Internal Amplitude, Structure and Identification of CAEs and GAEs in NSTX

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Abstract. Fast-ions (e.g. fusion alphas and neutral beam ions) will excite a wide range of instabilities in ITER and a Fusion Nuclear Science Facility device. Among the possible instabilities are high frequency Alfvén eigenmodes (AE) excited through Doppler-shifted cyclotron resonance with beam ions. High frequency AEs cause fast-ion transport, correlate with enhanced electron thermal transport and are postulated to contribute to ion heating. These high frequency modes have historically been identified as a mixture of compressional (CAE) and global (GAE) Alfvén eigenmodes, but distinguishing between the CAEs and GAEs has sometimes proven difficult. Identification is essential to understanding the extent of their effect, since the two types of modes have very different effects on resonant particle orbits. The effect on plasma performance of high frequency AEs is investigated in NSTX, facilitated by a recently upgraded array of 16 fixed-frequency quadrature reflectometers. Detailed measurements of high frequency AE amplitude and eigenmode structure were obtained in a high power (6 MW), beam-heated H-mode plasma that is very similar to those in which high frequency AE activity is shown to correlate with enhanced electron thermal transport. These measurements, which extend from the plasma edge to deep in the core, can be used in modeling the effects of the modes on electron thermal transport. The observed modes are identified by comparison of their frequency and measured toroidal mode numbers with local Alfvén dispersion relations. The modes identified as CAEs have higher frequencies (predominantly $f > \sim 600$ kHz) and smaller toroidal mode numbers ($|n| \leq 5$) than the GAEs (predominantly $f < \sim 600$ kHz, $n = -6 - -8$). Also, they are strongly core localized, in contrast with the GAEs, which also peak toward the plasma center but have much broader radial extent.

1. Introduction

Fast-ions (e.g. fusion alphas and neutral beam ions) will excite a wide range of instabilities in ITER and a Fusion Nuclear Science Facility device. Among the possible instabilities are high frequency Alfvén eigenmodes (AE) excited through Doppler-shifted cyclotron resonance with beam ions [1]. High frequency AEs cause fast-ion transport [2,3,4], correlate with enhanced electron thermal transport [5] and are postulated to contribute to ion heating [6]. Historically, various attempts have been made to identify these high frequency modes, leading to the conclusion that they are a mixture of compressional (CAE) and global (GAE) Alfvén eigenmodes [7,2,1,3]. However, distinguishing CAEs from GAEs has sometimes proven difficult [3]. Identification is essential to understanding the extent of their effect, since the two types of modes have very different effects on resonant particle orbits. The effect on plasma performance of high frequency AEs is investigated in NSTX. A recently upgraded array of 16 fixed-frequency quadrature reflectometers has facilitated this investigation, yielding detailed measurements of high frequency AE amplitude and eigenmode structure (see Fig. 1) in a high power (6 MW), beam-heated H-mode plasma (shot 141398) [8]. The plasma investigated is very similar to those discussed in Ref. [5], where high frequency AE activity was observed to correlate with enhanced core electron thermal transport. The structure measurements help distinguish CAEs from GAEs and can also be employed in modeling the effects on electron thermal transport. The observed modes are identified by comparison of their frequency and measured toroidal mode numbers with local Alfvén dispersion relations (see Fig. 3; also see Ref. [8]). The modes identified as CAEs are observed to possess higher frequencies and smaller toroidal mode numbers (i.e. smaller $|n|$) than the GAEs. Also, they are strongly core localized, in contrast with the GAEs, which also peak toward the plasma center but have much broader radial extent.

In the following sections, these points are discussed in more detail. Section 2 describes the results of the mode structure and amplitude measurements, and the diagnostics and analysis techniques used. Section 3 describes the analysis used to identify the modes as either CAEs or GAEs and the results of the analysis. Section 4 discusses the significance of the measurements and mode identification.

2. Mode Structure Measurements

A spectrum of global coherent high frequency modes ($f \approx 400 - 800$ kHz; $f/f_{c0} \approx 0.17 - 0.33$ where $f_{c0} = 2.4$ MHz is the beam ion cyclotron frequency at the magnetic axis) is observed in a high power (6 MW), beam-heated H-mode plasma (shot 141398) [8]. The modes are observed simultaneously using an array of fixed frequency reflectometers and a toroidally distributed array of poloidal magnetic field sensing coils at the plasma edge. Radial structure and amplitude measurements of the modes are obtained using the reflectometers (discussed in significant detail in Ref. [8]). The reflectometers operate at frequencies distributed over 30 – 75 GHz, corresponding to cutoff densities of 1.1 – 6.9 x 10^{13} cm^{-3} when operating with ordinary-mode polarization. They measure the phase shift ($\delta\phi$) of the probing millimeter-waves caused by the density fluctuations associated with the modes. For modes with large radial extent, this phase is dominated by displacement of the cutoff location caused by density fluctuations there [9], so *effective displacement*, given by $\xi = \delta\phi/2k_0$ approximates the cutoff displacement (k_0 is vacuum millimeter-wave wavenumber). Toroidal mode numbers (n) are determined using the array of edge magnetic sensing coils, which consists of twelve coils irregularly toroidally distributed with separations ranging from 10° to 180°. The minimum spacing of 10° allows mode number to be determined for modes with $|n| \leq 18$ when a single toroidal mode number is dominant at the mode frequency, as is typically the case here.

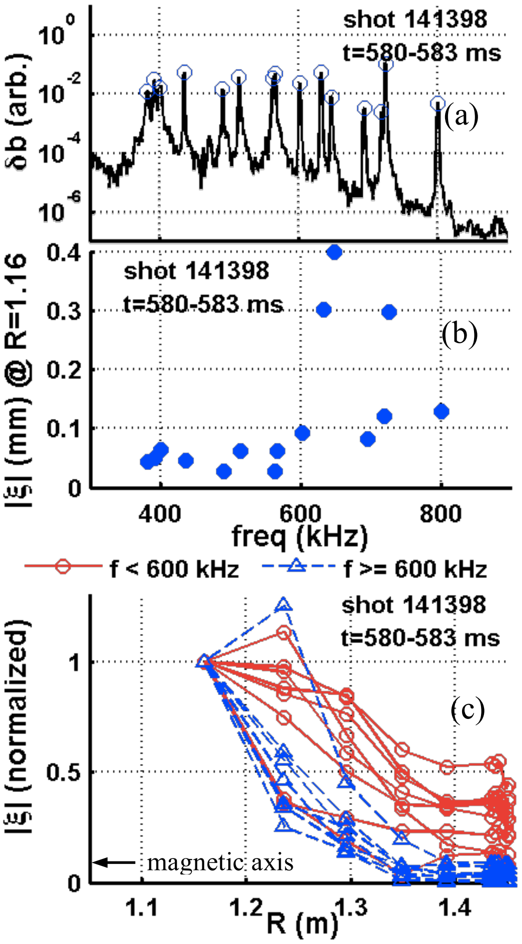

FIG. 1. (a) δb spectrum with modes marked; (b) Effective radial displacement (|ξ|) of modes at R=1.16 m; and (c) |ξ| vs. R, normalized by |ξ| at R=1.16 m.

Figure 1 shows structure and amplitude for modes with frequencies in the range $f \approx 400$ – 800 kHz. The measurements span a significant portion of the plasma minor radius in the midplane, from major radius R = 1.47 m, near the plasma edge, to R = 1.16 m. The magnetic axis is at R_0 = 1.05 m. Figure 1a shows the magnetic fluctuation spectrum (δb) at the edge of the plasma. The peaks in the spectrum corresponding to the modes are marked. Figure 1b shows $|\xi|$ vs. frequency at the deepest measurement location for the modes indicated in Fig. 1a. Figure 1c shows $|\xi|$ vs. cutoff radius for each mode. For each mode, $|\xi|$ is normalized by the value of $|\xi|$ at R =1.16 m so that the different mode structures may be easily compared.

Inspection of Fig. 1 shows that the modes can be divided into two categories based on their frequency, amplitude and structure. Figure 1c shows structures for all of the modes. As can be seen, there is a group of modes that peak sharply in the core ($R < \sim 1.3$ m), and a group of modes that have significantly more radially extended structure, peaking toward the magnetic axis, but with significant amplitude at the plasma edge. The modes that peak sharply in the core have predominantly $f > 600$ kHz, while those with broad structure have predominantly lower frequency. This is illustrated by the use of different line styles for the modes with $f > \sim$ 600 kHz (dashed lines) and those with $f <$ 600 kHz (solid lines). Figures 1a and 1b show that modes with $f > \sim$ 600 kHz also tend have larger $|\xi|$ at R =1.16 m, and smaller δb at the plasma edge, than those with $f < \sim$ 600 kHz.

The two groups of modes also have distinctly different ranges of n. Modes with $f > \sim$ 600 kHz modes have $|n| \leq 5$. In contrast, modes $f < \sim$ 600 kHz modes have $|n|$ = 6 – 8. All propagate counter to the neutral beams (i.e. $n < 0$).

3. Identification of Modes

The modes shown in Fig. 1 are identified using f and measured n. Modes in NSTX with n and f in the observed range, where f is a significant fraction of the beam ion cyclotron frequency, and the observed propagation is counter to the neutral beams, have historically been identified as a mixture of CAEs and GAEs excited by Doppler-shifted cyclotron resonance with beam ions [7,2,1,3]. However, distinguishing GAEs from CAEs has sometimes proven difficult [3]. In order to facilitate making this distinction, an analysis is presented that utilizes f and measured n in conjunction with local Alfvén dispersion relations (see also [8]). Using this analysis, the group of strongly core-peaking modes with predominantly $f > \sim$ 600 kHz are clearly identified as CAEs, while the group of broad structure modes with predominantly $f < \sim 600$ kHz are identified as GAEs.

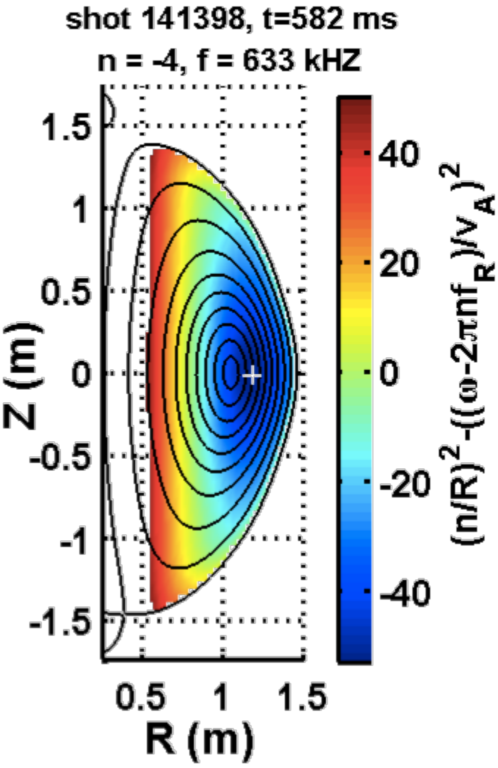

FIG. 2. CAE "well" for n = –4, f = 633 kHz for shot 141398, t = 582 ms. Deepest point of well marked by "+".

The local dispersion relationship for compressional Alfvén waves, $\omega^2 \sim k^2 V_A^2$, can be used to establish minimum requirements on ω and n for a CAE to exist. For $n \neq 0$, CAEs should be localized within a "well" in the R-Z plane where $(n^2/R^2)V_A^2 - \omega^2 < 0$, inside of which compressional Alfvén waves can propagate but outside of which they are evanescent. This requirement applies in the plasma frame, so in the presence of a Doppler shift due to toroidal rotation the requirement becomes $(n^2/R^2)V_A^2 - (\omega - 2\pi f_{ROT})^2 < 0$. Figure 2 illustrates the well for the mode with f = 633 kHz, n = –4 at t = 582 ms in shot 141398. Of course, for an eigenmode to fit inside the well, the R-Z "wavelength" inside the well, $\lambda_{R\text{-}Z} = [(\omega - 2\pi f_{ROT})^2/V_A^2 - n^2/R_0^2]^{-1/2}/2\pi$, should be comparable to or smaller than the well. The modes are compared with these expectations using equilibrium electron density from Multipoint Thomson Scattering [10] and magnetic field from LRDFIT equilibrium reconstruction [11,12,13] using Motional Stark Effect [14] measurements of magnetic field pitch. The toroidal rotation Doppler shift is compensated using Charge Exchange Recombination Spectroscopy [15]. Calculating $\lambda_{R\text{-}Z}$ with the most negative value of $(n^2/R^2)V_A^2 - (\omega - 2\pi f_{ROT})^2$ for each mode leads to the conclusion that the $f > \sim 600$ kHz modes easily meet this requirement because of their smaller $|n|$. In contrast, for some of the $f < \sim 600$ kHz modes, $(n^2/R^2)V_A^2 - (\omega - 2\pi f_{ROT})^2 > 0$ everywhere, while for the others, $\lambda_{R\text{-}Z}$ significantly larger than the width (in the major radial direction) of the region where $(n^2/R^2)V_A^2 - (\omega - 2\pi f_{ROT})^2 < 0$.

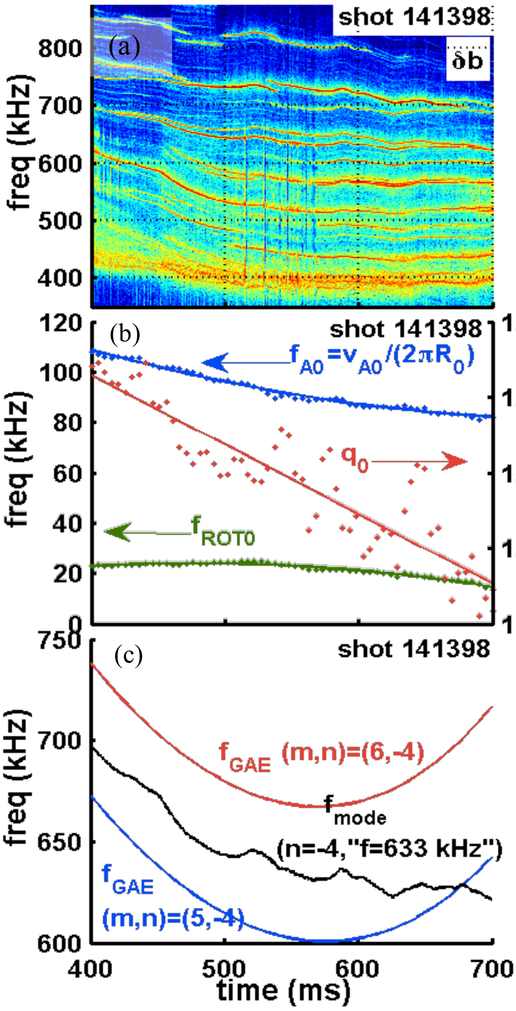

FIG. 3. (a) δb spectrum with CAEs and GAEs; (b) Alfvén frequency (f_{A0}), toroidal rotation (f_{ROT0}) and safety factor (q_0) at R_0; (c) frequency of "f = 633 kHz" mode (cf. Fig. 1) compared with GAE frequency (f_{GAE}).

Comparison of long-term frequency evolution (Fig. 3a) with expectation for GAEs (discussed in more detail in Ref. [8]) also proves informative. GAEs peak near a minimum of the Alfvén continuum (e.g. at R_0) and have $\omega \approx |k_{\|}|V_A$ (in the plasma frame), where $k_{\|} \approx (m/q - n)/R$. The modes in Fig. 1 peak near R_0, so the GAE frequency (f_{GAE}) is calculated using safety factor (q_0), Alfvén frequency ($f_{A0} = V_{A0}/2\pi R_0$) and toroidal rotation (f_{ROT0}) (Fig. 3b) at R_0. Figure 3c illustrates the analysis for the f = 633 kHz mode, showing that the mode frequency appears to evolve inconsistently with f_{GAE}. This is largely due to a high sensitivity to q_0 variation. The small $|n|$ mandates a large poloidal mode number, m, to fit the observed frequency. The frequencies of the other $f > \sim 600$ kHz modes also evolve inconsistently with f_{GAE}. In contrast, the lower frequency modes have frequency evolution consistent with f_{GAE}. They tend to have large $|n|$, leading to small m, so f_{GAE} is relatively insensitive to q_0 variation.

4. Discussion

The mode structure measurements presented here were obtained in plasmas very similar to those discussed by Stutman *et al.* in Ref. [5], in which high frequency AE activity is shown to correlate with enhanced core electron thermal transport. Stutman *et al.* [5] posited that the high frequency AEs caused the transport by stochasticization of resonant electron orbits in the presence of multiple modes (see also, [16]). They noted that the modes may "resonantly couple to the bulk (~1 keV energy), primarily trapped electron population", because they have frequencies comparable to the characteristic trapped electron bounce frequency of 0.6 MHz.

The structure measurements presented here show that the high frequency AEs tend to peak in the plasma core. This is consistent with a key element supporting the hypothesis of Stutman *et al.* [5], which was their expectation that the modes peaked in the plasma core where the enhanced transport occurred. Although structure measurements were not available, they identified the modes as GAEs—which are theoretically expected to peak in the plasma core—based on their frequencies and on signatures in the time-dependent mode spectrum.

In contrast with Stutman *et al.* [5], however, analysis presented here indicates that some of the modes are, in fact, CAEs instead of GAEs. The distinction is significant for several reasons. First, the two types of modes have very different effects on resonant particle orbits. For instance, in contrast with GAEs, CAEs localized in the core of the plasma will have a significant parallel magnetic field fluctuation in the core that will perturb the pitch angle of resonant electrons. Fully understanding the implications of this and other differences in orbital effects will require future investigation.

A second motivation for distinguishing CAEs from GAEs stems from the potential use of measured mode structure measurements to predict the resulting electron thermal transport. Gorlenkov, *et al.* showed in Ref. [16] how the magnetic perturbation structures of a spectrum of high frequency AEs could be used as input to the guiding center orbit code ORBIT to make such a prediction. In order to test the hypothesis of Stutman *et al.* [5], they calculated the transport that would result from a spectrum of GAEs. In performing the calculation, they used ad-hoc GAE structures motivated by phenomenological considerations as inputs for ORBIT. In principle, however, the mode-structure measurements presented here could be used to derive a more physically accurate set of magnetic perturbation structures, specific to the plasma under investigation, as inputs for ORBIT. It is here that the distinction between GAEs and CAEs become significant. The structure measurements are sensitive to density perturbations. The plasma flows that cause the density perturbations are primarily due to ExB motion, so the measurements can be used to determine the electric, and associated magnetic, perturbation of the modes. (Displacement parallel to the equilibrium magnetic field, which is largely due to the acoustic response of the plasma, is expected to be negligible at the high frequencies of the observed modes.) However, the relationship between the density and flow perturbations is different for CAEs and GAEs. For AEs with primarily shear Alfvén polarization, such as GAEs, the density perturbation results from radial displacement of the equilibrium density gradient in the region where the mode is localized. On the other hand, for AEs with primarily compressional Alfvén polarization, such as CAEs, compression also contributes significantly to the density perturbation in the region where the mode is localized. Consequently, interpretation of the measured structures to obtain inputs for ORBIT benefits from the determination of mode polarization that results from distinguishing CAEs from GAEs. Extending the analysis of Gorelenkov *et al.* [16] to incorporate the mode structures presented here is left as an exercise for future investigation.

A significant result of the transport modeling of Gorelenkov *et al.* [16] was a prediction of the total GAE fluctuation level necessary to cause the level of electron thermal transport observed by Stutman *et al.* [5]. Although the transport modeling assumed a spectrum of GAEs with ad-hoc structures differing significantly from the measured structures, it is informative to compare the measured mode amplitudes with this prediction. The predicted fluctuation level is expressed in terms of the parameter $\alpha = \delta A_{\|}/B_0 R_0$, where $\delta A_{\|}$ is the parallel component of the perturbed vector potential. The fluctuation level required to explain the observed electron thermal transport is $\alpha = 4 \times 10^{-4}$. For the modes identified here as GAEs, the total value of α at R = 1.16 m can be estimated by summing over values estimated for each of the individual modes. For GAEs, α is related to the radial plasma displacement, ξ_r, by the expression $\xi_r = \delta B_r / i k_{||} B_0 = \alpha R_0 k_\theta / k_{||}$ [16]. The measured effective displacement at R = 1.16 m (Fig. 1b) is used as an estimate of ξ_r. Values for $k_{||}$ are estimated from the GAE dispersion relation, $\omega \approx |k_{||}| V_A$, using measured V_A at R_0 (Fig. 3b) and correcting for a toroidal Doppler shift using the measured n and f_{ROT0} (Fig. 3b). Values for $k_\theta = m/r$ are determined by estimating effective values of m at R_0 via the relation $k_{||} \approx (m/q - n)/R$, using the measured values of n and q_0 (Fig. 3b). The resulting values have magnitudes in the range $|m| \approx 1/2 - 2\ 1/2$. The value for r is taken to be $r = (1.16\ \text{m} - R_0) = 0.11$ m. The result of this analysis gives a total GAE fluctuation level of $\alpha = 3.4 \times 10^{-4}$ at R = 1.16 m, which is comparable to the predicted value required to explain the observed electron thermal transport. There are, of course, significant approximations used in this analysis, so a more careful analysis is warranted as a part of future investigation. Also, an approach must be developed for predicting the contribution of the CAEs to the transport.

Another significant result of the transport modeling of Gorelenkov *et al.* [16] was a prediction that the level of transport caused the by the spectrum of modes was extremely sensitive to the number of modes. The modeling showed a sharp increase in the level of transport when the number of modes reached a threshold of ~ 15, which is equal to the total number of modes observed in the range $f \sim 400 - 800$ kHz. Of course, some of those modes are identified to be CAEs, while the modeling considered only GAEs. Further work is necessary to evaluate the mode number threshold for a spectrum including both CAEs and GAEs.

Another important area for future investigation will be to compare the mode structure measurements presented here with predictions from theory. E. Belova *et al.* [17] are currently engaged in theoretical modeling of the CAE/GAE spectrum expected for the plasma investigated here using the hybrid kinetic and MHD code, HYM. Modes from the simulations that have been validated by measurements can then be incorporated into the type of analysis used by Gorelenkov *et al.* [16] to further refine the prediction of electron thermal transport.

In conclusion, a spectrum of high frequency AEs is observed in a high power, beam-heated H-mode NSTX plasma [8] similar to those in which high frequency AE activity is shown by Stutman *et al.* [5] to correlate with enhanced core electron thermal transport. An analysis is presented using local dispersion relations in combination with mode frequency and measured toroidal mode number to show that some of the modes are GAEs and, in contrast with the identification of Stutman *et al.*, some of the modes are CAEs. Stutman *et al.* posited that the high frequency AEs caused the transport by stochasticization of resonant electron orbits in the presence of multiple modes. The observed spectrum is shown to be consistent with an analysis by Gorelenkov *et al.* [16] that predicts the number of modes and total fluctuation level required to cause the thermal transport observed in the plasmas investigated by Stutman *et al.* However, the analysis of Gorlenkov *et al.*, which was based on electron orbit modeling, assumed a spectrum of GAEs with phenomenologically motivated ad-hoc structures as inputs. GAEs and CAEs can have very different effects on resonant electron orbits, so further investigation is warranted extending the analysis to incorporate a mixed spectrum of CAEs and GAEs. Also, the measured structures reported here can, in principle, be used to provide a more physically accurate set of magnetic perturbation inputs to the orbit modeling.

5. Acknowledgements

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