Simulations of Energetic Particle-driven Instabilities with Source and Sink

G.Y. Fu[1], J. Lang[1], Y. Chen[2], H.L. Berk[3], E. Fredrickson[1], N. Gorelenkov[1]
[1]Princeton Plasma Physics Laboratory, Princeton, NJ 08543, USA
[2]University of Colorado, Boulder, Colorado, USA
[3]University of Texas at Austin, Austin, Texas, USA
e-mail: fu@pppl.gov

Abstract

The energetic particle-driven instabilities and energetic particle transport are important for burning plasmas such as ITER. Here we report new results of nonlinear simulations of energetic particle-driven Toroidal Alfven Eigenmodes(TAE) with particle collision, particle source and sink, as well as with plasma micro-turbulence-induced energetic particle radial diffusion. The results of TAE saturation level scaling with collisional frequency agree very well with analytic theory. The simulation results also show that the effects of plasma micro-turbulence-induced radial diffusion can be more important than the collisional effects for determining the saturation level of energetic particle-driven Alfven modes in the present day tokamaks and future reactors. Finally, the simulation of beam-driven TAEs in the National Spherical Torus Experiment(NSTX) demonstrated the importance of plasma toroidal rotation on TAE modes.

1. Introduction

In burning plasmas of fusion reactors such as ITER, the fusion product alpha particles can drive Alfvén instabilities by tapping the free energy associated with radial gradient of the distribution via wave particle resonances. The driven Alfvén instabilities can in turn cause energetic particle loss to the reactor wall and induce wall damage. Thus it is very important to investigate the nonlinear saturation of Alfvén instabilities and the energetic particle transport.

Here we report our recent results of nonlinear simulations of energetic particle-driven TAE with particle collision, particle source and sink [1]. The kinetic/MHD hybrid code M3D-K [2] is used. The calculated TAE saturation level scaling with collisional frequency agrees well with the analytic theory [3].

We have also investigated the effects of plasma micro-turbulence on the TAE saturation. It has been shown recently that plasma micro-turbulence can induce small but finite radial diffusion of energetic particles [4-6]. Here we will show that the effects of the radial diffusion on TAE saturation are similar to that of particle Coulomb collision. We have made an estimate for the importance of the diffusion relative to that of collision. We will show that the effects of micro-turbulence-induced diffusion can be more important than that of Coulomb collision for present day tokamaks and burning plasma experiments such as ITER.

Finally we have investigated the effects of plasma toroidal rotation on the beam-driven TAEs in the NSTX plasmas. The results show the importance of the rotation on the TAE modes.

2. Simulation of Energetic Particle-driven TAE with Source and Sink

It is known that particle collisions and particle source and sink are important for the nonlinear saturation process of energetic-particle-driven Alfven instabilities. Without collision, the driven mode initially saturates due to trapping of resonant particles in the finite amplitude waves and the resultant flattening of the energetic particle distribution function near the resonances. After initial saturation, the mode then decays to zero due to plasma background damping. However, with particle collision and particle source and sink, the distribution tends to relax back to the equilibrium distribution. The balance of this collisional relaxation and nonlinear flattening yields a steady state saturation at sufficiently large collision rates [3].

In order to study these energetic particle collisional effects on TAE saturation, we have recently implemented a simple collision operator (pitch angle scattering and slowing-down) together with energetic particle source and sink in the M3D-K hybrid code. For neutral beam ions, the particles are injected at an initial velocity. The particles are then evolved through pitch angle scattering and slowing-down due to collisions with background thermal plasma. The pitch angle scattering is implemented using a standard

Monte-Carlo method. We have carried out nonlinear simulations of energetic particle-driven TAE for various collision frequencies. First, for cases well above marginal stability, the mode saturation is approximately steady state at sufficiently large collision frequencies. Figure 1 shows the evolution of the n=1 TAE mode amplitude for several collision frequencies with both pitch angle scattering and slowing down. The results were obtained at a fixed energetic particle pressure. The pitch angle scattering rate ν_d is related to the slowing down rate ν as $\nu_d = (v_c^3/2v^3)\nu$ with v_c being the critical velocity. It is observed that the saturation level increases with the collision frequency. Figure 2 shows that the calculated saturation level A_{sat} scales with collision frequency ν as $A_{sat} \sim \nu^{0.73}$ which agrees well with the analytic $\nu^{2/3}$ scaling[3]. For cases with pitch angle scattering only, a scaling of $A_{sat} \sim \nu^{0.65}$ is obtained, reproducing the analytic $\nu^{2/3}$ scaling. Second, for cases near marginal stability and sufficiently low collision frequencies, the mode saturation exhibits pulsation behavior with frequency chirps up and down. The pulsation is associated with dynamic relaxation of particle distribution function in phase space, possibly due to bounce motion of resonant particles trapped in the waves. The details of these results have recently published in Physics of Plasmas [1].

3. Effects of turbulence-induced radial diffusion on TAE saturation

Recent work has shown that plasma background micro-turbulence can cause anomalous radial diffusion of energetic particles when energetic particle energy is not too high [4-6]. It is expected that effects of radial diffusion are similar to that of Coulomb collision on nonlinear saturation because resonant particles can move around resonant region due to radial diffusion just as collisions do. Here we investigate the effects of radial diffusion on nonlinear saturation of energetic particle-driven TAE using the M3D-K code. The effect of micro-turbulence is simulated in the code with a simple radial diffusion operator for the energetic particles. We consider an energetic particle-driven n=3 TAE mode in a circular tokamak plasma with aspect ratio R/a=3 and zero thermal beta. The safety factor profile is $q = 1.25 + \Psi$ with Ψ being the the normalized poloidal flux variable ($\Psi = 0$ at magnetic axis and $\Psi = 1$ at plasma edge). The plasma density profile is uniform. The energetic particle parameters are $v_0/v_A = 1.7$ and $\rho_h/a = 0.028$ with v_0 being the injection speed, ρ_h the gyroradius at injection velocity. Using these parameters and profiles, linear and nonlinear hybrid simulations have been carried with energetic particle radial diffusion and without collisions. In the linear regime, an unstable n=3 TAE mode is found with frequency $\omega/\omega_A = 0.33$ and growth rate $\gamma/\omega_A = 0.9\%$. Figure 3 shows the contour of the velocity stream function U of the corresponding eigenmode. In the nonlinear regime, the mode saturates at a steady state level approximately with sufficient large radial diffusion coefficient. Figure 4 shows the nonlinear evolution of the mode amplitude at three diffusion rates. It is observed that the saturation level increases with radial diffusion. Figure 5 plots the saturation level as a function of the radial diffusion rate. The calculated saturation level scales as $U \sim D_r^{0.64}$ in agreement with analytic scaling of $U \sim D_r^{2/3}$. Here D_r is the radial diffusion coefficient of the energetic particles. This numerical result shows that the effects of diffusion are similar to that of collision. In order to estimate quantitatively the equivalent values of radial diffusion, we have also carried nonlinear simulations of the n=3 TAE with collisonal pitch angle scattering only. The simulation results show that the radial diffusion is equivalent to pitch angle scattering (with respect to their effects on nonlinear saturation) when the following condition is satisfied for the n=3 TAE mode:

$$\frac{D_r/a^2}{\nu_d} = 0.07 \tag{1}$$

where ν_d is the rate of collisional pitch angle scattering. We now derive analytically the equivalence condition for the radial diffusion and estimate the importance of radial diffusion relative to Coulomb collision for present day tokamaks and future burning plasmas. We start from the following kinetic equation for energetic particles with radial diffusion and pitch angle scattering:

$$\frac{df}{dt} = \frac{1}{rJ}\frac{1}{\partial r} rJD_r \frac{1}{\partial r} f + \nu_d \frac{1}{\partial \lambda}(1-\lambda^2)\frac{1}{\partial \lambda} f + S \tag{2}$$

where r is radius, J is the coordinates jacobian, $\lambda = v_\parallel/v$ is pitch angle and S is the source term. The diffusion operator and the pitch angle scattering operator can be simplified by expanding the operators near resonances. Thus, we define a new variable $\Omega = \omega - k_\parallel v_\parallel$ with $k_\parallel = (n - m/q)$ and n and m are toroidal and poloidal mode number respectively. Note that $\Omega = 0$ corresponds to wave particle resonance. Equation 2 can then be rewritten as

$$\frac{df}{dt} = D_r(\frac{\partial\Omega}{\partial r})^2 \frac{\partial^2}{\partial\Omega^2} f + \nu_d(1-\lambda^2)(\frac{\partial\Omega}{\partial\lambda})^2 \frac{\partial^2}{\partial\Omega^2} f + S \tag{3}$$

where

$$\frac{\partial\Omega}{\partial r} = \frac{m v_\parallel}{q^2 R} q' \tag{4}$$

and

$$\frac{\partial\Omega}{\partial\lambda} = k_\parallel v + \frac{\partial\Omega}{\partial r}\frac{qR\rho}{r} \tag{5}$$

Note that in Eq. (3), we have only kept the dominant second order derivative terms because the resonance regions tend to be very narrow for TAE instability. Then, the equivalence condition between the radial diffusion and collisional pitch angle scattering is given by

$$D_r(\frac{\partial\Omega}{\partial r})^2 = \nu_d(1-\lambda^2)(\frac{\partial\Omega}{\partial\lambda})^2 \tag{6}$$

or

$$\frac{D_r/a^2}{\nu_d} = \frac{(1+\frac{2maq'}{q}\frac{qR\rho\lambda}{ar})^2}{(\frac{2maq'}{q})^2}\frac{1-\lambda^2}{\lambda^2} \tag{7}$$

where we have used $k_\parallel = 1/(2qR)$ for TAE modes. We can now compare our analytic estimate given by Eq. (7) and the numerical result given by Eq. (1). Using the parameters for the case of n=3 TAE described above, the right hand side can be estimated to be

$$\frac{(1+\frac{2maq'}{q}\frac{qR\rho\lambda}{ar})^2}{(\frac{2maq'}{q})^2}\frac{1-\lambda^2}{\lambda^2} \approx 0.12 \tag{8}$$

where we have taken $v_\parallel = v_A$ for resonant particles and $(1-\lambda^2)/\lambda^2 \sim 1$. We note that the analytic estimate of Eq. (8) is fairly close to the numerical result of Eq. (1) considering the simplification made for the $(1-\lambda^2)/\lambda^2$ term.

The analytic estimate can be simplified further for TAEs with large mode number. In particular, in the limit of $mq\rho/r \sim 1$ valid for most unstable TAEs and $2q'R/q >> 1$, the Eq. (7) then reduces to

$$\frac{D_r/a^2}{\nu_d} = (1-\lambda^2)(\frac{qR\rho}{ar})^2 \tag{9}$$

or

$$R_d = \frac{1}{1-\lambda^2}\frac{ra}{qR\rho}\frac{D_r/a^2}{\nu_d} = 1 \tag{10}$$

where we have defined the ratio R_d. Therefore when $R_d = 1$, the effect of radial diffusion on TAE saturation is equivalent to that of collisional pitch angle scattering. For $R_d > 1$, the effect of radial diffusion is more important. We now proceed to estimate this ratio R_d for present day tokamaks as well as for ITER. We first need to determine the effective radial diffusion D_r of energetic particles induced by plasma background turbulence. Following the work of Zhang et al [6], the radial diffusion of energetic particles can be described by the following formula;

$$D_r = \frac{5T}{E} D_{r,th} \tag{11}$$

where T is ion temperature, E is the energetic particle energy (resonant particles), and $D_{r,th}$ is the diffusion coefficient of thermal ions. First, we consider the case of beam-driven TAEs in TFTR. The main parameters are: plasma density $n_e = 3.0 \times 10^{13} cm^{-3}$, plasma temperature $T = 1.5kev$, $q = 1.5$, $R = 240cm$, $a = 80cm$, and $B = 1.0T$. We also assume $D_{r,th} \sim 4.0m^2/sec$. The Eq. (10) then gives $R_d \sim 2.0$. Second, we consider the case of alpha particle-driven high-n TAEs in ITER. The main parameters are: plasma density $n_e = 1.0 \times 10^{14} cm^{-3}$, plasma temperature $T = 15kev$ at $r/a = 0.5$, $q = 1.5$, $R = 620cm$, $a = 200cm$, and $B = 5.3T$. Furthermore, we also assume $D_{r,th} \sim 1m^2/sec$. The corresponding ratio is $R_d \sim 5.0$. From these results, we have shown that the micro-turbulence-induced radial diffusion can be more important than Coulomb collision for nonlinear saturation of energetic particle-driven TAE instability in present day tokamaks and future burning plasma experiments.

4. Simulation of Beam-driven TAE with Toroidal Rotation in NSTX

In the NBI-heated NSTX plasmas, the beam-driven Alfven instabilities are routinely observed with rich mode spectrum and nonlinear behavior [7]. For typical NSTX plasmas, the plasmas have significant toroidal rotation up to $40kHz$ because NBI-injection is usually in co-direction only. Thus, the toroidal rotation is expected to affect the Alfven continuum spectrum and TAE mode structure substantially. Indeed, this has been confirmed by a recent analysis of continuum mode spectrum and TAE mode structure using NOVA code for a NSTX plasma[7]. In this work, the M3D-K code is used to carry out linear simulations of beam-driven TAEs in a NBI-heated NSTX plasma (discharge #124781 at $t = 0.285sec$) where the toroidal rotation is significant. The calculated mode frequency of the n=3 TAE is $f = 103kHz$ which agrees well with the experimental value of $f_{exp} \sim 100kHz$ at $t = 0.285sec$. Furthermore, our numerical results show that the eigenmode radial structure is significantly affected by rotation. These results are consistent with the NOVA analysis for the same NSTX plasma. It should be noted that our simulation model is more self-consistent than the ideal MHD analysis of NOVA. In particular, the continuum resonances, continuum damping, energetic particle drive and mode structure are all non-perturbatively treated. The mode structure near the continuum resonances are resolved physically.

5. Summary

In summary, we have carried out nonlinear simulations of energetic particle-driven Toroidal Alfven Eigenmodes with energetic particle collision, and energetic particle source and sink, as well as with plasma micro-turbulence-induced energetic particle radial diffusion. The results of TAE saturation level scaling with collisional frequency agree very well with the analytic theory[3]. The simulation results with plasma micro-turbulence-induced diffusion show that the effects of the radial diffusion can be more important than the collisional effects for determining the saturation level of energetic particle-driven Alfven modes in the present day tokamaks and future burning plasma experiments such as ITER. Finally, the linear simulations of beam-driven TAEs in a NSTX plasma demonstrate the importance of plasma toroidal rotation on TAE modes.

Acknowledgements

We gratefully thank Dr. B. N. Breizman for stimulating discussions. This work was supported by the U.S. Department of Energy under contract DE-AC02-09CH11466.

[1] J. Y. Lang et. al., Phys. Plasmas 17, 042309 (2010).
[2] G. Y. Fu et. al., Phys. Plasmas 13, 052517 (2006).
[3] H. L. Berk and B. N. Breizman, Phys. Fluids B 2, 2235 (1990).
[4] C. Estrada-Mila et. al., Phys. Plasmas 13, 112303 (2006).
[5] T. Dannert et. al., Phys. Plasmas 15, 062508 (2008).
[6] W. Zhang et. al., Phys. Rev. Lett. 101, 095001 (2008).
[7] E.D. Fredrickson et. al., Phys. Plasmas 16, 122505 (2009).

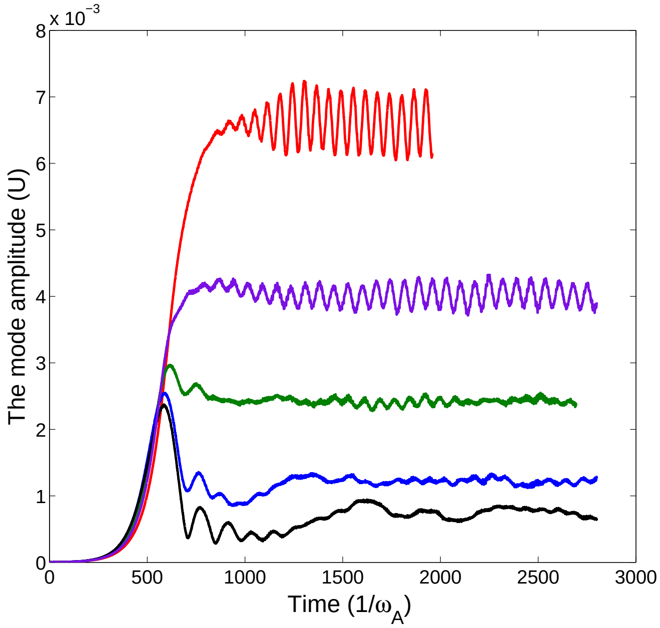

Figure 1: The time evolution of the mode amplitude with both pitch angle scattering and slowing down at different collision rates. From the top to the bottom, the slowing down rates are $\nu = 1.0 \times 10^{-3}\omega_A$, $\nu = 5.0 \times 10^{-4}\omega_A$, and $\nu = 2.5 \times 10^{-4}\omega_A$, $\nu = 1.0 \times 10^{-4}\omega_A$, and $\nu = 5.0 \times 10^{-5}\omega_A$.

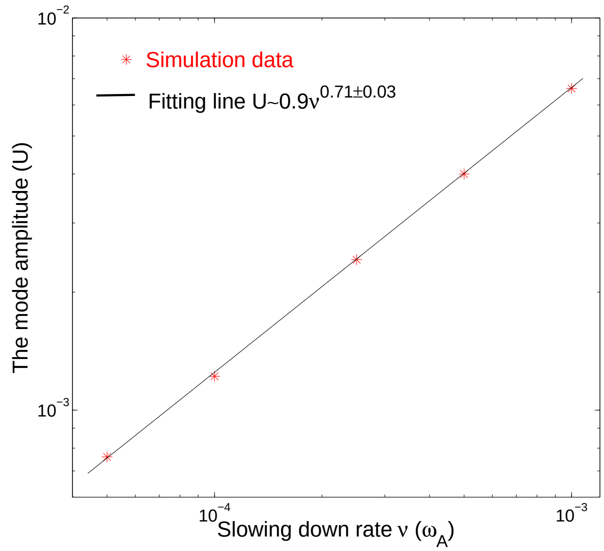

Figure 2: The nonlinear saturation level of the mode amplitude as a function of collision rate in the presence of both pitch angle scattering and slowing down

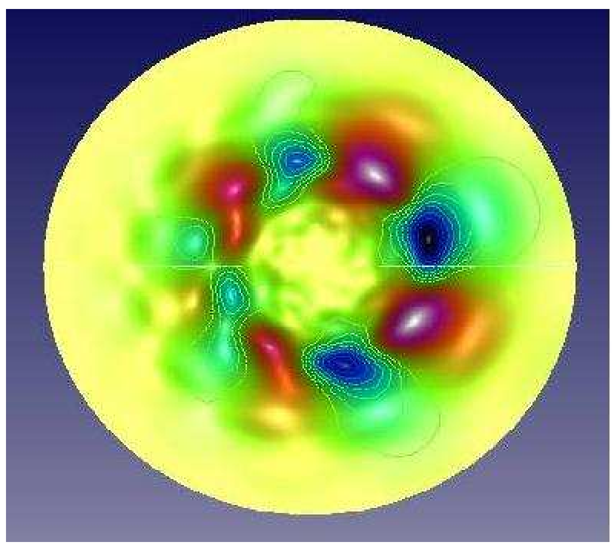

Figure 3: The contour of the velocity stream function U for an n=3 TAE.

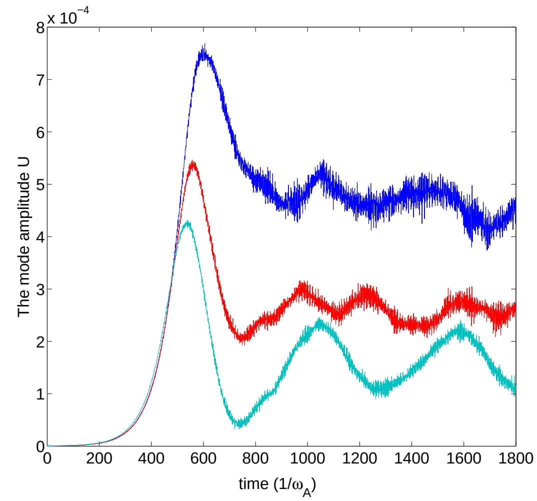

Figure 4: The amplitude of the velocity stream function U as a function of time obtained at the radial diffusion coefficient of $D_r/a^2 = 2.5 \times 10^{-5}\omega_A$ (blue),$D_r/a^2 = 1.0 \times 10^{-5}\omega_A$ (red), and $D_r/a^2 = 0.5 \times 10^{-5}\omega_A$ (cyan).

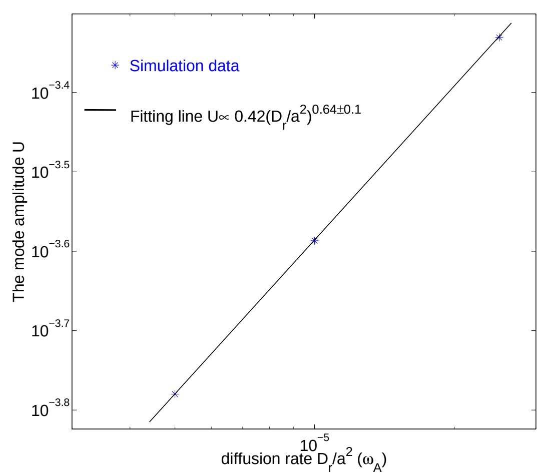

Figure 5: The saturation level of the velocity stream function U as a function of radial diffusion coefficient D_r/a^2.