPs, in spheromaks, for laboratory, solar, and astrophysical applications

Hyper-resistivity can be used to set the finest resolved radial scale in simulations

✓The hyper-Lundquist number

$$S_H = \mu_0 R^3 v_A / \eta_H = S/\alpha_H,$$

with a dimensionless hyper-Lundquist parameter

$$\alpha_H = \eta_H/(R^2\eta).$$

✓For a collisional electron viscosity,

$$\alpha_H = \mu_e/(R^2 \nu_{ei}).$$

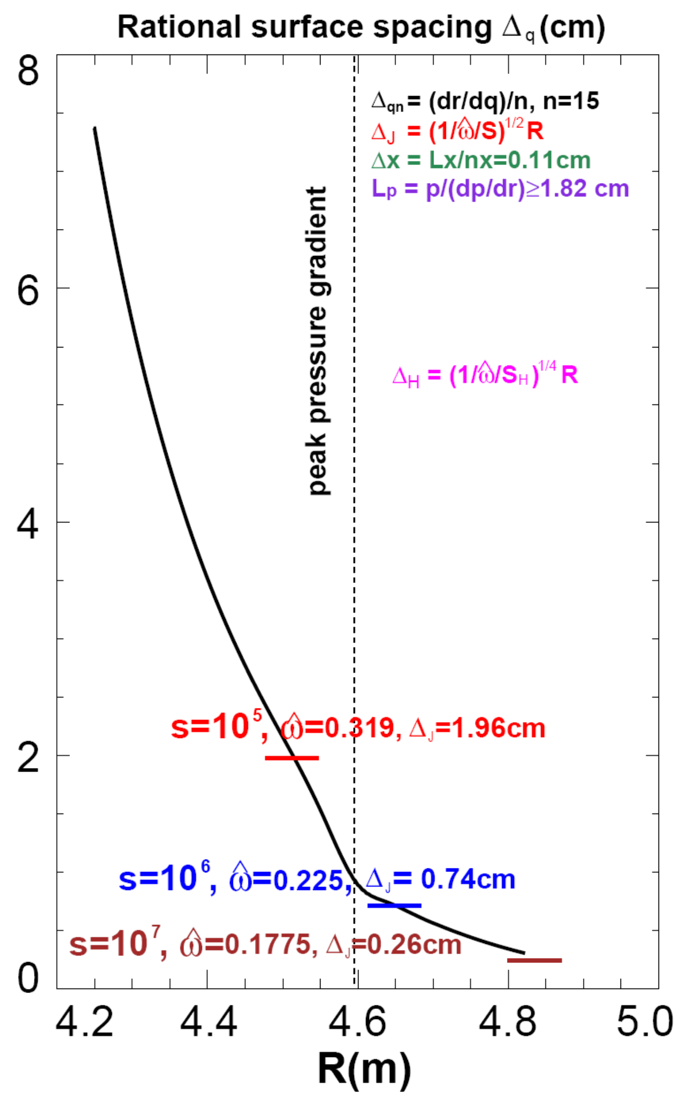
✓Assuming $\mu_e \approx \chi_e \approx 1\ m^2/s$ and $\nu_{ei} \approx 10^5$, we can estimate the amplitude of the hyper-Lundquist parameter to be

$$\alpha_H \approx 10^{-4} - 10^{-6}.$$

✓**For real pedestal plasmas $S_H = 10^{12}$, the viscous layer width can be estimated as**

$$\Delta_H = \left(\frac{1}{\hat{\omega} S_H}\right)^{1/4} R \approx 1.78mm$$

$$L_p > \Delta_q > \Delta_H > \Delta_x > \rho_i(\delta_i, \delta_e) > \Delta_J$$



Nonlinear simulations of peeling-ballooning modes with anomalous electron viscosity and their role in ELM crashes

- **We do not completely understand anomalous electron viscosity μ_e/hyper-resistivity η_H**

- **but we will use the parameter to model ELM dynamics**
  - ✓ **assuming $\mu_e \sim \chi_e$**

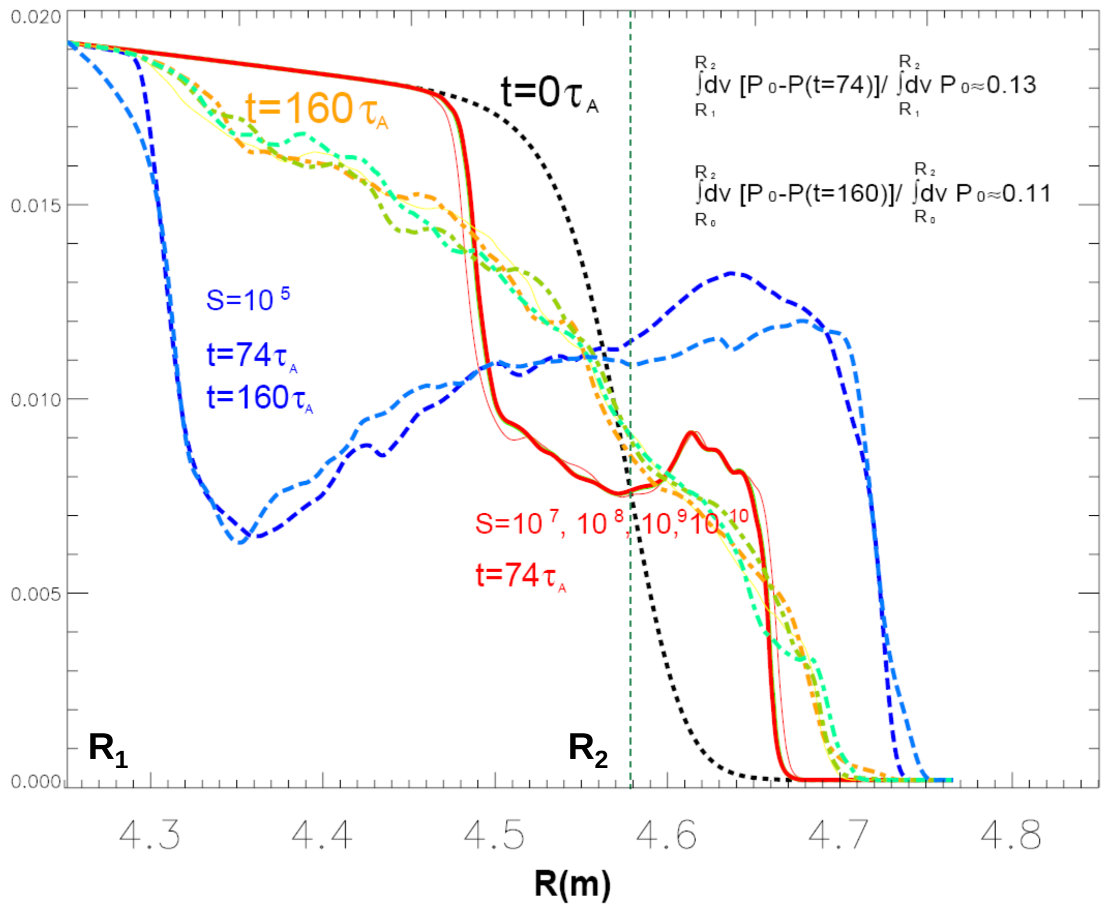
- **In our present model, the frozen-in flux condition of ideal MHD theory is broken by**
  - ✓ **resistivity**
  - ✓ **hyper-resistivity**

During the campaign of FES Joint Theory and Experiment Research Target 2011

We may use GYRO to compute χ_e and μ_e, and assess the assumption

Flux-surface-averaged pressure profile $2\mu_0 <P>/B^2$ vs S with $S_H=10^{12}$

low S -> large ELM size, ELM size is insensitive when $S>10^7$



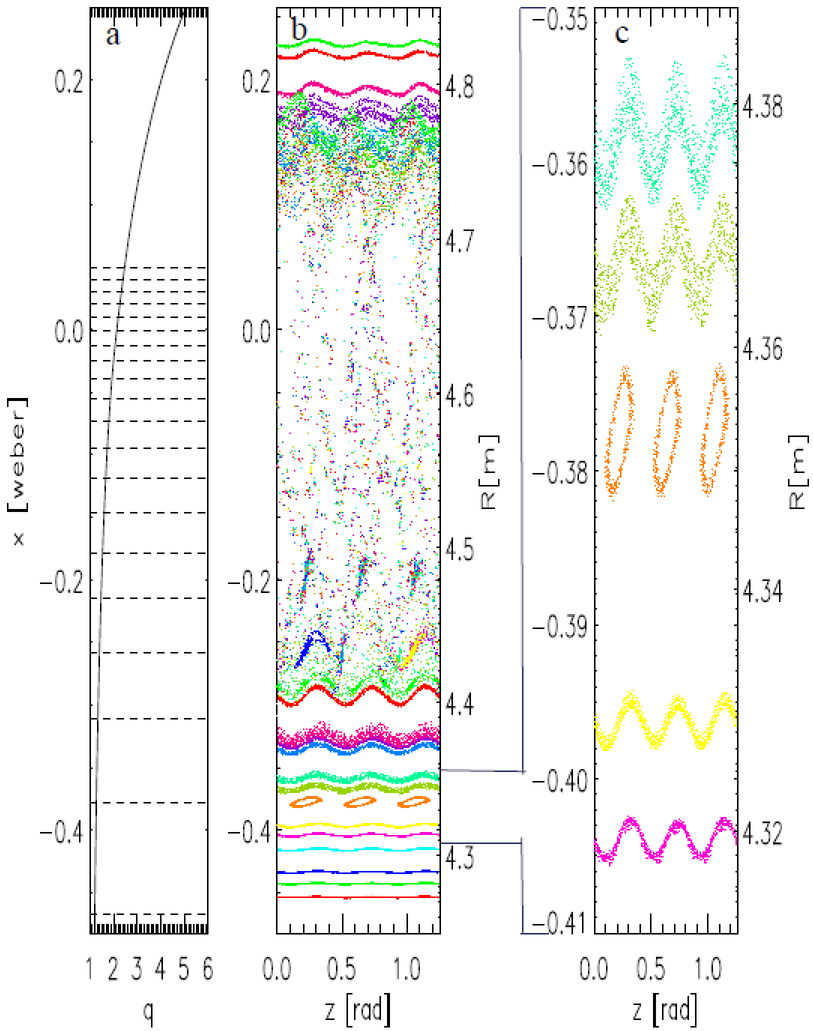
ELM size= $\Delta W_{ped}/ W_{ped}$

ΔW_{ped}= the ELM energy loss

W_{ped} =pedestal stored energy

(1) a sudden collapse as: P-B modes -> magnetic reconnection -> bursting process
(2) a slow backfill as a turbulence transport process

Line trace during pedestal pressure crash shows a novel feature of ideal MHD characteristics in peak gradient region and island formation on top of the pedestal



(a) radial distance x vs safety factor q, dashed lines show rational surfaces q=m/n with n=15;

(b) line trace for $S = 10^8$ and $S_H = 10^{12}$ during pedestal pressure crash in field-aligned coordinate (x,y,z);

(c) a zoom-in view of small region x=[-0.41,-0.35] in (b) to show the island formation.

CDBM Transport Model

Itoh, Fukuyama, Yagi, et al. PPCF 1993, PPCF 1994

- Thermal diffusivities of the CDBM model is based on the theory of self-sustained turbulence due to the ballooning mode driven by the turbulent current diffusivity.
  - Drive: Inclusion of the electron viscosity allows the electromagnetic fluctuation
    - to enhance electron viscosity
    - to make instabilities more unstable in a short-wavelength mode
  - Sink: As the fluctuation amplitude increases, the stabilizing effect due to the anomalous thermal diffusivity χ and the ion viscosity μ_{ii} eventually overcomes the destabilizing effect of the current diffusivity μ_e.
  - Pedestal width: The steady pedestal profile is determined by the balance of these effects.
- By solving an eigen-value problem for the ballooning mode, a general expression of the turbulent thermal diffusivity can be obtained as

$$\chi_{\mathrm{TB}} = C F(\hat{s}, \alpha)\alpha^{3/2}\frac{c^2}{\omega_{\mathrm{pe}}^2}\frac{v_{\mathrm{A}}}{qR}. \quad \hat{s} \equiv \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}, \quad \alpha \equiv -q^2 R\frac{\mathrm{d}\beta}{\mathrm{d}r}, \quad \beta = \frac{P_e + P_i}{2B_0^2\mu_0}$$

$$F = \begin{cases} \dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}}, & \text{for } s' = \hat{s} - \alpha < 0, \\ \dfrac{(1+9\sqrt{2}s'^{5/2})}{\sqrt{2}(1-2s'+3s'^2+2s'^3)}, & \text{for } s' = \hat{s} - \alpha > 0. \end{cases}$$

The fitting parameter, *C = 12, is determined by* comparing the energy confinement time For the Standard plasma parameters with the ITER-89P L-mode scaling law

- **BOUT++ simulations can be used to verify the transport model.**
- **An similar analysis to EPED for KBM can be done to get pedestal width.**

The brief outline for the CDBM modes, Itoh, Fukuyama, Yagi, et al.

$$\frac{\partial \varpi}{\partial t} + v_E \cdot \nabla \varpi = B_0^2 \nabla_{||}\left(\frac{j_{||}}{B_0}\right) + 2b_0 \times \kappa \cdot \nabla P + \mu_{ii} \nabla_\perp^2 \varpi,$$

$$\frac{\partial P}{\partial t} + v_E \cdot \nabla P = \chi \nabla_\perp^2 P,$$

$$\frac{\partial A_{||}}{\partial t} = -\nabla_{||}(\phi + \Phi_0) + \frac{\eta}{\mu_0}\nabla_\perp^2 A_{||} - \frac{\eta_H}{\mu_0}\nabla_\perp^4 A_{||},$$

$$\varpi = \frac{n_0 M_i}{B_0}\left(\nabla_\perp^2 \phi + \nabla_\perp^2 P\right),$$

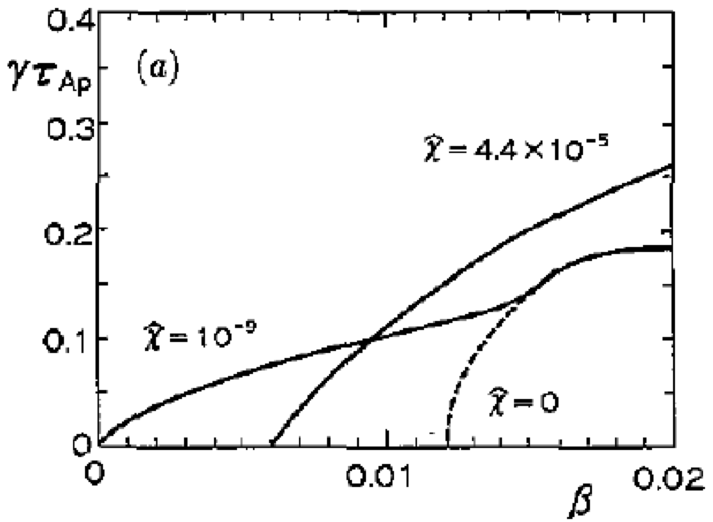
$$j_{||} = J_{||0} - \frac{1}{\mu_0}\nabla_\perp^2 A_{||}, v_E = \frac{1}{B_0} b_0 \times \nabla(\phi + \Phi_0)$$

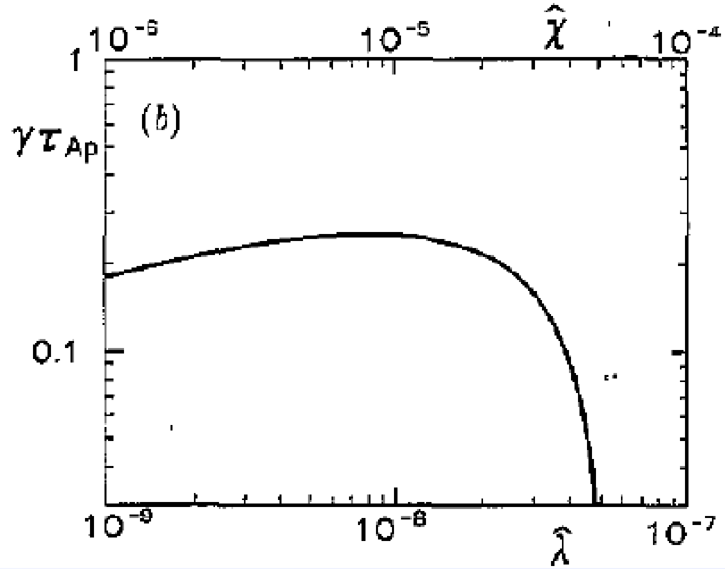
- **The parameters μ_{ii}, χ, η_H are anomalous transport coefficients**

- **The marginal stability condition of the linearized set of equations yields the constrain between the plasma pressure gradient and the fluctuation-induced transport coefficients**

- **Assuming the Prandtl number is unity,**

$$\mu_{ii}/\chi \approx 1, \mu_e/\chi \approx 1$$

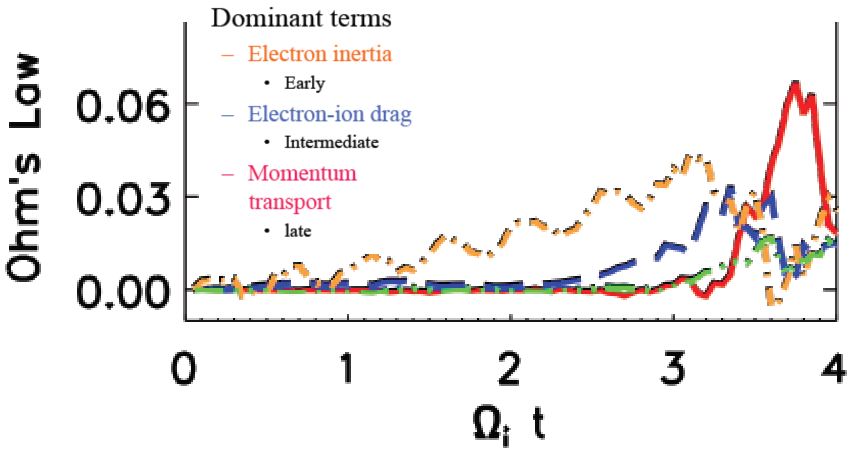


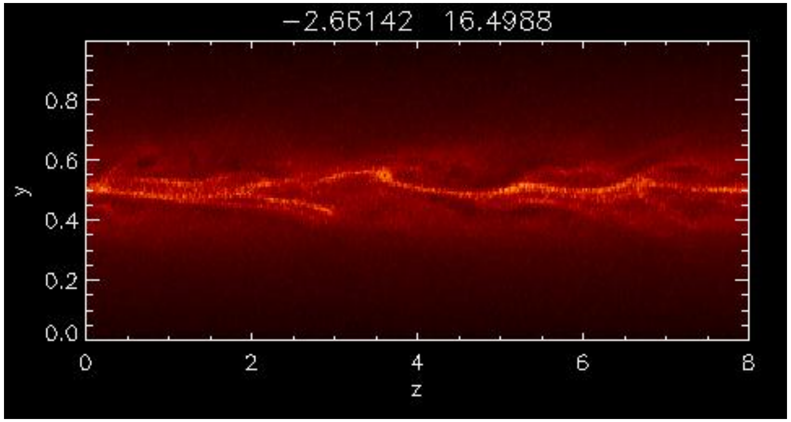


Breaking field lines during reconnection: it's anomalous viscosity not anomalous resistivity

- For earth's magnetosphere application, the 3D PIC simulations show:
- Turbulence is driven by the electron current during low-β_e reconnection with a guide field
- Current driven instabilities such as Buneman or the lower hybrid instability develop and produce anomalous resistivity and electron heating but do not stop the electrons from running away
- The continued thinning of the current layer continues until an electromagnetic electron sheared-flow instability breaks up the current layer
  – The resulting anomalous momentum transport is sufficient to balance the reconnection electric field
  – The rate of reconnection undergoes a modest jump as the shear-flow instability onsets

Evolution of Ohm's law as current layer is thinning





Summary

- **Non-ideal physics effects are essential for pedestal plasma modeling**
  - **Hyper-resistivity can be used to set the finest radial scale in high S simulations**
  - **From nonlinear simulations, ELM dynamics can be described as**
    - **P-B modes -> magnetic reconnection-> pedestal collapse**

- **With addition of the anomalous electron viscosity μ_e under the assumption that $\mu_e \sim \chi_e$, it is found from simulations using a realistic high Lundquist number S**
  - **the pedestal collapse is limited to the edge region**
  - **the ELM size is about 5-10% of the pedestal stored energy.**

  These are consistent with observations of large ELMs.

- **CDBM transport model can be one of possible mechanisms for pedestal plasmas**
  - **To provide anomalous energy transport and yield pedestal width when pedestal height is below Peeling-Ballooning instability threshold**
  - **To facilitate the ELM crash when pedestal height is above Peeling-Ballooning instability threshold**

- **Validated BOUT++ simulations can yield a self-consistent capability to determine pedestal height and width as the EPED model**
  - **Model KBM and ETG transport can be added as well.**