Study of neoclassical toroidal viscosity in tokamaks with a δf particle code and resonant nature of magnetic braking

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Abstract:

This paper reports a study of neoclassical toroidal viscosity (NTV) with a new δf particle code (POCA) and the improved understanding of magnetic braking in perturbed tokamaks. Calculation of accurate δf and δB is essential to improve prediction of the non-ambipolar transport and NTV. POCA calculates the NTV by computing δf from guiding-center particle motion with δB from analytic model and IPEC. Application of an analytic single harmonic perturbation shows a shift of peak NTV responding to the applied mode, which clearly indicates the strong resonant nature of NTV. For experimental application of POCA, a fitting technique is developed to smooth nonphysical peaks of δB by IPEC. NTV torque profiles in the NSTX magnetic braking experiments are inferred from toroidal rotation damping rate measured by CHERS. POCA calculations show good agreements with the observations for NTV torques and provides an improved prediction. It is implied that calculation of self-consistent δB by a general perturbed equilibria is necessary since non-ideal plasma response can change the penetration of the perturbed field throughout the plasma and impact the NTV torque.

1 Introduction

Symmetry braking in perturbed tokamaks can fundamentally change neoclassical transport in tokamaks [1]. Magnetic perturbations, induced by intrinsic error fields, MHD activities, and externally applied non-axisymmetric fields, distort particle orbits on deformed or broken flux surfaces, and modify the neoclassical transport of particles. The transport by the symmetry breaking, often called neoclassical toroidal viscosity (NTV) transport in tokamaks, is one of the essential effects of the non-axisymmetric magnetic perturbations. The NTV transport provides an additional channel for toroidal momentum transport in tokamaks, and modifies toroidal plasma rotation in experiments, which is called magnetic braking.

In order to achieve a precise and self-consistent descriptions for NTV transport, two physics components should be combined; δf the perturbed distribution function and δB the non-axisymmetric magnetic field perturbation. Substantial progresses by various analytical attempts have been made for calculating δf and NTV [2, 3], but they are largely limited by heavy approximations. Analytic theories generally calculate δf based on the large aspect-ratio expansion, trapped particle effects, and a regime separation by collisionality with a strongly simplified δB.

Therefore, a numerical approach is eventually required for NTV calculation in the practical experiments, which should make use of the self-consistent δf and δB models. This paper reports the study of NTV with a new δf guiding-center particle code, POCA (Particle Orbit Code for Anisotropic pressures) [4]. POCA calculates δf by guiding-center particle motions in the realistic geometry with δB provided by Ideal Perturbed Equilibrium Code (IPEC) considering an ideal plasma response. In this paper, a resonant nature of magnetic braking driven by NTV is indicated, and applications of POCA to NSTX magnetic braking experiments are described.

2 Particle Orbit Code for Anisotropic pressures

POCA solves the Fokker-Planck equation with a δf Monte Carlo method [5]. The Fokker-Planck equation is written as

$$\frac{d \ln f_M}{dt} + \frac{d\hat{f}}{dt} = C_m, \qquad (1)$$

where the distribution function f is approximated to $f \approx f_M(1+\hat{f})$ with the local Maxwellian f_M in the fusion plasmas, and $C_m = C/f$ with the collision operator C. Then, $\hat{f}$ is obtained by following equation,

$$\Delta \hat{f} = -\Delta\psi \frac{\partial \ln f_m}{\partial \psi} + 2\nu \frac{u}{v} \lambda \Delta t - \frac{e}{T}\frac{d\Phi}{d\psi}, \qquad (2)$$

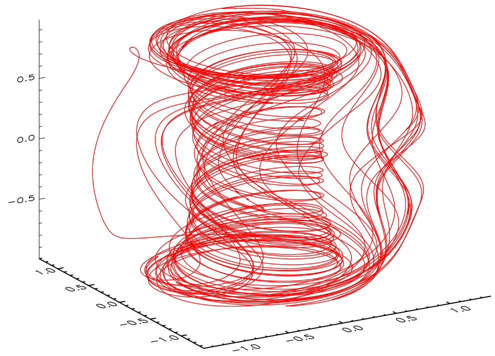

FIG. 1: 3D Trajectory of a single guiding-center particle motion in a NSTX plasma, calculated by POCA

where ν is the collision frequency, u is the parallel flow velocity, λ is the pitch angle, and Φ is the electric potential. The first term in the right hand side of Eq. (2) represents δf driven by particle drift motions, and the second term is the momentum correction term to preserve the toroidal momentum conservation. The last term represents the electric potential effect, which is closely related to the toroidal rotation.

The guiding-center orbit motion of a test particle is tracked by the Hamiltonian equations of motion on the Boozer coordinates. Thus, all kinds of guiding-center motion are producible with POCA [4] depending on the particle's energy and pitch, magnetic field, electric potential, and their combinations. One example of the guiding-center drift motion is presented in Fig. 1, which shows 3D trajectory of the guiding-center motion of a single

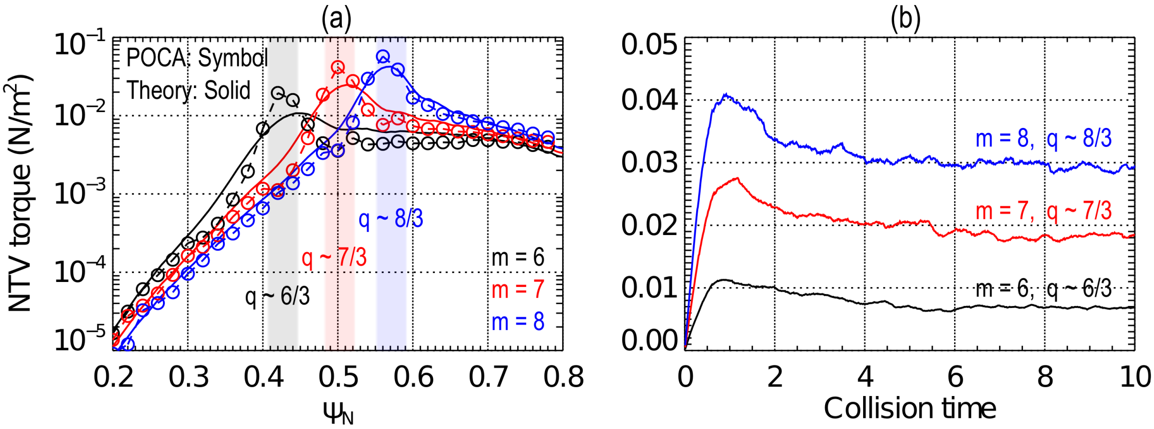

FIG. 2: (a) NTV torque profiles by $(m = 6, 7, 8, n = 3)$ modes. Strong NTV peaks appear at the resonant flux surfaces where $q = 6/3, 7/3, 8/3$ for each poloidal mode. (b) Time evolutions of the peak NTV torques. The calculated torques approach quasi-steady states in sufficient collision times.

particle in a NSTX plasma. As shown, the guiding-center motion reflects a typical feature of the magnetic field line structure in NSTX.

NTV torque can be estimated by calculating perturbed pressures and utilizing magnetic field spectrum [6, 7]. In the Boozer coordinates, the NTV is calculated by $\tau_\phi \equiv \langle \mathbf{e}_\phi \cdot \nabla \cdot \mathbf{P} \rangle = \left\langle \frac{\delta P}{B} \frac{\partial B}{\partial \phi} \right\rangle$, where δP is the perturbed pressures defined by $\delta P = \int d^3v (mv_\parallel^2 + 1/2mv_\perp^2)\delta f$. The brackets denote the flux surface average. POCA has been successfully benchmarked with neoclassical transport theories for diffusion, bootstrap current, and momentum conservation, and with the combined NTV theory [8] for the NTV torque in the various collisionalities within the high aspect-ratio and low β limit [4].

A well-known supposition in the magnetic braking is that it is mainly driven by non-resonant parts of the magnetic perturbations. However, we show a clear resonant nature of the NTV by applying an analytic perturbed field containing a single resonant mode. A single harmonic magnetic perturbation, expressed as $\delta B/B_0 = \delta_{mn}(\psi)\cos(m\theta - n\phi)$ with $\delta_{mn} = 0.02\psi_n^2$, is applied to a background plasma of $\nu_* \sim 1.0$ in Ref. [4]. Fixing toroidal mode as $n = 3$, poloidal mode number is varied as $m = 6, 7, 8$ to change the resonant mode. Here, $\vec{E} \times \vec{B}$ rotation is set to be zero so that the toroidal rotation effect such as bounce harmonic resonance is excluded in this study.

As shown in Fig. 2(a), strong NTV peaks appear nearby the resonant flux surfaces at $q = 6/3, 7/3, 8/3$ corresponding to the applied poloidal mode number. The resonant flux surface is shifted by the applied mode, indicating a clear resonant nature of the magnetic braking driven by NTV. On the other hand, NTVs rapidly drop at the off-resonant flux surfaces. Much stronger torques are found at the edge rather than the core in spite of off-resonance, which is consistent with the theoretical prediction $\tau_\phi \propto (\delta B)^2$. Overall trend of NTV profiles by POCA shows an excellent agreement with the combined theory in the high aspect-ratio. The amplitudes also agree well within a factor of 2, indicating a validity of δf by POCA. Fig. 2(b) presents the time evolutions of the peak NTV torques in Fig.

2(a). One can notice that the calculated NTVs reach quasi-steady states in sufficient collision times, which reveals a good convergence feature of POCA simulation. The NTV values obtained from the simulation are the long-time averages in the quasi-steady phase.

3 Reconstruction of perturbed magnetic field

As previously described, an accurate calculation of the perturbed magnetic field δB is essentially required for a precise estimation of the non-ambipolar transport and NTV torque. In a practical experiment, δB is expressed as multi-harmonic Fourier series, and is much more complicated than an analytic expression. In addition, δB can largely differs from a vacuum perturbed field δB_v, which is generally calculated by a vacuum field approximation, due to a plasma response. As a resolution, we use IPEC, a well-known ideal perturbed equilibrium code capable of computing δB with an ideal plasma response [9]. POCA is developed to read axisymmetric equilibrium from 20 different equilibrium types for experiment and analytic solution, and non-axisymmetric perturbation from IPEC.

IPEC calculates the perturbed magnetic field spectrum as

$$\delta B_{mn}(\psi_n) = \sum_m a_{mn} \cos(m\theta - n\phi) + b_{mn} \sin(m\theta - n\phi), \tag{3}$$

where a_{mn} and b_{mn} are the coefficients of Fourier harmonics for each mode. However, a direct application of IPEC δB is not efficient in the particle simulation since δB in Eq. (3) contains strong nonphysical peaks at the rational flux surfaces, as will be shown in Fig. 3. The nonphysical peaks are due to the ideal plasma response excluding islands, and can impact the drift motion of the particles crossing the rational flux surfaces. Thus, we develop a fitting technique using Chebyshev polynomials to smooth the peaks. The fitting function of the IPEC δB is given as

$$\delta B_{mn}(\psi_n) = \sum_m \sum_j a_j \cos[j\cos^{-1}(x)] \cos(m\theta - n\phi) + b_j \cos[j\cos^{-1}(x)] \sin(m\theta - n\phi), \tag{4}$$

where a_j and b_j is the Chebyshev coefficients for each degree j, and $x = 2\psi_n - 1$.

Fig. 3 shows comparisons of δB profiles between IPEC and fitting in the NSTX discharges. The radial profiles are taken at $(\theta, \phi) = (0, 0)$. One can find that the fitted profiles reflect overall trend of complex field structure by IPEC very well, even though the nonphysical peaks at the rational flux surfaces are efficiently smoothed. The fitted δB is practically supplied to the POCA simulation for an experimental NTV analysis.

4 Application of POCA to NSTX experiments

Two NSTX discharges, 124439 and 132729 are simulated with POCA. Axisymmetric equilibriums are reconstructed using LRDFIT, and the electric potential profiles are obtained from a radial force balance equation $\nabla p = eN(\vec{E} + \vec{u} \times \vec{B})$. The toroidal rotation speed u_ϕ is measured by charge exchange recombination spectroscopy (CHERS) based on carbon

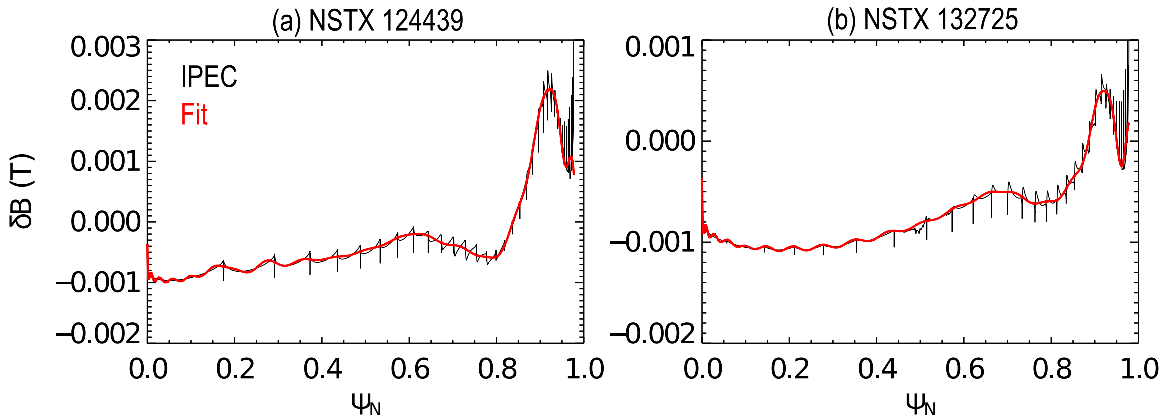

FIG. 3: Comparison of non-axisymmetric magnetic field profiles between IPEC and fitting with Chebyshev polynomials for NSTX discharge (a) 124439 *and (b)* 132725*. Profiles are taken at* $(\theta, \phi) = (0, 0)$*. Fittings efficiently smooth the nonphysical peaks of* δB *but reflect overall features of the original IPEC calculation.*

impurities, assuming the CHERS represents the main ion rotation. The poloidal rotation is neglected here due to fast toroidal rotation in the selected discharges.

Practically, NTV torque can be inferred from a toroidal flow damping rate and/or a toroidal angular momentum change by the magnetic braking. For the selected discharges, it is convenient to estimate the damping rate using reference shots with the almost identical plasmas but without magnetic braking. In the discharge 124439, where $\kappa = 2.3$, $I_p = 0.8MA$, and $B_{T0} = 0.45$ T in lower single null configuration, the toroidal rotation was observed to damp and relax to a different rotational equilibrium after $n = 3$ magnetic braking was fully applied by EF/RWM coils with $600A$ for each. Comparing with the reference shot, the particle transport was not changed even though the non-axisymmetric fields produce a strong momentum transport. It is clearly indicated that the damping is purely driven by the $n = 3$ magnetic braking [10].

We estimate the rotational damping rate ν_{damp} from the measurement of the toroidal rotation changes between the reference and the selected discharge. A short time period after turning on EF/RWM coils is considered so that an exponential decay of the rotation can be linearized [10]. Then, the NTV is calculated from the damping rate using a following relation

$$\tau_\phi \approx \nu_{damp} u_N^\phi RMN, \tag{5}$$

where M is the mass of a species, N is the density, R is the major radius, and u_N^ϕ is the neoclassical toroidal flow. The neoclassical toroidal flow is defined by

$$u_N^\phi \equiv u_\phi + C_N \left| \frac{1}{eZ} \frac{dT}{d\chi} \right|, \tag{6}$$

where χ is the poloidal flux function. The second term of Eq. (6) represents a neoclassical offset flow with $C_N \approx 3.5$ for $1/\nu$ regime, $C_N \approx 0.92$ for $\nu_{-}\sqrt{\nu}$ regime, and $C_N \approx 2.0$ for combined regime. The offset flow has not been measured in NSTX, so it is calculated by theory.

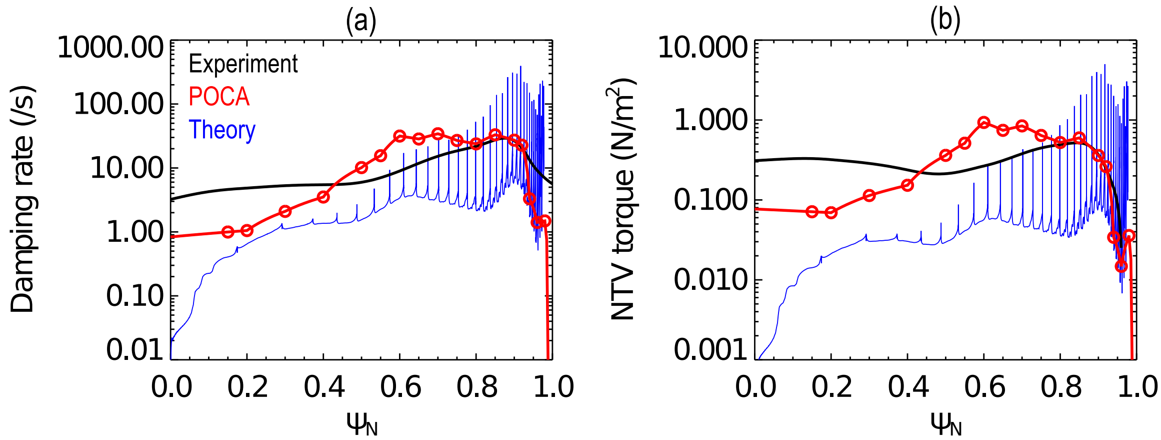

FIG. 4: Comparison of (a) rotational damping rate and (b) NTV torque profiles between measurement (black) and POCA (red) for NSTX discharge 124439. Theory predictions (blue) are drawn together. Total NTV torque inferred from measurement is $3.5Nm$, and agrees well with $4.5Nm$ by POCA.

Fig. 4(a) shows a comparison of the rotational damping rate between the measurement and POCA, drawn together with a prediction by the combined theory. One can find that POCA shows a good agreement with the observation for the profile even though ν_{damp} by POCA is weaker at $\psi_N < 0.4$ and stronger at the outer region. Theory prediction is only valid within an order of magnitude. Fig. 4(b) shows the NTV profiles calculated by POCA and inferred from the measurement. Similarly, POCA indicates weaker NTV at inner region and stronger at outer region. POCA gives $4.5Nm$ for total NTV torque that agrees very well with the experimental value $3.5Nm$. The combined theory gives $0.55Nm$, which might be due to a large aspect-ratio expansion. POCA provides a better prediction than the theory, and this is promising for the experimental NTV analysis.

NTV in the NSTX discharge 132729 is calculated in the same manner, where $I_p = 1.1MA$ and $B_{T0} = 0.55T$. Series of error field correction experiment have been performed by changing error field correction coil (EFC) current, and the selected discharge is the case of $I_{EFC} = 750A$, which produced a strong magnetic braking [11]. Fig. 5 presents comparisons of the damping rate and NTV torque profiles between POCA and measurement. Discrepancies are found in both profiles, where POCA predicts weaker NTV at the inner and edge region and stronger NTV elsewhere. However, one can still find good agreement for the total NTV torque; POCA gives $4.66Nm$ and the experiment $5.1Nm$.

There are uncertainties in estimating the NTV from the toroidal rotation measurements. First, the damping rate measured by carbon ions with CHERS can be different from that of main ion due to a different responding time to the non-axisymmetric perturbations. Second, when estimating NTV torque profile from the measured damping rate, theoretically calculated neoclassical offset flow was used rather than measurement. Generally, the offset flow can be strong at the H-mode edge due to a steep temperature gradient, so it can greatly enhance the NTV value at the edge. This implies the important role of offset rotation in NTV transport. If one ignores the offset flow in the neoclassical

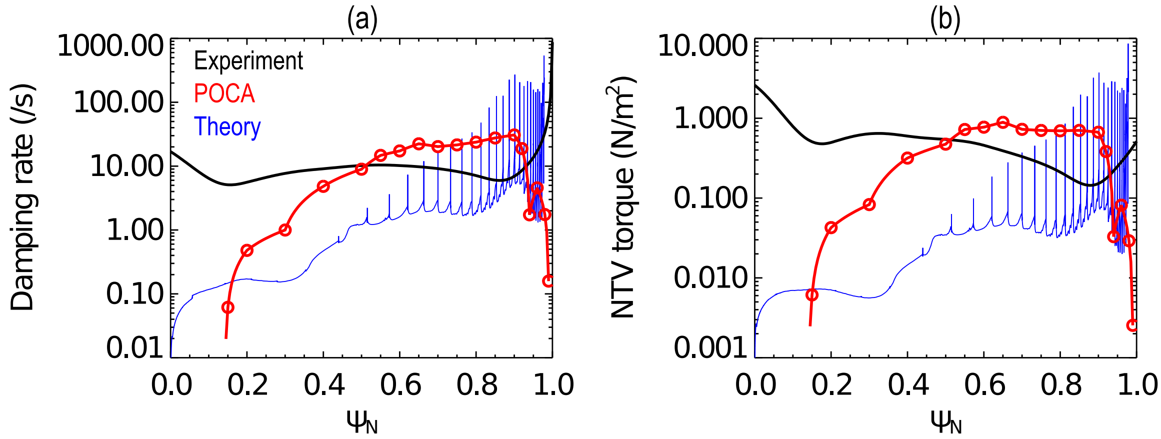

FIG. 5: Comparison of (a) rotational damping rate and (b) NTV torque profiles among measurement (black), POCA (red), and theory(blue) for NSTX discharge 132729

toroidal flow in Eq. (6), the interpretation gives a moderated torque particularly at the edge, and reduced total NTV at $1.5Nm$ for 124439 and $2.36Nm$ for 132729. Therefore, more robust diagnostics for the toroidal rotation and offset flow are required to eliminate such uncertainties.

It should be also noted that the ideal perturbed equilibria can fail in the high β plasmas and strong NTV braking, as in the NSTX discharges. The validity of the ideal perturbed equilibria has been theoretically discussed using dimensionless parameters [12]; $s \equiv -\delta W/\delta W_v$ and $\alpha \equiv -T_\phi/2N\delta W_v$ with δW the total energy, δW_v the required energy to produce the same perturbation without plasma, and T_ϕ the total NTV torque. Theory indicates that the ideal perturbed equilibria can be valid, when $|s| > |\alpha|$ thus the NTV is weak. Otherwise, a shielding effect associated with the toroidal toque becomes crucial to prevent a penetration of the perturbed magnetic fields. In this sense, the δB from the ideal perturbed equilibria would give an inconsistent NTV due to an ignorance of shielding effect. Since $|s| \sim 0.5 > |\alpha| \sim\ 0.2$ for the NSTX discharges in this study, the NTV effect on the perturbed equilibria cannot be ignored. Remembering the strong resonant nature of the NTV, the local NTV effect on the perturbed equilibria also should be considered particularly at the edge, which is dense with the rational surfaces. The self-consistent calculation of δB including the non-ideal plasma response will be eventually required to improve the prediction of the perturbed equilibrium and thereby the NTV transport. It can be achieved from a general perturbed equilibrium code solving 3D force balance with the perturbed anisotropic tensor pressure.

5 Concluding remarks

A particle simulation model has been developed for the accurate calculation of δf by guiding-center drift and δB by ideal perturbed equilibria. NTV analyses have been carried out using analytic and practical non-axisymmetric magnetic field perturbations. Simulation with the analytic perturbation clearly indicates the strong resonant NTV torque as well as good agreements with the combined NTV theory. POCA calculations in NSTX magnetic braking experiments also show good agreements for the toroidal rotation damping rate and NTV torque. In the high β plasma and strong magnetic braking as NSTX, the ideal perturbed equilibria can be broken. Penetration of the perturbed field into plasmas can be significantly changed by the non-ideal plasma response. A self-consistent δB by a general perturbed equilibria is required to achieve more precise NTV prediction. The self-consistent calculation of δB and thereby NTV can be accomplished throughout an integrated calculation of the transport and the general perturbed equilibrium codes, which is actively under way.

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