Interaction between Turbulence and Neoclassical Dynamics and Its Effect on Tokamak Transport: Gyrokinetic Simulations and Theory

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in collaboration with

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Outline and Primary Results

I. Gyrokinetic model for rotating plasmas
addressing both turbulence and neoclassical physics

II. Toroidal momentum transport from turbulence simulation experiments
- Observations of inward non-diffusive momentum flux
- Discovery of residual stress due to $k_{\|}$ symmetry breaking induced by quasi-stationary ZF shear
- Effect of zonal flow damping

III. Neoclassical momentum transport
- Off-diagonal momentum flux driven by ∇T_i
- Overall neoclassical contribution is small

IV. Residual turbulence with strong $\mathbf{E} \times \mathbf{B}$ flow shear
- It may lead to anomalous momentum flux, while driving ion heat flux only on the order of neoclassical level

V. Global nonlinear ETG simulations of NSTX experiments

I. Simulation Model for Rotating Plasmas

- Gyrokinetic Tokamak Simulation (GTS) code: generalized gyrokinetic simulation model; PIC approach

Turbulence fluctuation part δf:

$$\frac{D\delta f}{Dt} \equiv \frac{\partial \delta f}{\partial t} + (v_\parallel \hat{b} + \vec{v_{E_0}} + \vec{v_E} + \vec{v_d}) \cdot \nabla \delta f - \hat{b^*} \cdot \nabla(\mu B + \frac{e}{m_i}\Phi_0 + \frac{e}{m_i}\bar{\phi})\frac{\partial \delta f}{\partial v_\parallel}$$

$$= -\vec{v_E} \cdot \nabla f_0 + \hat{b^*} \cdot \nabla(\frac{e}{m_i}\bar{\phi})\frac{\partial f_0}{\partial v_\parallel} + C_i^l(\delta f).$$

Neoclassical equilibrium f_0:

$$\frac{\partial f_0}{\partial t} + (v_\parallel \hat{b} + \vec{v_{E_0}} + \vec{v_d}) \cdot \nabla f_0 - \hat{b^*} \cdot \nabla(\mu B + \frac{e}{m_i}\Phi_0)\frac{\partial f_0}{\partial v_\parallel} = C_i(f_0, f_0).$$

Lowest order equilibrium solution for rotating plasma:

$$f_0 = f_{SM} = n(r,\theta)(\frac{m_i}{2\pi T_i})^{3/2} e^{-\frac{m_i}{T_i}[\frac{1}{2}(v_\parallel - U_i)^2 + \mu B]}$$

parallel flow: $U_i = I\omega_t/B$, density: $n(r,\theta) = N(r)e^{\frac{m_i U_i^2}{2T_i} - \frac{e\tilde{\Phi}_0}{T_i}}$

Simulation model for rotating plasmas

$$\frac{D\delta f}{Dt} == \left\{ - \left[\frac{m}{T_i} \left(\frac{1}{2}(v_\parallel - U_i)^2 + \mu B \right) - \frac{3}{2} \right] \vec{v_E} \cdot \nabla \ln T - \vec{v_E} \cdot \nabla \ln n(r, \theta) \right.$$

$$\left. - \frac{m(v_\parallel - U_i)}{T_i} \vec{v_E} \cdot \nabla U_i(r, \theta) + \frac{mU_i}{T_i v_\parallel} \vec{v_E} \cdot \mu \nabla B - \frac{1}{T_i}(v_\parallel \hat{b} + \vec{v_d}) \cdot \nabla(e\bar{\Phi})(1 - \frac{U_i}{v_\parallel}) \right\} f_0.$$

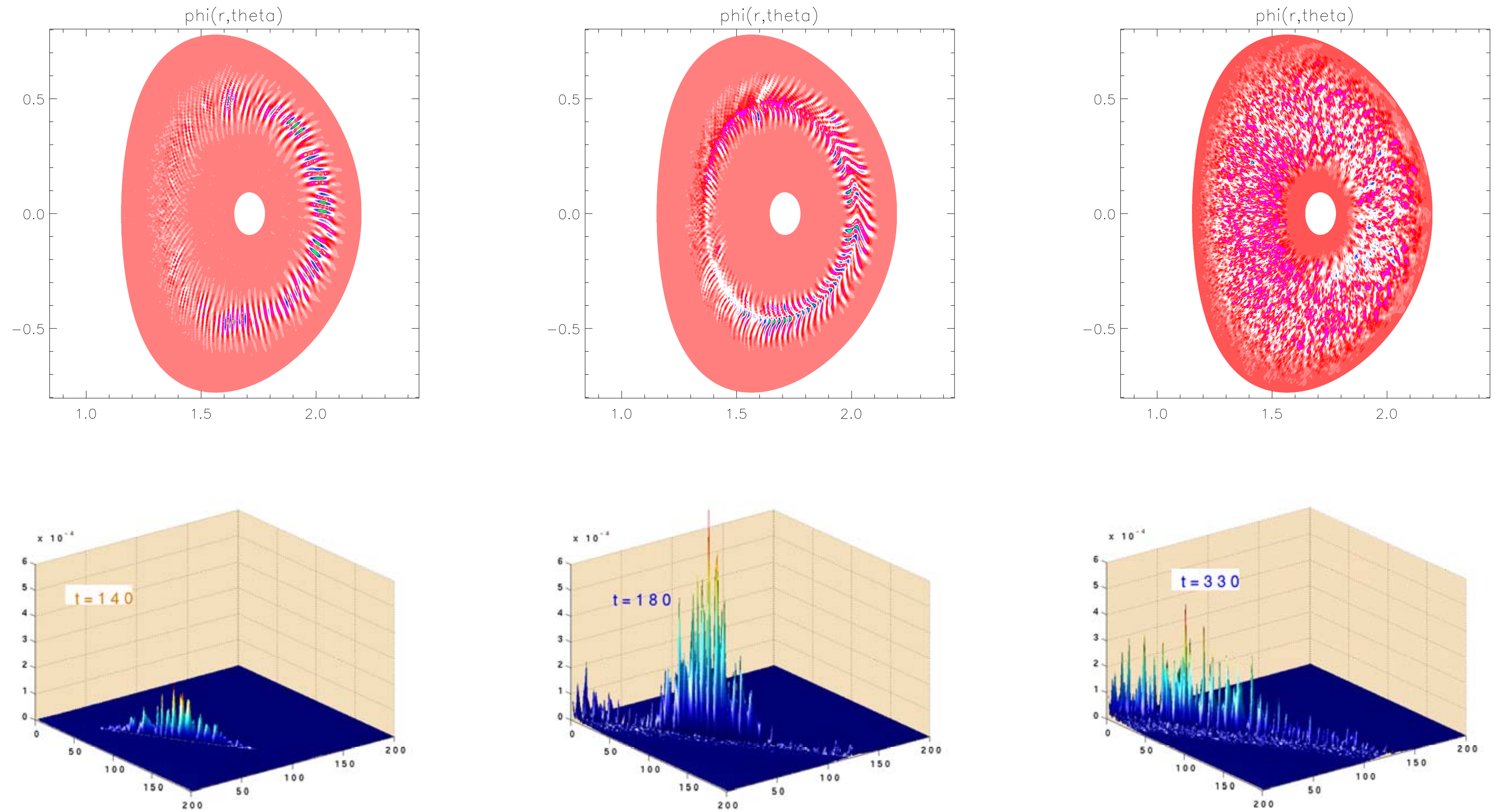
$\{\langle n(r, \theta)\rangle$, $T(r)$, $\Phi_0(r)$, and $\omega_t(r)\} \Longrightarrow$ turbulence & transport

(energy, particle and momentum flux)

- Interfaced with MHD equilibrium codes (based on ESI interface by Zakharov and White) and TRANSP data base

Global gyrokinetic turbulence is characterized by distinguishable dynamical phases

- from linear, to nonlinear transient, to a well developed turbulent state



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Dynamics of gyrokinetic turbulence should not be sensitive to numerical techniques

Two different ways to solve gyrokinetic Poisson equation in the code:

- Solver I. in simple geometry equations for $\delta\Phi$ and $\langle\Phi\rangle$ can be decoupled

$$\left(1+\frac{T_i}{T_e}\right)\frac{e\delta\Phi}{T_i}-\frac{e\widetilde{\delta\Phi}}{T_i}=\frac{\delta\bar{n}_i-\langle\delta\bar{n}_i\rangle}{n_0}-\frac{\delta n_e^{(1)}-\langle\delta n_e^{(1)}\rangle}{n_0},$$

$$\frac{1}{\mathcal{V}_r'}\frac{d}{dr}\left[\frac{d\Phi_{00}}{dr}\mathcal{V}_r'\langle g^{rr}\rangle\right]=\frac{1}{\mathcal{V}_r'}\frac{d}{dr}\left\{\frac{d}{dr}\left[\frac{T_i}{e}\left(\frac{\langle\delta\bar{n}_i\rangle}{n_0}-\frac{\langle\delta n_e^{(1)}\rangle}{n_0}\right)\right]\mathcal{V}_r'\langle g^{rr}\rangle\right\}$$

$$-\langle\frac{1}{\rho_i^2}\rangle\frac{T_i}{e}\left(\frac{\langle\delta\bar{n}_i\rangle}{n_0}-\frac{\langle\delta n_e^{(1)}\rangle}{n_0}\right),$$

using approximations:

i) $\left\langle\tilde{\Phi}\right\rangle\approx\widetilde{\langle\Phi\rangle}$ – not justified in general geometry!

ii) Pade approximation $\Gamma_0(b)\equiv I_0(b)e^{-b}\approx 1/(1+b)$

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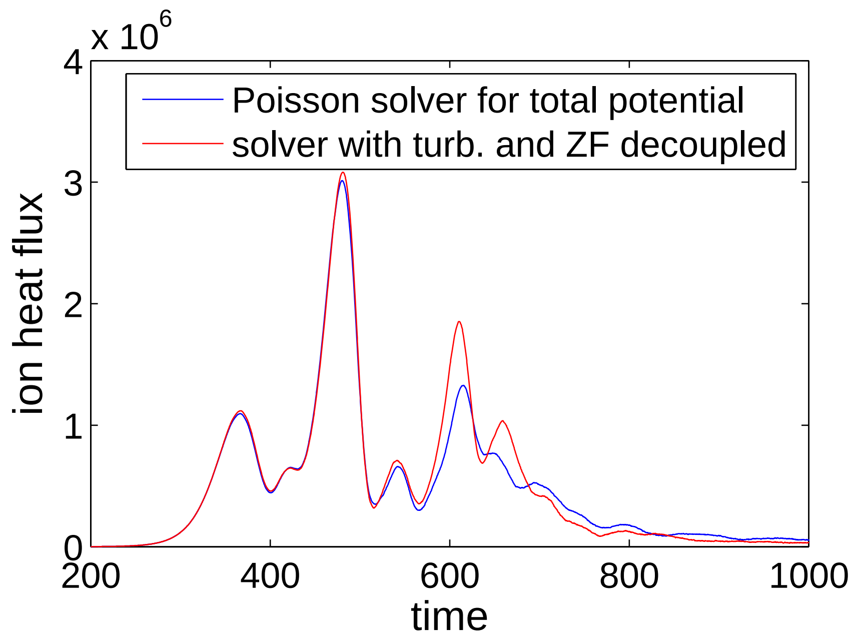
Dynamics of gyrokinetic turbulence should not be sensitive to numerical techniques

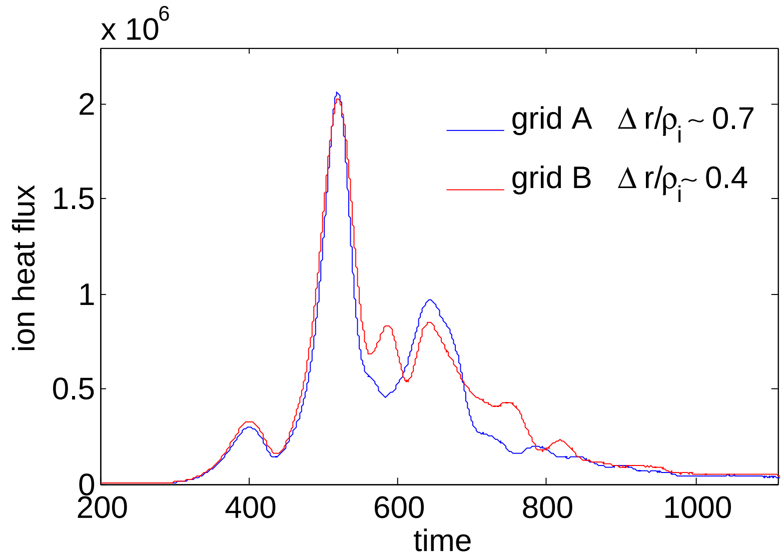
- II. Generalized Poisson Solver to solve integral equation for total potential $\Phi = \delta\Phi + \langle\Phi\rangle$

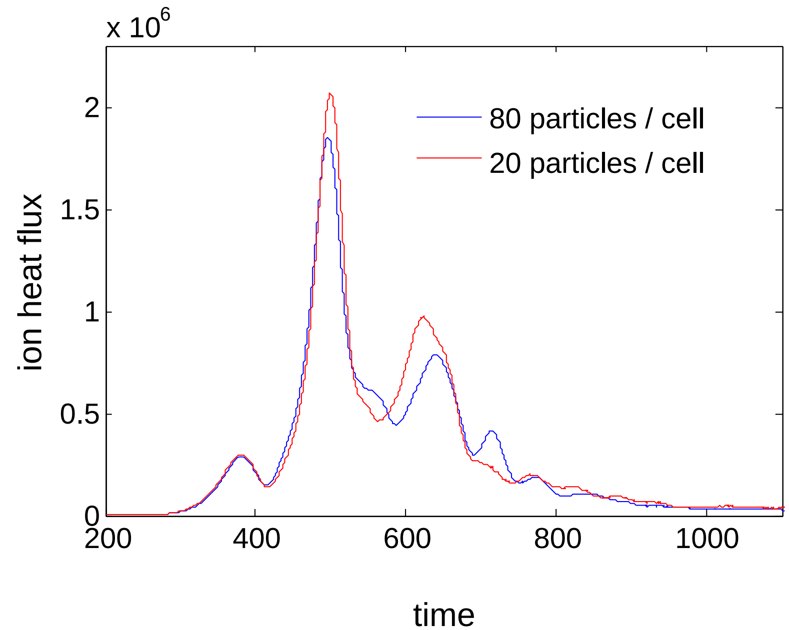
$$\left(1+\frac{T_i}{T_e}\right)\frac{e\Phi}{T_i} - \frac{e\widetilde{\Phi}}{T_i} - \frac{e\langle\Phi\rangle}{T_e} = \frac{\delta\bar{n}_i}{n_0} - \frac{\delta n_e^{(1)}}{n_0}$$



In simple geometry case (large aspect ratio and weak shaping), two highly different solvers give almost the same results!

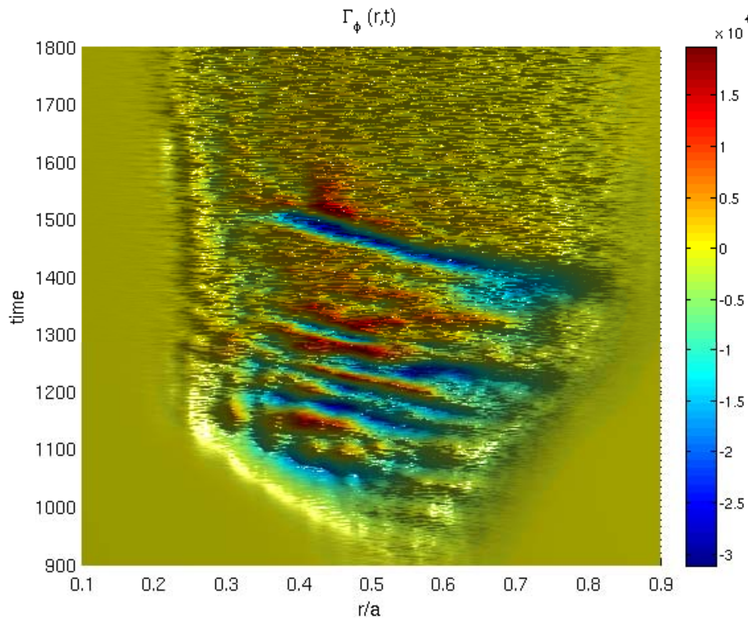
Dynamics of gyrokinetic turbulence should not be sensitive to numerical techniques

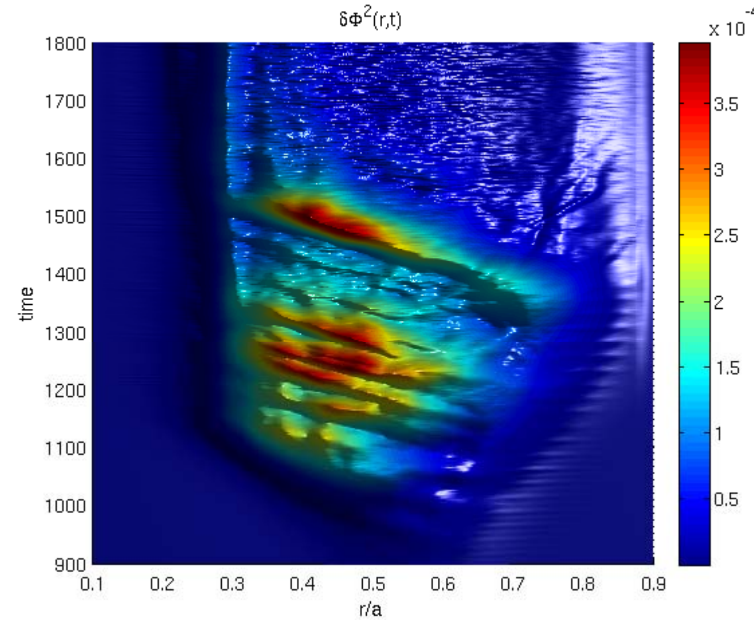


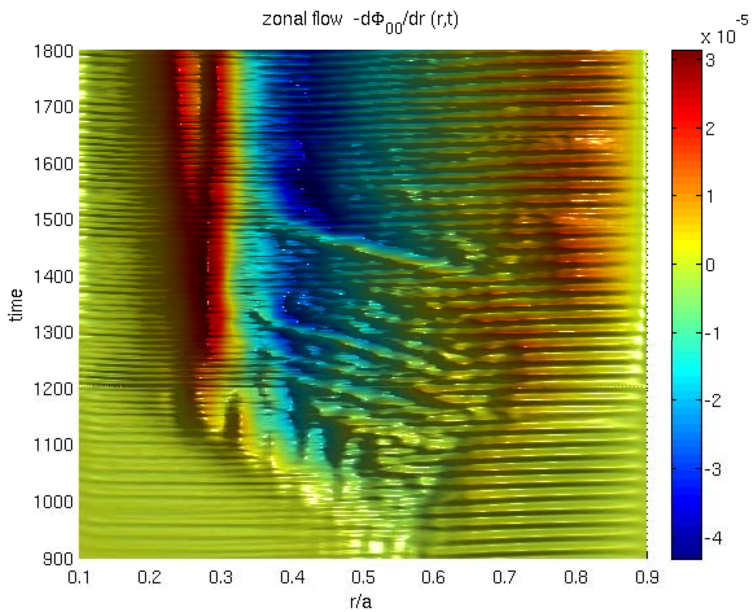


- the nonlinear transient phase is rather robust and not artificial
- not sensitive to how we solve Poisson equation (in simple geometry case)
- not sensitive to how simulation grids are set up
- not sensitive to the numbers of simulation particles

Global turbulence and fluxes are characterized by spatio-temporal bursting with radially inward propagation







- coherent spatio-temporal bursting shown in $\langle\delta\Phi^2\rangle$, q_i, Γ_ϕ and zonal flows
- radial propagation likely non-diffusive
- underlying nonlinear physics ?: tur. spreading, tur. and ZF interplay, change of local plasma gradients ...
  connection with the nature of turbulence transport?

II. Turbulent Momentum Transport: various features of toroidal momentum flux and definitions

- Fluid

  toroidal momentum: $L_\phi \equiv mnRu_\phi$

  toroidal momentum flux (radial):

  $\Gamma_\phi \equiv \langle \delta L_\phi \delta v_r \rangle \approx mnR \langle \delta u_\phi \delta v_r \rangle + mRu_\phi \langle \delta n \delta v_r \rangle$

- Kinetic

  $$\vec{\Gamma}_\phi \equiv \int d^3 v m R v_\phi \vec{v}_{\delta E} \delta f \approx -mn\chi_\phi^{eff} R^2 \nabla \omega_\phi$$

  radial flux (in general geometry):

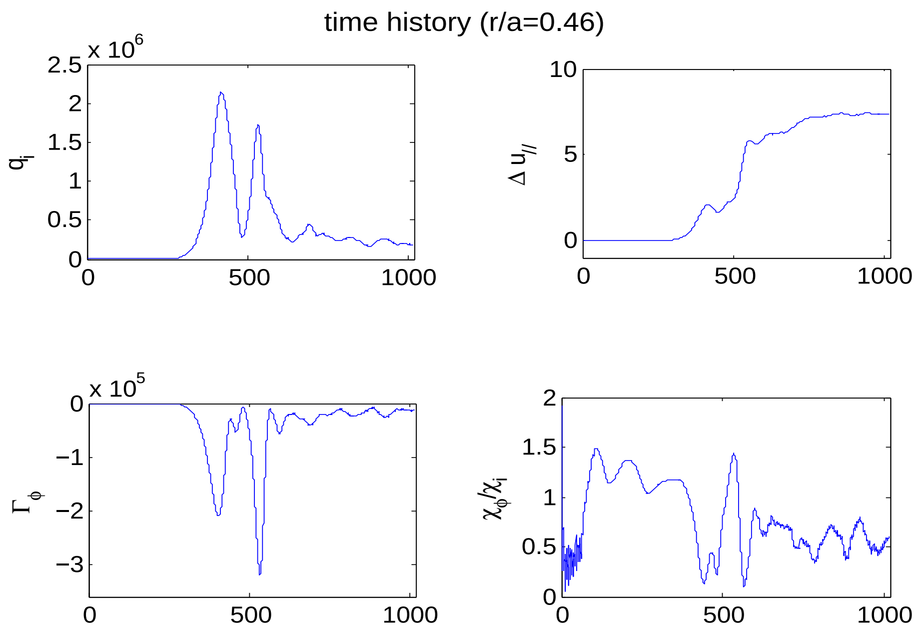
  $$\langle \vec{\Gamma}_\phi \cdot \hat{\rho} \rangle \equiv \langle \int d^3 v m R v_\phi \vec{v}_{\delta E} \cdot \nabla \rho / |\nabla \rho| \delta f \rangle \equiv -mn\chi_\phi^{eff}(\rho) \langle R^2 |\nabla \rho| \rangle \frac{d\omega_\phi}{d\rho}$$

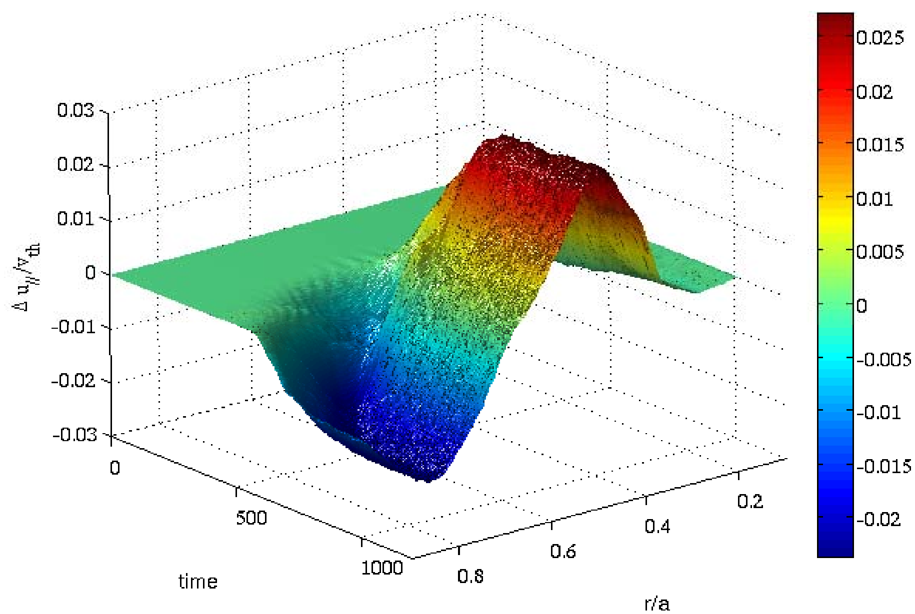
- Characteristics of momentum transport: diffusion, pinch effects, off-diagonal (residual stress), ...

  Recently, there have been a number of theoretical studies to address various mechanisms: symmetry-breaking, TEP, resonance, non-resonance, ... (Hahm, Diamond, Gurcan, Peeters, Coppi, ...)

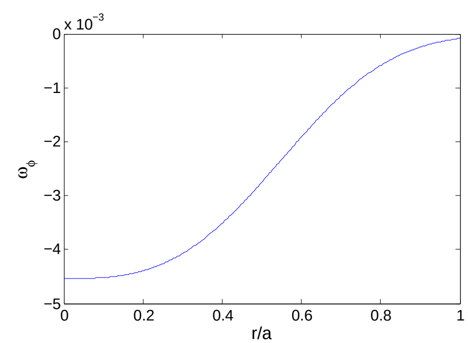
  and intensive experimental investigations ...

Large inward toroidal angular momentum flux found in post-saturation phase





- Large inward toroidal momentum flux driven in post-saturation phase
- It pumps toroidal momentum from edge to core $\Rightarrow \Delta u_{\|} \sim 3\%\ v_{th}$ (a significant fraction of rotation speed)
- global momentum conservation roughly maintained



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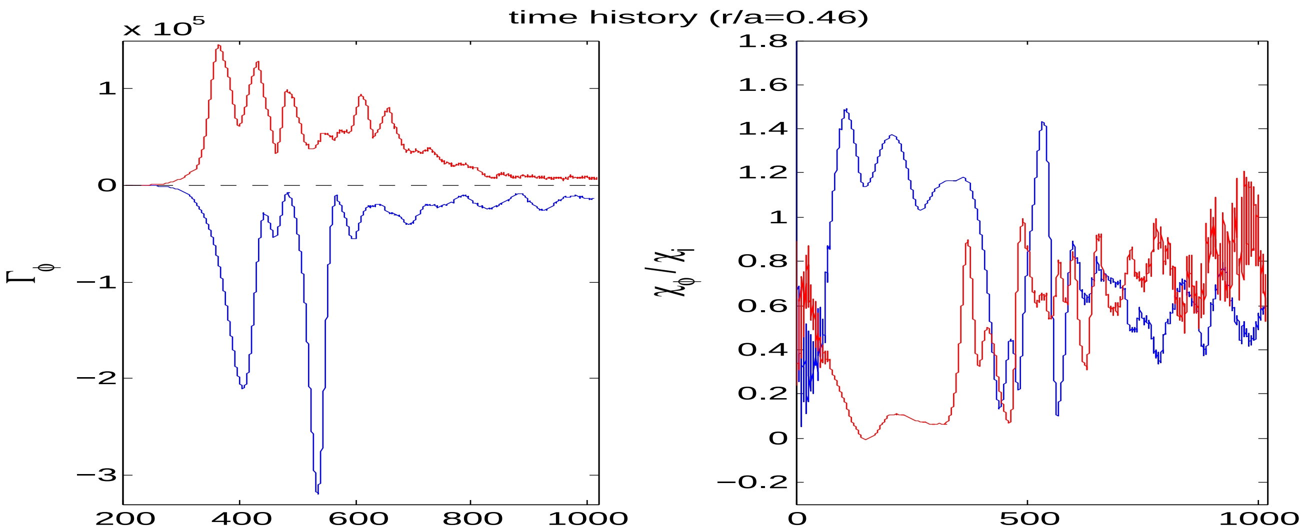
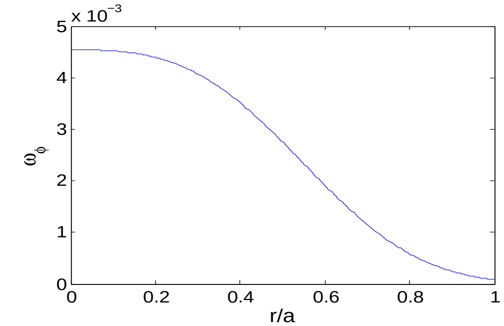
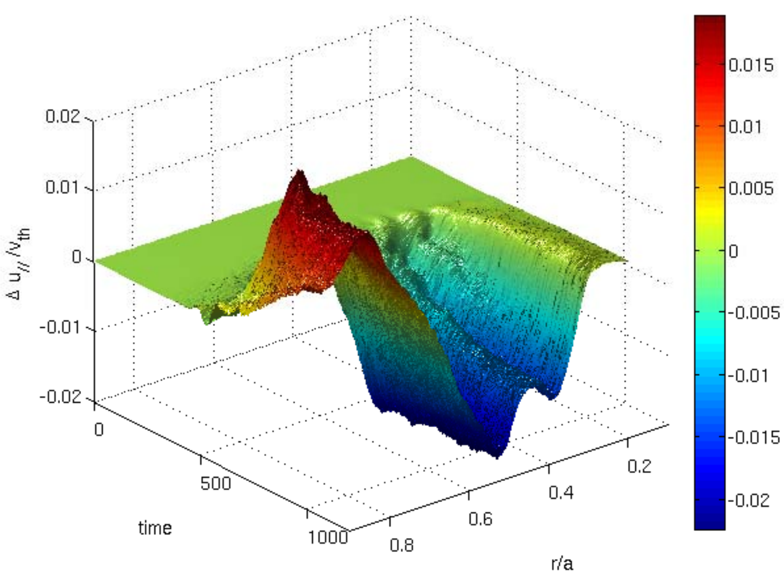
Simulation experiments of toroidal momentum transport in various situations



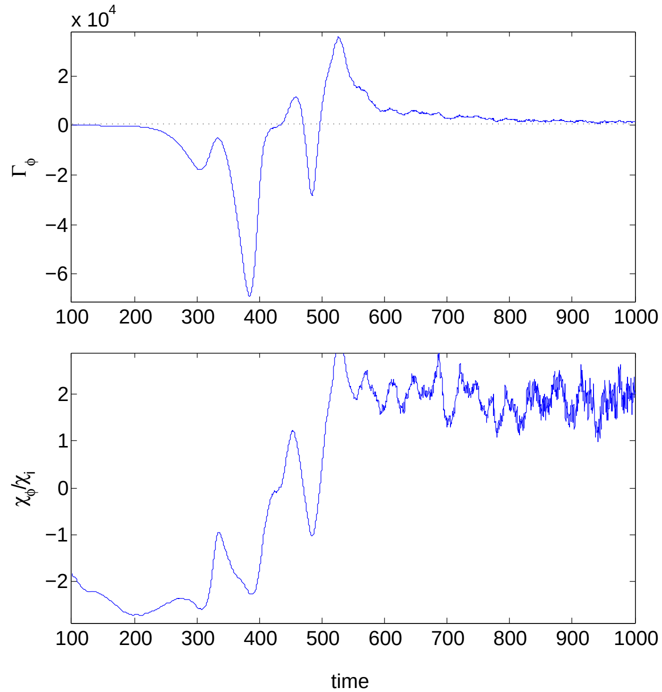


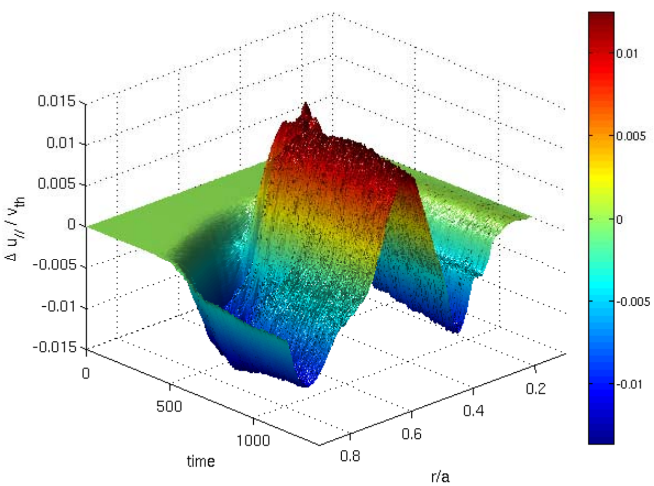
- smaller Γ_ϕ and $\Delta u_{\|}/v_{th}$ for $\nabla\omega_\phi < 0$ case



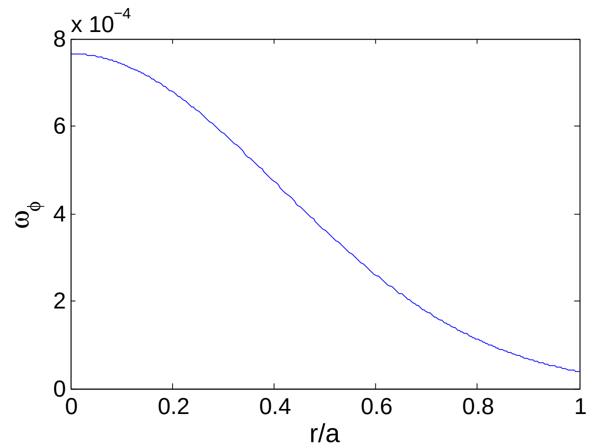
- Γ_ϕ in long-time steady state settles down to a lower level in direction of momentum diffusion
- effective χ_ϕ/χ_i is on the order of unity, in broad agreement with observations in conventional tokamaks and theory predictions for low-k fluctuation driven transport [Mattor-Diamond, PF'88]

Large transient inward momentum flux may lead to core rotation spin up – smaller ω_ϕ and smaller $\nabla\omega_\phi$ case

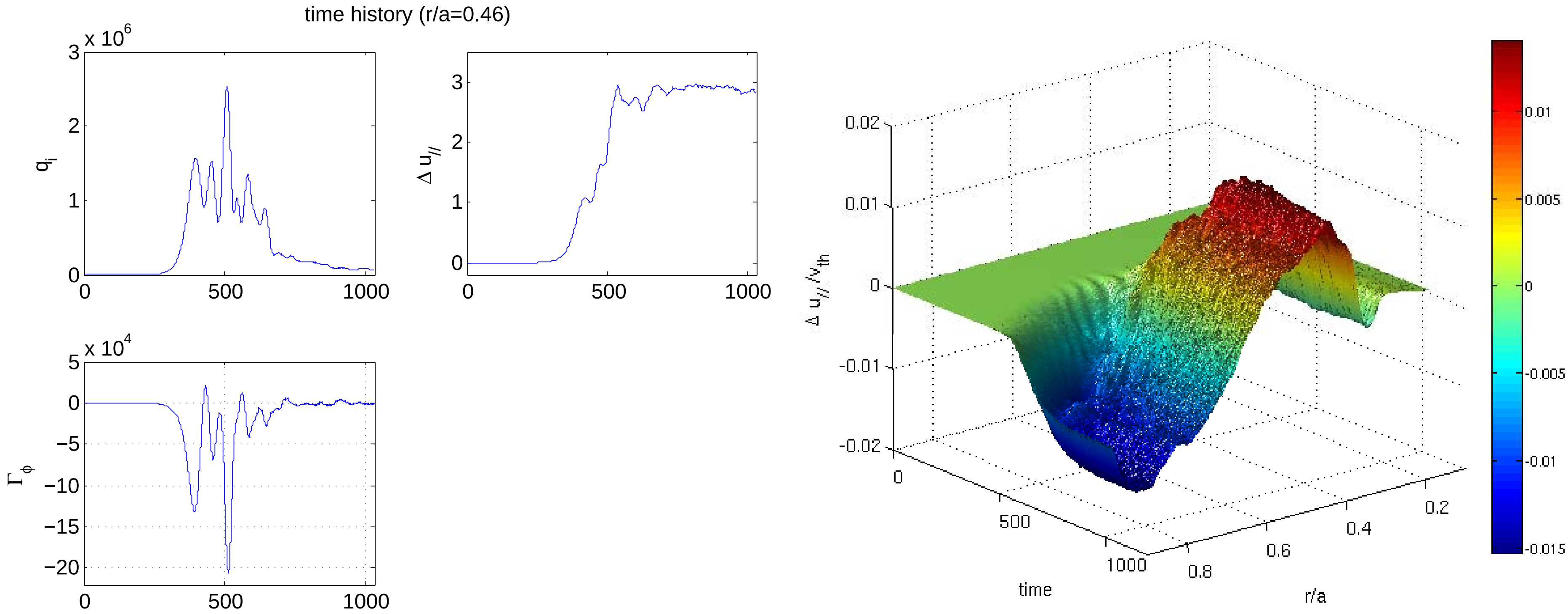




- large inward transient momentum flux is observed in the direction opposite to momentum diffusion (outward for this case) $\Rightarrow$ core rotation spins up
  pinch? off-diagonal (residual stress)?
  or ... ?
- however, long-time steady-state Γ_ϕ, smaller but finite, returns to diffusive direction



Large transient inward momentum flux found in various situations – rigid rotation with $\omega_\phi \neq 0$



- large inward flux remains in post-saturation phase – no rotation gradient necessary
- Γ_ϕ in long-time steady-state is vanishing or small

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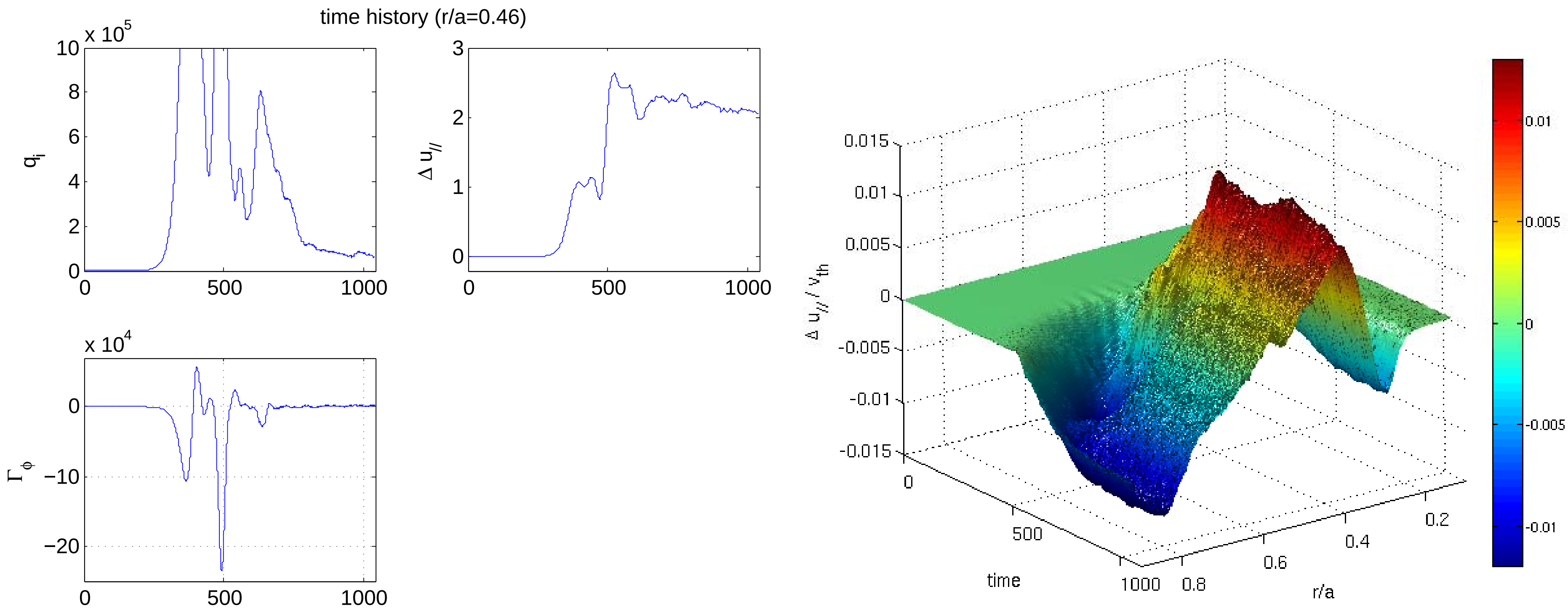
Large transient inward momentum flux observed in various situations – rigid rotation with $\omega_\phi = 0$



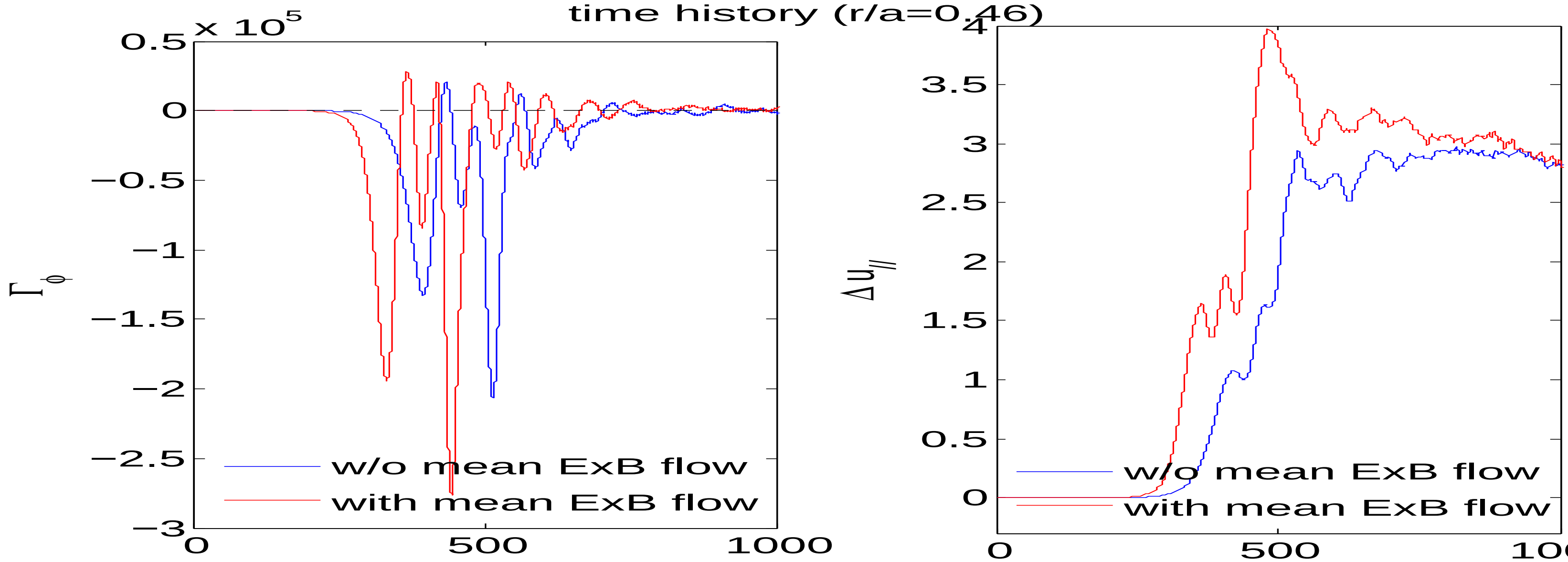
- no rotation even necessary for the large transient inward flux!

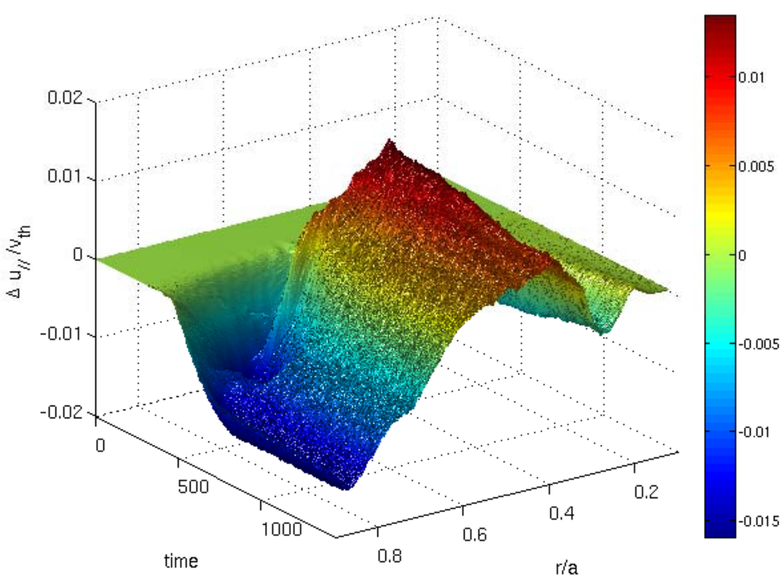
  $\longrightarrow$ existence of off-diagonal (residual stress) momentum flux

  (this does NOT necessarily indicate that momentum pinch is unimportant in general case)

- Γ_ϕ in long-time steady-state is vanishing or small – likely diffusive

Large transient inward momentum flux observed in various situations – rigid rotation with $\omega_\phi \neq 0$ and mean $\mathbf{E} \times \mathbf{B}$ shear flow





- transient inward flux slightly enhanced
- Γ_ϕ in long-time steady-state is vanishing or small – likely diffusive
- symmetry-breaking effect due to $\mathbf{E} \times \mathbf{B}$ shear flow [Gurcan, Diamond, Hahm, Singh, PoP'07] does not make significant difference in this particular case

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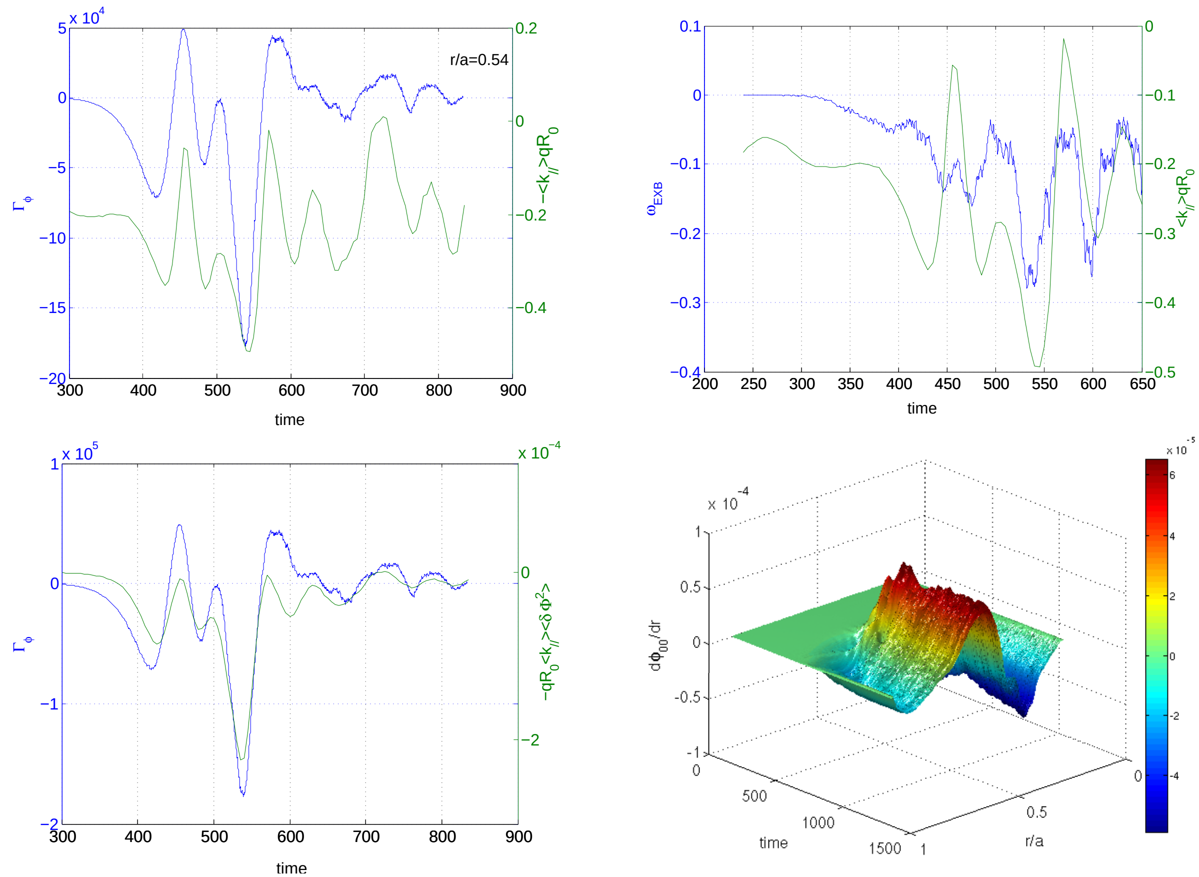
Underlying physics for ITG driven off-diagonal momentum transport is identified



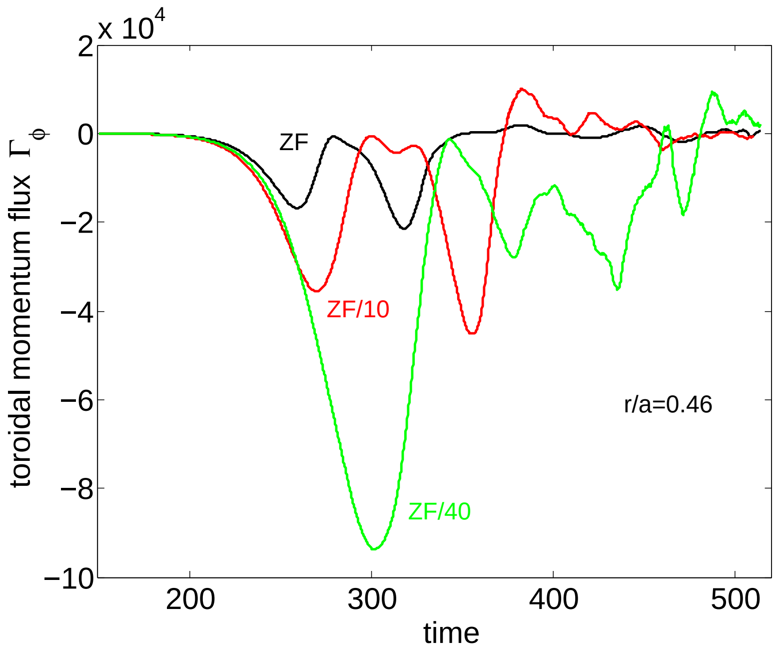
- Self-generated zonal flow is quasi-stationary in global ITG simulations

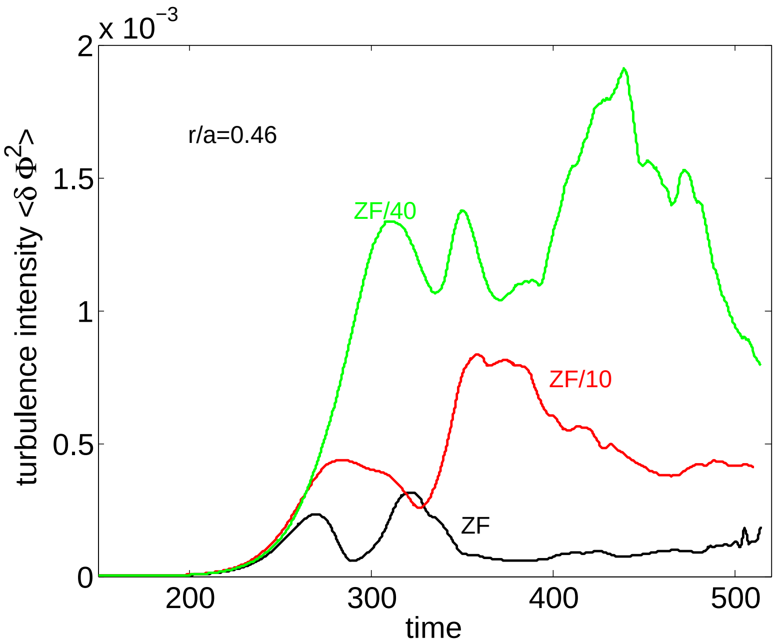
  → showing existence of toroidal zonal flow

- Discovery of residual stress generation due to $k_\parallel$ symmetry breaking induced by self-generated quasi-stationary ZF shear – an universal mechanism [along with mean $\mathbf{E} \times \mathbf{B}$ shear (Gurcan et al., '07)]

$$\langle k_\parallel \rangle (r) \equiv \frac{1}{qR_0} \frac{\sum (nq - m)\delta\Phi^2_{mn}}{\sum \delta\Phi^2_{mn}}$$

Effect of zonal flow damping on turbulence driven momentum transport

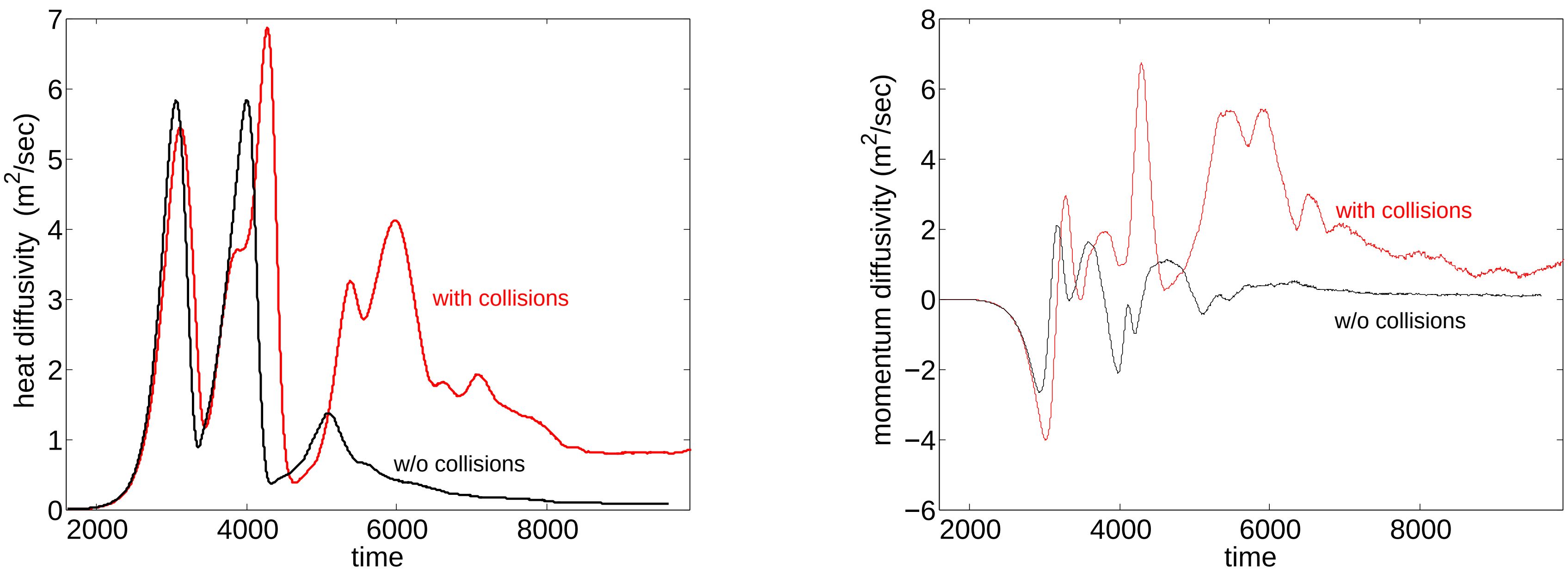




- Two competing effects of zonal flows on momentum transport: i) reduces turbulence; ii) break up $k_{||}$ symmetry
- Primary effect of zonal flow damping is found to be though its effect on turbulence saturation level:

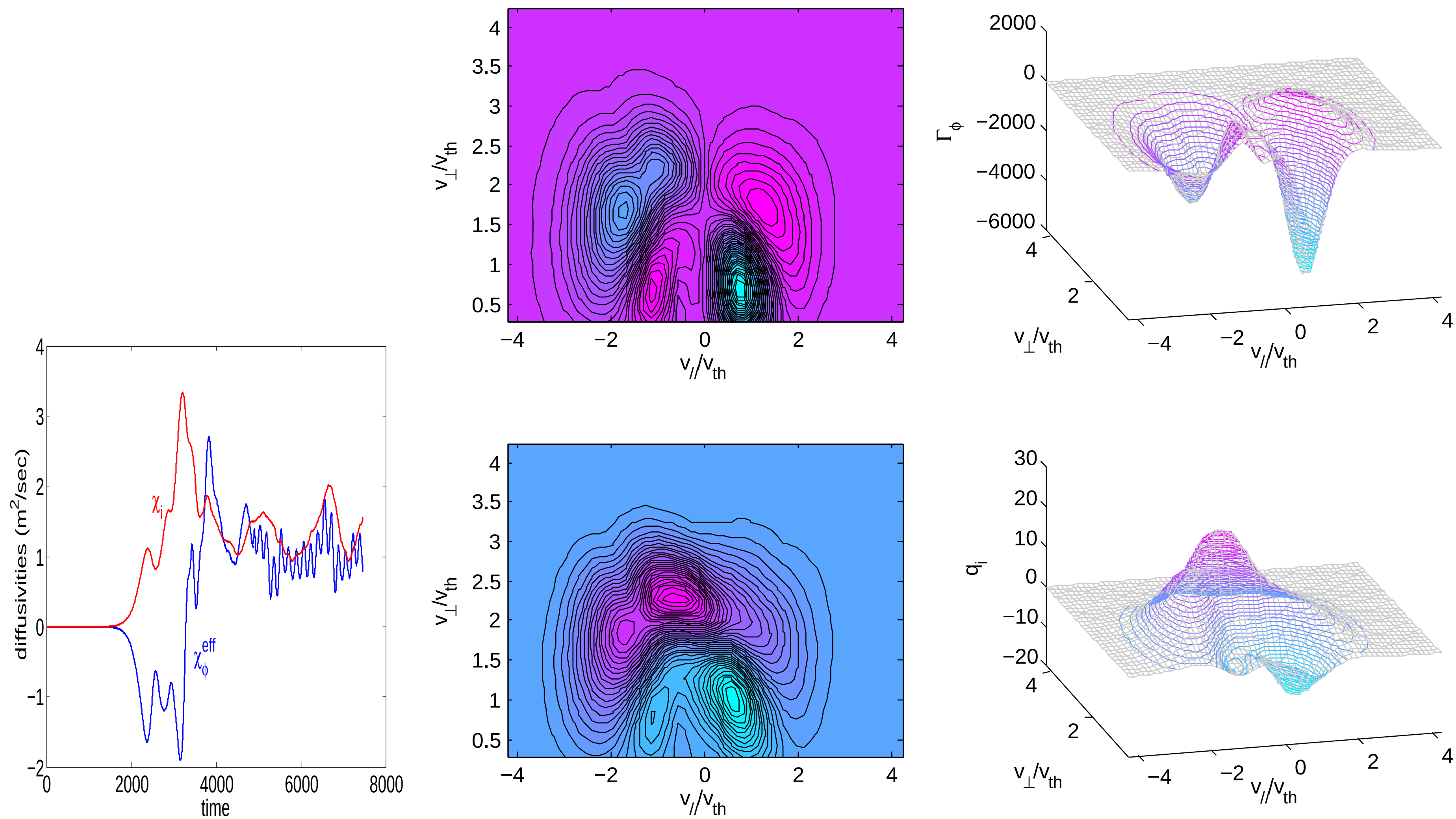
  increased ZF damping $\rightarrow$ increased turbulence intensity $\rightarrow$ increased Γ_ϕ associated with residual stress

Effect of zonal flow damping by collisions



- very large ZF and GAM (1, 0) is found to develop in long term steady state to reduce turbulence to very low level in collisionless case
- not clear yet about what causes ZF saturation in collisionless plasma; collisionless zonal flow damping seems to be weak
- collision dissipation seems to be most critical damping mechanism in determining ZF and GAM level, and then long term fluctuations and associated transport in simulations

Which and how particles contribute to momentum and energy transport: resonance and non-resonance



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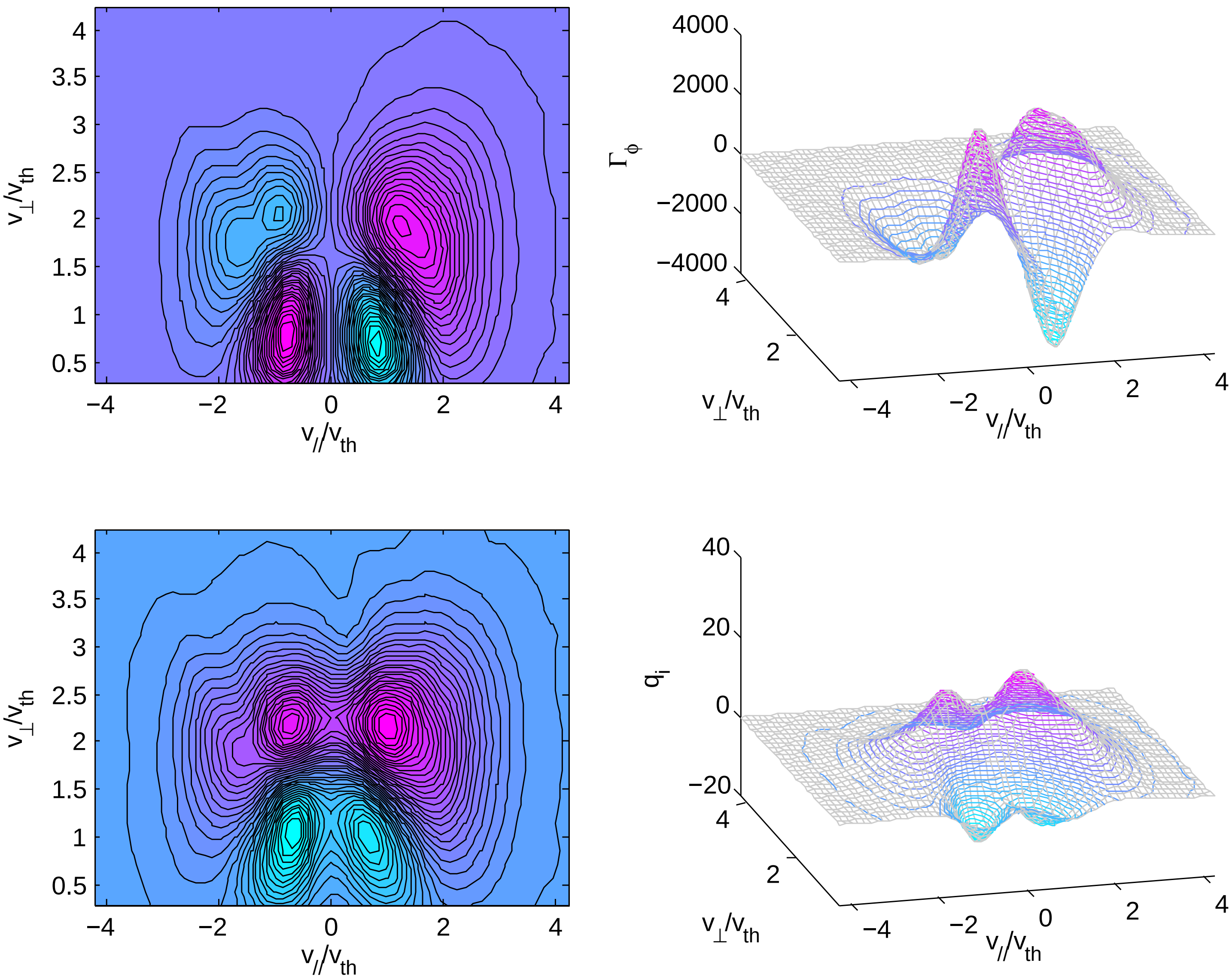
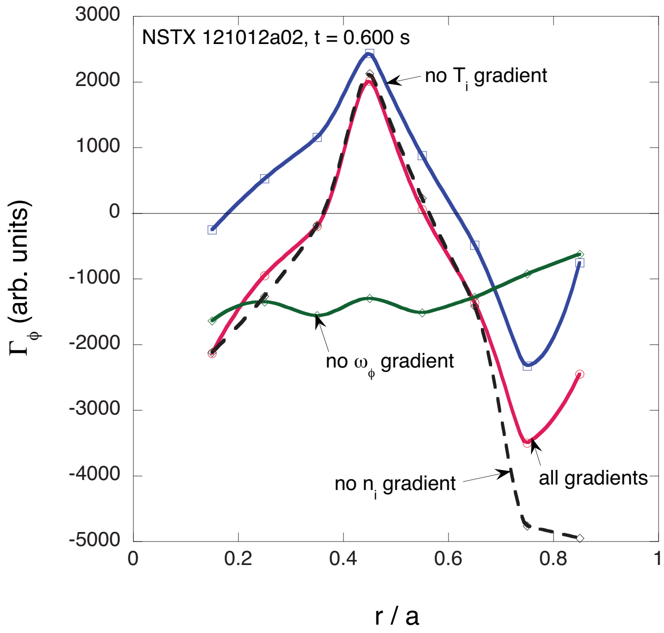
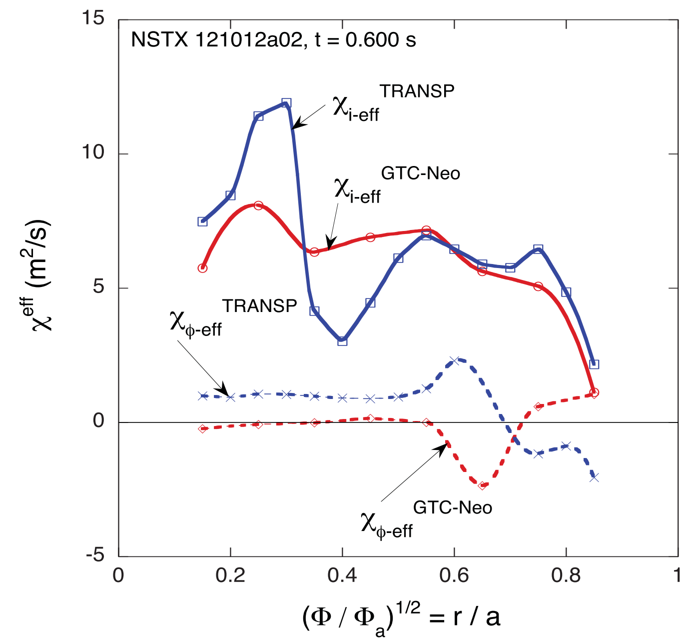
Which and how particles contribute to momentum and energy transport: resonance and non-resonance



Resonance condition: $\omega - \omega_{di}(v_\parallel^2) - \omega_{\nabla B}(\mu) - k_\parallel v_\parallel = 0$

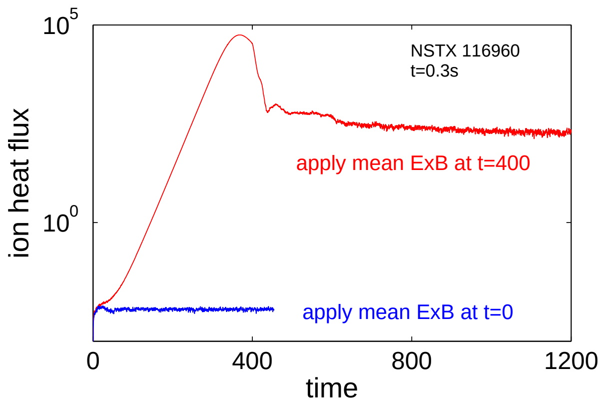
III. Neoclassical momentum transport can be non-diffusive; overall NC contribution is negligibly small

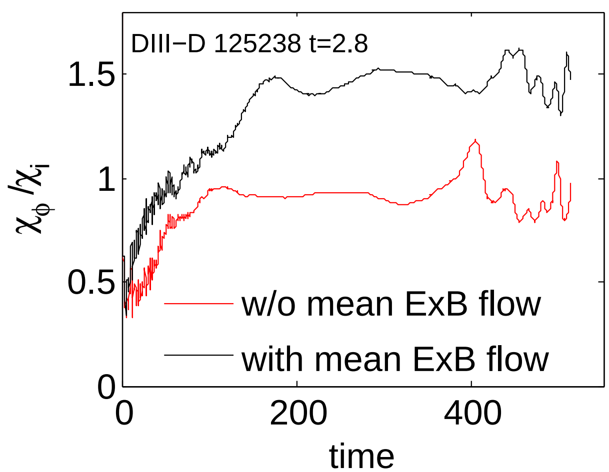




- Simulations show inward non-diffusive momentum flux associated with temperature gradient
- Simulations applied to NSTX and DIII-D discharges show that the NC contribution to momentum transport is negligibly small due to the absence of banana orbit enhancement
- Momentum transport is mostly anomalous, even when ion heat transport is reduced to neoclassical level

VI. Residual turbulence may account for puzzling co-existence of neoclassical-level ion heat and anomalous momentum transport





- Neoclassically $\chi_\phi/\chi_i \sim 0.01 - 0.1$, and neoclassical contribution to momentum transport is negligibly small
- Existence of strong coupling between ion momentum and heat transport for ITG turbulence, $\chi_\phi \sim \chi_i$, is verified
- Finite residual turbulence is found to survive strong mean $\mathbf{E} \times \mathbf{B}$ shear flow induced damping, and drive an insignificant ion heat flux and a finite momentum flux significantly higher than neoclassical level

- Equilibrium $\mathbf{E} \times \mathbf{B}$ flow shear is found to reduce turbulence driven transport for energy more efficiently than for momentum

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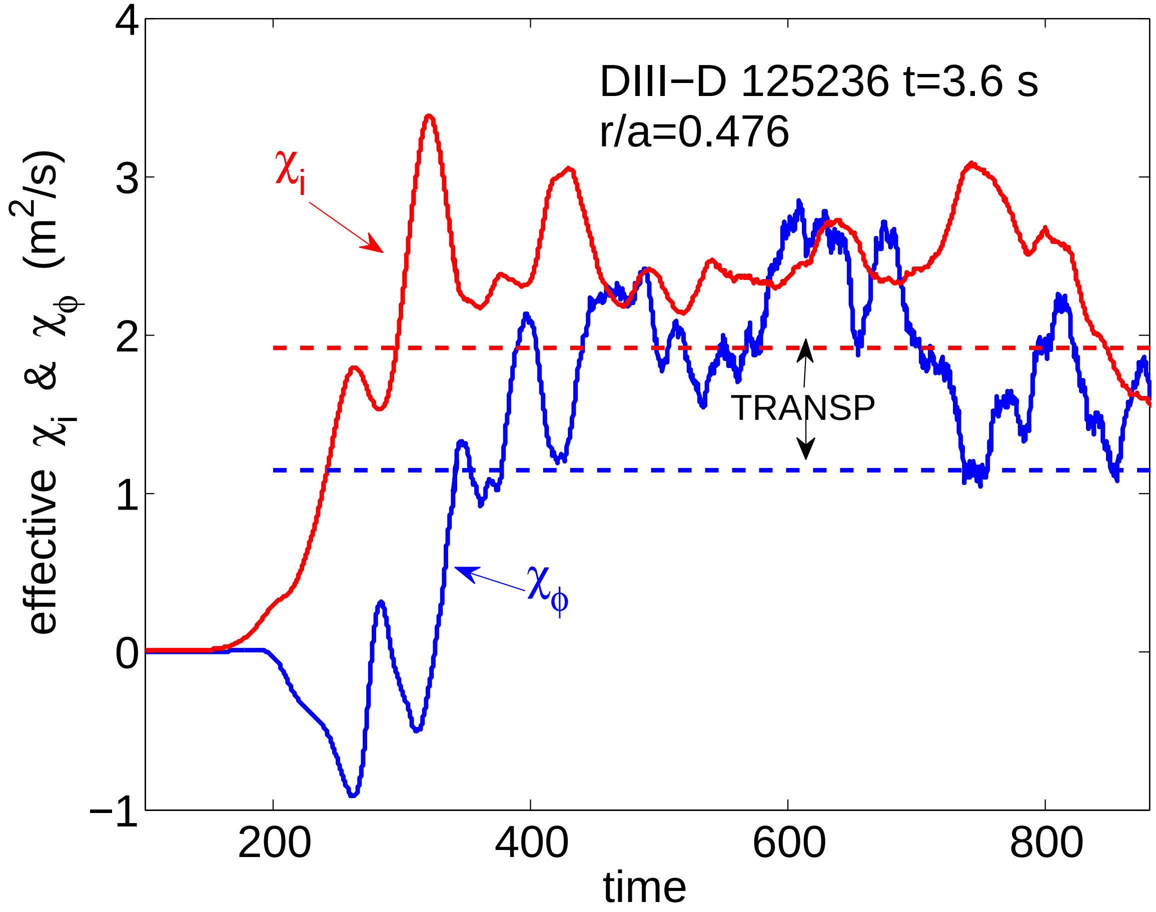
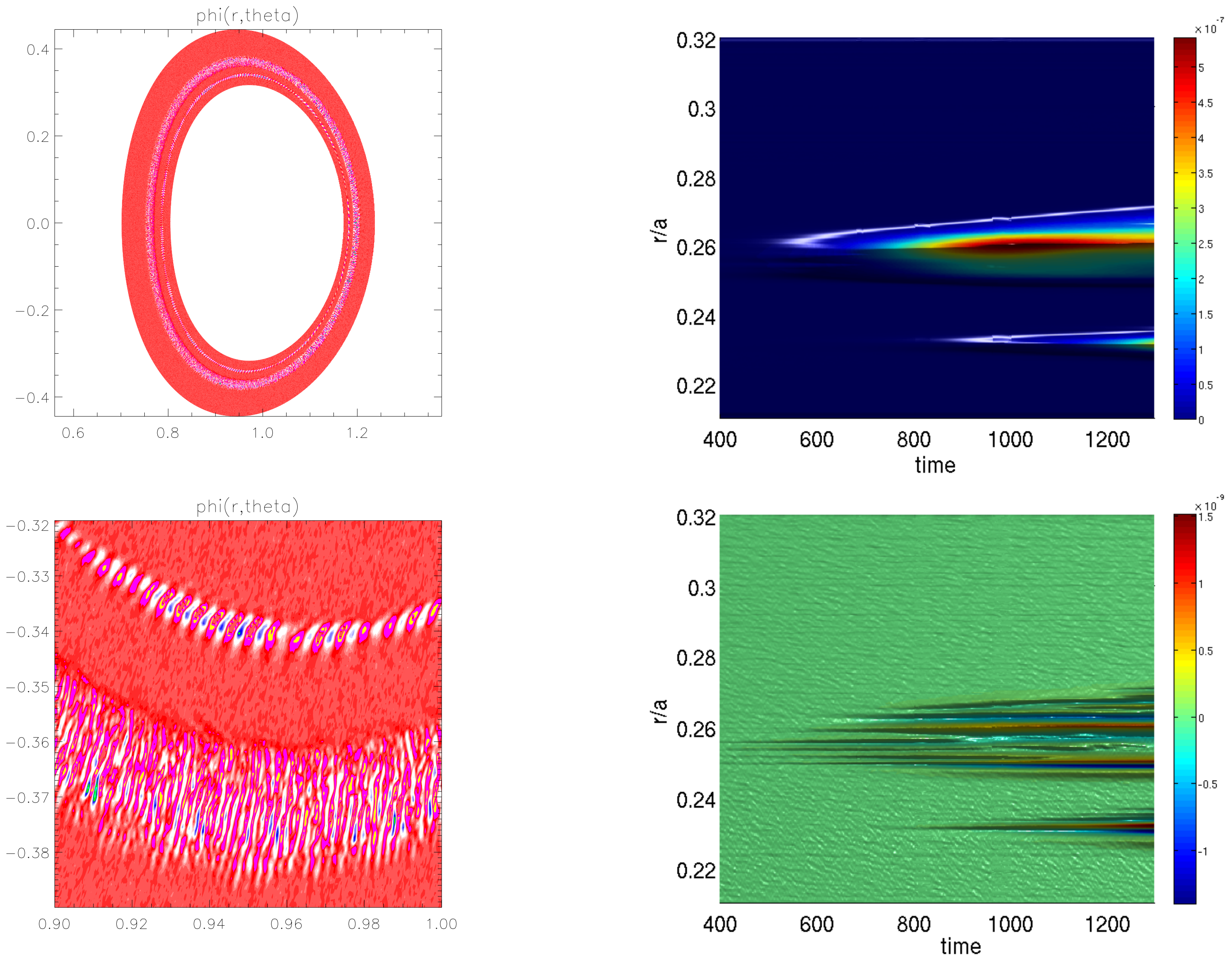
Validation of GTS turbulence simulation of momentum and energy transport against DIII-D measurements



VI. Global nonlinear ETG simulations of NSTX discharge support experimental observations of electron gyro-radius scale fluctuations (with E. Mazzucato ...)



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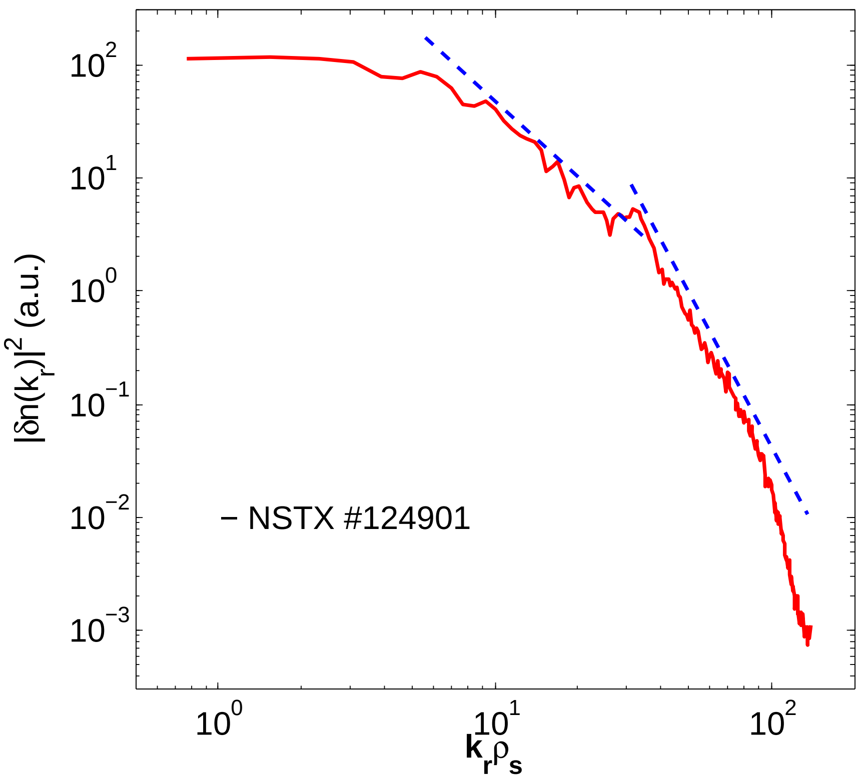
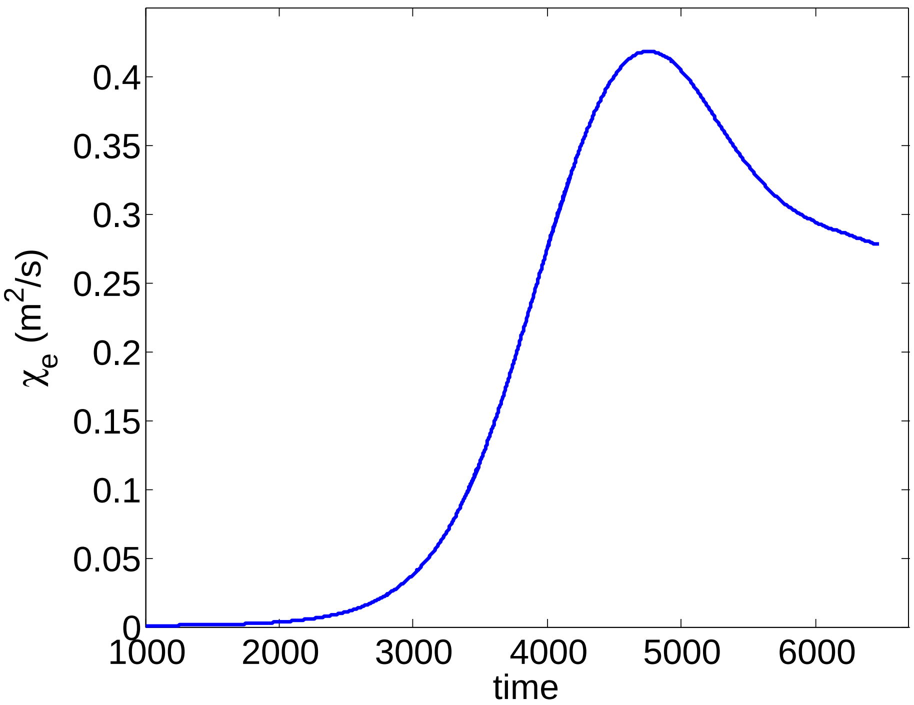
Global nonlinear ETG simulations of NSTX discharge support experimental observations of electron gyro-radius scale fluctuations





Summary

- A large inward flux of toroidal momentum is driven in the post-saturation phase of ITG turbulence. It is rather robust and mostly off-diagonal, which may lead to core rotation spin up (resulting in $\Delta u_{\|} \sim$ a few percent of v_{th} in the case of no momentum source at the edge)
- Underlying physics for the inward flux is related to the turbulence generated quasi-stationary zonal flow shear which breaks $k_{\|}$-symmetry and generates residual stress
- The relatively low level momentum flux in the long-time steady state appears to be approximately diffusive, with effective χ_ϕ/χ_i on the order of unity, in broad agreement with experimental observations and theory prediction for low-k fluctuations

Summary

- A significant inward, non-diffusive neoclassical momentum flux is observed associated with ion temperature gradient. However, the overall neoclassical contribution to momentum transport is negligibly small.
- Residual turbulence found to survive the dissipation of a strong mean $\mathbf{E}\times\mathbf{B}$ flow shear and drive anomalous momentum flux
- Equilibrium $\mathbf{E} \times \mathbf{B}$ flow shear reduces turbulence driven transport for energy more efficiently than for momentum

$\Longrightarrow$ one possible explanation to the puzzle of co-existence of neoclassical-level ion heat and anomalous momentum transport in experiments

- First global, nonlinear ETG simulation for experimental discharge has been carried out for direct validation against high-k measurements of electron gyro-radius scale fluctuations in NSTX