Towards a self consistent evaluation of the RF wave-field and the ion distribution functions in tokamak plasmas

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NSTX-U

TORIC v.5 code

- The TORIC v.5 code solves the wave equation for the electric field $\mathbf{E}$:
- TORIC v.5 uses a Maxwellian plasma dielectric tensor

$$\varepsilon \equiv \mathbf{I} + \frac{4\pi i}{\omega}\sigma = \mathbf{I} + \chi$$

- Two TORIC v.5's versions:
  - TORIC: IC minority regime
    - FLR corrections only up to the $\omega = 2\omega_{ci}$
  - TORIC-HHFW: High Harmonic Fast Wave regime
    - Full hot-plasma dielectric tensor employed
    - The k^2 value in the argument of the Bessel functions is obtained by solving the local dispersion relation for FWs
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ICRH minority regime: Beyond Maxwellian

FLR non-Maxwellian susceptibility in a local coordinate (Stix) frame $(\hat{\mathrm{x}}, \hat{\mathrm{y}}, \hat{\mathrm{z}})$, with $\hat{\mathrm{z}} = \hat{\mathrm{b}}$, $\mathrm{k}\cdot\hat{\mathrm{y}} = 0$, to second order in $k_\perp v_\perp/\omega_c$

$$\begin{aligned}
\chi_{xx} &= \frac{\omega_{p,s}^2}{\omega}\left[\frac{1}{2}(A_{1,0}+A_{-1,0}) - \frac{\lambda}{2}(A_{1,1}+A_{-1,1}) + \frac{\lambda}{2}(A_{2,1}+A_{-2,1})\right]\\
\chi_{xy} &= -\chi_{yx} = i\frac{\omega_{p,s}^2}{\omega}\left[\frac{1}{2}(A_{1,0}-A_{-1,0}) - \lambda(A_{1,1}-A_{-1,1}) + \frac{\lambda}{2}(A_{2,1}-A_{-2,1})\right]\\
\chi_{xz} &= +\chi_{zx} = -\chi_{yx} = \frac{\omega_{p,s}^2}{\omega}\left(\frac{1}{2}\frac{k_\perp}{\omega}\right)\left[(B_{1,0}+B_{-1,0}) - \lambda(B_{1,1}+B_{-1,1}) + \frac{\lambda}{2}(B_{2,1}+B_{-2,1})\right]\\
\chi_{yy} &= \frac{\omega_{p,s}^2}{\omega}\left[2\lambda A_{0,1} + \frac{1}{2}(A_{1,0}+A_{-1,0}) - \frac{3\lambda}{2}(A_{1,1}+A_{-1,1}) + \frac{\lambda}{2}(A_{2,1}+A_{-2,1})\right]\\
\chi_{yz} &= -\chi_{zy} = i\frac{\omega_{p,s}^2}{\omega}\left(\frac{k_\perp}{\omega}\right)\left[B_{0,0} - \lambda B_{0,1} - \frac{1}{2}(B_{1,0}+B_{-1,0}) - \lambda(B_{1,1}+B_{-1,1})\right.\\
&\quad\left. - \frac{\lambda}{4}(B_{2,1}+B_{-2,1})\right]\\
\chi_{zz} &= \frac{2\omega_p^2}{k_\parallel w_\perp^2}\left[(1-\lambda)B_{0,0} + \int_{-\infty}^{+\infty}\mathrm{d}v_\parallel\int_0^{+\infty}\mathrm{d}v_\perp v_\perp\frac{v_\parallel}{\omega}f_0(v_\parallel,v_\perp)\right]\\
&\quad + \frac{\lambda}{2}\frac{\omega_p^2}{\omega}\left[2\frac{\omega-\omega_c}{k_\parallel w_\perp^2}B_{1,0} + 2\frac{\omega+\omega_c}{k_\parallel w_\perp^2}B_{-1,0}\right] \qquad \lambda \equiv \frac{1}{2}\left(\frac{k_\perp v_\perp}{\omega_c}\right)^2
\end{aligned}$$

Evaluations of the FLR susceptibility requires computation of two functions $A_{n,j}$ $B_{n,j}$, for $n = -2\ldots 2$, $j = 0, 1$, which are $v_\perp$ moments of resonant integrals of $f_0(\psi, \frac{B}{B_{\min}}, v_\parallel, v_\perp)$

$$\left\{\begin{array}{c} A_{n,j}\\ B_{n,j}\end{array}\right\} = \int_{-\infty}^{\infty}\mathrm{d}v_\parallel\left\{\begin{array}{c}1\\ v_\parallel\end{array}\right\}\frac{1}{\omega - k_\parallel v_\parallel - n\omega_c}\int_0^{+\infty}2\pi v_\perp \mathrm{d}v_\perp H_j(v_\parallel, v_\perp)$$

with

$$\begin{aligned}
H_0(v_\parallel,v_\perp) &= \frac{1}{2}\frac{k_\parallel w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_\parallel} - \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)f_0(v_\parallel,v_\perp)\\
H_1(v_\parallel,v_\perp) &= \frac{1}{2}\frac{k_\parallel w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_\parallel}\frac{v_\perp^4}{w_\perp^4} - \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)f_0(v_\parallel,v_\perp)\frac{v_\perp^2}{w_\perp^2}
\end{aligned}$$

and

$$w_\perp^2 \equiv \int_{-\infty}^{\infty}\mathrm{d}v_\parallel\int_0^{+\infty}2\pi\mathrm{d}v_\perp v_\perp^2 f_0(v_\parallel,v_\perp)$$

HHFW regime: Beyond Maxwellian

Local coordinate frame $(\hat{\mathrm{x}}, \hat{\mathrm{y}}, \hat{\mathrm{z}})$ with $\hat{\mathrm{z}} = \hat{\mathrm{b}}$ and $\mathrm{k}\cdot\hat{\mathrm{y}} = 0$ (Stix)

$$\begin{aligned}
\chi_s &= \frac{\omega_{ps}^2}{\omega}\int_0^{+\infty}2\pi v_\perp\mathrm{d}v_\perp\int_{-\infty}^{+\infty}\mathrm{d}v_\parallel \hat{\mathrm{z}}\hat{\mathrm{z}}\frac{v_\parallel^2}{\omega}\left(\frac{1}{v_\parallel}\frac{\partial f}{\partial v_\parallel} - \frac{1}{v_\perp}\frac{\partial f}{\partial v_\perp}\right)_s +\\
&\quad + \frac{\omega_{ps}^2}{\omega}\int_0^{+\infty}2\pi v_\perp\mathrm{d}v_\perp\int_{-\infty}^{+\infty}\mathrm{d}v_\parallel\sum_{n=-\infty}^{+\infty}\left[\frac{v_\perp U}{\omega - k_\parallel v_\parallel - n\Omega_{cs}}\mathrm{T}_n\right]
\end{aligned}$$

where

$$U \equiv \frac{\partial f}{\partial v_\perp} + \frac{k_\parallel}{\omega}\left(v_\perp\frac{\partial f}{\partial v_\parallel} - v_\parallel\frac{\partial f}{\partial v_\perp}\right) \qquad \text{and}$$

$$\mathrm{T}_n = \begin{pmatrix} \frac{n^2 J_n^2(z)}{z^2} & \frac{in J_n(z)J_n'(z)}{z} & \frac{n J_n^2(z) v_\parallel}{z v_\perp}\\ -\frac{in J_n(z)J_n'(z)}{z} & (J_n'(z))^2 & -\frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp}\\ \frac{n J_n^2(z) v_\parallel}{z v_\perp} & \frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} & \frac{J_n^2(z) v_\parallel^2}{v_\perp^2}\end{pmatrix}, \quad z \equiv \frac{k_\perp v_\perp}{\Omega_{cs}}$$

- Integrals in the $v_\parallel$-space with the singularity function $(\omega - k_\parallel v_\parallel - n\Omega_{cs})^{-1}$
- Sum over the harmonic number n and the $k_\perp$ dependence in the argument of the Bessel functions
- Evaluate six components of T_n

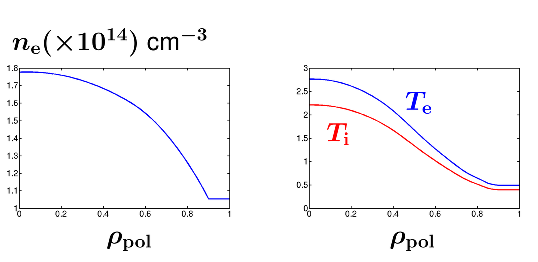
χ is pre-computed to reduce TORIC runtime

- A set of N_ψ files is constructed, each containing the principal values and residues of χ for a single species on a uniform mesh ($(v_\parallel, \theta)$ and $(v_\parallel, \theta, N_\perp)$ for ICRH and HHFW regimes, respectively), for a specified flux surface
- $f(v_\parallel, v_\perp)$, is specified in functional form at the minimum field strength point $B(\theta) = B_{\min}$ on ψ_j
- An interpolator returns the components of χ .

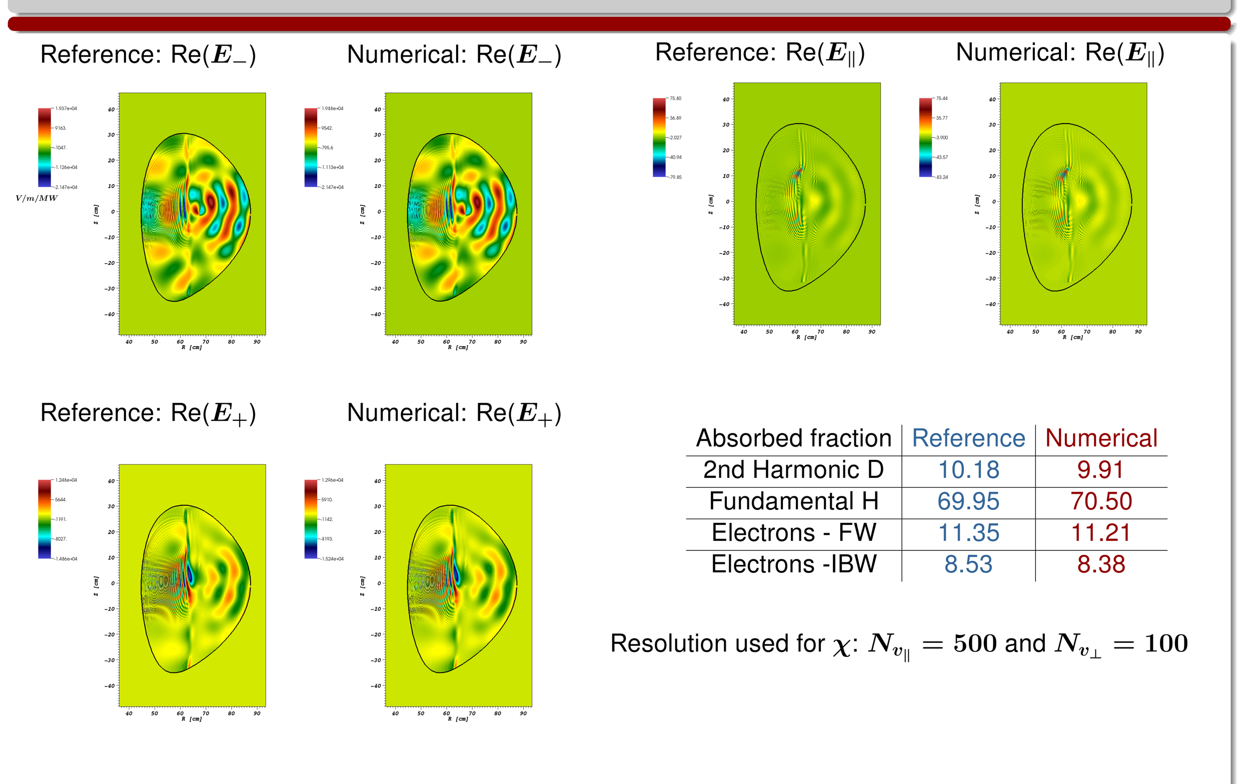
Alcator C-Mod case

Main parameters:

- Plasma species: electron, D, and minority H (4%)
- $B_T = 5$ T
- $I_p = 1047$ kA
- $T_e(0) = 2.76$ keV
- $n_e(0) = 1.78 \times 10^{14}$ cm^{-3}
- $T_{D,H}(0) = 2.2$ keV
- TORIC resolution: $n_{mod} = 255$, $n_{elm} = 480$



Test ICRH: Maxwellian reference case



Absorbed fraction	Reference	Numerical
2nd Harmonic D	10.18	9.91
Fundamental H	69.95	70.50
Electrons - FW	11.35	11.21
Electrons -IBW	8.53	8.38

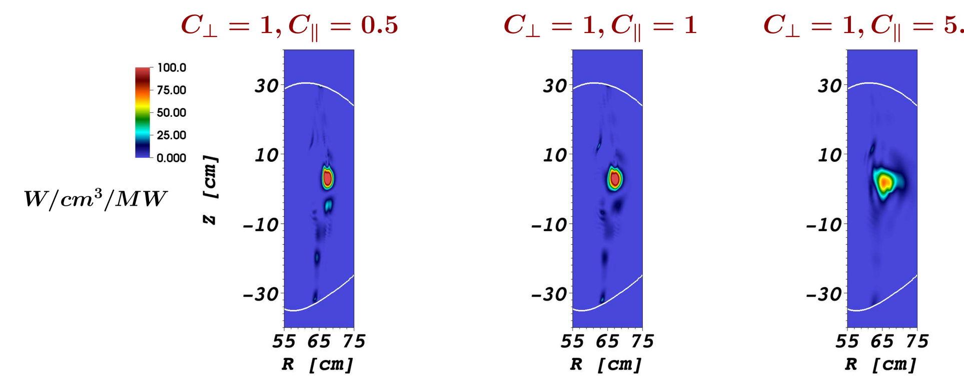
Resolution used for χ: $N_{v_\parallel} = 500$ and $N_{v_\perp} = 100$

ICRH application: Bi-Maxwellian distribution

$$f_H(v_\parallel, v_\perp) = (2\pi)^{-3/2}(v_{th,\parallel}v_{th,\perp}^2)^{-1}\exp[-(v_\parallel/v_{th,\parallel})^2 - (v_\perp/v_{th,\perp})^2]$$

with $v_{th,\parallel} = \sqrt{2C_\parallel T(\psi)/m_H}$, $v_{th,\perp} = \sqrt{2C_\perp T(\psi)/m_H}$, with constants $C_\parallel$ and $C_\perp$

- For $C_\perp = 1$ and $C_\parallel = \{.5, 1., 3., 5.\}$, $P_H = \{61.27\%, 70.50\%, 90.46\%, 94.18\%\}$
  - for small $C_\parallel$, the absorption profile tends to be localized to the resonant layers



- For $C_\parallel = 1$ and $C_\perp = \{.5, 1., 3., 5.\}$, P_H varies by less than 2%

TORIC-CQL3D coupling

Quasilinear diffusion coefficients with FLR approximation are derived and implemented in TORIC

- Power absorption by $\dot{W}$ and $\langle \mathbf{J}\cdot\mathbf{E}\rangle$

$$\mathbf{J}\cdot\mathbf{E} = \dot{\mathbf{W}} + \nabla\cdot\mathbf{T}$$

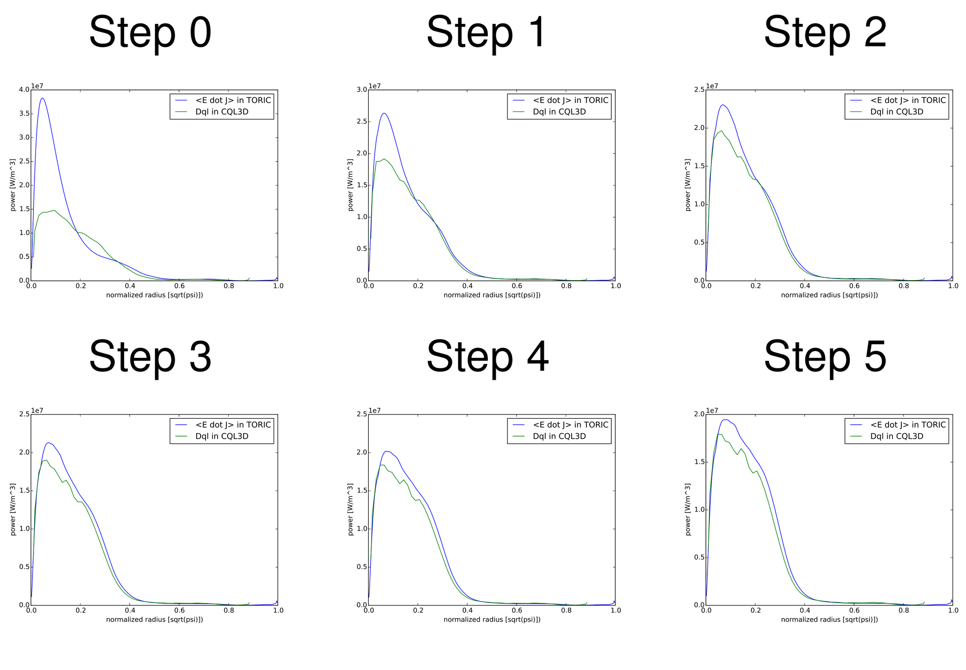
- TORIC uses FLR expansion
- $\dot{W} = \langle \mathbf{J}\cdot\mathbf{E}\rangle$ at the zero order in FLR

$$\begin{aligned}
\dot{W} &= \int_0^{2\pi}\frac{\mathrm{d}\theta}{\mathrm{B}\cdot\nabla\theta}\frac{1}{2}\mathrm{Re}(\mathrm{E}^*\cdot\mathrm{J})|_{\omega=\Omega_{ci}}\\
&= \frac{\omega}{4\pi}\sum_{m_1}\sum_{m_2}\int_0^{2\pi}\frac{\mathrm{d}\theta}{\mathrm{B}\cdot\nabla\theta}e^{i(m_2-m_1)\theta}\left[E_+(m_1)\mathrm{Im}(\hat{L})E_+(m_2)\right]\\
&= \frac{\omega}{4\pi}\sum_{m_1}\sum_{m_2}\int_0^{2\pi}\frac{\mathrm{d}\theta}{\mathrm{B}\cdot\nabla\theta}e^{i(m_2-m_1)\theta}E_+(m_1)\left[\sum_s\frac{\omega_{p,s}^2}{n_s\omega^2}\int \mathrm{d}^3u\frac{u_\perp}{2\gamma}\frac{\partial f_{0,s}}{\partial u_\perp}\frac{\omega}{\omega - k_\parallel v_\parallel - \Omega_s}\right]E_+(m_2)
\end{aligned}$$

Initial results on TORIC-CQL3D iteraction

Similar power profiles between TORIC and CQL3D over iterations are found

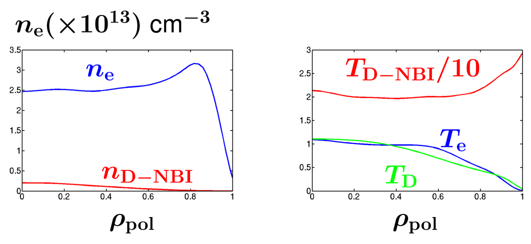
- for Alcator C-Mod shot 1110217027 with $P_{RF} = 1$ MW



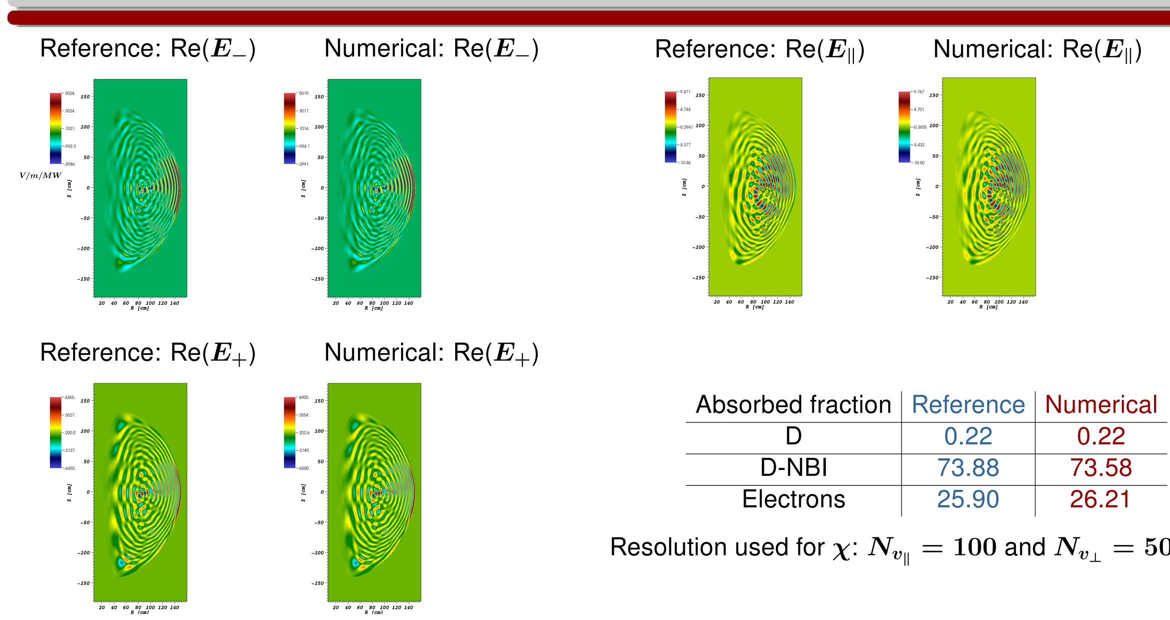
NSTX case

Main parameters:

- $B_T = 0.53$ T, $I_p = 868$ kA
- $T_e(0) = 1.1$ keV, $n_e(0) = 2.5 \times 10^{13}$ cm^{-3}
- $T_D(0) = 1.1$ keV, $T_{D-NBI}(0) = 21.4$ keV
- $n_{D-NBI}(0) = 2.01 \times 10^{12}$ cm^{-3}
- TORIC resolution: $n_{mod} = 31$, $n_{elm} = 200$



Test HHFW: Maxwellian reference case



Absorbed fraction	Reference	Numerical
D	0.22	0.22
D-NBI	73.88	73.58
Electrons	25.90	26.21

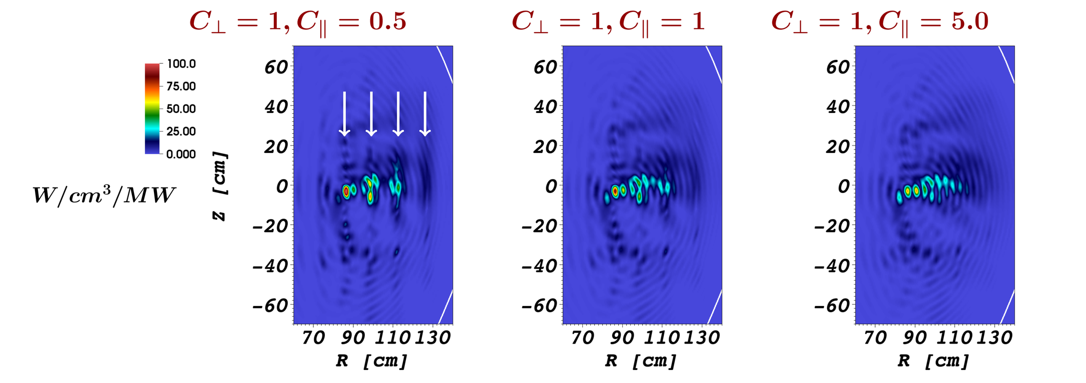
Resolution used for χ: $N_{v_\parallel} = 100$ and $N_{v_\perp} = 50$

HHFW regimes: Bi-Maxwellian distribution

$$f_D(v_\parallel, v_\perp) = (2\pi)^{-3/2}(v_{th,\parallel}v_{th,\perp}^2)^{-1}\exp[-(v_\parallel/v_{th,\parallel})^2 - (v_\perp/v_{th,\perp})^2]$$

with $v_{th,\parallel} = \sqrt{2C_\parallel T(\psi)/m_D}$, $v_{th,\perp} = \sqrt{2C_\perp T(\psi)/m_D}$, with constants $C_\parallel$ and $C_\perp$

- For $C_\perp = 1$ and $C_\parallel = \{.5, 1., 3., 5.\}$, P_{D-NBI}, varied by less than 1%
  - for small $C_\parallel$, the absorption profile tends to be localized to the resonant layers



- For $C_\parallel = 1$ and $C_\perp = \{.5, 1., 3., 5.\}$, the corresponding $P_{D-NBI} = \{70.06, 73.56, 62.84, 48.48\}$

Slowing-down distribution

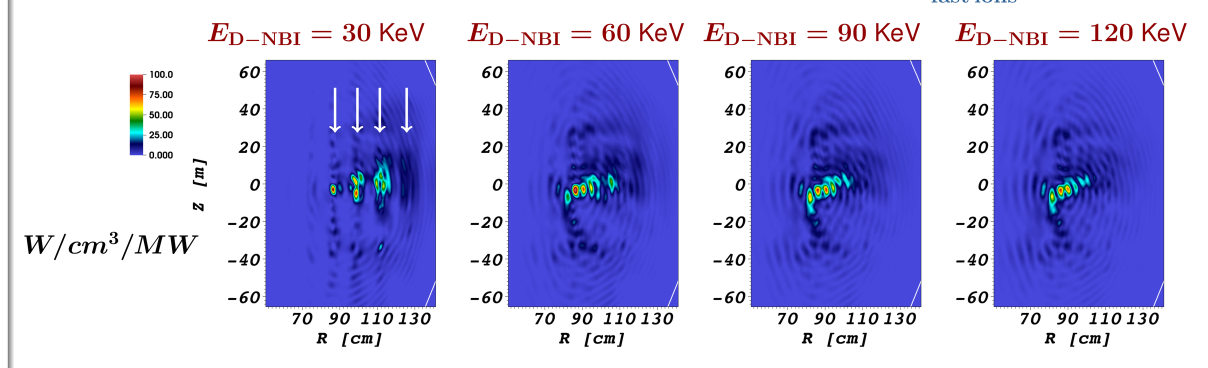
$$f_D(v_\parallel, v_\perp) = \begin{cases}\frac{A}{v_c^3}\frac{1}{1+(v/v_c)^3} & \text{for } v < v_m,\\ 0 & \text{for } v > v_m\end{cases} \qquad v_m \equiv \sqrt{2E_{D-NBI}/m_D}$$

$$A = 3/[4\pi\ln(1+\delta^{-3})], \quad \delta \equiv \frac{v_c}{v_m}, \quad v_c^3 = 3\sqrt{\pi}(m_e/m_D)Z_{eff}v_{th}^3, \quad Z_{eff} \equiv \sum_{ions}\frac{Z_i^2 n_i}{n_e}$$

For $Z_{eff} = 2$ and $E_{D-NBI} = 30, 60, 90, 120$ keV $\Longrightarrow$

$P_{D-NBI} = \{77.84\%, 75.85\%, 70.97\%, 64.71\%\}$

- Similar behavior when varied $C_\perp$ in the bi-Maxwellian case
- Fast ions absorption should decrease with something like $T_{\text{fast ions}}^{-3/2}$ (?)

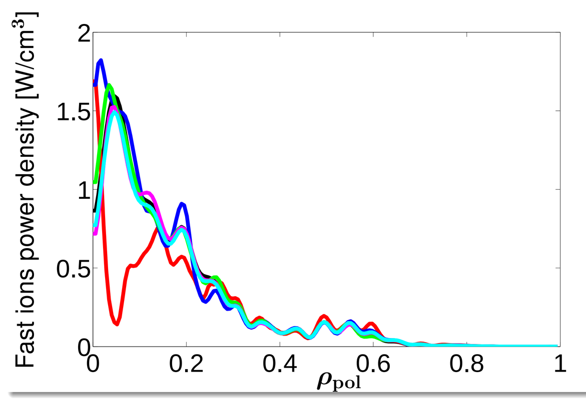


TORIC + P2F code: Maxwellian case

Procedure:

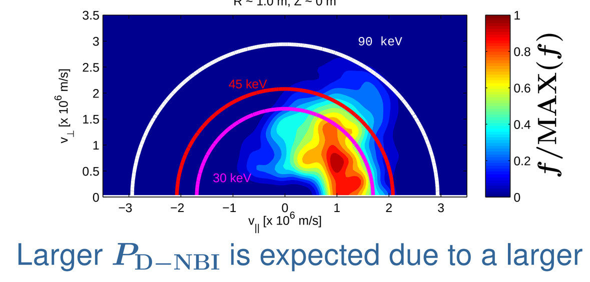
1. generate particle list representing a Maxw.
2. run P2F to obtain a distribution function
3. pre-compute χ with f above
4. run TORIC with pre-computed χ
5. compare TORIC with reference

$n_e(\rho = 0) = 2.5 \times 10^{13}$ cm^{-3}
$n_e(\rho = 1) = 2.5 \times 10^{12}$ cm^{-3}
$T_e(\rho = 0) = 1$ keV; $T_e(\rho = 1) = 0.1$ keV
$n_{FI}(\rho = 0) = 2.0 \times 10^{12}$ cm^{-3}
$n_{FI}(\rho = 1) = 2.0 \times 10^{11}$ cm^{-3}
$T_{FI}(\rho = 1) = 20$ keV; $T_e(\rho = 1) = 5$ keV
Parabolic profiles for n_e, T_e, and n_{FI}
$T_{FI}(\rho) = (T_{FI,0} - T_{FI,1})\left(1 - \rho^2\right)^5 + T_{FI,1}$



Total power differs by less than 1% among 10k, 100k, and 1M cases

NUBEAM particles list (NSTX shot 141711 WITHOUT HHFW)



Abs. fraction	f Maxw.	f non-Maxw.
Electrons	41.80 %	37.99 %
D-NBI	53.94 %	58.12 %

Larger P_{D-NBI} is expected due to a larger f tail formed by the RF application