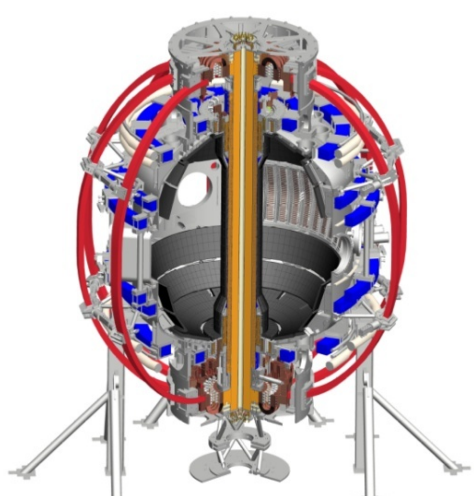
Study of neoclassical toroidal viscosity in tokamaks with a δf particle code and Resonant nature of magnetic braking

Kimin Kim[1], Jong-Kyu Park[1], Gerrit J. Kramer[1], Allen H. Boozer[2], Jonathan E. Menard[1], Stefan P. Gehardt[1] and the NSTX Team

[1]Princeton Plasma Physics Laboratory
[2]Columbia University

2012 IAEA Fusion Energy Conference
San Diego, CA, USA
October 8 – 13, 2012

Coll of Wm & Mary
Columbia U
CompX
General Atomics
FIU
INL
Johns Hopkins U
LANL
LLNL
Lodestar
MIT
Lehigh U
Nova Photonics
ORNL
PPPL
Princeton U
Purdue U
SNL
Think Tank, Inc.
UC Davis
UC Irvine
UCLA
UCSD
U Colorado
U Illinois
U Maryland
U Rochester
U Tennessee
U Tulsa
U Washington
U Wisconsin
X Science LLC



Culham Sci Ctr
York U
Chubu U
Fukui U
Hiroshima U
Hyogo U
Kyoto U
Kyushu U
Kyushu Tokai U
NIFS
Niigata U
U Tokyo
JAEA
Inst for Nucl Res, Kiev
Ioffe Inst
TRINITI
Chonbuk Natl U
NFRI
KAIST
POSTECH
Seoul Natl U
ASIPP
CIEMAT
FOM Inst DIFFER
ENEA, Frascati
CEA, Cadarache
IPP, Jülich
IPP, Garching
ASCR, Czech Rep

Motivation to new δf particle code for NTV calculation

- Neoclassical Toroidal Viscosity plays an important role for control of plasma rotation, stability and performance in perturbed tokamaks
  - Non-axisymmetric magnetic perturbations can fundamentally change the neoclassical transport by distorting particle orbits on deformed or broken flux surfaces
  - Application of non-axisymmetric perturbations drive significant magnetic braking by NTV, thus change plasma rotation impacting on tokamak performance
  - Non-axisymmetric perturbations are important control elements to actively stabilize locked modes, edge localized modes, and resistive wall modes
- A new δf particle code has been developed to calculate neoclassical transport in non-axisymmetric configurations
  - How to accurately calculate $\underline{\delta f}$ and $\underline{\delta B}$?
  - Analytic studies are limited in narrow regimes or strong approximations on particle orbits, geometries, and collisions
    - Large aspect-ratio approximation, trapped particle only, simplified δB, ignoring orbit width
    - $1/\nu$ theory: pitch angle scattering, but narrow regime, w/o magnetic precession

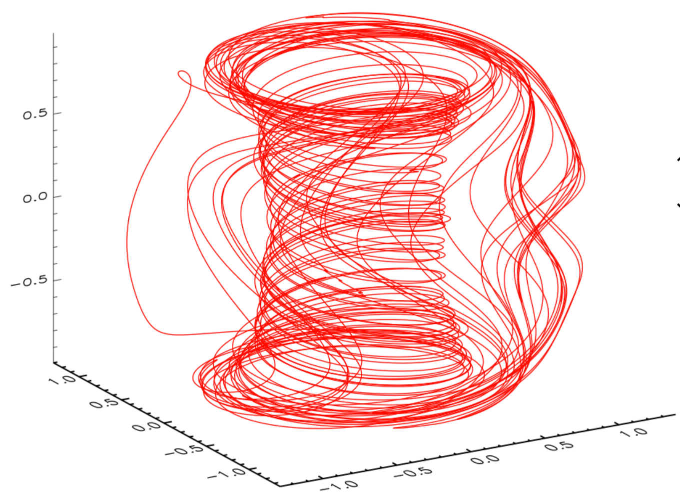
      [K.C. Shaing, Phys. Plasmas 10, 1443 (2003)]
    - Combined theory: combined regime, bounce harmonics, but Krook collision

      [J.-K. Park et al., Phys. Rev. Lett. 102, 065002 (2009)]
  - New code calculates *δf from guiding center particle motion* with momentum conserving collision operator using *δB from 3D perturbed equilibrium solver (IPEC)*

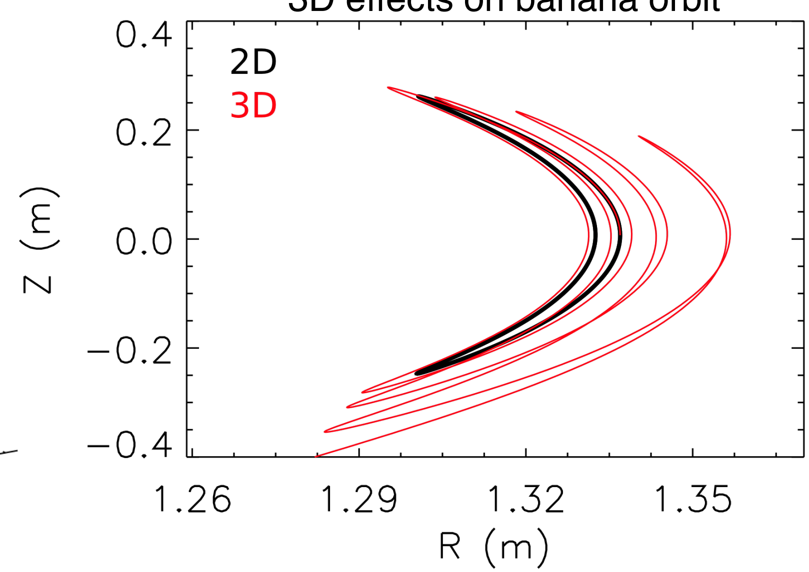
POCA is a drift-kinetic δf particle code for neoclassical transport with non-axisymmetric magnetic perturbations

- POCA (Particle Orbit Code for Anisotropic pressures)
  - Follows guiding center orbit motions in (ψ, θ, ϕ, $v_{||}$) space **[K. Kim et al., Phys. Plasmas 19, 082503 (2012)]**
  - Solves Fokker-Plank equation with modified pitch-angle scattering collision operator conserving toroidal momentum
  - Calculates local neoclassical quantities: Diffusion, flux, bootstrap current
  - Directly calculates anisotropic tensor pressure and NTV torque
  - Uses DCON/IPEC type routines and parallelized with MPI
  - Reads 2D equilibrium from 20 equilibrium types (exp/analytic), and 3D perturbation from IPEC and analytic model

Guiding center motion in NSTX



3D effects on banana orbit



POCA tracks guiding center orbit motions by Hamiltonian equations of motion

- Guiding center motion is described by Hamiltonian equations of motion
  - Boozer coordinates is used

$$\vec{B} = \frac{\mu_0}{2\pi}\left[G(\psi)\nabla\phi + I(\psi)\nabla\theta + \beta_*(\psi,\theta,\phi)\nabla\psi\right]$$

[A.H. Boozer, Phys. Fluids 23, 904 (1980)]

  - Drift Hamiltonian is expressed as

$$H = \frac{1}{2}mv_{\parallel}^2 + \mu B + q\Phi = \frac{q^2B^2}{2m}\rho_{\parallel}^2 + \mu B + q\Phi \qquad \rho_{\parallel} = \frac{mv_{\parallel}}{qB}, \qquad p_\theta = \frac{q}{2\pi}(\rho_{\parallel}i + \psi), \qquad p_\phi = \frac{q}{2\pi}(\rho_{\parallel}g - \chi)$$

  - Hamiltonian equations of motion are derived by coordinate transformations

$$\dot{\theta} = -\frac{2\pi}{q}\frac{1}{g+\iota i}\left\{\frac{q^2B^2}{m}\rho_{\parallel}\left[\rho_{\parallel}\frac{\partial g(\psi)}{\partial\psi} - \iota(\psi)\right] - g(\psi)\left[\left(\frac{q^2\rho_{\parallel}^2B^2}{m} + \mu B\right)\frac{1}{B}\frac{\partial B}{\partial\psi} + q\frac{\partial\Phi}{\partial\psi}\right]\right\}$$

$$\dot{\phi} = -\frac{2\pi}{q}\frac{1}{g+\iota i}\left\{i(\psi)\left[\left(\frac{q^2\rho_{\parallel}^2B^2}{m} + \mu B\right)\frac{1}{B}\frac{\partial B}{\partial\psi} + q\frac{\partial\Phi}{\partial\psi}\right] - \left(\rho_{\parallel}\frac{\partial i(\psi)}{\partial\psi} + 1\right)\frac{q^2B^2}{m}\rho_{\parallel}\right\}$$

$$\dot{\psi} = -\frac{2\pi}{q}\frac{1}{g+\iota i}\left[g(\psi)\left\{\left(\frac{q^2\rho_{\parallel}^2B^2}{m} + \mu B\right)\frac{1}{B}\frac{\partial B}{\partial\theta} + q\frac{\partial\Phi}{\partial\theta}\right\} - i(\psi)\left\{\left(\frac{q^2\rho_{\parallel}^2B^2}{m} + \mu B\right)\frac{1}{B}\frac{\partial B}{\partial\phi} + q\frac{\partial\Phi}{\partial\phi}\right\} - \frac{q}{2\pi}V(t)i(\psi)\right]$$

$$\dot{\rho}_{\parallel} = \frac{2\pi}{q}\frac{1}{g+\iota i}\left[\left(\rho_{\parallel}\frac{\partial g(\psi)}{\partial\psi} - \iota(\psi)\right)\left\{\left(\frac{q^2\rho_{\parallel}^2B^2}{m} + \mu B\right)\frac{1}{B}\frac{\partial B}{\partial\theta} + q\frac{\partial\Phi}{\partial\theta}\right\} - \left(\rho_{\parallel}\frac{\partial i(\psi)}{\partial\psi} + 1\right)\left\{\left(\frac{q^2\rho_{\parallel}^2B^2}{m} + \mu B\right)\frac{1}{B}\frac{\partial B}{\partial\phi} + q\frac{\partial\Phi}{\partial\phi}\right\} - \frac{q}{2\pi}V(t)\left(\rho_{\parallel}\frac{\partial i(\psi)}{\partial\psi} + 1\right)\right]$$

μ_0: permeability of free space, B : magnetic field, ψ: toroidal flux, G: poloidal current, I: toroidal current
μ: magnetic moment, q: charge of particle, ι: rotational transform, Φ: potential

[R.B. White, Phys. Fluids B 2, 845 (1990)]

POCA solves Fokker-Planck equation for δf

- δf is calculated from Fokker-Planck equation
  - Fokker-Planck equation is written as

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\vec{F}}{m} \cdot \frac{\partial f}{\partial \vec{v}} = C(f), \quad f = f_M \exp(\hat{f}) \approx f_M (1 + \hat{f}) \longrightarrow \frac{d \ln f_M}{dt} + \frac{d\hat{f}}{dt} = C_m(f) \equiv \frac{C(f)}{f}$$

  - Fokker-Planck equation is reduced to

$$\frac{d\hat{f}}{dt} - C_m(f) = -\vec{v} \cdot \nabla\psi \frac{\partial \ln f_M}{\partial \psi} - \vec{F} \cdot \frac{\partial \ln f_M}{\partial \vec{v}}$$

  - By using local Maxwellian, δf can be obtained as

$$f_M = \frac{N}{\left(\sqrt{\pi} v_t\right)^3} \exp\left(-\frac{U - e\Phi}{T}\right) \longrightarrow \Delta\hat{f} = -\left[\frac{1}{n}\frac{\partial n}{\partial \psi} + \left(\frac{3}{2} - \frac{E}{T}\right)\frac{1}{T}\frac{\partial T}{\partial \psi}\right]\Delta\psi - \frac{e}{T}\frac{d\Phi}{d\psi}\Delta\psi$$

- Collision operator without momentum conservation
  - Lorentz collision operator for pitch angle scattering is expressed as

$$C(f) = \frac{\nu}{2}\frac{\partial}{\partial \lambda}\left[(1 - \lambda^2)\frac{\partial f}{\partial \lambda}\right] \quad \text{with} \quad \lambda = \frac{v_{\parallel}}{v}$$

  - Monte Carlo equivalent of Lorentz collision operator is used to update particle's pitch

$$\lambda_{n+1} = \lambda_n (1 - \nu \Delta t) \pm \sqrt{(1 - \lambda_n^2)\nu \Delta t}$$

[A.H. Boozer and G. Kuo-Petravic, Phys. Fluids 24, 851 (1981)]

Modified pitch-angle collision operator is used to preserve toroidal momentum conservation

- Lorentz collision operator conserves momentum by a correction term
  - Original Lorentz collision operator does not conserve momentum
  - One form of momentum conserving operator is given by

[M.N. Rosenbluth, R.D. Hazeltine and F.L. Hinton, Phys. Fluids 15, 116 (1972)]
[A.H. Boozer and H.J. Gardner, Phys. Fluids B 2, 2408 (1990)]

$$C_{m.c.}(f) = \nu \frac{m}{B} v_{\parallel} \frac{\partial}{\partial \mu}\left[\mu\left(v_{\parallel}\frac{\partial f}{\partial \mu} + \frac{uB}{T} f\right)\right] \quad \text{with} \quad \mu = \frac{m v_{\perp}^2}{2B}$$

$$\longrightarrow \quad C_{m.c.}(f) = \frac{\nu}{2}\frac{\partial}{\partial \lambda}\left[(1-\lambda^2)\left(\frac{\partial f}{\partial \lambda} - 2\frac{u}{v} f\right)\right], \qquad \text{u : mean flow velocity}$$

  - This can be rewritten with previously used non-conserving Lorentz operator $C_m(f)$

$$C_{m.c.}(f) = \frac{\nu}{2}\frac{\partial}{\partial \lambda}\left[(1-\lambda^2)\frac{\partial \hat{f}}{\partial \lambda}\right] + 2\nu\frac{u}{v}\lambda = C(\hat{f}) + 2\nu\frac{u}{v}\lambda$$

$$\longrightarrow \quad \frac{d\hat{f}}{dt} - C_m(f) = -\vec{v}\cdot\nabla\psi\frac{\partial \ln f_M}{\partial \psi} - \vec{F}\cdot\frac{\partial \ln f_M}{\partial \vec{v}} + 2\nu\frac{u}{v}\lambda$$

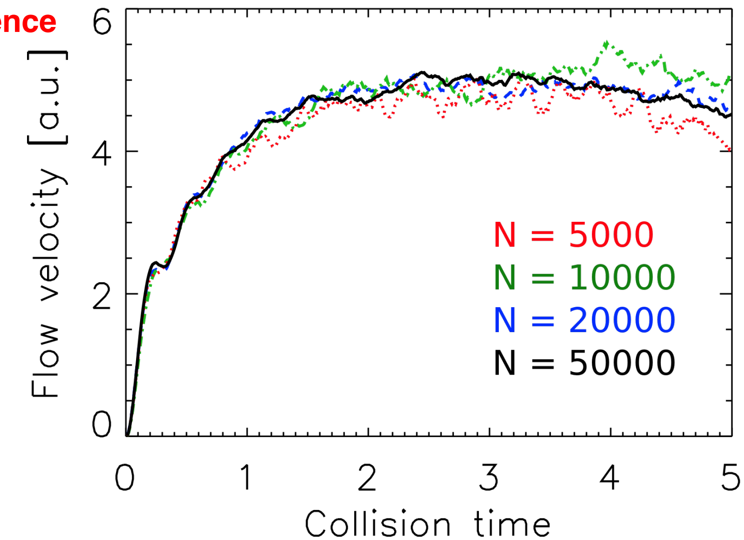
Momentum restoring term

  - Momentum conserving collision operator can be implemented by adding a correction term in δf calculation as [M. Sasinowski and A.H. Boozer, Phys. Plasmas 4, 3509 (1997)]

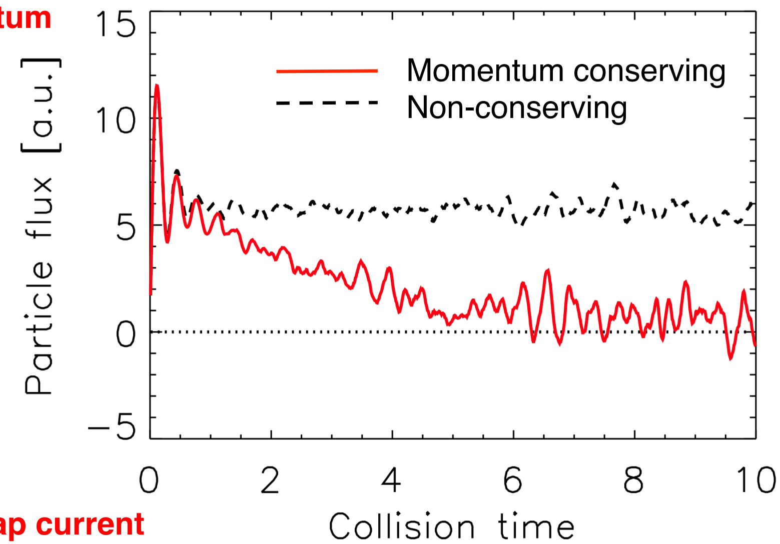
$$\Delta\hat{f} = -\left[\frac{1}{n}\frac{\partial n}{\partial \psi} + \left(\frac{3}{2} - \frac{E}{T}\right)\frac{1}{T}\frac{\partial T}{\partial \psi}\right]\Delta\psi + 2\nu\frac{u}{v}\lambda\Delta t - \frac{e}{T}\frac{d\Phi}{d\psi}\Delta\psi$$

POCA was successfully benchmarked in axisymmetry

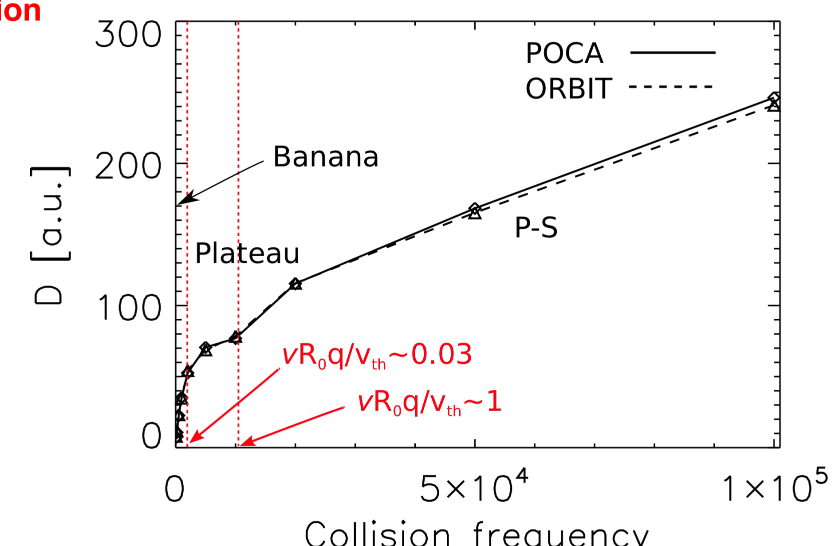
Convergence



Momentum

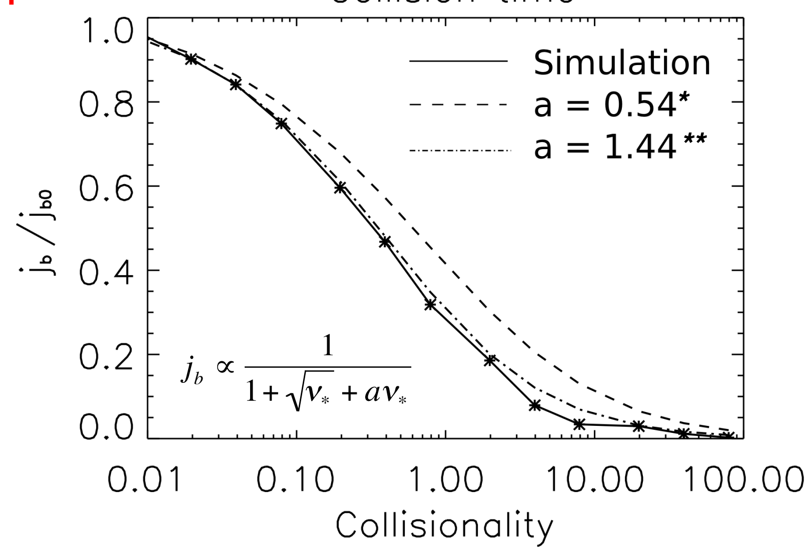


Diffusion



[R.B. White and M.S. Chance, Phys. Fluids B 27, 2455 (1984)]

Bootstrap current



*[F.L. Hinton and M.N. Rosenbluth, Phys. Fluids 16, 836 (1973)]

**[M. Sasinowski and A.H. Boozer, Phys. Plasmas 2, 610 (1995)]

Neoclassical Toroidal Viscosity is calculated using perturbed pressures and magnetic field spectrum

- POCA calculates NTV torque using
  - Perturbed pressures defined as

$$\delta P = \delta P_{\perp} + \delta P_{\parallel} = \int d^3 v \left(\frac{1}{2} m v_{\perp}^2 + m v_{\parallel}^2 \right) \delta f$$

  - Magnetic field spectrum decomposed to Fourier series assumed as

$$\frac{\delta B}{B_0} = \sum_{m,\, n \neq 0} \delta_{mn}(\psi) \cos(m\theta - n\phi)$$

  - Then, NTV torque is calculated using $J \propto 1/B^2$ in Boozer coordinates

$$\tau_{\phi} = \left\langle \hat{e}_{\phi} \cdot \nabla \cdot \overleftrightarrow{P} \right\rangle = \left\langle \frac{\delta P}{B} \frac{\partial B}{\partial \phi} \right\rangle = B \sum_{m,\, n \neq 0} n \delta_{mn} \left\langle \frac{\delta P}{B} \sin(m\theta - n\phi) \right\rangle$$

[J.L.V. Lewandowski et al., Phys. Plasmas 8, 2849 (2001)]
[J.D. Williams and A.H. Boozer, Phys. Plasmas 10, 103 (2003)]

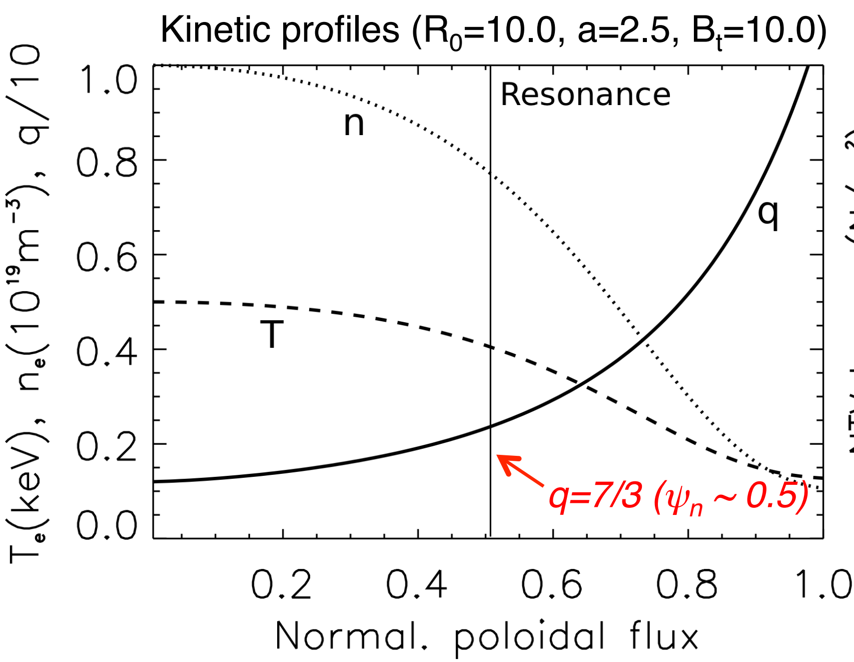
- Analytic perturbation model is used for basic study
  - An analytic magnetic perturbation model is prescribed as

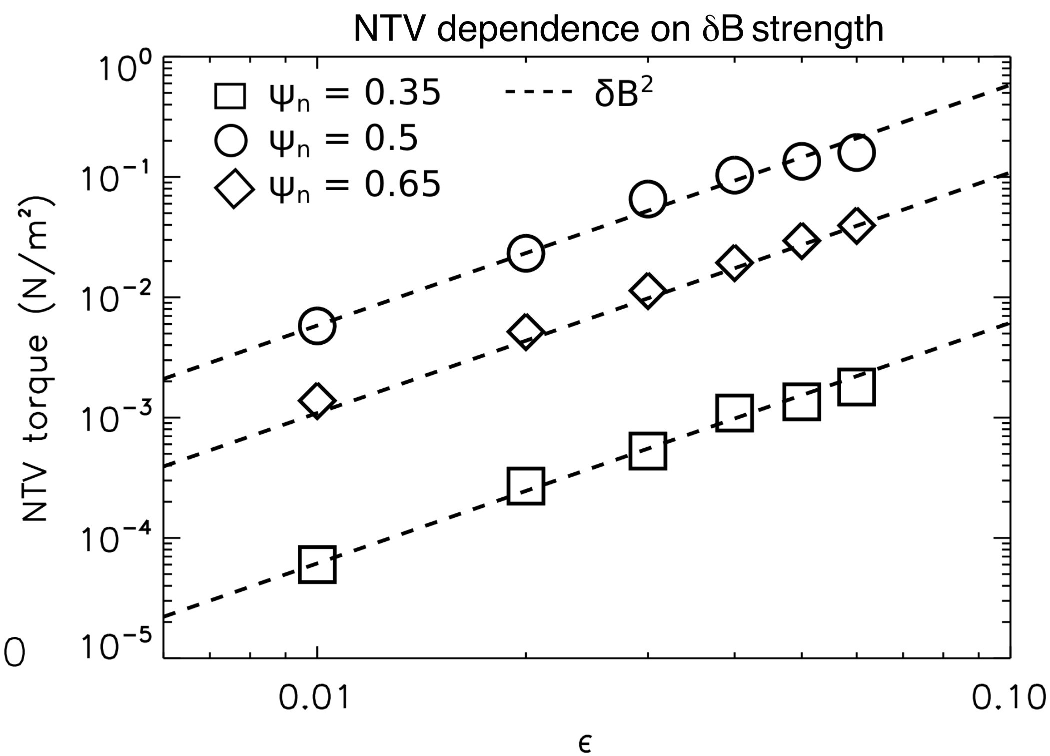
$$\frac{\delta B}{B_0} = \varepsilon \rho^2 \sum \cos(m\theta - n\phi) \quad \text{with assumed (m,n) and } \rho = \sqrt{\psi_n}$$

  - q profile is prescribed to have a single resonance at q=m/n

POCA confirms δB^2 dependence of NTV torque

- NTV from POCA indicates a theoretically predicted δB^2 dependence
  - ν_*~2.0 ($n_0=10^{19}$) and (m=7,n=3) are selected for δB scan
  - NTV is scanned by varying the strength of magnetic perturbation for resonant surface (ψ_n=0.5) and non-resonant surface (ψ_n=0.35, 0.65)
  - Clear δB^2 dependencies are found for both resonant and non-resonant flux surfaces, confirming theory prediction





[K. Kim et al., Phys. Plasmas 19, 082503 (2012)]

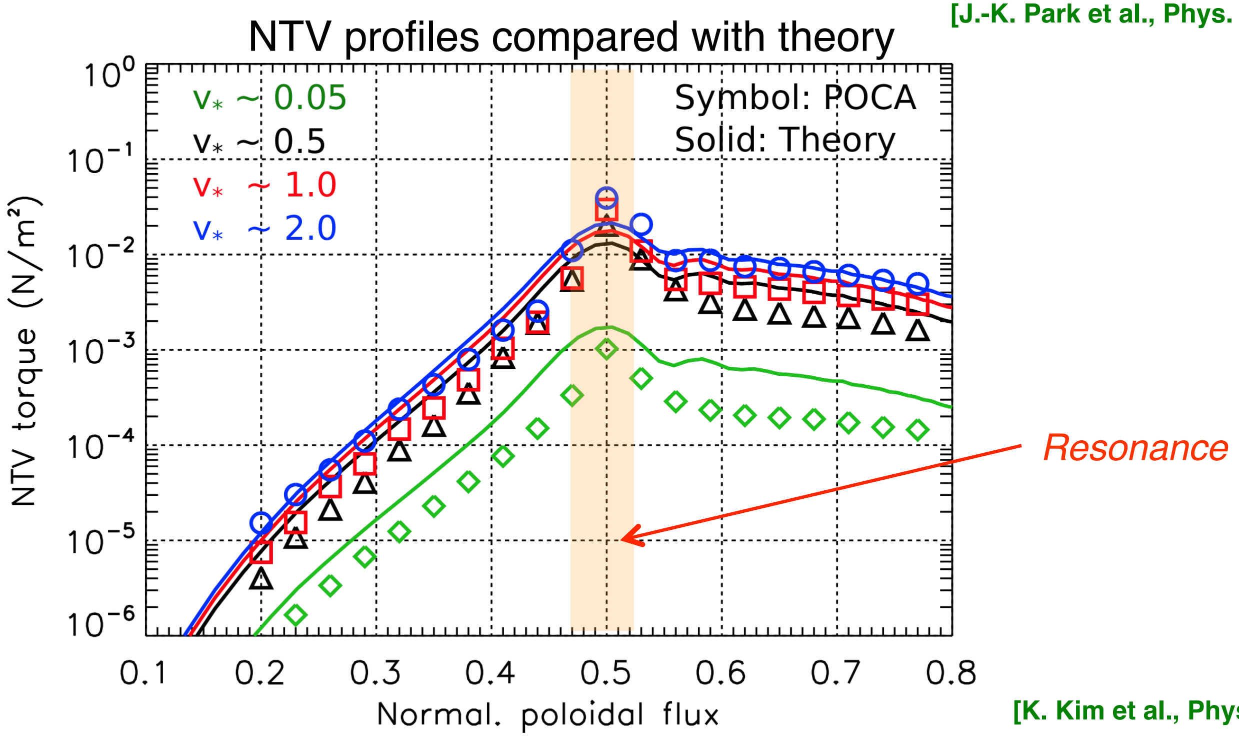
NTV torque was benchmarked with combined theory

- NTV is calculated by perturbed pressures and magnetic field spectrum
  - Analytic perturbation model is given to resonate with q=7/3 surface by

$$\frac{\delta B}{B_0} = \varepsilon \rho^2 \cos(m\theta - n\phi) \quad \text{with } \varepsilon = 0.02,\ (m,n) = (7,3)$$

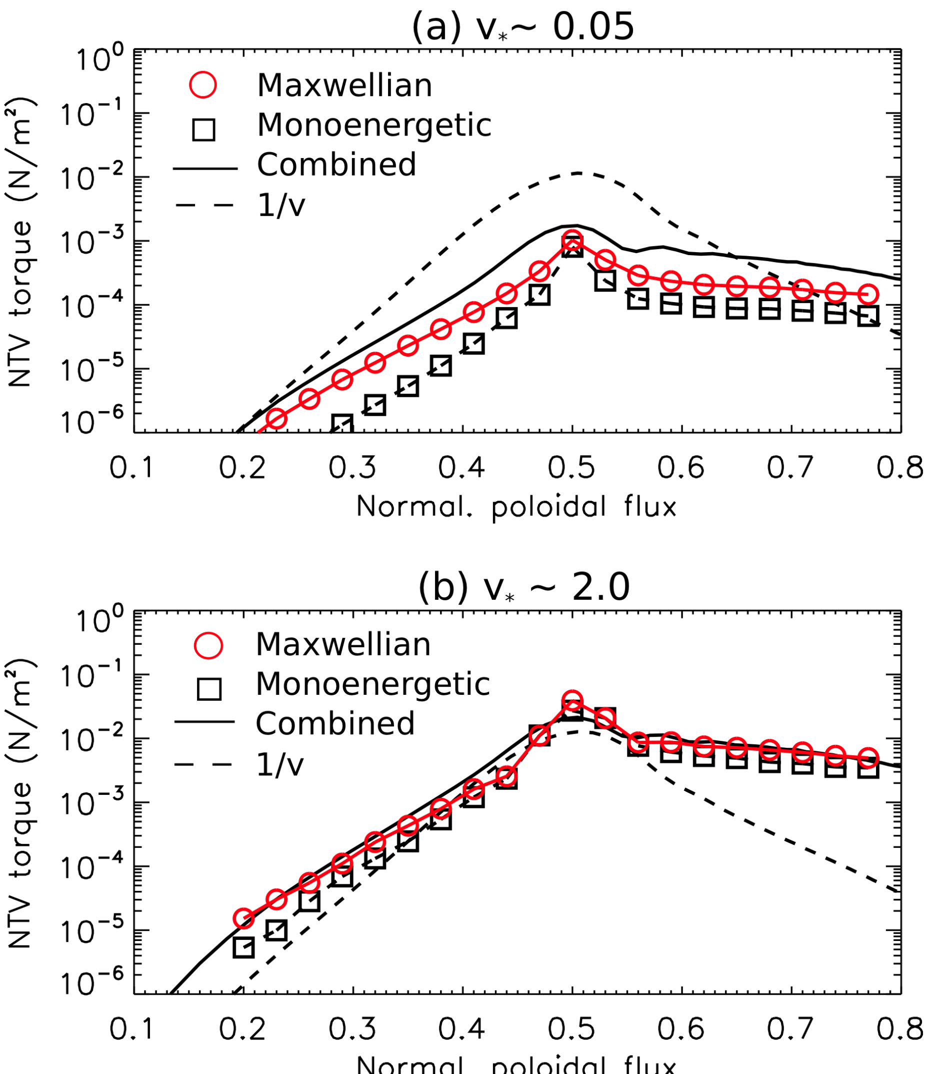
  - Calculated NTV torque shows very similar profile with theory revealing strong resonant features, but discrepancies exist depending on collisionality
  - Krook collision operator in theory may cause discrepancies in the low collisionality

[J.-K. Park et al., Phys. Rev. Lett. 102, 065002 (2009)]



[K. Kim et al., Phys. Plasmas 19, 082503 (2012)]

NTV torque was compared with 1/ν theory



- NTV approaches 1/ν regime as collisionality increases
  - 1/ν formula indicates stronger resonance but weaker non-resonance
  - Magnetic precession and regime overlapping by Maxwellian energy distribution in POCA and combined formula cause broader NTV profiles than 1/ν formula

[K.C. Shaing, Phys. Plasmas 10, 1443 (2003)]

- High energy particle impacts on NTV
  - High energy particles in the Maxwellian tails strongly impact at the non-resonant flux surfaces
  - In the high collisionality, collisions are found to become more dominant than the high energy particle effects

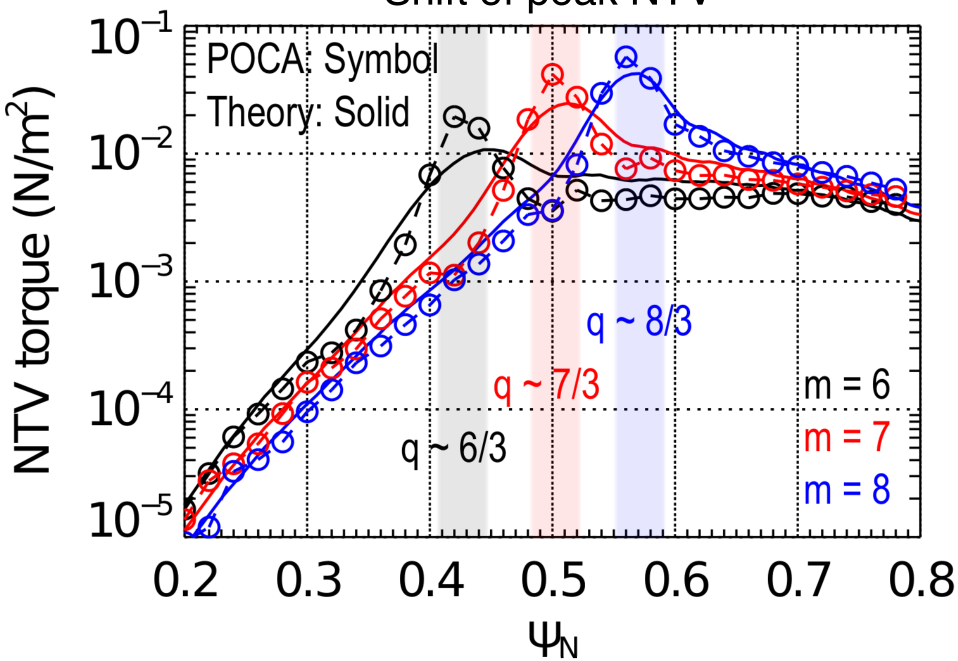
Shift of NTV peak indicates resonant nature of NTV transport

- Resonant flux surface for NTV is shifted by externally applied mode
  - Analytic δB is applied with changing poloidal mode number and fixing toroidal mode

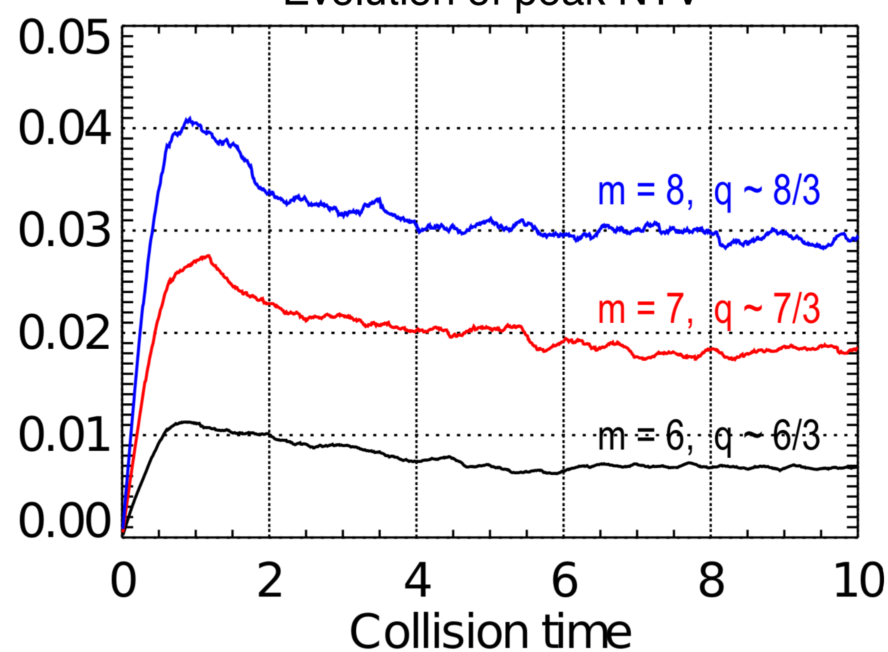
$$\frac{\delta B}{B_0} = 0.02\rho^2 \cos(m\theta - n\phi) \quad \text{with } (m = 6,7,8,\ n = 3)$$

  - NTV peak is shifted by the applied mode, and rapidly drops at the off-resonant surface; Strong resonant nature of magnetic braking
  - Simulation approaches steady states in sufficient collision times: Good Convergence

Shift of peak NTV

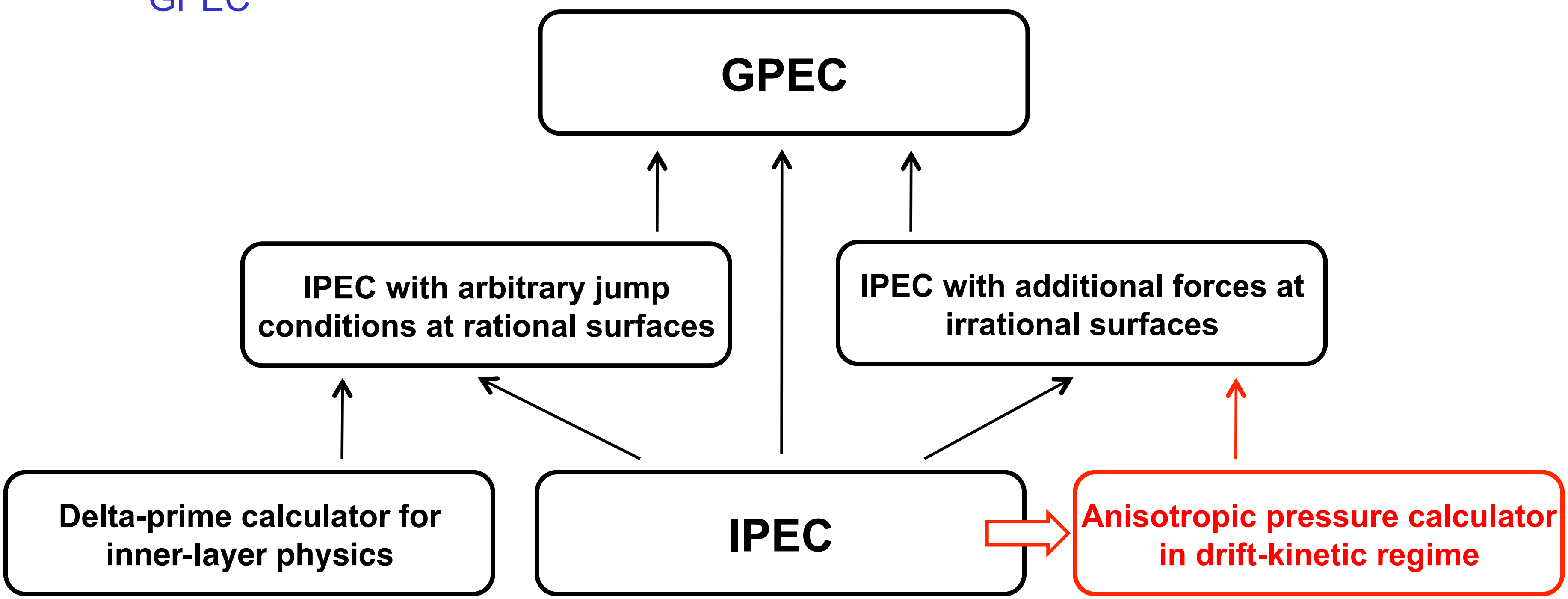


Evolution of peak NTV



General Perturbed Equilibrium Code (GPEC)

- Perturbed equilibrium codes are efficient to study 3D field physics in tokamaks with non-axisymmetric perturbations
  - IPEC solves ideal force balance with ideal constraints
  - GPEC will solve non-ideal force balance with arbitrary jump conditions, which will be matched with inner-layer solver
  - POCA will use 3D perturbations from IPEC, and provide anisotropic pressure tensor to GPEC



POCA is now applied to experimental analysis

- POCA uses 3D field spectrums calculated by IPEC

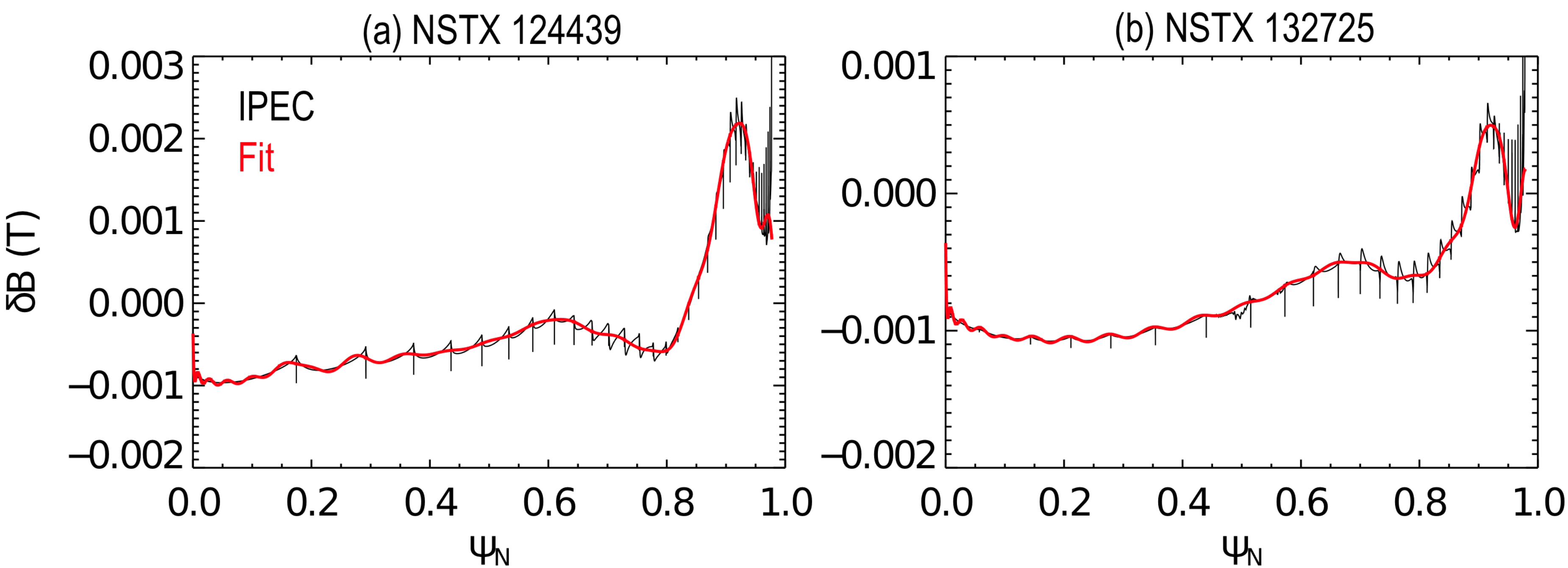
[J.-K. Park et al., Phys. Plasmas 14, 052110 (2007)]

  - Original IPEC output contains nonphysical peaks at the rational surfaces
  - Fitting technique (i.e. Chebyshev polynomials) is used as

$$\delta B_{mn}(\psi_n) = \sum_m a_{mn}(\psi_n)\cos(m\theta - n\phi) + b_{mn}(\psi_n)\sin(m\theta - n\phi) \quad \longleftarrow \quad \textit{IPEC}$$

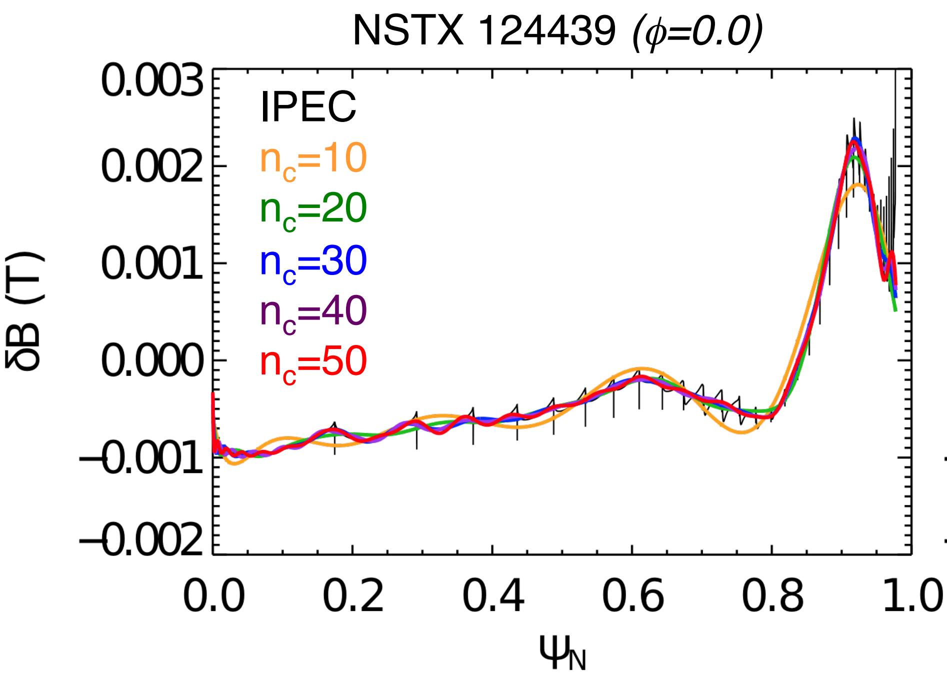
$$\delta B_{mn}(\psi_n) = \sum_m \left[\sum_j^{n_c} a_j \cos\left(j\cos^{-1}(x)\right)\cos\left(m\theta - n\phi\right) + b_j \cos\left(j\cos^{-1}(x)\right)\sin\left(m\theta - n\phi\right) \right] \quad \longrightarrow \quad \textit{POCA}$$

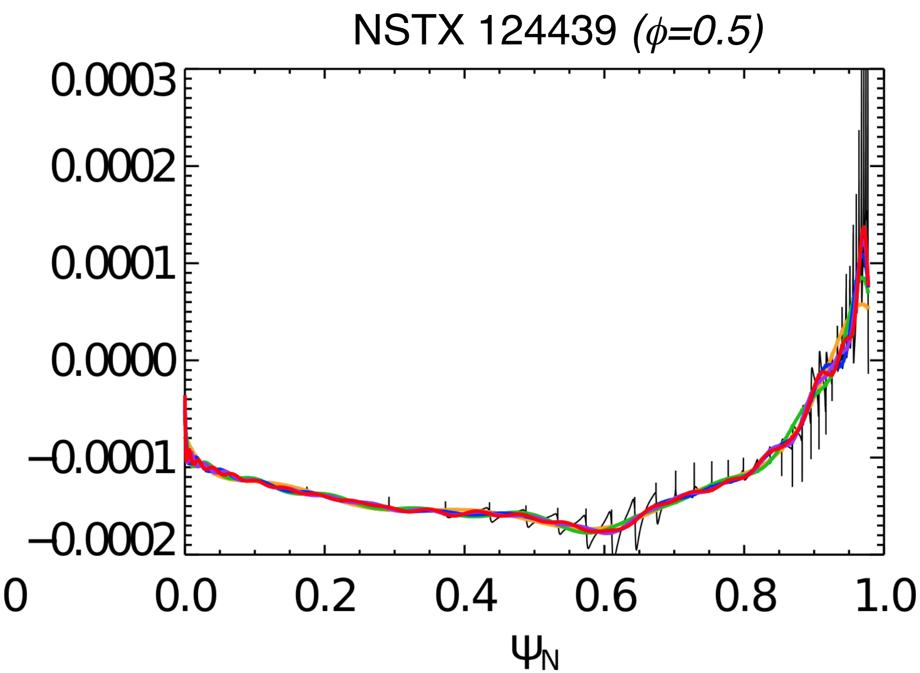
  - Fitting follows overall features of IPEC δB, and effectively smoothes the peaks



Sensitivity of Chebyshev Polynomials

- Fitting is sensitive to the degree of Chebyshev polynomials
  - Lower degree roughly follows IPEC δB with wobbling, and is poor at the edge of dense rational flux surfaces
  - High degree shows good agreement with IPEC δB, and fitting follows the rapid δB changes at the rational flux surfaces well
  - n_c>20 provides a good resolution for fitting, but this is case-dependent
  - Higher degree is more accurate but requires longer computing time

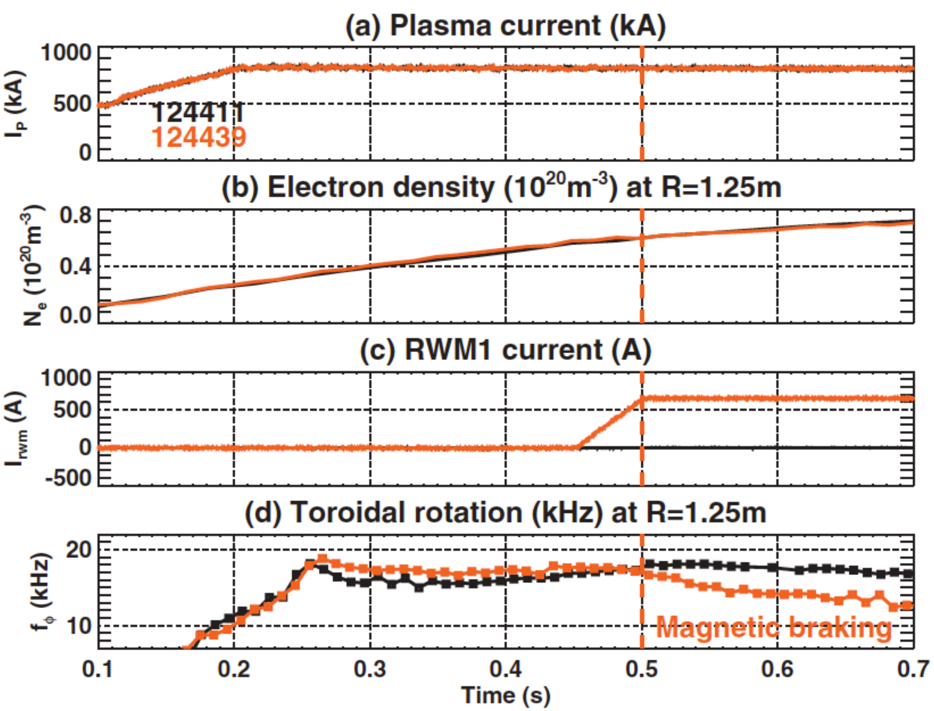


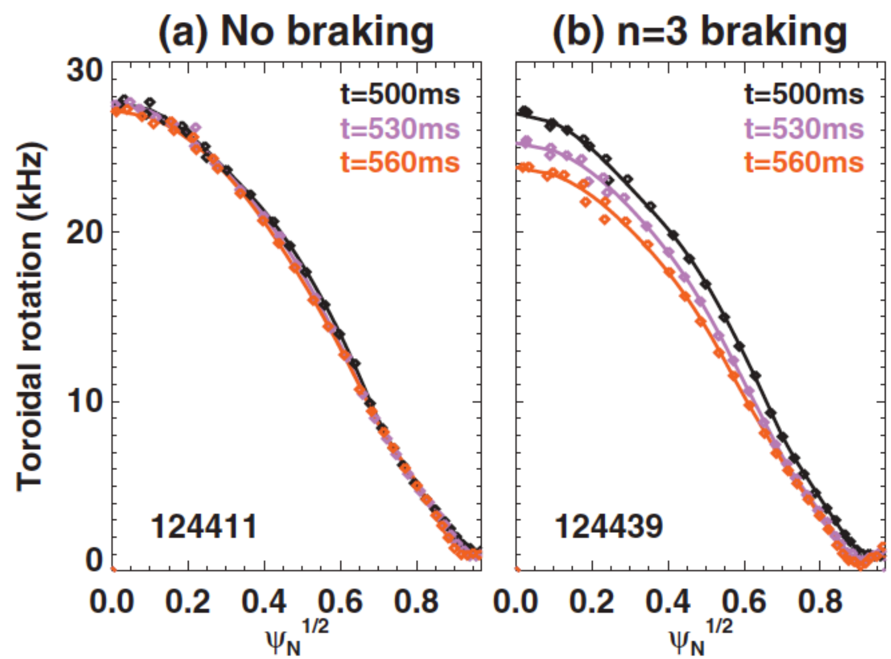


NTV can be inferred from toroidal rotation damping

- Rotational damping rate and NTV are estimated from CHERS
  - Damping rate is calculated by toroidal rotation change compared to a reference discharge without magnetic braking (κ=2.3, I_p=0.8MA, B_{T0}=0.45T, <u>*n=3*</u> for 124439)
  - NTV torque is interpreted from the damping rate by $\tau_\phi \approx \nu_{damp} u_N^\phi RMN$
  - Theoretically calculated neoclassical offset rotation is used for toroidal flow by

$$u_N^\phi = u_\phi + C_N \left| \frac{1}{eZ} \frac{dT}{d\chi} \right| \quad \text{with } C_N \approx 3.5\ (1/\nu),\ C_N \approx 0.92\ (\nu\_\sqrt{\nu}),\ C_N \approx 2.0\ (\text{combined})$$

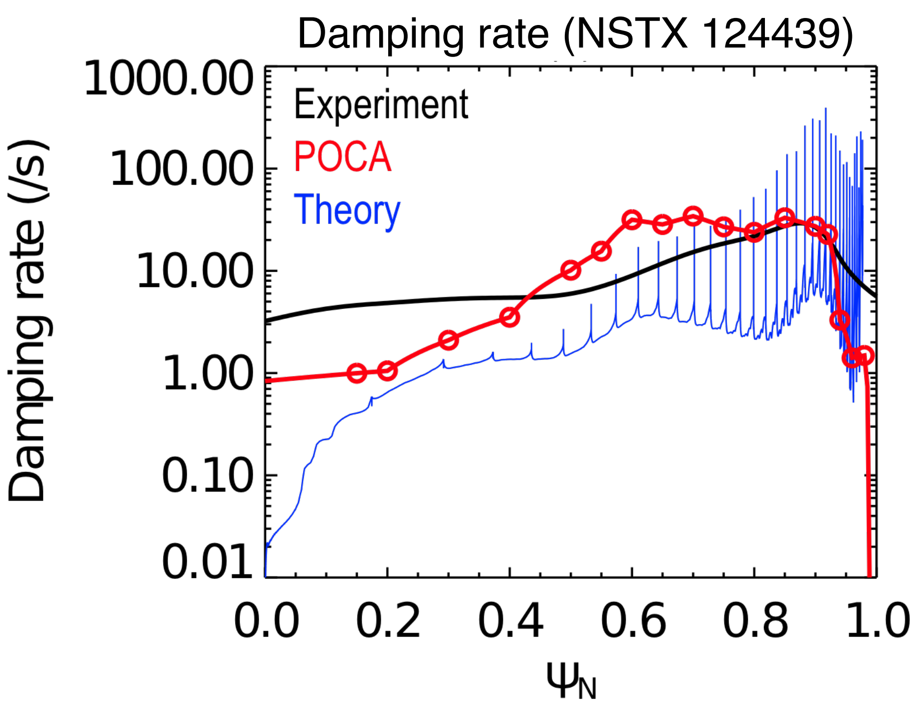


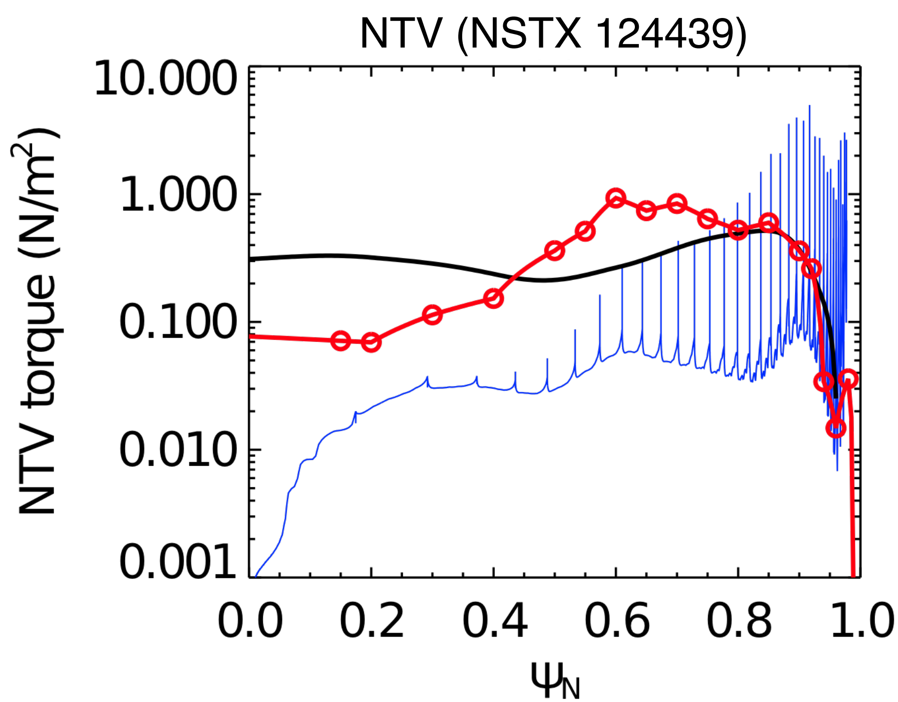


[J.-K. Park et al., Phys. Plasmas 16, 056115 (2009)]

First numerical application of POCA in NSTX indicates good agreements in NTV profile and total torque

- NTV by POCA gives a good agreement with measurement in NSTX
  - POCA calculates damping rate from NTV / Experiment gives NTV from damping rate
  - Both damping and NTV torque profiles show good agreements with measurements while POCA predicts weaker NTV at central region and stronger at outer region
  - Combined theory is valid only within an order of magnitude, which might be due to the large aspect-ratio expansion
  - Total NTV torque agrees very well; Experiment 3.5 Nm / POCA 4.5 Nm

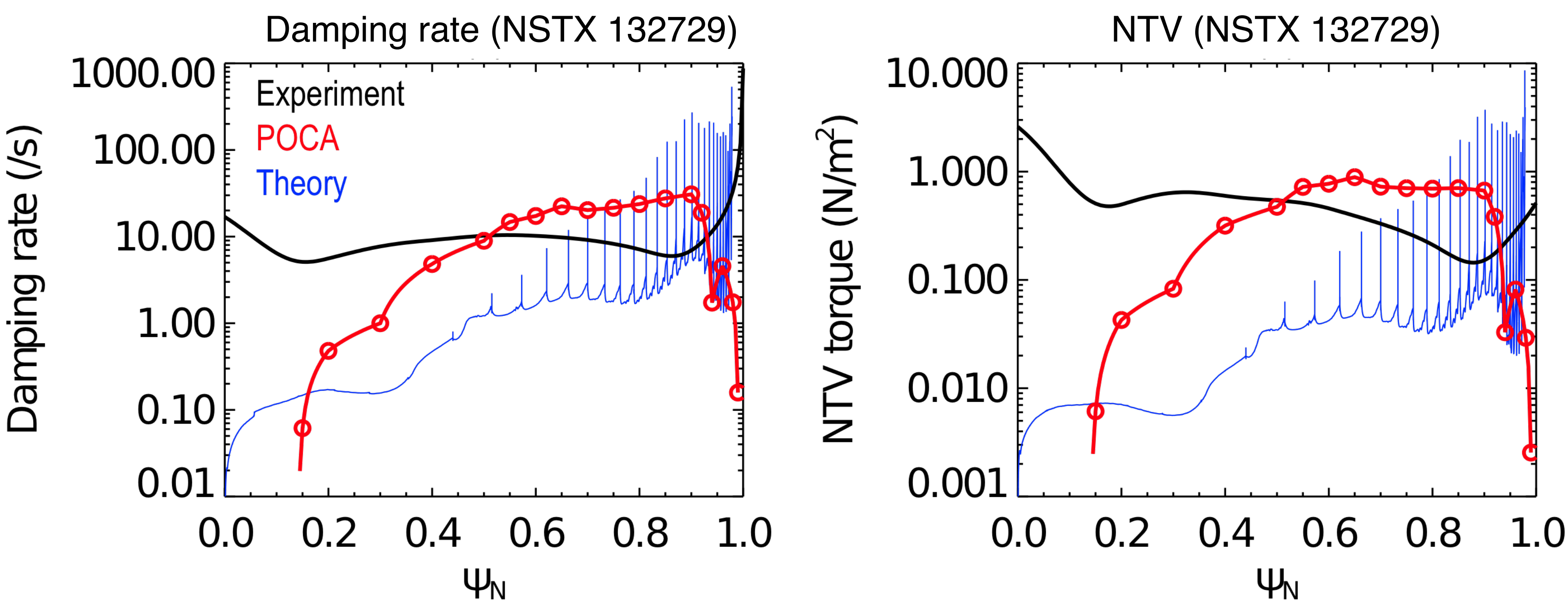




POCA provides an improved prediction for NTV in the NSTX magnetic braking experiment

- NTV is calculated for NSTX error field correction experiment
  - Selected discharge 132729 is a case of I_{EFC}=750A, which produced a strong magnetic braking (I_p=1.1MA, B_{T0}=0.55 T)
  - Discrepancies are found in damping and NTV profiles: POCA predicts weaker NTV at inner and edge region and stronger NTV elsewhere
  - Total NTV torque still agrees well; Experiment 5.1 Nm / POCA 4.66 Nm

[S.P. Gerhardt et al., Plasma Phys. Control. Fusion 52, 104003 (2010)]



Uncertainties in Measurement and Calculation of NTV

- Robust diagnostic is required for NTV analysis
  - Damping rate is inferred from CHERS assuming CHERS represents the main ion rotation: Different responding time to the non-axisymmetric perturbations
  - Neoclassical offset flow is critical but difficult to measure
    - Offset flow can be strong at the H-mode edge due to a steep temperature gradient, so it can greatly enhance the NTV value at the edge
    - Ignoring offset, measurement gives a moderated torque at the edge, and reduces total NTV: 3.5 Nm → 1.5 Nm for 124439 / 5.1 Nm → 2.35 Nm for 132729

- Ideal perturbed equilibria can fail in high β and strong NTV braking
  - Ideal perturbed equilibria neglects a shielding effect associated with toroidal toque
  - NTV effect on the perturbed equilibria cannot be ignored, since |*s*|~*0.5* > |α|~*0.2* in the NSTX discharges, where $s \equiv -\dfrac{\delta W}{\delta W_v}$ $\quad \alpha \equiv -\dfrac{T_\phi}{2N\delta W_v}$ [A.H. Boozer., Phys. Rev. Lett. 86, 5059 (2001)]
  - local NTV effect on the perturbed equilibria should be considered particularly at the edge, which is dense with the rational surfaces.
  - Self-consistent calculation of δB including non-ideal plasma response will be eventually required, and can be achieved from a general perturbed equilibrium code solving 3D force balance with the perturbed anisotropic tensor pressure

Summary

- A new δf particle code (POCA) has been developed
  - Optimized to calculate neoclassical transport in non-axisymmetric configurations
  - Benchmarked with neoclassical and NTV theories
- Strong resonant nature of magnetic braking by NTV
  - Application of single harmonic magnetic perturbation shifts the peak NTV at the resonant flux surface corresponding to the applied mode, indicating strong resonant nature of magnetic braking
- POCA was applied to NSTX magnetic braking experiments
  - IPEC provides perturbed magnetic field throughout Chebyshev polynomials
  - POCA gives good agreements on the rotation damping and NTV torque profiles
  - Excellent agreements are found for total NTV torque
- Improved measurements and self-consistent δB are necessary
  - Robust measurement of toroidal rotation and neoclassical offset is critical
  - Self-consistent δB and thereby NTV can be accomplished throughout General Perturbed Equilibrium Code (GPEC) coupled with transport

POCA demonstrates bounce harmonic resonance

- Bounce harmonic resonance can significantly enhance NTV
  - Resonant ExB creates a new type of bounce orbit defining ℓ class, which resonates with bounce frequency
  - Numerous modified orbits exist depending on energy and pitch of particle, magnetic field configuration, and ExB rotation
    - Even in small rotation, small fraction of particles can have bounce harmonic resonances
  - The modified orbit prevents phase mixing of bounce motion, thus enhances radial transport and NTV