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Resistive DCON

Computation of Resistive Instabilities in Tokamaks with Full Toroidal Geometry and Coupling using DCON

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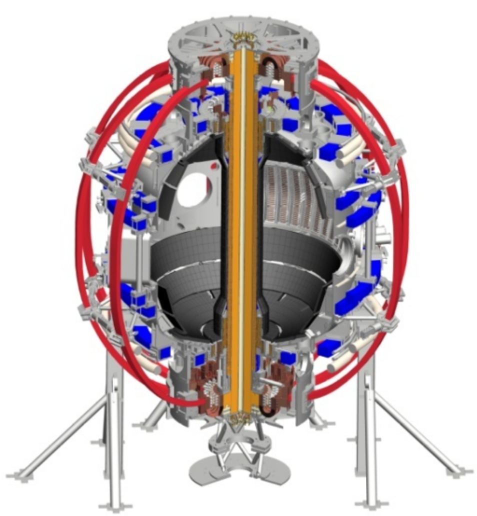

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Motivation

- Resistive instabilities, and their boundary in operational or parametric space, have never been fully addressed for tokamaks or ITER, unlike ideal instabilities
  - Due to strong coupling with steep eigenfunctions near the resonant surfaces

- No alternatives has been offered except the poor cylindrical approximation with Δ' (tearing mode index) for linear resistive instability
  - Non-linear theories for tearing modes and neoclassical tearing modes are all based on the scalar index Δ', but toroidally each layer is all coupled resulting an index matrix

- Two approaches:
  - MARS-F directly solved resistive MHD equations through core to edge, but is difficult to resolve stable mode structures that can be important to couple to external fields; difficult to extend inner layer physics
  - PEST-III uses asymptotic matching, but limited to low β and weak shaping

- Asymptotic matching code applicable to high β and strong shaping is necessary to resolve resistive instabilities : **Resistive DCON**

Outline

- New DCON with resonant Galerkin method
  - Non-homogeneous Euler-Lagrange equation with big solutions
  - Resonant Galerkin method
- Improved convergence of resistive DCON
- Numerical benchmark with MARS-F
  - Mode structures
  - Growth rates with GGJ inner region model
- Summary and Discussion

Formulation for DCON in outer region

- In the approach of asymptotic matching method, outer region is solved independently of inner region of the layer
  - **This separation enables flexible implementation of inner-layer models**

- Outer region is modeled by zero-frequency ideal MHD :

$$\vec{j} \times \vec{B} + \vec{J} \times \vec{b} - \nabla p = 0$$

$$p = -\vec{\xi} \cdot \nabla P - \Gamma P \nabla \cdot \vec{\xi}$$

$$\vec{b} = \nabla \times (\vec{\xi} \times \vec{B})$$

$$\vec{j} = \nabla \times \vec{b}$$

Euler-Lagrange Equation:

$$\mathbf{L}\left[\Xi\right] = -\left(\mathbf{F}\Xi' + \mathbf{K}\Xi\right)' + \left(\mathbf{K}^{\dagger}\Xi' + \mathbf{G}\Xi\right) = 0$$

$$where \quad \Xi = \{\xi_{mn}(\psi)\} \quad and \quad \vec{\xi} \cdot \vec{\nabla}\psi(\psi,\vartheta,\varphi) = \sum_{mn} \xi_{mn}(\psi)\exp(im\vartheta - in\varphi)$$

- This system of the second-order ODE is singular near the layer, and difficult to solve when coupled with regular non-resonant solutions

Euler-Lagrange equation is solved with resonant solutions by Frobenious expansion

- Surprisingly a great deal of difficulty for resistive instability calculations come from outer region, rather than inner region
  - Since the small or non-resonant solutions are almost invisible by big resonant solutions near the resonant surfaces

- Approaches in PEST-III: **[Pletzer and Dewar, J. Plasma Physics (1991)]**
  - Compute big and small solutions by Frobenious expansion
  - Solve non-homogeneous E-L equation with big solutions

$$\mathbf{L}\left[\Xi\right] = -\sum_{i,p}\mathbf{L}\left[\Xi_{ip}^{b}\right]$$

RHS denotes big solutions from i=1~N resonance surfaces, and p=R or L representing the right or the left side of the resonant surfaces

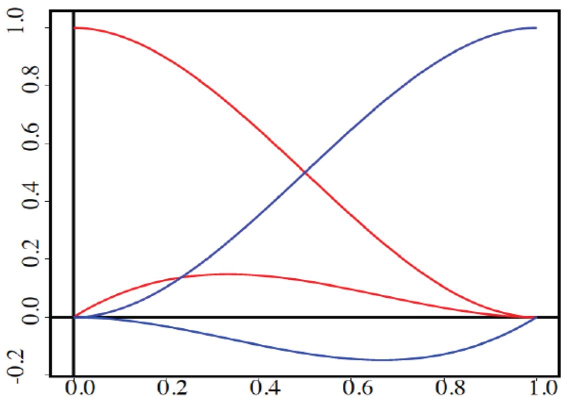
  - The 2N solutions by big resonant solution on the right and left side of N resonant surfaces are new “resistive” solutions in addition to M ideal solutions
  - Coefficients for M+2N solutions can be determined by M external boundary and 2N matching conditions

Resistive DCON uses C^1 Hermite cubics and precise Frobenious expansion up to arbitrary order

- New DCON uses finite element method modified with Frobenious expansion near the resonant surfaces
  - Previous shooting method was highly successful for ideal DCON, but has been concluded numerically unstable for resistive DCON (Backup slides)

- Boundary value problem with the system of singular ODE is solved with:

C^1 Hermite Cubics



- **Cubic polynomials on (0,1)**
- **C^1 continuity: function values and first derivatives**
- **Used for non-resonant solutions across the layer**

Convergent Power Series Expansion

$$\Xi = z^r \sum_{k=0}^{K} \Xi^{(k)} z^k \qquad z = \begin{cases} \psi - \psi_r & (\psi \geq \psi_r) \\ \psi_r - \psi & (\psi < \psi_r) \end{cases}$$

$$r = -\frac{1}{2} \pm \sqrt{-D_I}$$

- Solved to arbitrarily high order N;

Automated using matrix formulation

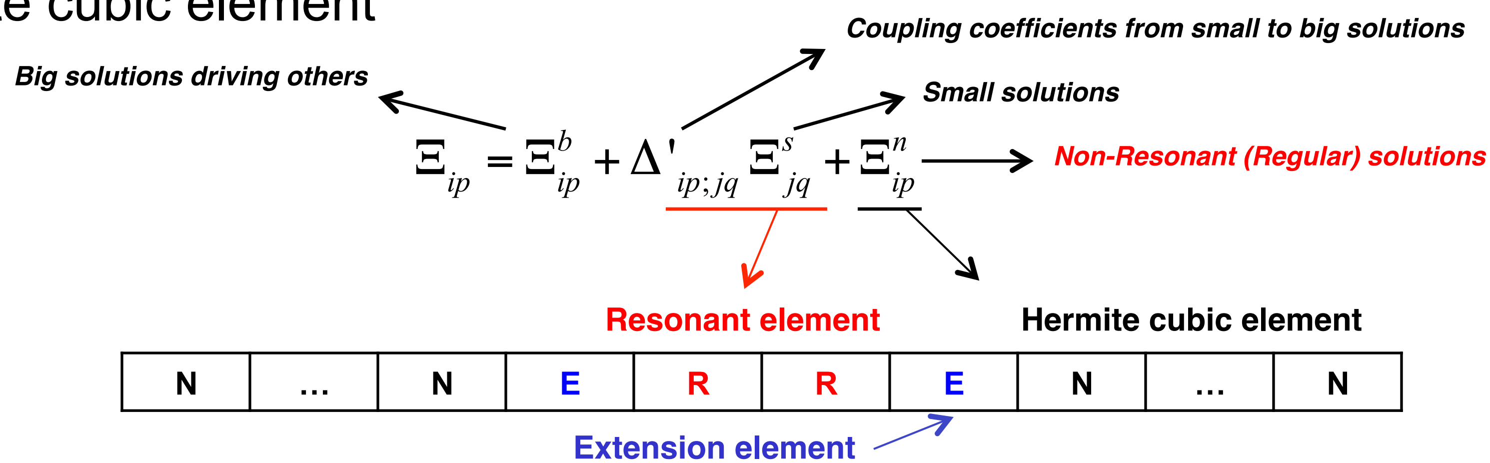
- Essential for larger values of $|D_I|$;

Generally improves convergence

Resonant and extension elements are additionally used to precisely treat solutions near resonant surfaces

- Resonant element includes small resonant solutions as extra basis functions and extension element connects solutions smoothly to normal Hermite cubic element

$$\Xi_{ip} = \Xi_{ip}^{b} + \Delta'_{ip;jq}\,\Xi_{jq}^{s} + \Xi_{ip}^{n}$$



- With adjustable grid packing methods, new DCON solves Euler-Lagrange equation by modified "resonant" Galerkin method

$$W = \frac{1}{2}\left(\bar{\bar{\Xi}}, \mathbf{L}\left[\bar{\bar{\Xi}}\right]\right) - \left(\bar{\bar{\Xi}}, \mathbf{L}\left[\Xi_{ip}^{b}\right]\right)$$

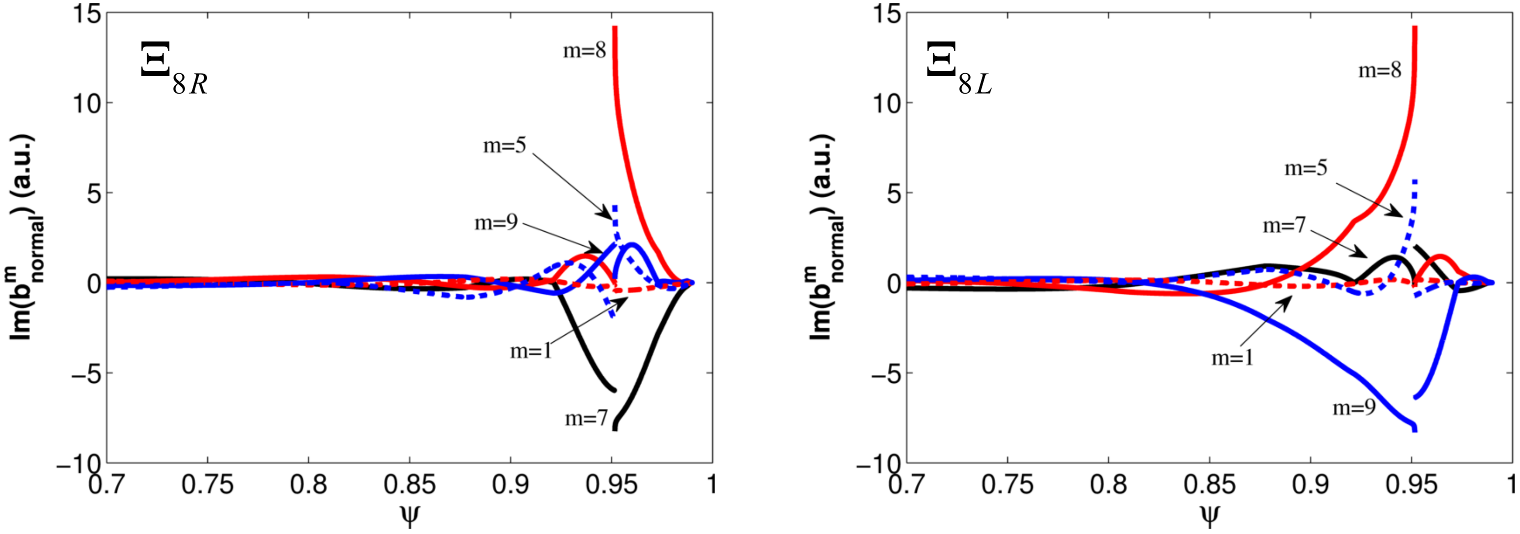
$$\delta W = \left(\delta\bar{\bar{\Xi}}, \mathbf{L}\left[\bar{\bar{\Xi}}\right]\right) - \left(\delta\bar{\bar{\Xi}}, \mathbf{L}\left[\Xi_{ip}^{b}\right]\right) = 0$$

Big solution can drive small solutions in all other surfaces as well as non-resonant solutions

- Each of 2N big resonant solution can drive all other 2R small resonant solutions as well as non-resonant solutions

$$\Xi_{ip} = \Xi^{b}_{ip} + \Delta'_{ip;jq}\,\Xi^{s}_{jq} + \Xi^{n}_{ip}$$

NSTX high-β example using Resistive DCON

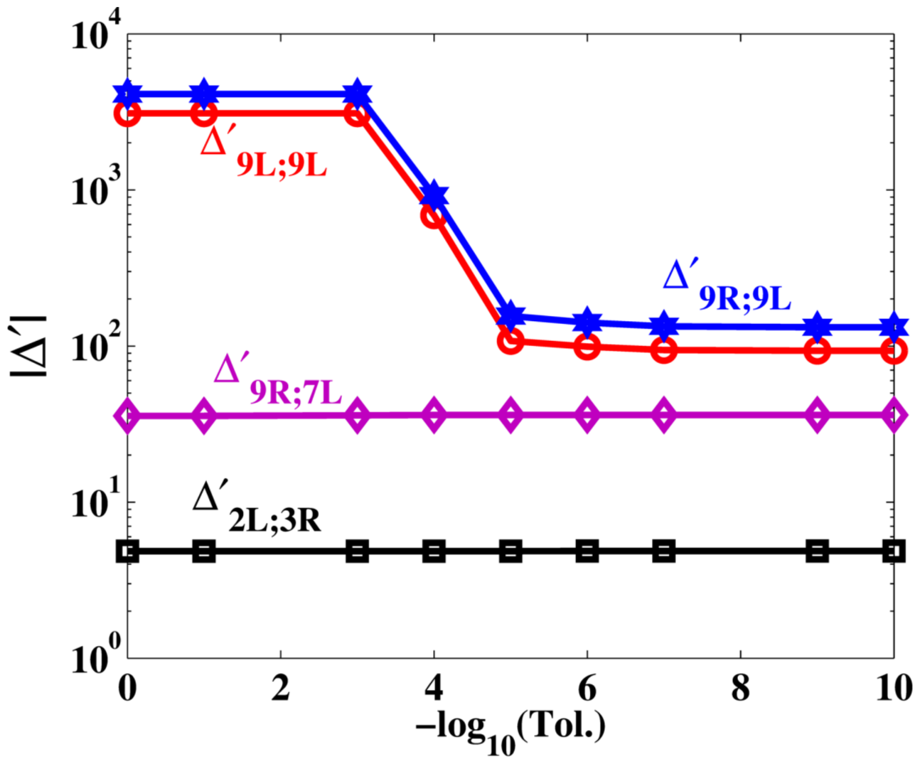


- Note resistive instability is determined by the stability index matrix $\Delta'_{ip;jq}$ rather than a scalar tearing mode index Δ'

Convergence has been greatly improved in newly developed DCON

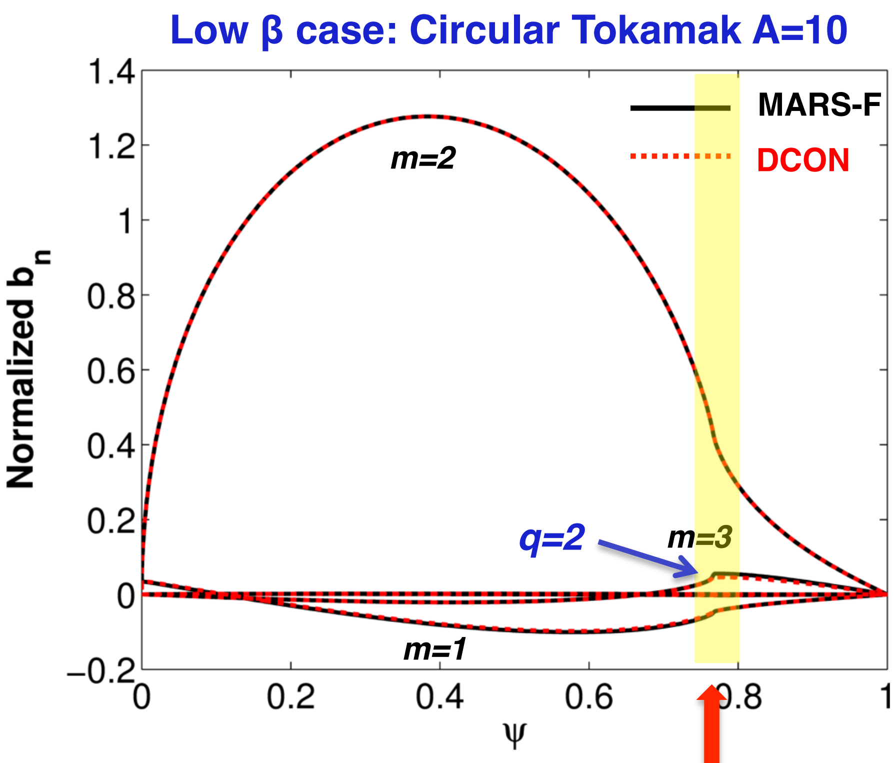
- A very robust convergence of matching data is achieved even in a challenging NSTX case

NSTX case (q_0=2.13, q_a=14.9, β_N=3.29)



The scan of integral tolerance shows the quick and stable convergence of Δ' value among q=2,3,7,9 resonant surfaces

Benchmark with MARS-F for outer region solutions

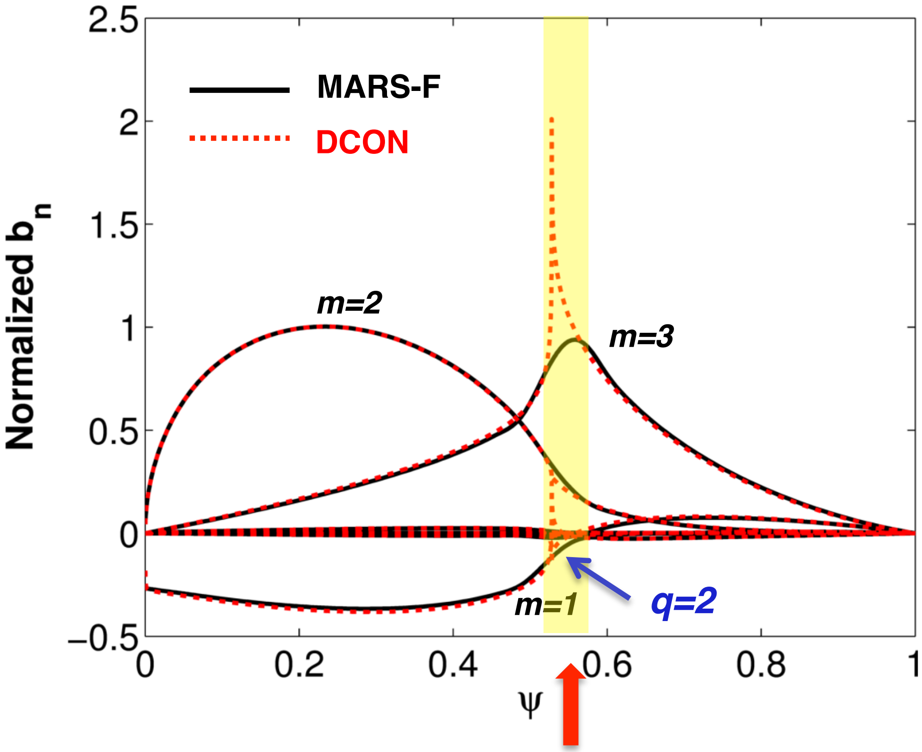


Constant-ψ approximation is used to construct the outer region solution in DCON.

- In low β case, The constant-ψ approximation is valid in the construction of outer region solution in DCON
- Mode coupling is weak due to low β and large aspect ratio
- The outer region solutions of DCON agree with MARS-F

Benchmark with MARS-F for outer region solutions

High β case: Circular Tokamak A=10



Solution near resonant surface should be resolved by inner layer model in DCON

- In higher β case, the outer region solutions of DCON also agree with MARS-F
- Mode coupling is strong due to high β
- Constant-ψ approximation fails to reproduce MARS-F in high β case, but DCON solution can be reconstructed with adjustable coefficients
- To determine coefficients consistently in the code, inner region model is needed

Δ' benchmark with MARS-F

- A cylindrical Δ' can be obtained ignoring other surfaces by

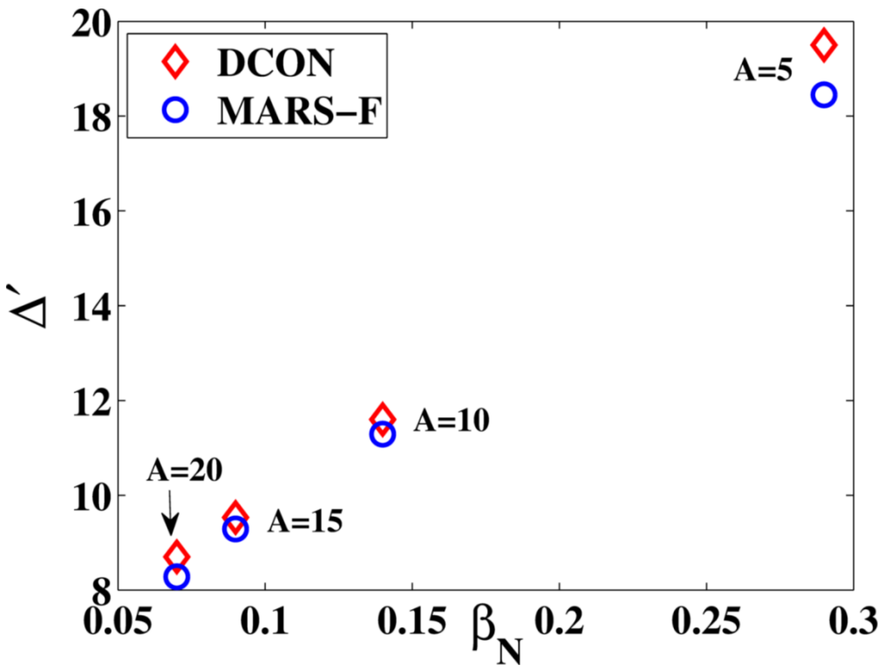
PEST 3 ←

$$\Gamma' = \Delta_{RR} + \Delta_{RL} - \Delta_{LR} - \Delta_{LL}$$
$$\Delta' = \Delta_{RR} - \Delta_{RL} - \Delta_{LR} + \Delta_{LL}$$

→ DCON

MARS-F calculates Δ' back from computed γ and GGJ model

- Quantitative agreement has been obtained in various cases



Vacuum included

Aspect ratio	β_N	(DCON)	(MARS)
5	0.29	19.5	18.45
10	0.14	11.6	11.29
15	0.09	9.53	9.29
20	0.07	8.7	8.28

Present layer model is based on Glasser-Greene-Johnson theory for toroidal geometry

- GGJ theory with inertia and resistivity produces the even and odd inner region solutions as **[Glasser, Greene, and Johnson, Phys. Fluids (1975)]**

$$\Xi_{i\pm}(x) = \Xi^b_{i\pm}(x) + \Delta_{i\pm}(Q)\Xi^s_{i\pm}(x)$$

Q: Resistive scaled growth rate
x: Scale length across layer

- Outer region formulation with the left and right side solutions can be replaced by even and odd, and matching conditions are given by

$$\lim_{z\to 0}\sum_{ip} c_{ip}\left(\delta_{ij}\delta_{pq}\Xi^b_{ip}(z) + \sum_{jq}\Delta'_{ip;jq}\,\Xi^s_{jq}(z)\right) = \lim_{x\to\infty} d_{ip}\left(\Xi^b_{ip}(x) + \Delta_{ip}(Q)\Xi^s_{ip}(x)\right)$$

- Matching the big and small solutions gives

$$d_{ip} = c_{ip} \quad and \quad \sum_{ip}\left[\Delta'_{ip;jq} - \delta_{ip}\Delta_{jq}\left(Q\right)\right]c_{ip} = 0$$

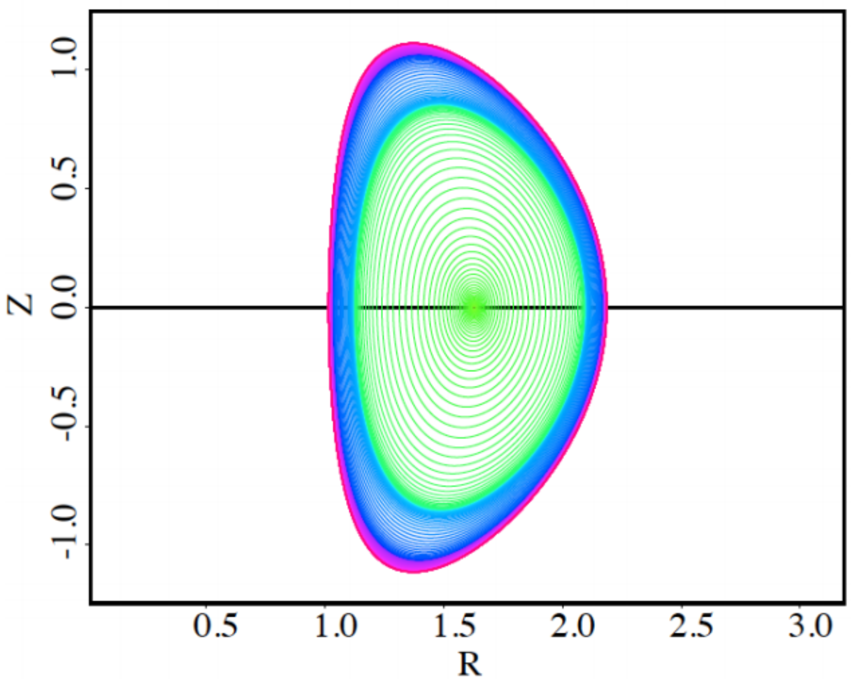
- So resistive instability and growth rate is determined by

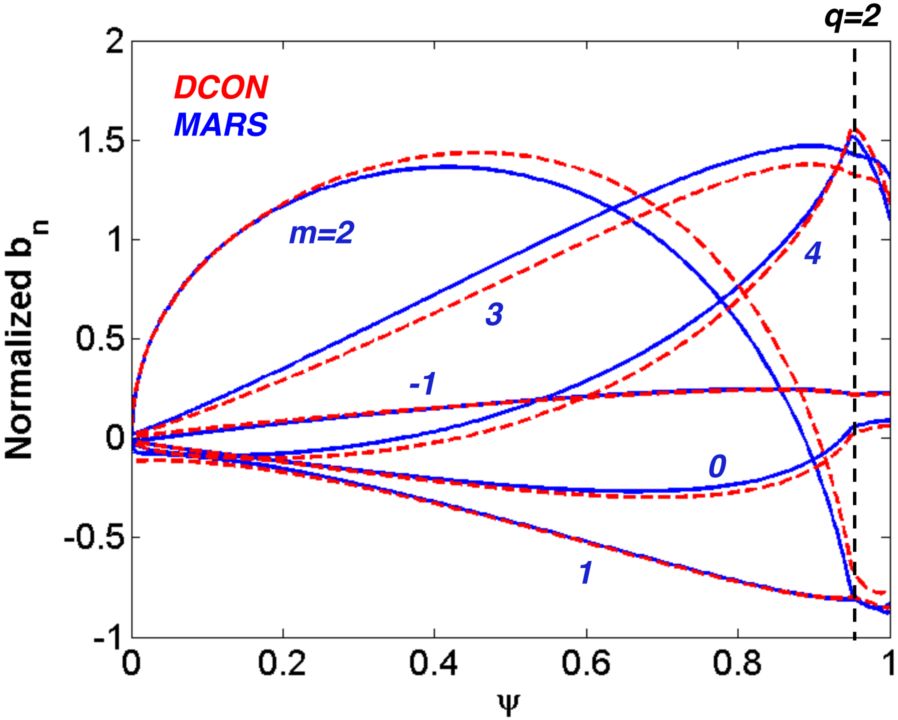
$$\det\left|\Delta'_{ip;jq} - \delta_{ip}\Delta_{jq}\left(Q\right)\right| = 0$$

Resistive DCON solutions with GGJ model and comparison with MARS-F

- Eigenfunctions calculated by DCON with GGJ model and by MARS-F for the D-shaped plasma showed a good agreement for a single resonant surface case

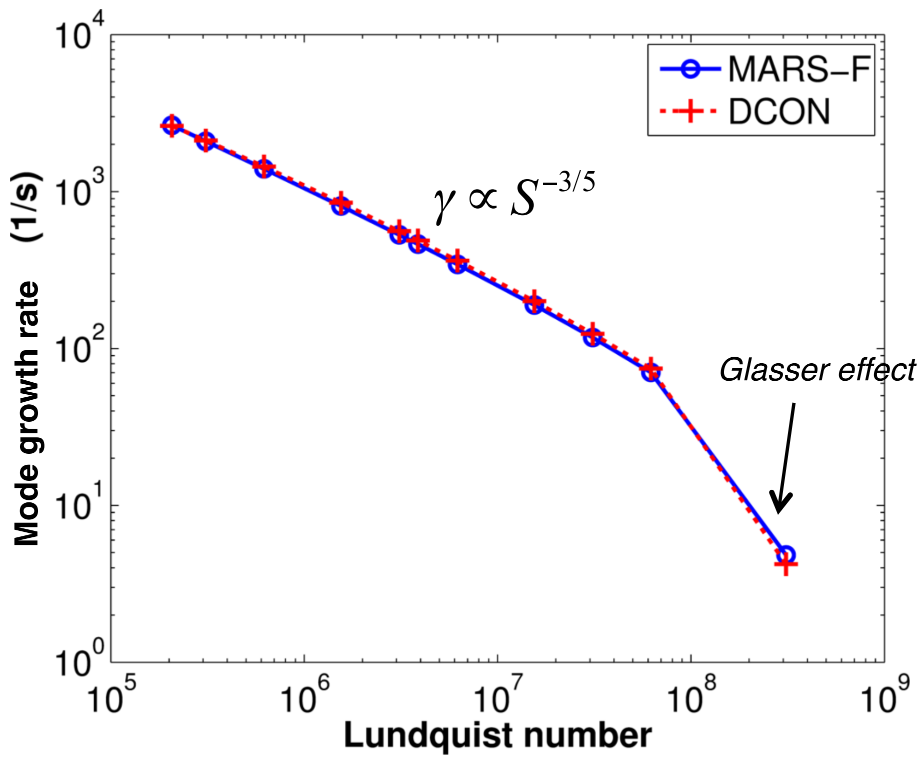
$q_0 = 1.05, q_a = 2.31, \beta_N=0.77$ Aspect ratio=2.734





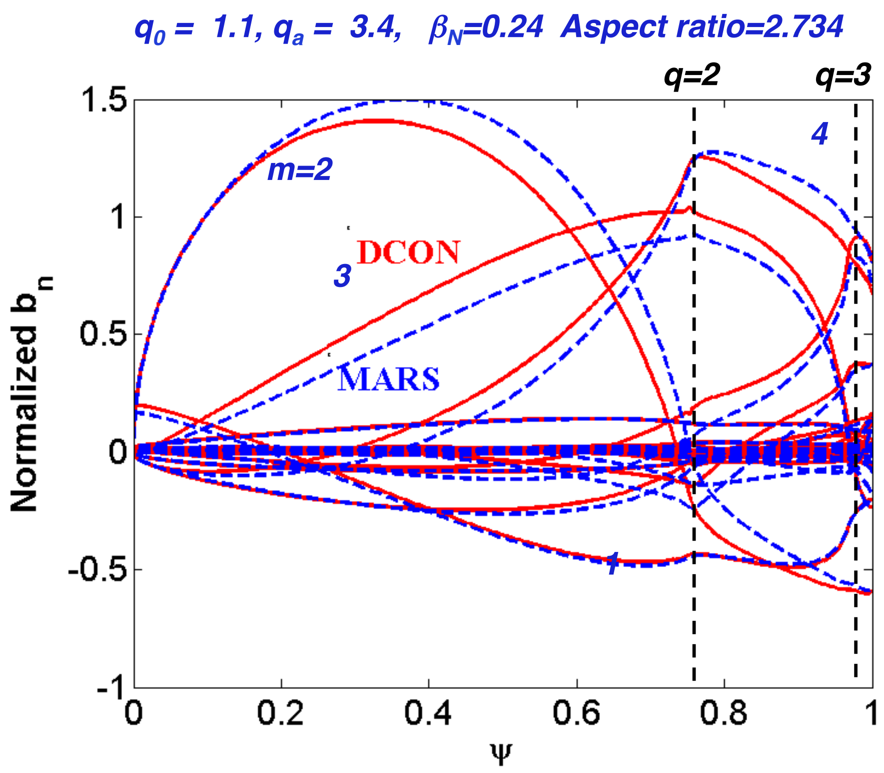
Growth rate comparison with MARS-F

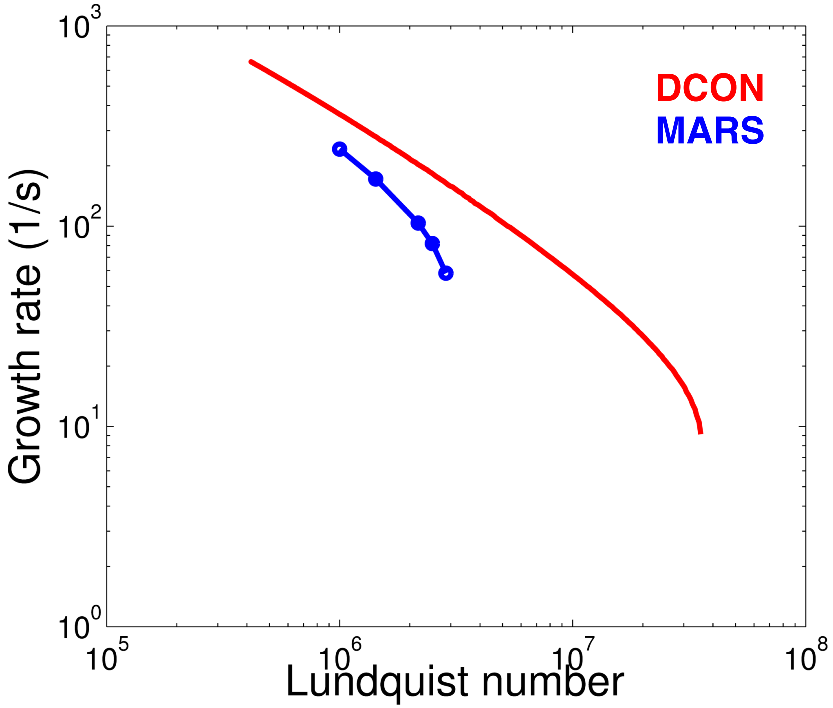
- Growth rates calculated by DCON with GGJ model and by MARS-F showed an excellent quantitative agreement, with expected scaling for tearing mode instability



Further numerical benchmark is still needed between DCON and MARS-F for multiple rational surfaces

- The structure of eigenfunction also shows a good agreement after the matching, but disagreement of growth rate should be further resolved





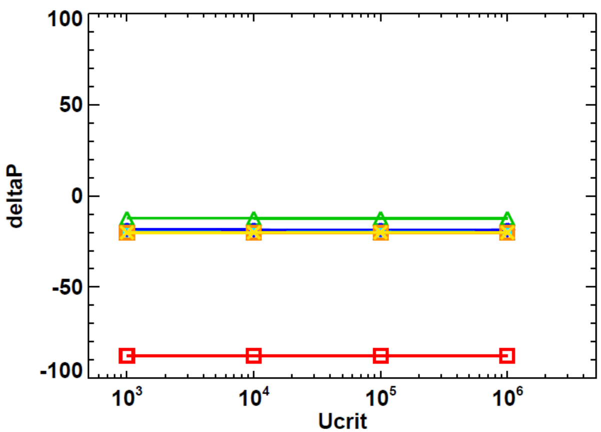
Summary and Discussion

- Resistive DCON has been successfully developed with resonant Galerkin method by replacing the previous shooting method
- Resistive DCON shows an excellent converging property and numerical stability in various cases including high-β NSTX plasmas
- Eigenfunctions, tearing mode index matrix, growth rates were successfully compared with MARS-F, but disagreement for the growth rates was found with multiple rational surfaces
- Resistive DCON has no limitation and can be applied to highly advanced scenarios in tokamaks and ITER
- Resistive DCON can resolve the stable mode and resistive perturbed equilibrium, and also can extend the layer model flexibly

Backup: the convergence issue in shooting method while solving outer region

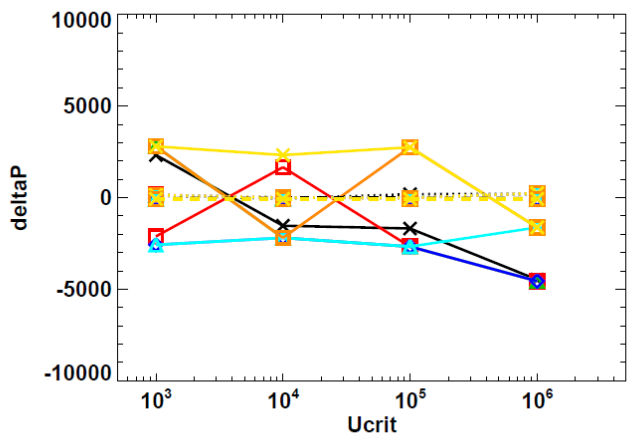
Shooting method is used to solve Δ' of n=1 tearing mode

Circular tokamak case (A=3)



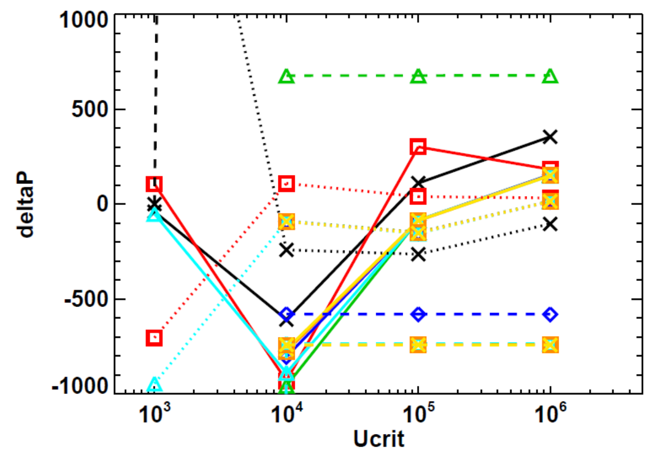
The convergence of Δ' is good

DIII-D low β case: g097741.01605



The value of Δ' starts to diverge

DIII-D high β case: g126006.03600



The convergence of Δ' value is bad

The value of Δ' becomes sensitive to the numerical parameters (fixing up criteria and integral tolerance) and diverge