Rotation and kinetic effects on ideal and resistive-wall modes in NSTX

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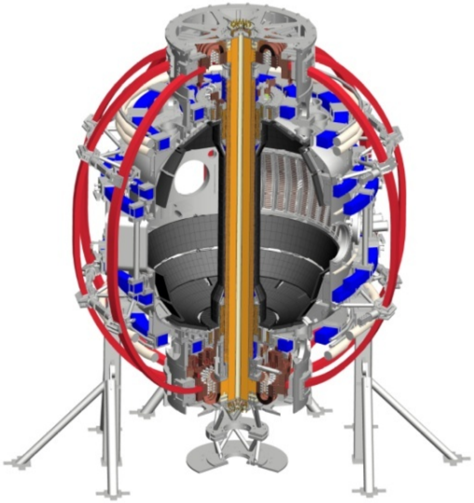

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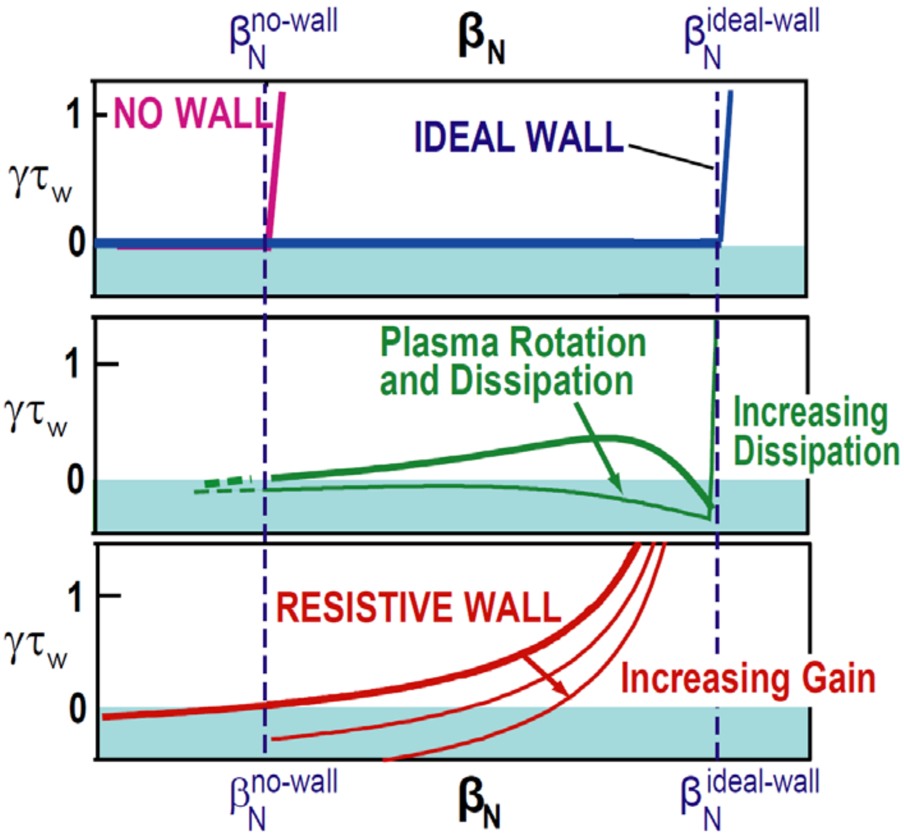
Pressure-driven kink limit is strong physics constraint on maximum fusion performance

$$\mathbf{P_{fusion}} \propto n^2\langle\sigma v\rangle \propto p^2 \propto \beta_T{}^2 B_T{}^4 \propto \boldsymbol{\beta_N{}^4 B_T{}^4} (1+\kappa^2)^2 / A\, f_{BS}{}^2$$



M. Chu, et al., Plasma Phys. Control. Fusion 52 (2010) 123001

- Modes grow rapidly above kink limit:
  - $\gamma \sim$ 1-10% of $\tau_A{}^{-1}$ where $\tau_A \sim 1\mu s$
- Superconducting "ideal wall" can increase stable β_N up to factor of 2
- Real wall resistive → slow-growing "resistive wall mode" (RWM)
  - $\gamma\, \tau_{wall} \sim 1$ →
  - ms instead of μs time-scales
- RWM can be stabilized with:
  - kinetic effects (rotation, dissipation)
  - active feedback control

Here we focus on <u>ideal-wall mode</u> (IWM), also treat RWM vs. Ω_ϕ

Background

- Characteristic growth rates and frequencies of RWM and IWM
  - RWM: $\gamma\tau_{wall}$ ~ 1 and $\omega\tau_{wall} < 1$
  - IWM: $\gamma\tau_A$ ~ 1-10% ($\gamma\tau_{wall} >> 1$) and $\omega\tau_A \sim \Omega_\phi\tau_A$ (1-30%) ($\omega\tau_{wall} >> 1$)
- Kinetic effects important for RWM (see Berkery poster)
  - Publications: Berkery, et al. PRL 104 (2010) 035003, Sabbagh, et al., NF 50 (2010) 025020
- Rotation and kinetic effects largely unexplored for IWM
  - Such effects generally higher-order than fluid terms (∇p, $J_{||}$, $|\delta B|^2$, wall)
- **Calculations for NSTX indicate both rotation and kinetic effects can modify both IWM and RWM stability limits**
  - High toroidal rotation generated by co-injected NBI in NSTX
    - Fast core rotation: $\Omega_\phi / \omega_{sound}$ up to ~1, $\Omega_\phi / \omega_{Alfven}$ ~ up to 0.1-0.3
  - Fluid/kinetic pressure is dominant instability drive in high-β ST plasmas

MARS-K: self-consistent linear resistive MHD including toroidal rotation and drift-kinetic effects

- **Perturbed single-fluid linear MHD:**

Y.Q. Liu, et al., Phys. Plasmas 15, 112503 2008

$$(\gamma+in\Omega)\xi = \mathbf{v}+(\xi\cdot\nabla\Omega)R^2\nabla\phi$$

$$\rho(\gamma+in\Omega)\mathbf{v} = \mathbf{j}\times\mathbf{B}+\mathbf{J}\times\mathbf{Q}-\nabla\cdot\mathbf{p}$$

$$\boxed{+\rho\left[2\Omega\hat{\mathbf{Z}}\times\mathbf{v}-(\mathbf{v}\cdot\nabla\Omega)R^2\nabla\phi\right]-\nabla\cdot(\rho\xi)\Omega\hat{\mathbf{Z}}\times\mathbf{V}_0}$$

$$(\gamma+in\Omega)\mathbf{Q} = \nabla\times(\mathbf{v}\times\mathbf{B})+(\mathbf{Q}\cdot\nabla\Omega)R^2\nabla\phi-\nabla\times(\eta\mathbf{j})$$

$$(\gamma+in\Omega)p = -\mathbf{v}\cdot\nabla P-\Gamma P\nabla\cdot\mathbf{v} \qquad \mathbf{j}=\nabla\times\mathbf{Q}$$

- **Drift-kinetic effects in perturbed anisotropic pressure *p*:**

$$\mathbf{p}=p\mathbf{I}+p_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}}+p_{\perp}(\mathbf{I}-\hat{\mathbf{b}}\hat{\mathbf{b}})$$

$$p_{\|}e^{-i\omega t+in\phi}=\sum_{e,i}\int d\Gamma M v_{\|}^2 f_L^1$$

$$p_{\perp}e^{-i\omega t+in\phi}=\sum_{e,i}\int d\Gamma \frac{1}{2} M v_{\perp}^2 f_L^1$$

$$f_L^1=-f_\epsilon^0\epsilon_k e^{-i\omega t+in\phi}\sum_{m,l,u}X_m^u H_{ml}^u \lambda_{ml} e^{-in\tilde{\phi}(t)+im\langle\dot{\chi}\rangle t+il\omega_b t}$$

$$H_L=\frac{1}{\epsilon_k}[Mv_{\|}^2\vec{\kappa}\cdot\xi_{\perp}+\mu(Q_{L\|}+\nabla B\cdot\xi_{\perp})]$$

- Rotation and rotation shear effects:
- Mode-particle resonance operator:

Diamagnetic

$$\lambda_{ml}=\frac{n[\omega_{*N}+(\hat{\epsilon}_k-3/2)\omega_{*T}+\omega_E]-\omega}{n(\langle\omega_d\rangle+\omega_E)+[\alpha(m+nq)+l]\omega_b-i\nu_{\text{eff}}-\omega}$$

Precession *ExB* *Transit and bounce* *Collisions*

- Fast ions: analytic slowing-down f(v) model – isotropic or anisotropic

This poster

- Include toroidal flow only: $\mathbf{v}_\phi = \mathbf{R}\Omega_\phi(\psi)$ and $\omega_E = \omega_E(\psi)$

Real part of complex energy functional provides equation for growth-rate useful for understanding instability sources

Dispersion relation

$$\delta K + \delta W = 0$$

Kinetic energy

$$\delta K = \frac{1}{2}\int d^3x\rho\left(\gamma + in\Omega\right)^2\left|\vec{\xi}_\perp\right|^2$$

Potential energy

$$\delta W = -\frac{1}{2}\int d^3x\,\mathbf{F}\cdot\xi_\perp^*$$

$$\delta K_1 = -\frac{1}{2}\int d^3x\rho\left|\vec{\xi}_\perp\right|^2$$

Growth rate equation: mode growth for $\delta W^{re} < 0$

$$\left(\gamma^{re}\right)^2 = \left(\delta W_K^{re} + \delta W_F^{re} + \delta W_{vb} + \delta W_{rot}^{re}\right) / \delta K_1$$

$$\delta W_K = -\frac{1}{2}\int d^3x\mathbf{F}^K\cdot\xi_\perp^* \qquad \mathbf{F}^K = -\nabla\cdot\mathbf{p}^{\text{kinetic}}$$

$$\delta W_{rot} = \delta W_\Omega + \delta W_{d\Omega} + \delta W_{cf} + \delta K_2$$

$$\delta W_F^p = -\frac{1}{2}\int d^3x\mathbf{F}^p\cdot\xi_\perp^*$$

$$= \frac{1}{2}\int d^3x\left[(\xi_\perp\cdot\nabla P)\nabla\cdot\xi_\perp^* + \Gamma P|\nabla\cdot\xi|^2 - \Gamma P(\nabla\cdot\xi)(\nabla\cdot\xi_\parallel^*)\right] + S_F^p$$

$$\delta W_F^j = -\frac{1}{2}\int d^3x\mathbf{F}^j\cdot\xi_\perp^* \;=\; \frac{1}{2}\int d^3x|Q|^2 + S_F^j$$

$$\delta W_F^Q = -\frac{1}{2}\int d^3x\mathbf{F}^Q\cdot\xi_\perp^* \;=\; \frac{1}{2}\int d^3x\left[J_\parallel\hat{\mathbf{b}}\cdot\xi_\perp^*\times\mathbf{Q}_\perp - \frac{Q_\parallel}{B}(\xi_\perp^*\cdot\nabla P)\right]$$

$$S_F^p = -\frac{1}{2}\int\left[(\xi_\perp\cdot\nabla P) + \Gamma P\nabla\cdot\xi\right]\xi_\perp^*\cdot d\mathbf{s}$$

$$S_F^j = \frac{1}{2}\int BQ_\parallel\xi_\perp^*\cdot d\mathbf{s}$$

Coriolis - Ω

$$\delta W_\Omega = \frac{1}{2}\int d^3x\left[-2\rho\Omega\left(\gamma + in\Omega\right)\mathbf{Z}\times\vec{\xi}_1\cdot\vec{\xi}_\perp^*\right]$$

Coriolis - dΩ/dρ

$$\delta W_{d\Omega} = \frac{1}{2}\int d^3xR\left(2\rho\Omega\left(\vec{\xi}_1\cdot\nabla\Omega\right)\vec{\xi}_{\perp\mathbf{R}}^*\right)$$

Centrifugal

$$\delta W_{cf} = \frac{1}{2}\int d^3xR\Omega^2\nabla\bullet\left(\rho\vec{\xi}_1\right)\vec{\xi}_{\perp\mathbf{R}}^*$$

Differential kinetic

$$\delta K_2 = -\frac{1}{2}\int d^3x\rho\left(\omega + n\Omega\right)^2\left|\vec{\xi}_\perp\right|^2$$

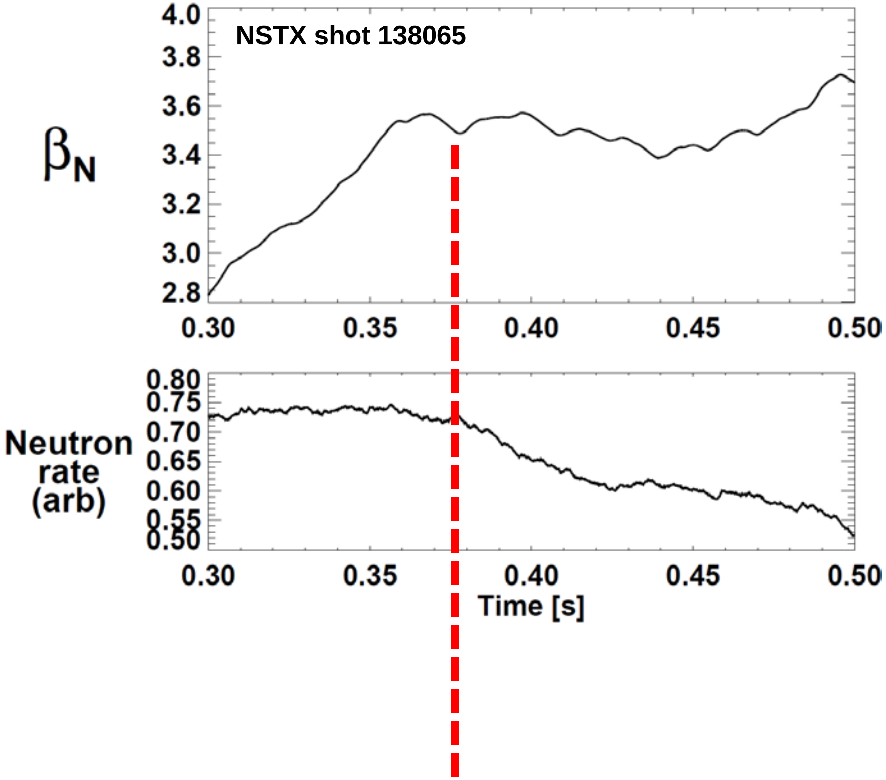
Note: n = -1 in MARS

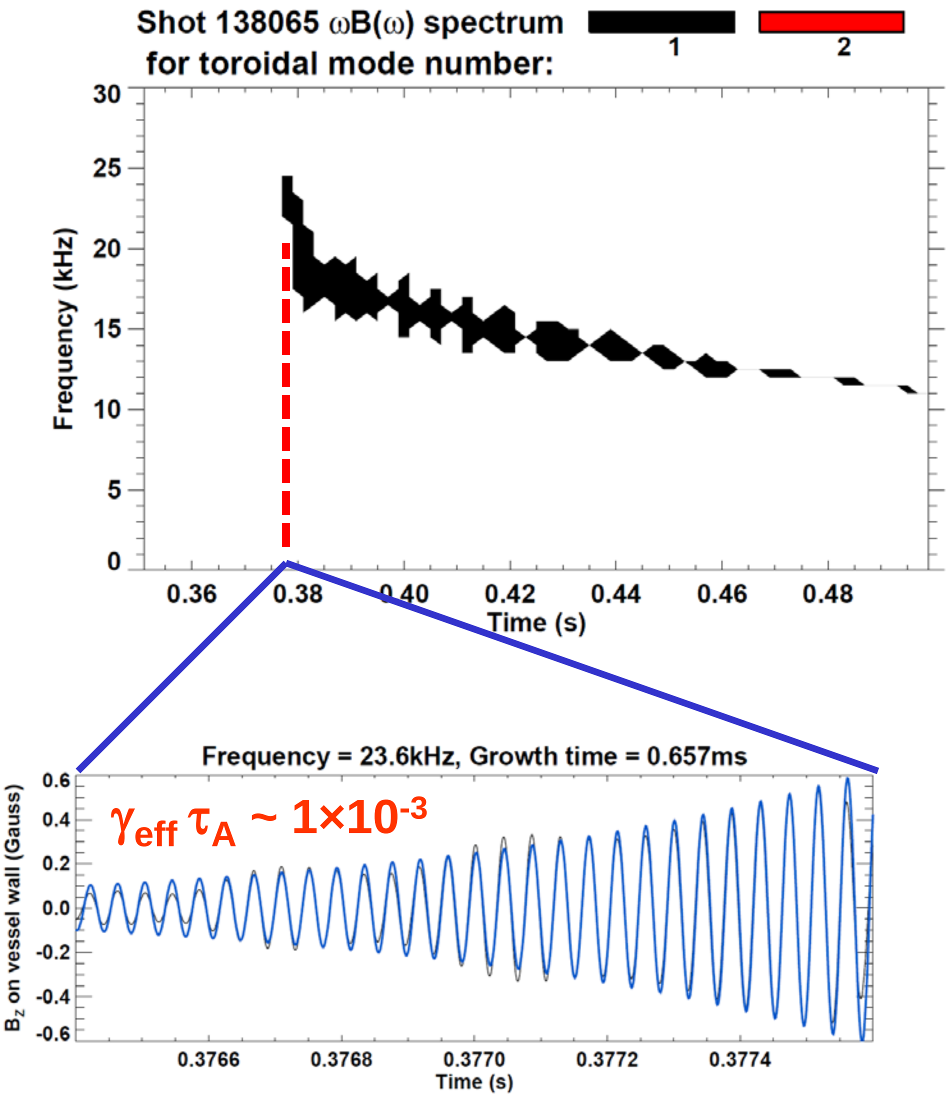
Study 3 classes of IWM-unstable plasmas spanning low to high β_N

- Low β_N limit ~3.5, often saturated/long-lived mode
  - q_{min} ~ 2-3
  - Common in early phase of current flat-top
  - Higher fraction of beam pressure, momentum (lower n_e)
- Intermediate β_N limit ~ 5
  - q_{min} ~1.2-1.5
  - Typical good-performance H-mode, H_{98} ~ 0.8-1.2
- Highest β_N limit ~ 6-6.5
  - q_{min} ~ 1
  - "Enhanced Pedestal" H-mode → high H_{98} ~ 1.5-1.6
  - Broad pressure, rotation profiles, high edge rotation shear

Low β_N limit ~ 3.5: Saturated f=15-30kHz n=1 mode common during early I_P flat-top phase

Fixed P_{NBI} = 3MW, I_P = 800kA, β_T=10-15%



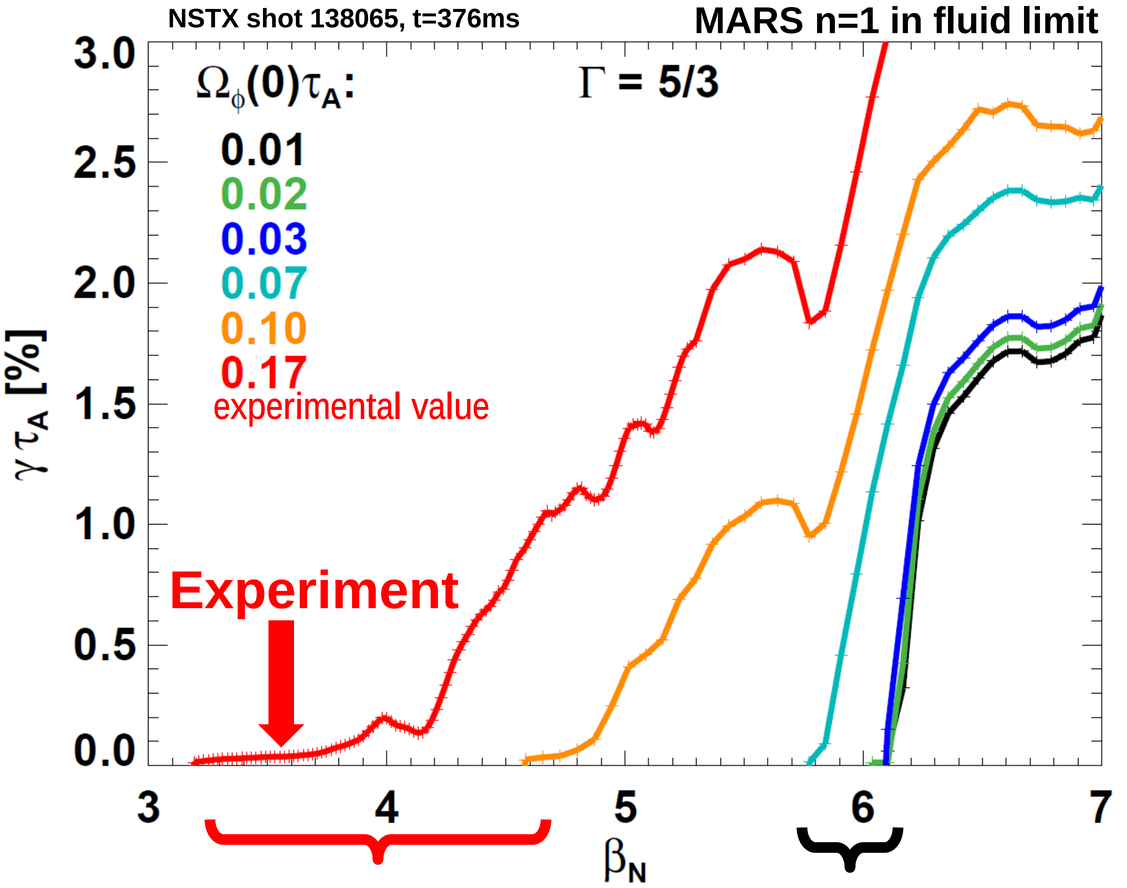


$\gamma_{eff}\,\tau_A$ ~ 1×10^{-3}

Mode clamps β_N to ~3.5, reduces neutron rate ~20%

sometimes slows → locks → disrupts

Fluid (non-kinetic) MARS-K calculations find: Rotation reduces IWL β_N = 6 → 3-3.5



High rotation β_N limit ~ 4.5 → 3.2

for $\Omega_\phi(0)\tau_A$ = 10 → 17% (experimental)

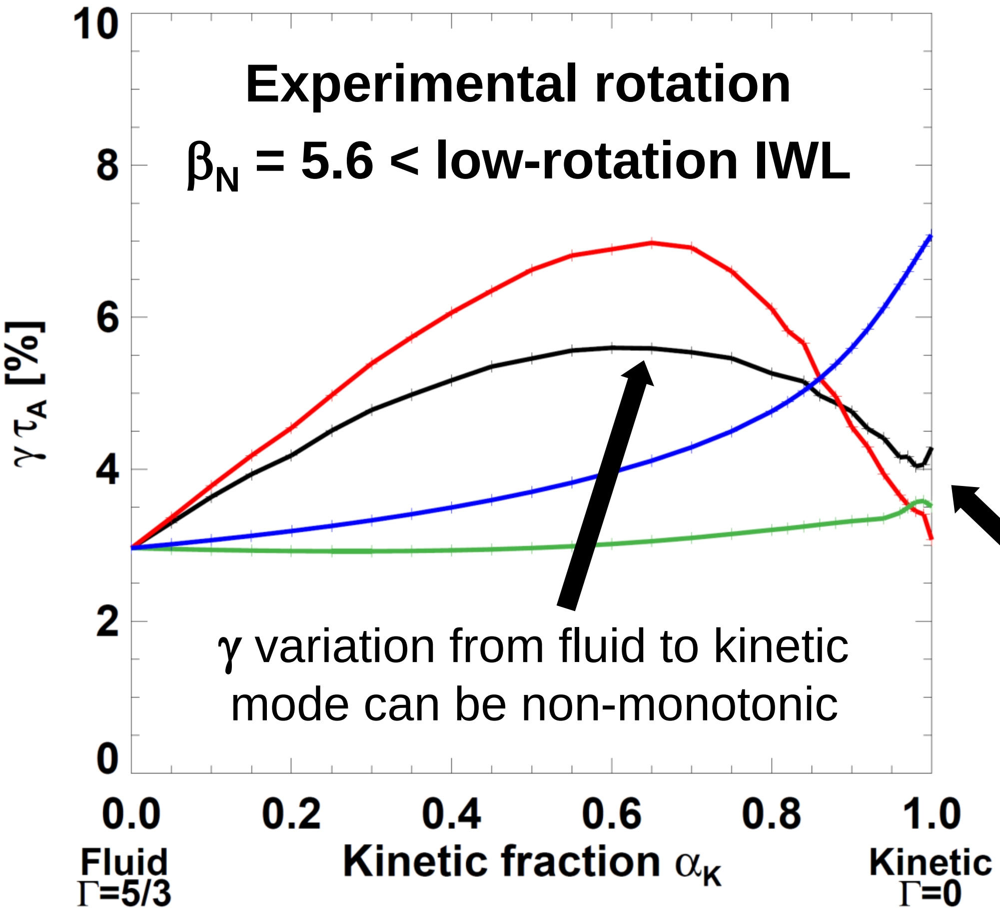
Low rotation β_N limit ~ 6

for $\Omega_\phi(0)\tau_A$ = 1 → 7%

Fluid MARS marginal β_N ~ 3 – 4 consistent with experiment

Kinetic mode also destabilized by rotation

Kinetic mode tracked numerically by starting from fluid root and increasing kinetic fraction $\alpha_K = 0 \rightarrow 1$ as $\Gamma = 5/3 \rightarrow 0$



Experimental rotation

β_N = 5.6 < low-rotation IWL

γ variation from fluid to kinetic mode can be non-monotonic

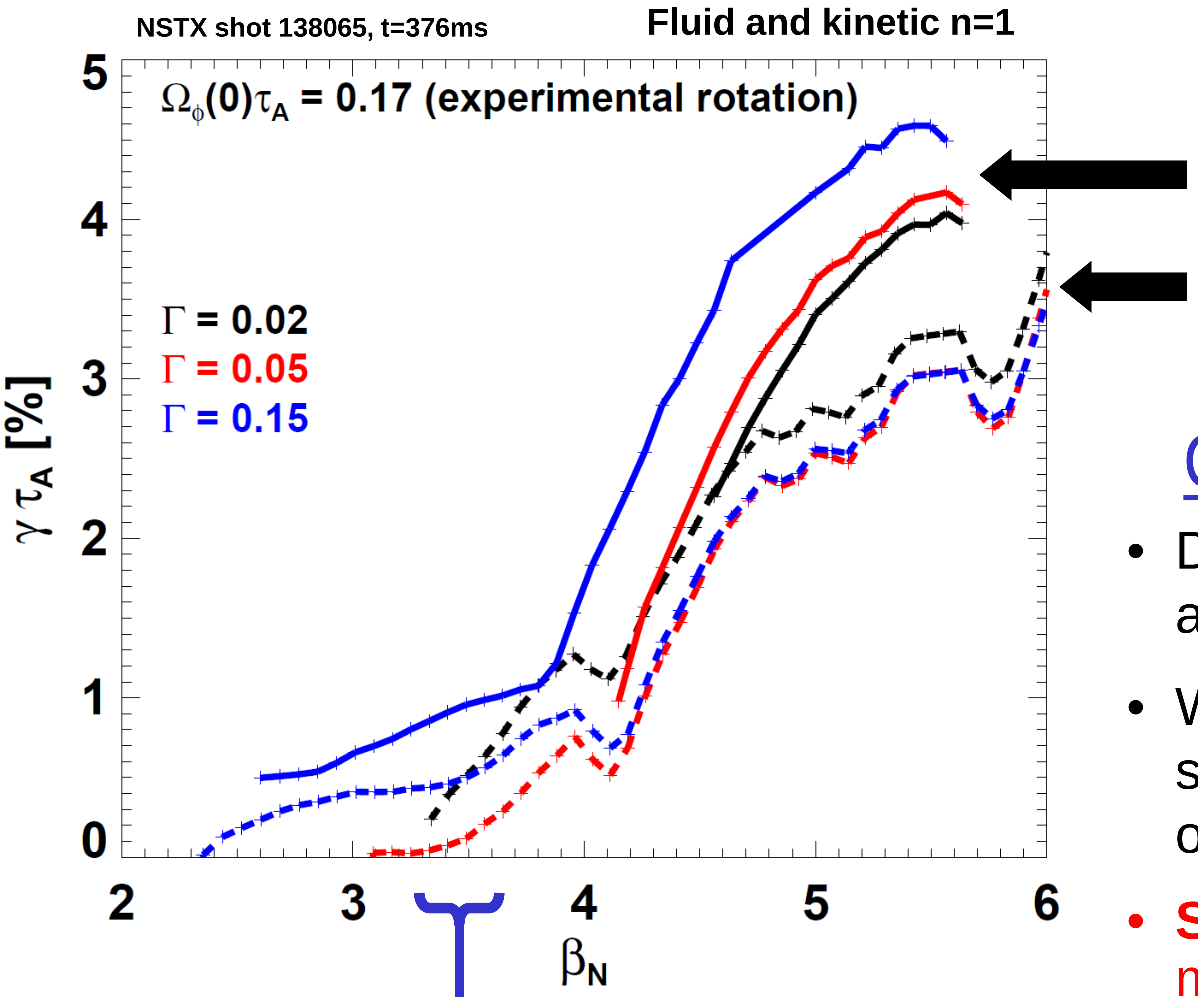
Non-adiabatic $\delta W_{kinetic}$ fraction varied (α_K=0 → 1) including:

- ◀ **No thermal or fast**
- ◀ **Thermal and fast**
- ◀ **Fast only**
- ◀ **Thermal only**

Kinetic mode $\gamma \approx$ fluid γ implies destabilization results from rotation

Kinetic stability limit similar to fluid limit:

Marginal $\beta_N < 3.5$ far below low-rotation β_N limit of ~6



Solid: Kinetic γ

Dashed: Fluid γ

Experimental β_N for n=1 mode onset

Convergence issues:

- Drift-kinetic MHD model assumes fluid $\Gamma = 0$
- When $\Gamma = 0$, MARS can sometimes track wrong root or find spurious root
- **Solution:** take $\Gamma \rightarrow 0$ limit, monitor eigenfunctions for continuous trend vs. α_k, β, Γ

Real part of complex energy functional consistent with rotational destabilization ($\delta W_{rot} \le 0$) across minor radius

$$\delta K + \delta W = 0 \qquad (\gamma^{re})^2 = \left(\delta W_K^{re} + \delta W_F^{re} + \delta W_{vb} + \delta W_{rot}^{re}\right)/\,\delta K_1 \qquad \delta K_1 = -\frac{1}{2}\int d^3x\rho\left|\vec{\xi}_\perp\right|^2 < 0$$

$$\delta W_{rot} = \delta W_\Omega + \delta W_{d\Omega} + \delta W_{cf} + \delta K_2$$

Coriolis - Ω

$$\delta W_\Omega = \frac{1}{2}\int d^3x\left[-2\rho\Omega(\gamma + in\Omega)\mathbf{Z}\times\vec{\xi}_1\cdot\vec{\xi}_\perp^*\right]$$

Coriolis - dΩ/dρ

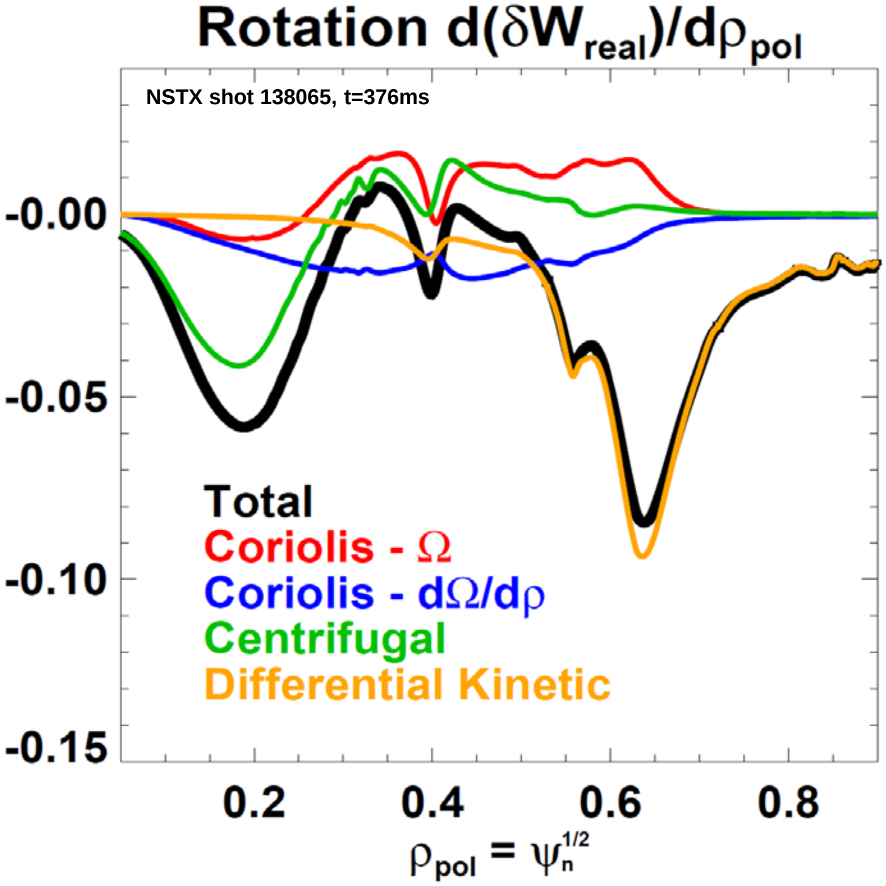
$$\delta W_{d\Omega} = \frac{1}{2}\int d^3xR\left(2\rho\Omega\left(\vec{\xi}_1\cdot\nabla\Omega\right)\vec{\xi}_{\perp\mathbf{R}}^*\right)$$

Centrifugal

$$\delta W_{cf} = \frac{1}{2}\int d^3xR\Omega^2\nabla\bullet\left(\rho\vec{\xi}_1\right)\vec{\xi}_{\perp\mathbf{R}}^*$$

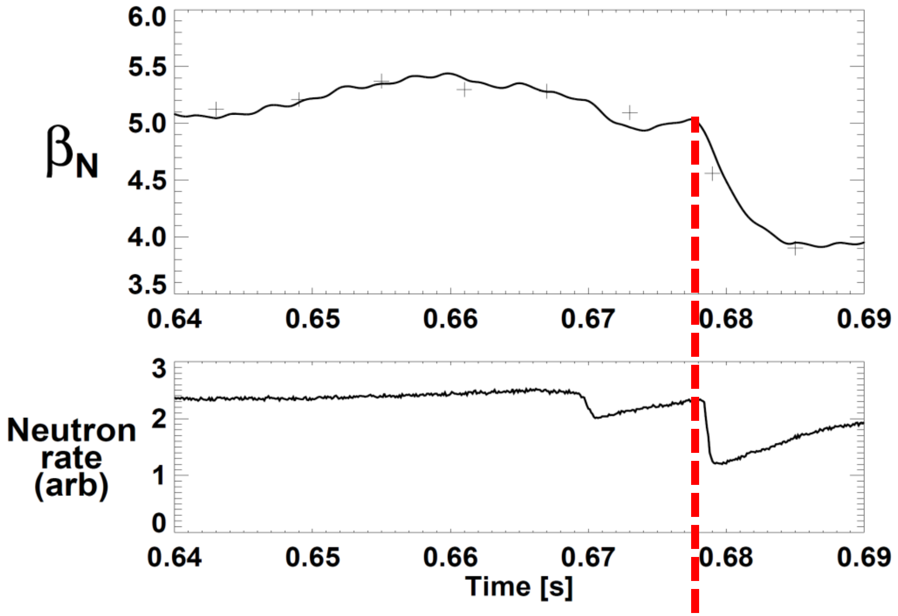
Differential kinetic (always destabilizing)

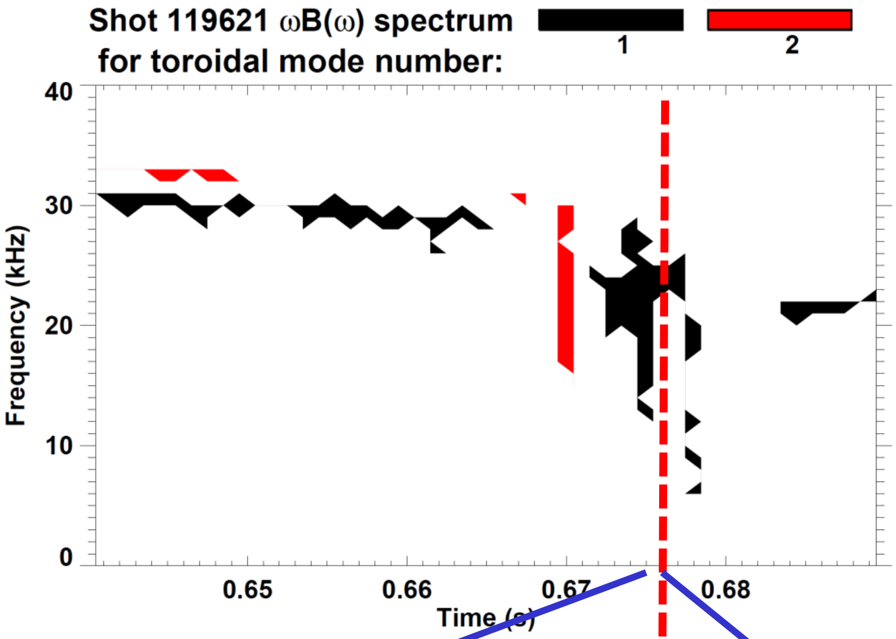
$$\delta K_2 = -\frac{1}{2}\int d^3x\rho(\omega - n\Omega)^2\left|\vec{\xi}_\perp\right|^2$$

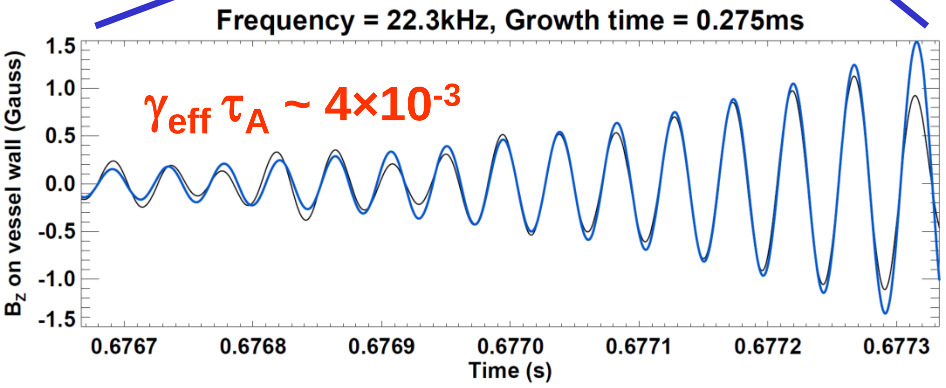


Destabilization from: Coriolis **(dΩ/dρ)**, centrifugal, differential kinetic

Intermediate β_N limit ~ 5: Small f=30kHz continuous n=1 mode precedes larger 20-25kHz n=1 bursts







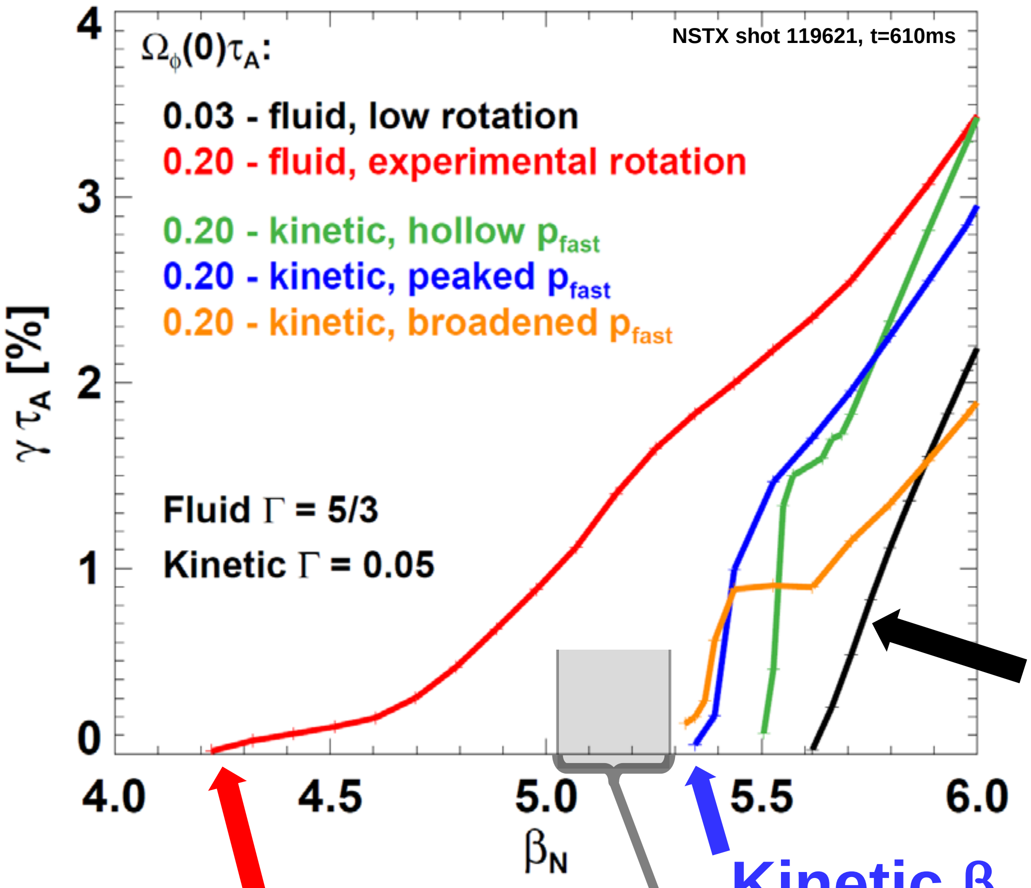
$\gamma_{eff}\,\tau_A$ ~ 4×10^{-3}

First large n=1 burst → 20% drop in β_N

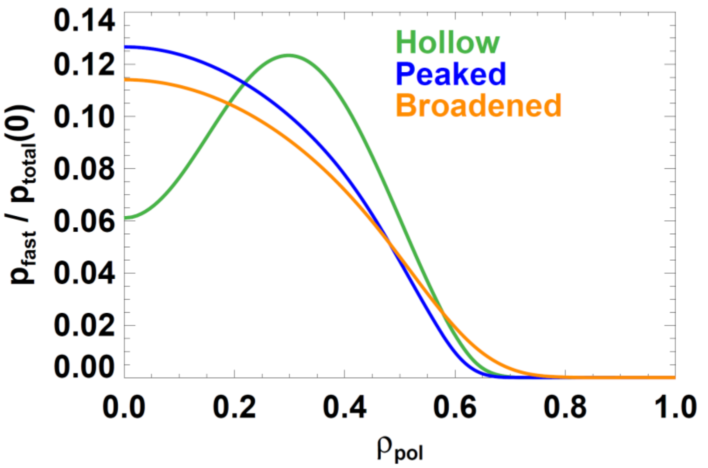
50% neutron rate drop

Later n=1 modes → full disruption

Kinetic IWM β_N limit consistent with experiment, fluid calculation under-predicts experimental limit



Fast-ion p profiles used in kinetic calculations



Fluid β_N limit for low Ω_ϕ

Kinetic β_N limit for experimental Ω_ϕ

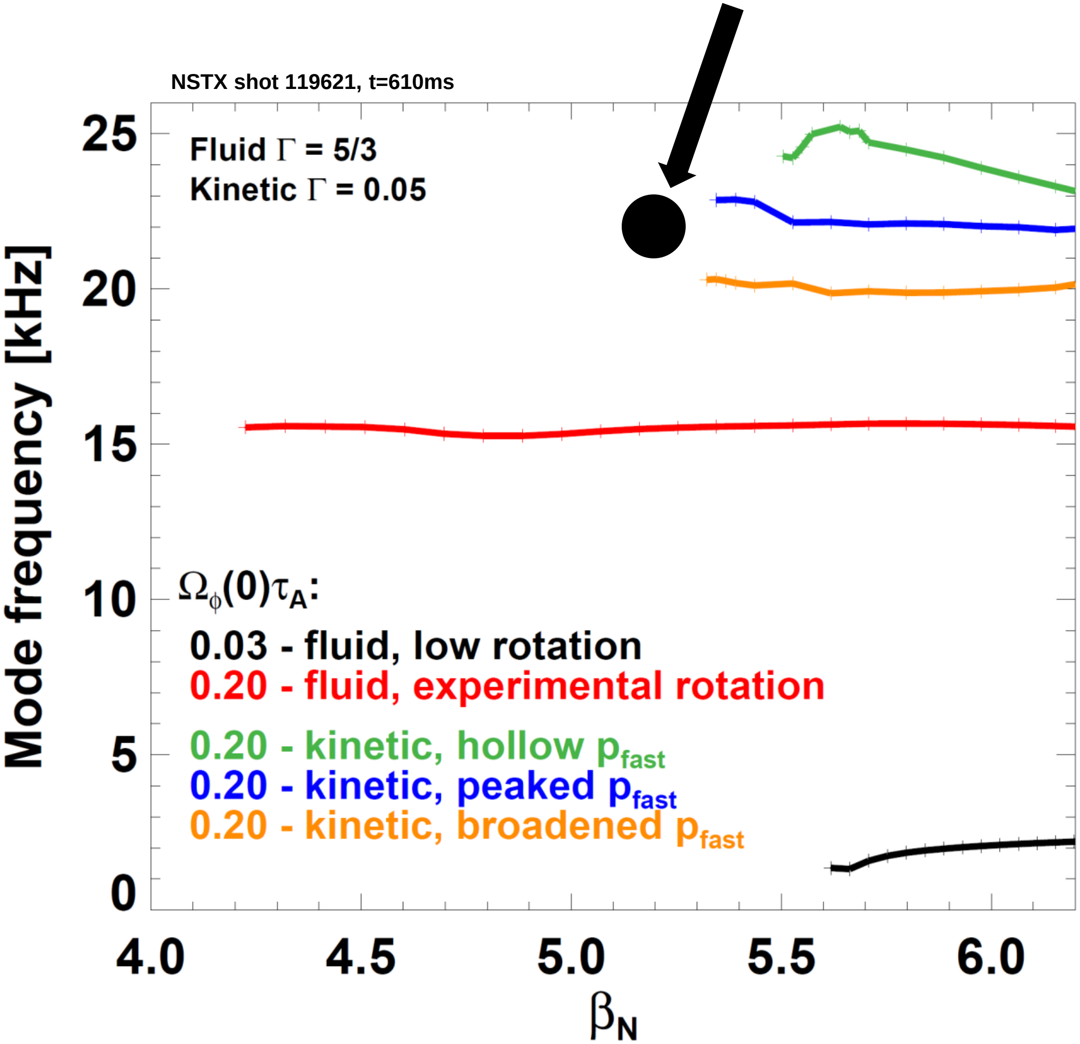
(weakly dependent on fast-ion peaking factor)

Experimental β_N for n=1 mode onset

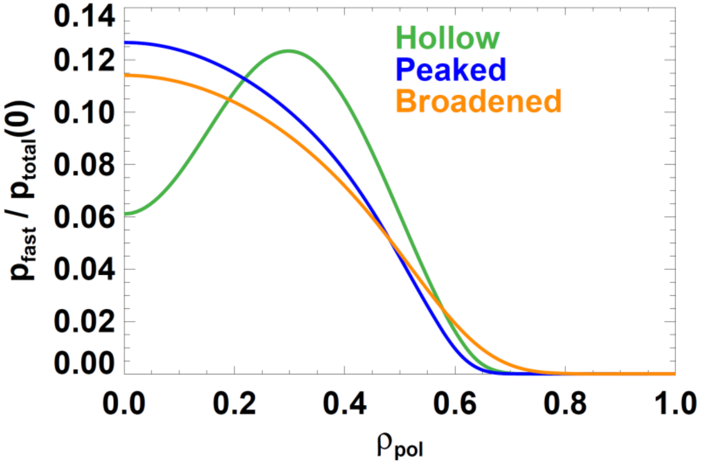
Fluid β_N limit for experimental Ω_ϕ ($\Delta\beta_N$ = -1.4 vs. low rotation)

Measured IWM real frequency more consistent with kinetic model than fluid model

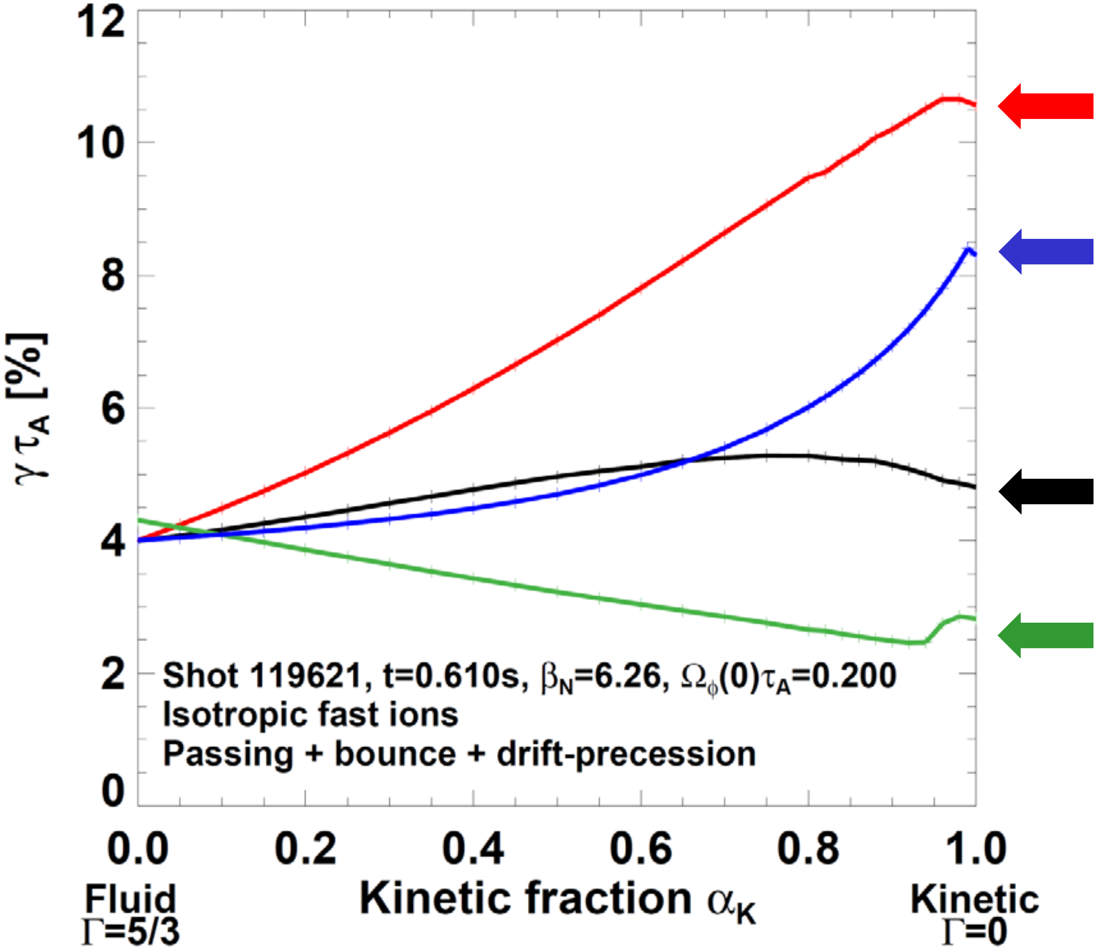
Measured mode frequency



Fast-ion p profiles used in kinetic calculations



IWM: Kinetic fast-ions destabilizing, thermals stabilizing



Fast ions only: kinetic γ exceeds all other γ values

Kinetic γ with no damping

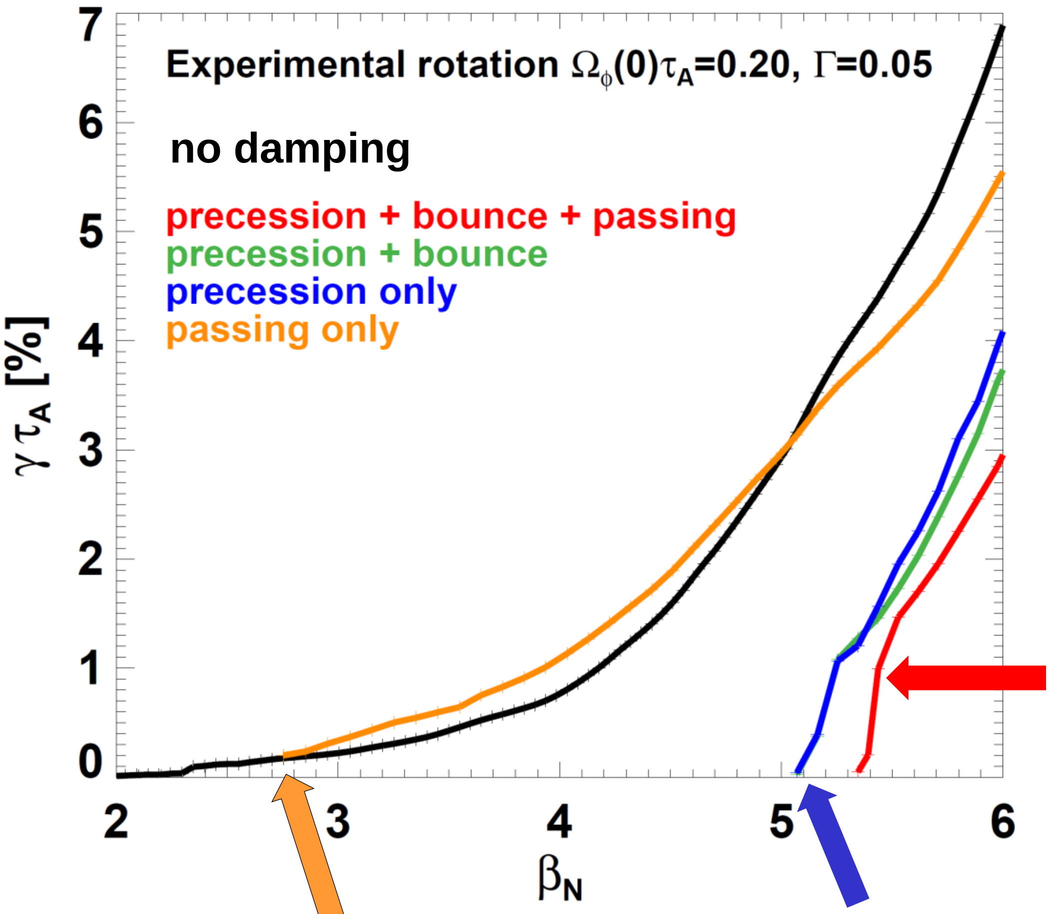
(non-adiabatic δW_k = 0 limit)

Kinetic γ with thermal + fast: similar to fluid γ

Thermals only: stabilizing

Implication: thermal damping stabilizes rotation-driven mode

IWM: Precession resonance dominates damping, highest β_N requires inclusion of passing resonance



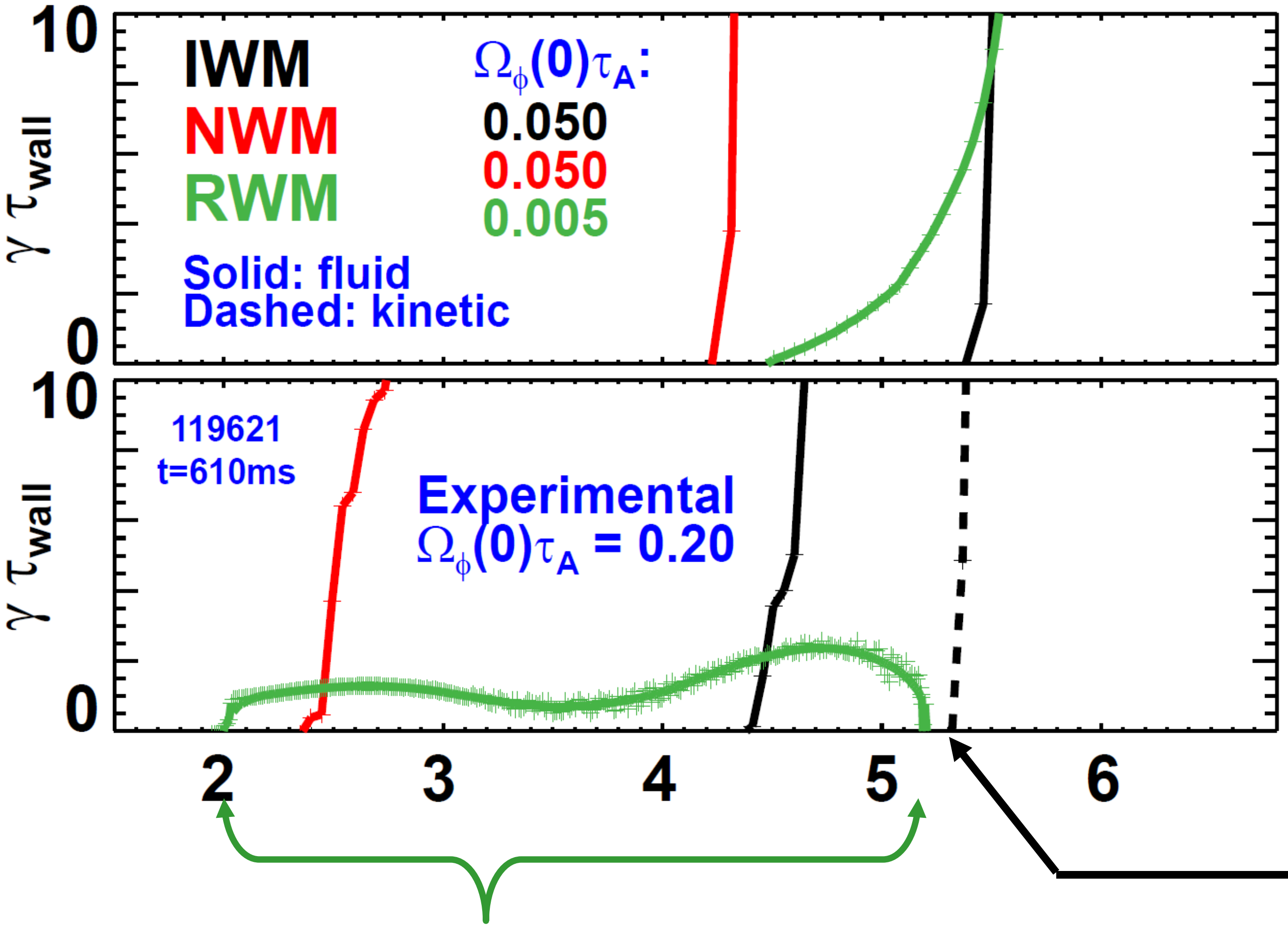
Passing more important than bounce resonance after precession

Precession resonance provides most stabilization

Resonance occurs near location of $\omega_r = \omega_E$ in core (not shown)

Passing resonance alone provides little stabilization: $\gamma \approx \gamma_{\text{no-damp}}$

High rotation reduces β_N limits for ideal wall mode (IWM), no-wall mode (NWM), and resistive wall mode (RWM)

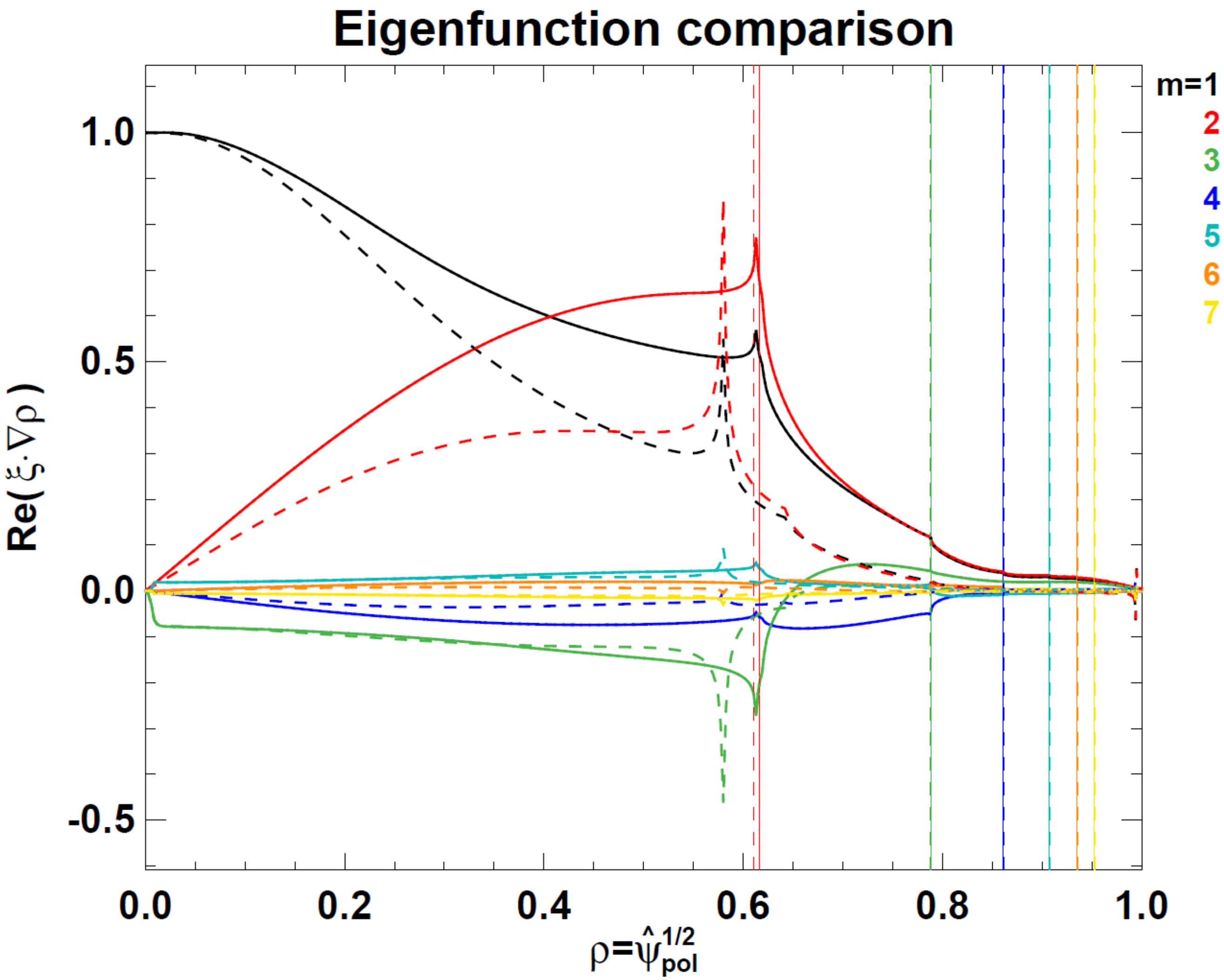


- At high rotation, RWM marginal β_N limit can extend below fluid NWM limit, above fluid IWM limit, **near kinetic IWM limit**

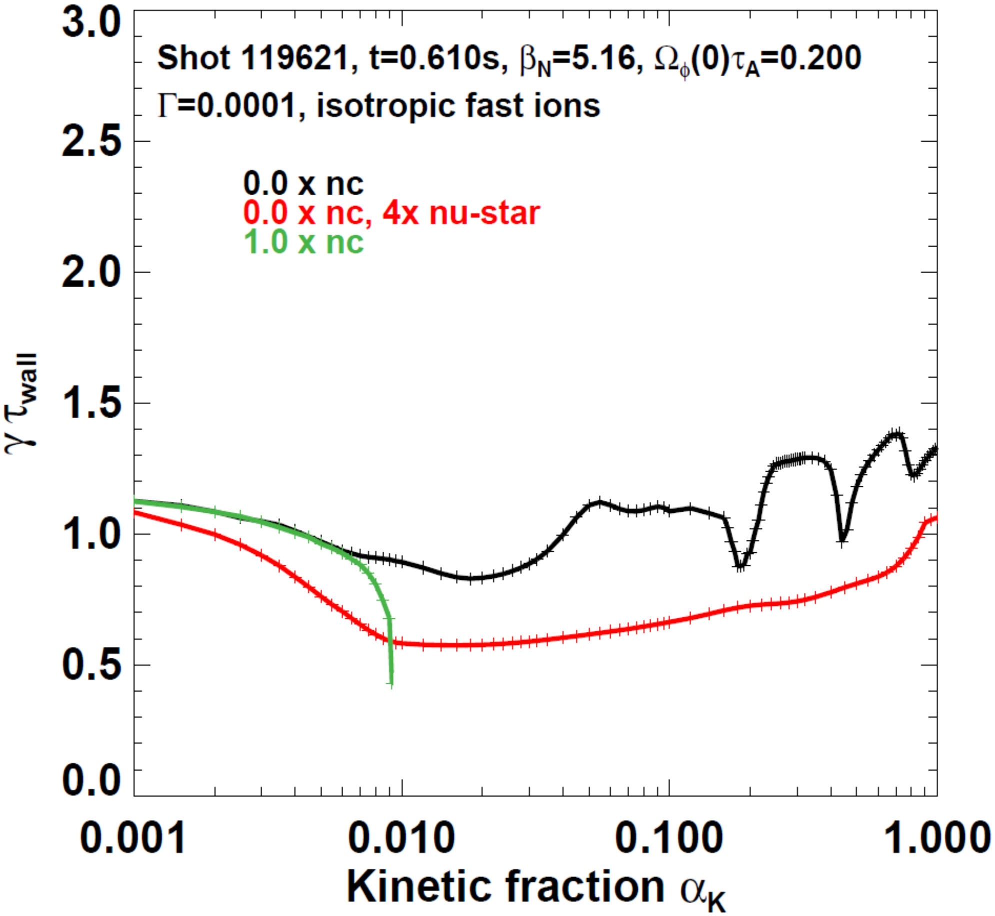
RWM: Rotation can change fluid RWM eigenfunction and move regions of singular displacement away from rationals

Solid: $\Omega_\phi(0)\tau_A = 0.005, \Gamma=5/3$ **Dashed: $\Omega_\phi(0)\tau_A = 0.20, \Gamma=0.0001$**

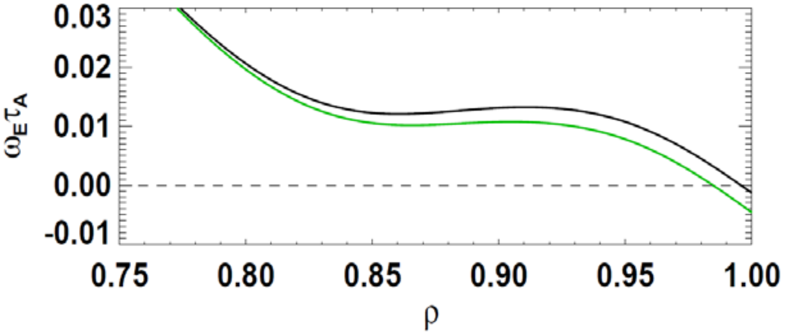
Note: β_N values are not identical



RWM kinetic stabilization for this case is sensitive to rotation profile near edge

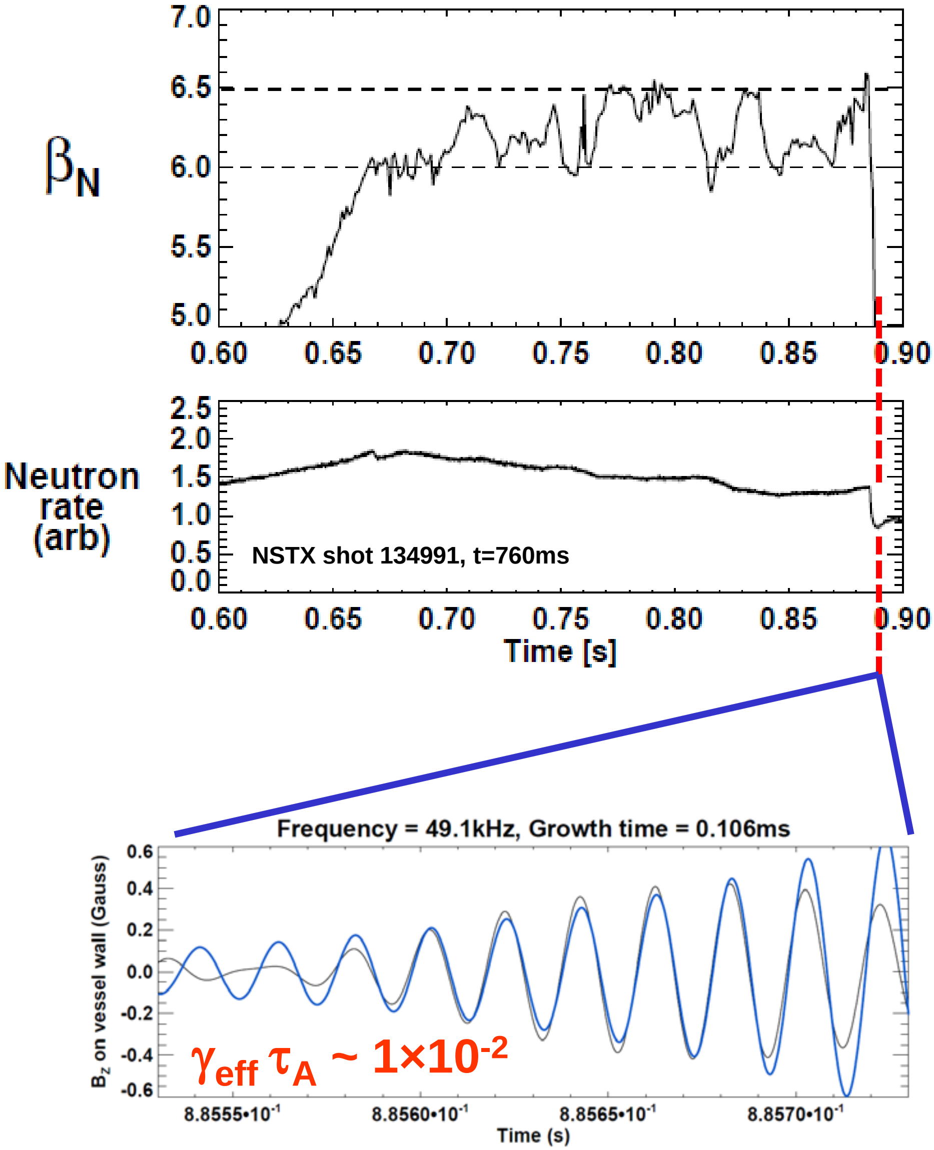


- RWM predicted to be unstable at experimental collisionality and ignoring poloidal rotation
- Increased collisionality (4x) decreases growth rate, but does not stabilize the mode
- Inclusion of neoclassical v_θ in Ω_E stabilizing for this case
  - Poloidal rotation effects only influence outer 20% of ρ = r/a
  - RWM stability consistent w/ expt



Results highlight sensitivity of RWM stability to rotation profile details – analysis ongoing…

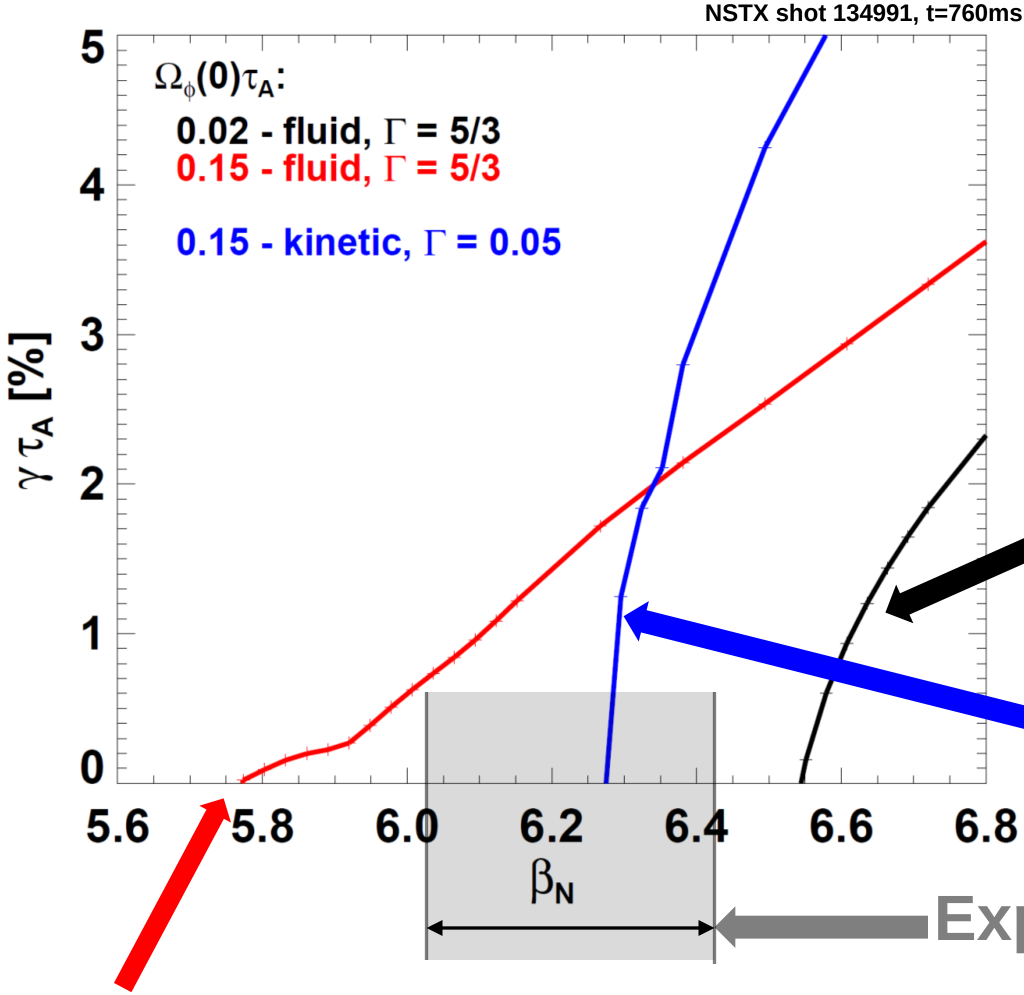
Highest β_N limit ~ 6-6.5: Experimental characteristics



$\gamma_{eff}\,\tau_A$ ~ 1×10^{-2}

- β_N = 6-6.5 sustained for 2-3τ_E
  - Oscillations from ELMs and bottom/limiter interactions
  - Possible small RWM activity
  - Only small core MHD (steady neutron rate)
- f = 50kHz mode causes 35% β_N drop ending high-β phase
  - Mode grows very fast (~100μs)
  - n-number difficult to determine
  - Possible that mode has n > 1

Kinetic IWM stability consistent with access to $\beta_N > 6$ Fluid calculation under-predicts experimental β_N



Fluid β_N limit ~6.55 for low Ω_ϕ

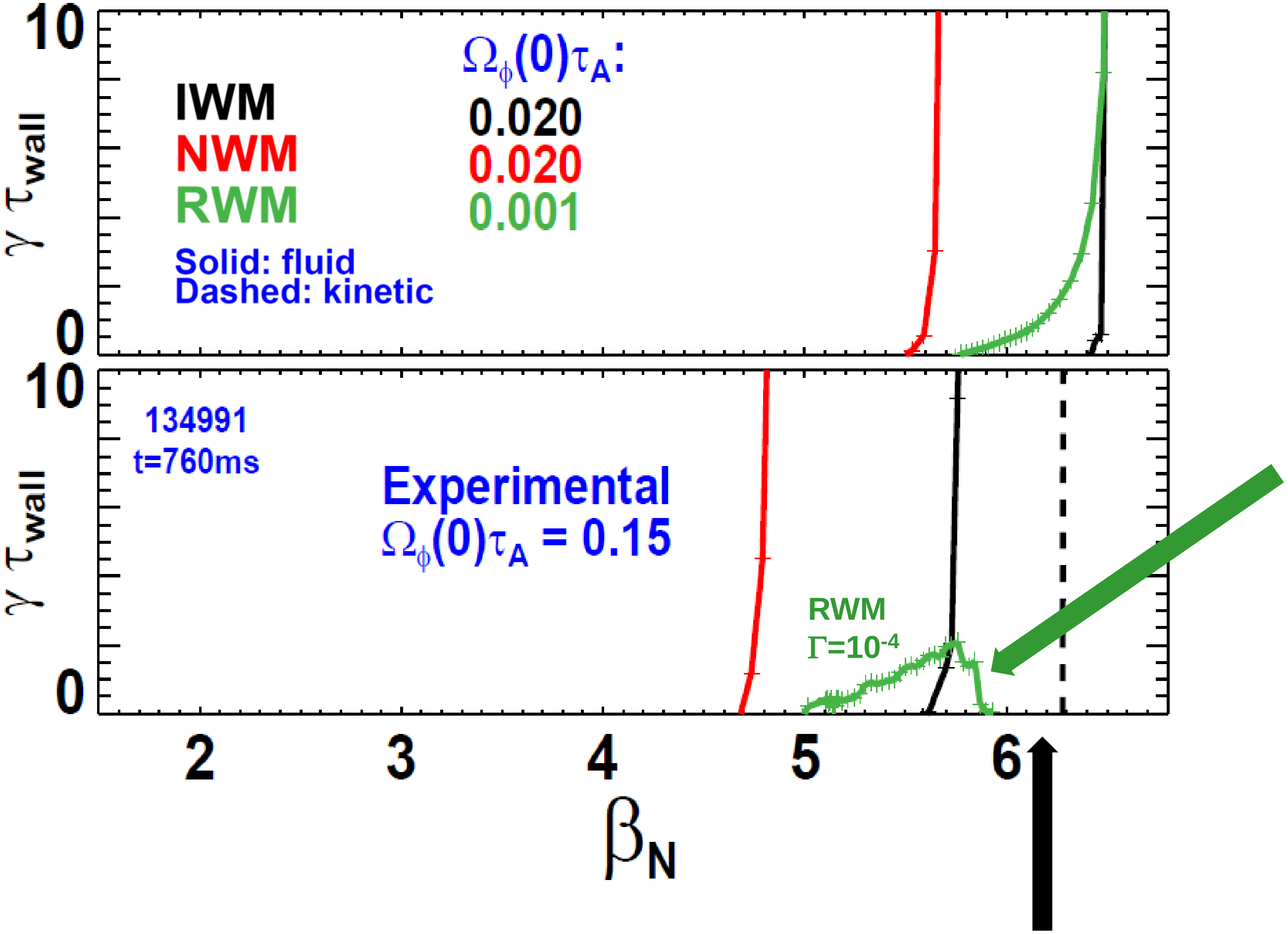
Kinetic β_N limit ~ 6.3 for experimental Ω_ϕ

Experimental β_N range

Fluid β_N limit ~ 5.7 for experimental Ω_ϕ

Rotational de-stabilization weaker ($\Delta\beta_N$ = -0.8 vs. low rotation)

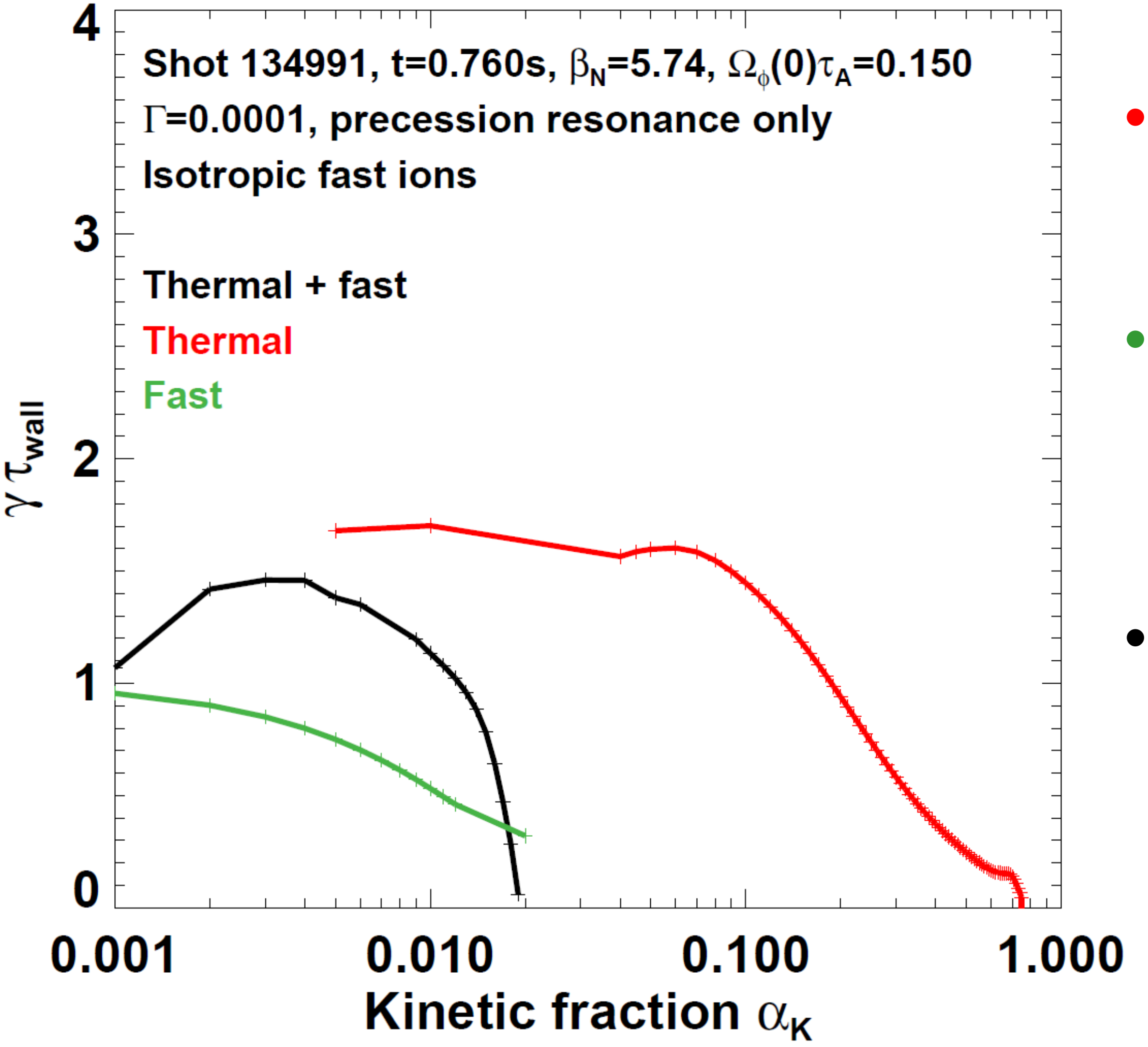
High rotation again reduces β_N limits for IWM, NWM, & RWM



Fluid RWM β_N limit can extend above the fluid IWM limit

- Note: Fluid RWM β_N limit is lower than kinetic IWM limit
  - Possible operating window at very high β_N where fluid RWM stable

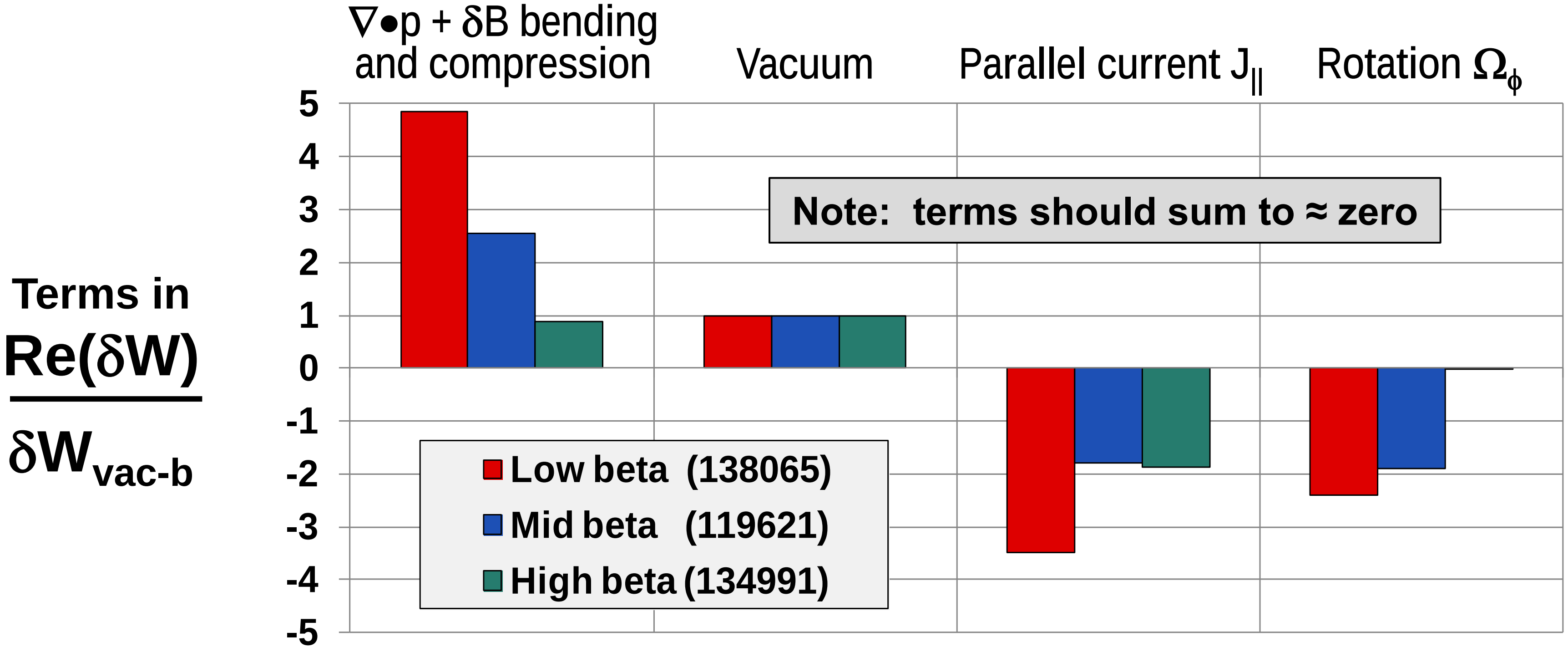
Precession resonance alone provides RWM passive stabilization (passive stability consistent with experiment)



- Thermals marginally stabilizing: $\alpha_{k\text{-crit}} \sim 0.75$
- Fast ion contribution also stabilizing relative to fluid γ
- Fast + thermal contributions strongly stabilizing: $\alpha_{k\text{-crit}} \sim 0.02$

IWM energy analysis near marginal stability elucidates trends from growth-rate scans

- All cases: field-line bending+compression balances primarily ∇p



Note: terms should sum to ≈ zero

- **Low β: $J_{||}$ (low q shear) and high Ω_ϕ strongly destabilizing**
- **Mid β: Reduced destabilization from $J_{||}$ & Ω_ϕ increases β limit**
- **High β: Large Ω_ϕ' at edge minimizes Ω_ϕ drive → highest β**

Summary

- Rotation, kinetic effects can modify IWM & RWM at high Ω_ϕ, β
  - Rotation effects most pronounced for plasmas near rotation-shear enhanced interchange/Kelvin-Helmholtz (KH) threshold
  - High rotation shear near edge is most stable in theory and experiment
- Kinetic damping from thermal resonances can be sufficient to suppress rotation-driven IWM → access low-rotation IWL
- Fluid RWM β limits follow fluid NW and IW limits **with rotation**
- Kinetic IWM β limits closer to experiment than fluid limits
- Future work:
  - Understand kinetic damping of rotation-driven modes in more detail
  - Test more realistic fast-ion distribution functions – anisotropic / TRANSP
  - Assess finite orbit width effects – for fast, edge thermal ions
  - Assess modifications to RWM stability from rotation/rotation shear
  - Utilize off-axis NBI, NTV in NSTX-U to explore IWM, RWM limit vs. rotation