Computation of Resistive Instabilities in Tokamaks with Full Toroidal Geometry and Coupling using DCON*

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Precise determination of resistive instabilities is an outstanding issue in tokamaks, remaining unsatisfactory for a long time despite its importance for advanced plasma control. A poorly estimated tearing mode index Δ' is commonly used in lieu of rigorously computed quantities. A scalar Δ' is inadequate to properly incorporate inner region dynamics, but nevertheless used in many extended physics studies for classical and neoclassical tearing modes because no alternatives were offered. This paper presents the first successful computation of such resistive instabilities by upgrading DCON [1], including full mode coupling, multiple singular surfaces, and a vacuum region, which will routinely provide fast and reliable prediction of resistive instabilities in tokamaks as well as robust basis of other extended tearing mode studies in the future.

Incorporating the resistive layer model of Glasser-Greene-Johnson (GGJ) [2] and matching the inner-layer solutions into full outer-layer solutions in DCON, a complete picture of resistive instabilities in tokamaks can be obtained and studied. Quantitative agreement with the MARS-F code [3], for both growth rate and outer-layer solutions, has been achieved as an example is shown in Figure 1. Many examples have been studied with DCON and MARS-F, but DCON is faster and more robust. Convergence is also a distinguished property in DCON, as Figure 2 shows convergence of Δ' -like quantities along with numerical tolerance for a challenging NSTX equilibrium with strong shaping, high-β, and multiple rational surfaces up to 10. Note that Δ' reduces to the conventional tearing mode index in a cylinder with constant-ψ approximation. However, in general tokamaks the tearing mode stability cannot be determined merely by a scalar index Δ', but by a matrix of such quantities.

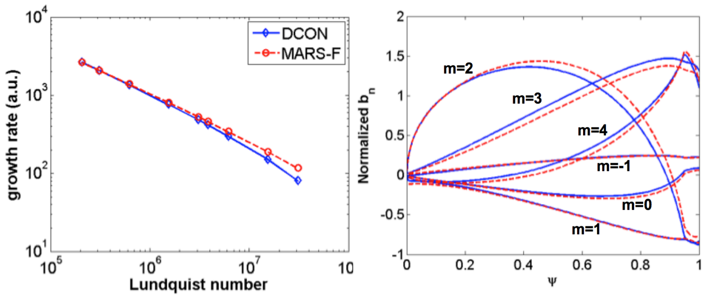

Figure 1. Comparison between DCON and MARS-F for growth rates (Left) and eigenfunctions (Right) of a resistive tearing mode, studied with a D-shape finite β target. Note the strong coupling between poloidal modes of eigenfunctions.

The difficulty in determining resistive instabilities is due to the stiffness of outer-layer solutions nearby rational surfaces, not from inner-layer where physics is typically more complicated than ideal MHD. The minimization of potential energy δW leads to a system of coupled ordinary differential equations for a vector Ξ of Fourier components of the normal displacements in the ideal outer region, with regular singular points at rational surfaces where the safety factor $q = m/n$ and the solution varies rapidly. For ideal stability calculations, DCON uses an adaptive ODE solver to integrate these equations from the magnetic axis to the plasma-vacuum interface, where it couples to a vacuum region. This method is fast, accurate, and widely used in the tokamak community.

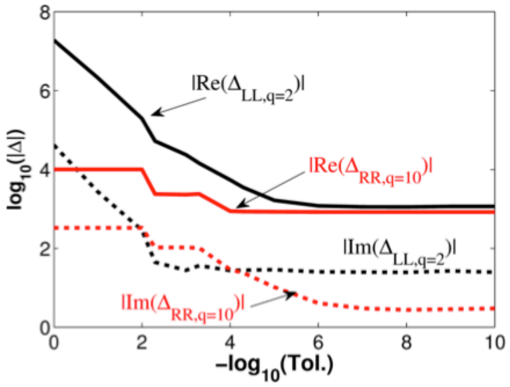

Figure 2. Convergence of Δ as a function of numerical integration tolerance. Δ_{Xq1Yq2} is defined by the coefficient ratio of small resonant solution at q1 surface on X to large resonant solution at q2 surface on Y (X,Y=Left or Right).

When applied to the determination of resistive instability, this initial value problem becomes a shooting method to determine the solution to a 2-point boundary value problem, which suffers from a well-known numerical instability. To avoid this, the shooting method is replaced with a stable approach introduced by Pletzer and Dewar for PEST-3 code [4], using a Galerkin method, expanding in linear finite elements along with the radial variable ψ. The PEST-3 implementation of this method is greatly improved by better choice of basis functions: C^1 Hermite cubics to resolve the nonresonant components of the solution, supplemented by a high-order Frobenius expansion of the small resonant solutions. These are distributed on a nonuniform grid, using an improved packing algorithm to concentrate the grid near the rational surfaces while leaving adequate grid to resolve the nonresonant regions. This enables treatment of high-β, strongly-shaped equilibria with a large range of q values, strong poloidal coupling, and large Mercier criterion values $|D_I| >> 1$.

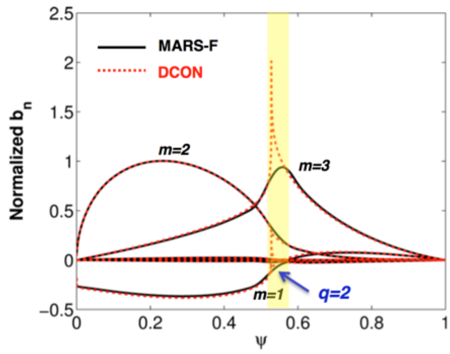

Figure 3. Reconstruction of MARS-F resistive solutions by linearly combining the complete set of DCON outer-layer solutions.

Another important advantage in DCON comes with the separation of the inner-layer from the outer-layer regions. The complete set of outer-layer solutions is determined independently of the inner-layer model and can be used to match two independent inner-layer solutions. Figure 3 shows the excellent reconstruction of MARS-F resistive solutions using DCON outer-layer solutions even without inner-layer matching, illustrating the completeness of the DCON solutions. This separation will allow us to extend inner-layer modeling to more advanced fluid equations, including drift kinetic effects with the Fokker-Planck collision operator, as long as the governing equations can be expressed in flux-source form, $M^{i,j}\partial_t u^j + \nabla \cdot F^i = S^i$. On the other hand, the outer-layer equations can be maintained with a reduced form, which will enable efficient handling of complicated external boundary conditions such as a conducting wall or external 3D coil currents and thus more precise calculations on non-ideal stability and 3D perturbed equilibria.

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