Nyquist analysis of kinetic effects on the plasma response in NSTX and DIII-D experiments*

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Abstract. Nyquist analysis, a powerful tool in stability theory, has been used to study the plasma response to applied non-axisymmetric magnetic fields in tokamaks. The technique, combined with Padé approximation, can provide a deep physical understanding of the plasma response. Since the plasma response is the result of the linear combination of stable eigenmodes, this method clearly indicates how kinetic effects fundamentally change these eigenmodes in DIII-D and NSTX plasmas. The Nyquist technique can also quantify the contribution of each eigenmode in order to characterize details of the multi-modal plasma response. A modified Nyquist method, including both coil phase and frequency information, is developed. The potential of using this new method to optimize the plasma response for edge localized modes' suppression is presented.

1. Introduction

Externally applied non-axisymmetric magnetic perturbations can strongly modify tokamak plasmas. It leads to the plasma response, which has been systematically observed in tokamaks. Many important subjects, such as the resonant field amplification and the Edge Localized Mode (ELM) control using 3D magnetic coils [1-5], closely relate to plasma response. Therefore, it is essential to better understand the behavior of plasma response and improve its controllability. For instance, the importance of the drift kinetic effects on modifying the plasma response has been demonstrated via the quantitative modeling of NSTX and DIII-D high beta experiments [6, 7]. Nyquist analysis, a powerful tool in stability theory, has been used to study the RWM stability [8,9]. In this work, this Nyquist method is extended to further study the underlying physics of the stable plasma response. When combined with Padé approximation, this method can directly extract the plasma transfer functions from the magnetic measurement of plasma response. Since the linear theory assumes the plasma response is the linear combination of certain stable eigenmodes responding to the external 3D magnetic fields, the method can be used to investigate how the damping rates of these stable eigenmodes in NSTX and DIII-D plasmas are changed by the drift kinetic effects, without resorting to direct stability computations.

The Nyquist technique also provides a direct tool to analyze the so-called multi-mode plasma response, a phenomenon currently under extensive discussions [2]. The n=2 multi-mode response has been detected in DIII-D phase scan experiments. However, many open questions about the multi-mode response remain to be studied, including how many eigenmodes respond to the external fields and how each eigenmode contributes to the total plasma response. The Nyquist method with Padé approximation can be used to directly identify the multi-mode response and separate the contribution of each eigenmode through the extracted plasma transfer functions. A modified Nyquist analysis developed in this work, modifies the transfer function by including one more degree of freedom, namely the toroidal phase of the applied field, in addition to the coil frequency. The new transfer functions can be used to optimize the coil phase and frequency for the amplification of the preferred eigenmode with more edge-localized peeling structures, which can be important to ELM suppression in the experiments. The technique provides an integrated way to analyze magnetic field measurements from poloidally-separated sensor arrays.

2. Nyquist analysis of eigenmodes in plasma response

The standard way to form the Nyquist contour involves varying the field rotation frequency, f, generated by external magnetic perturbation coils from -∞ to +∞, where the '+' and '-' denotes the co and counter plasma current directions. The real and imaginary parts of the total radial perturbed magnetic fields δB^{tot} measured by toroidal arrays of magnetic sensors can be plotted in the complex plane to form the Nyquist contour. In this work, the simulation of Nyquist contour is performed by MARS-K code, which employs the hybrid drift kinetic-MHD model [10] including a thin resistive wall and the 3D coils. In the MARS-K simulation, a high resolution frequency scan is considered to capture the detailed variation of plasma response until the response becomes zero at high frequency due to the shielding effects of the wall.

To illustrate this approach, the procedure is applied to understand plasma response measurements in the NSTX device. Here, the external coil on the low field side (LFS) midplane is used to generate an n=1 magnetic perturbation. The plasma response modified by kinetic effects, the so-called kinetic plasma response, has been shown to be in quantitative agreement with DIII-D and NSTX experimental measurements and has explained the physics of linear increment of the plasma response when β_N approach and exceeds the ideal MHD no-wall limit β_N^{nw} [6,7]. However, it is still not clear how the different effects (e.g. the kinetic effects and the plasma rotation) change the plasma eigenmodes, particularly the least stable mode that closely relates to the global plasma stability. For instance, Fig. 1(a) presents the β dependence of simulated plasma response with or without the kinetic effects and the plasma rotation in MHD equations, where the fluid response is the solution of ideal MHD and a coil frequency f=30 Hz is applied. The simulated δB^{tot} is measured by the upper sensor at LFS in NSTX experiments. The fluid model results show a relatively small amplitude response. Typically, reduced amplification of the applied field is associated with an decrease in the proximity to marginal stability. However, the Nyquist analysis shows that the plasma is unstable to resistive wall mode if the fluid model is used without including rotation. With the other three models, the plasma is stable while $\beta_N > \beta_N^{nw}$. The smaller amplification of fluid response without rotation is due to the shielding effect of eddy current in the resistive wall [7].

Nyquist technique is particularly applicable to investigate the underlying plasma information in both experiments and the simulation. The simulated Nyquist contour of δB^{tot} is plotted in Fig. 1(b). The equilibrium at β_N/β_N^{nw}=1.05 is used in the simulation, where β_N^{nw}=4.75 is the conventional no-wall beta limit predict by ideal MHD with no rotation. In Fig. 1(b), the contours corresponding to the four cases of plasma response in Fig.1(a) are noted as the fluid contour and kinetic contour with or without rotation respectively. The red and blue diamonds are the markers of plasma response at f=-10 Hz and 10 Hz respectively. Clearly, Fig.1(b) shows the fluid contour with no rotation at the low frequency (e.g. $f = \pm 10$Hz) and the contour direction (counter clockwise) are different from other three cases. The differences indicate the kinetic effects can strongly modify the ideal response even without any plasma rotation. Moreover, the fluid contour with rotation is similar to the two kinetic contours since the physics of ion acoustic damping is dominant at high rotation in NSTX experiments. Note that the ion acoustic damping is the fluid description of the kinetic resonance of passing ions in ideal MHD theory.

Besides the aforementioned results, the Nyquist contour, associated with the Padé approximation, is powerful to learn the fundamental physics of plasma response and understand how kinetic effects and plasma rotation change the plasma stability. Considering the linear plasma response, the Nyquist contour measured by the magnetic sensor can be fitted by the plasma transfer function

$$P = \sum_{j=1}^{N} \frac{n_i}{i2\pi f - \gamma_i}, \tag{1}$$

Here, each term in the transfer function corresponds to one eigenmode. n_i is the coefficient of ith eigenmode (not the toroidal mode number). γ_i is the the eigenvalue of ith eigenmode where its real part γ_i^{re} represents the mode damping/growth rate. The fitted transfer function can be used to infer the plasma stability. Applying Eq.(1) to fit the four contours in Fig. 1(b), the transfer functions extracted from the fluid contour with no rotation finds one unstable eigenmode with γ_{max}^{re}=13.66 Hz > 0, which indicates the plasma is unstable while $\beta_N > \beta_N^{nw}$. In the fluid case with rotation, the extracted least stable mode has the damping rates γ_{max}^{re}=-28.97 Hz. The least stable modes extracted from the kinetic contours without and with rotation, gives γ_{max}^{re}=-34.3 Hz and γ_{max}^{re}=-13.22 Hz respectively, which clearly shows that kinetic effects significantly modify the plasma stability. Comparing γ_{max}^{re} values of two kinetic contours also indicate the flow rotation in the MHD equations slightly destabilizes the plasma due to the plasma inertia [7].

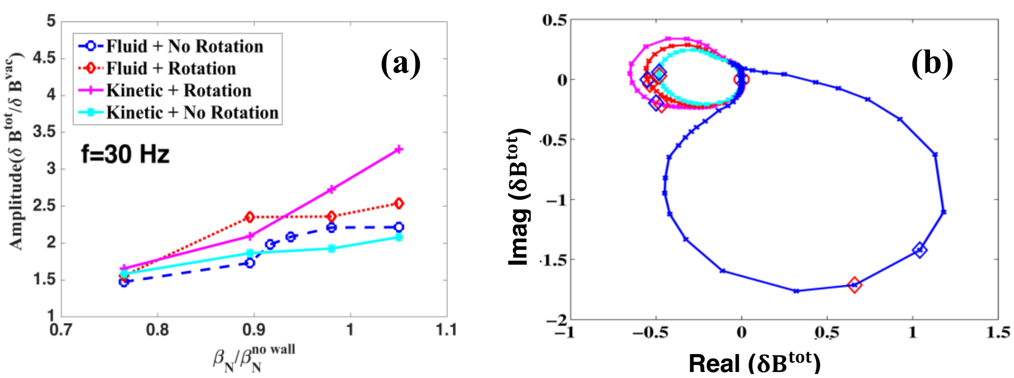

Fig.1. (a) β dependence n=1 plasma response and (b) Nyquist contours with β_N/β_N^{nw}=1.05 in NSTX plasma. The ideal MHD without plasma flow (blue), with flow (red) and kinetic-MHD without plasma flow (magenta) and with flow (cyan) are used to simulate the plasma response and Nyquist contours, respectively. δB^{tot} is measured by the upper sensor at LFS in NSTX plasmas.

Nyquist analysis is also applied to study DIII-D experiments based on the discharge 135759 used in [1,6], where β_N/β_N^{nw}=1.06. The Nyquist contours in terms of four cases as defined in Fig. 1(b) are also simulated with DIII-D equilibrium and presented in Fig.2(a), where the n=1 Nyquist contour in DIII-D case is formed by scanning the frequency of the applied field from the two-row I-coil, which was configured with a toroidal phase difference between the upper and lower rows of 240°. The radial magnetic perturbation is measured by radial sensor located on the LFS at the midplane. The Padé approximation is applied to extract the eigenmodes in four cases in Fig.2(a). The extracted unstable mode from the fluid contour without rotation has γ_{max}^{re}=79.24 Hz. The least stable mode in the fluid case with rotation has γ_{max}^{re}=-20.44 Hz. The cases of kinetic response without and with rotation, gives γ_{max}^{re}=-9.19 Hz and γ_{max}^{re}=-8.69 Hz respectively. Similar to NSTX results, the rotational and kinetic effects can stabilize the plasma. Due to the relatively low rotation in the discharge 135759 [6] (compared to NSTX), the two kinetic contours are quantitatively different from the fluid case with rotation. At low rotation, the kinetic effects contribute via the precession resonance and the bounce resonance of trapped ions. However, the ion acoustic damping in ideal MHD model cannot well describe these two kinetic effects. Previous work has shown that the hybrid kinetic-MHD model is needed to simulate the response in DIII-D and NSTX experiments [5,6] especially in the high β plasma regime. Moreover, the similar behavior of two kinetic contours means the effect of flow in MHD formulation is not significant in this DIII-D experiment even though neutral beam heating directed only in the plasma current direction was used.

Fig.2(b) further investigates the impact of EXB rotation on the kinetic stabilization of plasma through the Nyquist analysis. It shows a general decrease in the amplitude of the kinetic contour with rotation while EXB frequency ω_E increases. The damping rates of the extracted plasma transfer function show that the smaller rotation stabilizes the least stable model when the precession resonance of trapped ions contributes to the kinetic dissipation at low rotation.

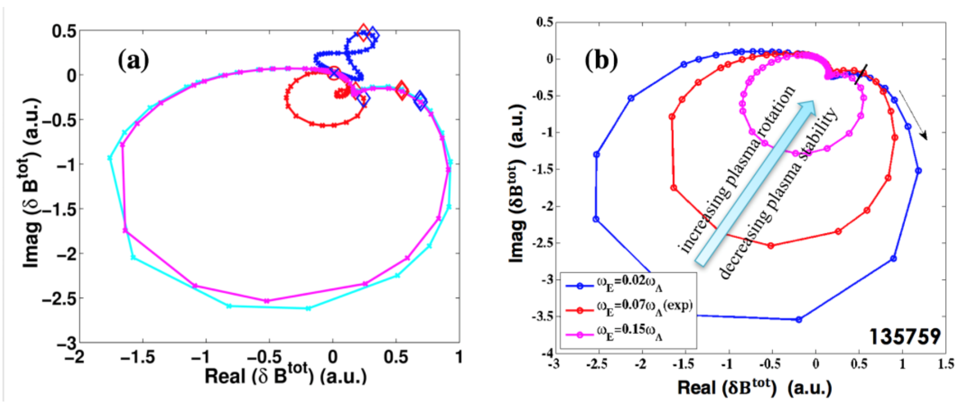

Fig.2. (a) n=1 Nyquist contours in DIII-D plasma for discharge 135759 with four different models defined as in Fig.1(a), where β_N/β_N^{nw}=1.06. (b) The kinetic contours with rotation are simulated with different EXB rotation. ω_E is normalized by the Alfvén frequency ω_A computed at the geometric plasma center.

3. Nyquist analysis of multi-mode response

It is interesting to note that increased plasma rotation causes the smaller contour in Fig.2(b) to deviate from a circular contour more than the other two cases. A circular contour is consistent with a single-mode response due to the existence of a single pole. Therefore, the contribution from sub-dominant modes increases with the plasma rotation and kinetic damping. To better understand this multi-mode response, the Padé approximation is applied to extract the eigenmodes from the case with $\omega_E = 0.15\omega_A$. The plotted contours in Fig. 3 correspond to each eigenmode. It finds the three major eigenmodes which contribute to the total kinetic response. The eigenvalue of the first least stable mode is γ_1=-13.34-0.28i Hz. The γ_2 of the secondary mode is γ_2=-16.85+26.9i Hz. The third mode is much more stable with γ_3=-77.7-12.24i Hz. Since the kinetic contour is contributed by the extracted transfer functions of three major eigenmodes, it directly characterizes the multi-mode response and quantifies the contribution from each mode. This allows the dominant contribution of the second least stable mode (red) to be identified. This multi-mode response is different from the conventional understanding that the least stable mode always plays the dominant role in the plasma response. Looking further into the details of the contours, the 1st and 2nd modes contribute similarly to the plasma response at the zero coil frequency (marked as 'o'). When the coil frequency is 10Hz (marked as '◊'), the amplitude of 1st mode decreases while the 2nd mode contribution increases until it becomes the dominant contribution. This suggests that changing the coil frequency can provide one more degree of freedom (in addition to the upper-lower coil toroidal phase difference) to amplify a preferred eigenmode for ELM suppression.

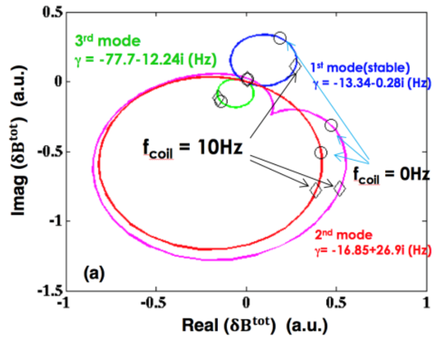

Fig. 3. Decomposition of Nyquist contours (magenta) for the case with $\omega_E = 0.15\omega_A$ *in Fig. 2 by Padé approximation. Three dominant poles (three circles) contributed to the Nyquist contour are plotted respectively.*

In DIII-D experiments, n=2 plasma response measurements have shown evidence of multi-mode response and ELM suppression thorough a coil phasing scan [2]. However, many open questions related to [2] remain to be resolved, e.g. how many and which major eigenmodes contribute to the multi-mode response, and how to optimize the coil configuration to achieve more reliable ELM suppression. A modified Nyquist analysis including the information of coil phasing is proposed in this work. The modified plasma transfer function depending on the coil phase and frequency can be developed by assuming the response is linear. The transfer function with respect to different sensor in the machine is defined as

$$\mathrm{P_j}(\phi_{up}, \phi_{\mathrm{low}}, \mathrm{f}) = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{\mathrm{a_i^j} e^{\mathrm{i}\phi_{\mathrm{up}}} + \mathrm{b_i^j} e^{\mathrm{i}\phi_{\mathrm{low}}}}{\mathrm{f}+\gamma_\mathrm{i}}, \tag{2}$$

where $\mathrm{P_j}$ is the plasma transfer function of jth sensors. γ_i is still the eigenvalue of ith eigenmode and must be the same on different sensors. $\mathrm{a_i^j}$ and $\mathrm{b_i^j}$ are the coefficients of each eigenmode for the jth sensor. ϕ_{up} and ϕ_{low} are the phases of upper and lower coils respectively. Since the phase difference between the upper and lower coils $\Delta\phi = \phi_{\mathrm{up}} - \phi_{\mathrm{low}}$, is the fundamental variable in the experiments, Eq.(2) can be rewritten as

$$\mathrm{P_j}(\Delta\phi, \mathrm{f}) = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{\mathrm{a_i^j} + \mathrm{b_i^j} e^{-\mathrm{i}\Delta\phi}}{\mathrm{f}+\gamma_\mathrm{i}}, \tag{3}$$

Eq.(3) can be used to fit the sensor measurements from both the experiment and the numerical simulation. One validation of the new transfer function is made by fitting the magnetic response measured in DIII-D n=2 plasma response experiments [2]. In this experiments, the coil phase is scanned from 0 to 360 degree with zero coil frequency. Therefore, in this special case, Eq.(2) is reduced to

$$\mathrm{P_j}(\Delta\phi) = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{\mathrm{a_i^j} + \mathrm{b_i^j} e^{-\mathrm{i}\Delta\phi}}{\gamma_\mathrm{i}}, \tag{4}$$

In the fitting procedure, three poloidal sensors are considered: MPID66M at low field side, MPID1A and MPID1B at high field side. The measurements of each sensor array are fit by the corresponding $\mathrm{P_j}(\Delta\phi)$ simultaneously, providing an integrated way to analyze poloidally-separated magnetic field measurements. Here, a one pole model (N=1) and a three pole model (N=3) are applied to fit to the experimental data respectively. The comparison of signal amplitude between the fitted plasma transfer functions and the experimental data, is reported in Fig. 4. It clearly shows the one pole case has a poor agreement with the experiments particularly at MPID1B sensor. On the other hand, the three pole case agrees well for both amplitude and phase of plasma response at each sensor. The two pole model is also tested and can be better than one pole model, but the fit is not as good as the three pole transfer function as shown in Fig. 4. Besides indicating the multi-mode response, the result further points out that three eigenmodes play a major role to respond to the coil perturbations. Furthermore, the results also confirm that the plasma response observed in the experiments is linear. However, due to the zero coil frequency, the value of γ_i is meaningless since the dimension of γ_i can be determined only when f is finite.

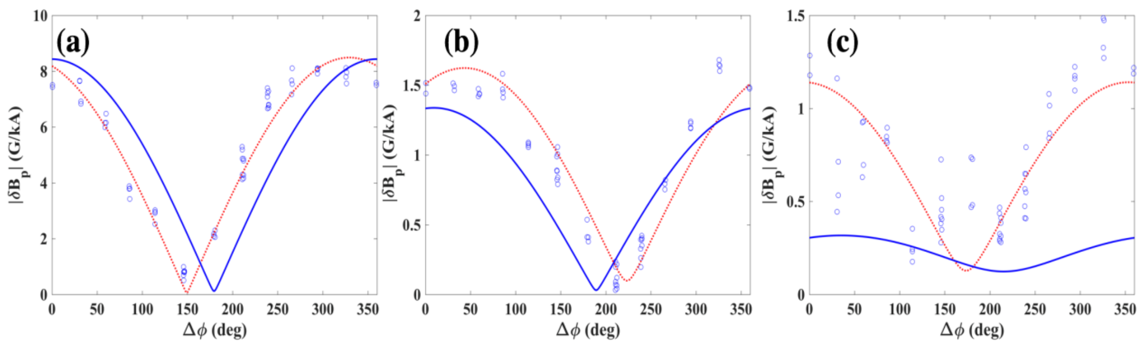

Fig.4. Phase scan of plasma response measured by the magnetic sensors (a) MPID66M at the low field side, (b) MPID1A and (c) MPID1B at the high field side. The amplitude of poloidal magnetic perturbation $|\delta B_p|$ *is compared among the experimental measurements('o'), the extracted one pole (solid line) and three pole (dotted line) transfer function.*

The transfer function of Eq.(3) is further verified with the n=1 fluid response simulated by MARS-F code, where one D-shape numerical equilibrium with β_N=0.97, aspect ratio A=0.24 is adopted. A radial sensor at the midplane of LFS is assumed to measure the fluid response. In the MARS-F simulation, the coil phasing scan is performed at f=10 Hz, 30 Hz and 60 Hz respectively. Then Eq.(3) is applied to fit these simulated sensor measurements as shown in Fig. 5, where the extracted transfer functions agree well with the MARS-F simulation.

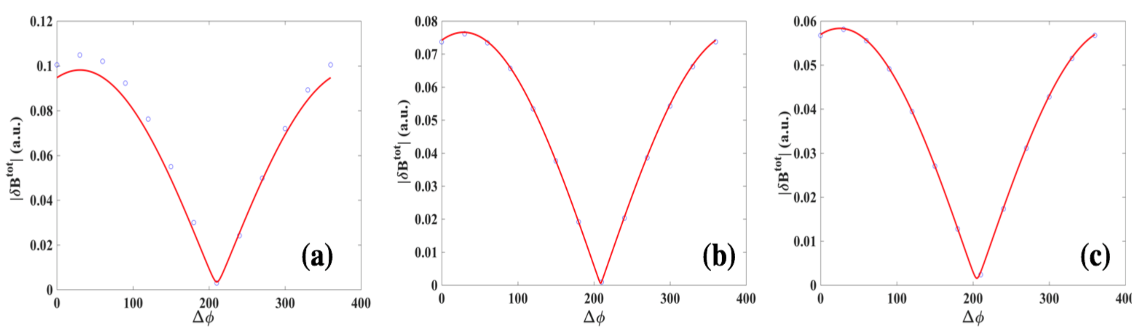

Fig.5. Phase scan of plasma response measured by the simulated radial sensors at the midplane of low field side. The radial magnetic perturbation is compared between the direct MARS-F simulation ('o') and the extracted transfer function ('solid line').

In this numerical simulation, we find the plasma response is contributed by the two major eigenmodes, where the transfer function measured at the simulated sensor is

$$\mathrm{P} = \frac{0.9+0.17i+(0.92-0.66i)e^{-i\Delta\phi}}{if(-30.71-2.22i)} + \frac{2.27-0.094i+(16.9+0.027i)e^{-i\Delta\phi}}{if-(-458.7+0.299i)}. \quad (5)$$

On the right hand side of Eq.(5), the first term and the second term represent the least stable mode λ_1 and the secondary mode λ_2. To verify the reliability of extracted transfer function, the Nyquist contour simulated by MARS-F is compared with the one given by Eq.(5), where the coil phase $\Delta\phi$=240 deg is adopted in Fig. 6. It shows the transfer function can well repeat the response in whole frequency region, though Eq.(5) is obtained from the phase scan with the three low frequencies.

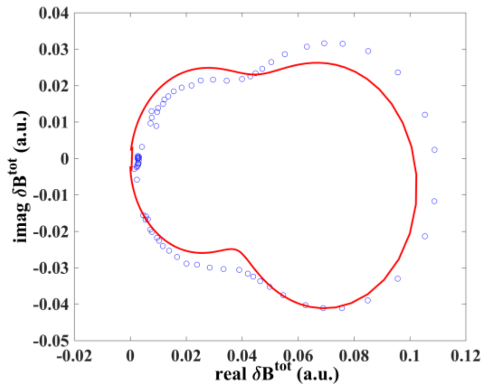

Fig.6. Comparison of Nyquist contour between the MARS-F direct simulation ('o') and the extracted transfer functions (solid line).

The DIII-D example and the numerical simulation indicate that the new method can be valid in both experiments and the simulation. The new method greatly reduces the requirement of extracting the plasma transfer function, whereas the conventional Nyquist analysis requires a scan of the coil frequency over a wide range. The plasma response in the high frequency can be too weak and hard to be measured with small signal noise. It can also be polluted by other high frequency mode e.g. ELM. The new method can avoid the high frequency scan and stay with the low coil frequency by introducing the coil phase information. The other advantage of this method is to optimize the coil phase and frequency to amplify the preferred eigenmode for ELM suppression. For instance, we can choose the coil phase and frequency through the transfer function to amplify the secondary mode, which may have more edge-localized peeling structures, leading to the sustainment of small islands and potentially to stochastic field lines near the plasma edge. Fig.7 illustrates this idea by calculating $|\lambda_1/\lambda_2|$ in Fig.7(a) and $|\lambda_2/\lambda_1|$ in Fig.7(b) respectively. It clearly shows the best phase to amplify least stable eigenmode is zero degree. The amplification of secondary mode is at 240 degrees. In particular, increasing the coil frequency can further amplify the secondary mode. This finite coil frequency may also be important to avoid the core mode locking.

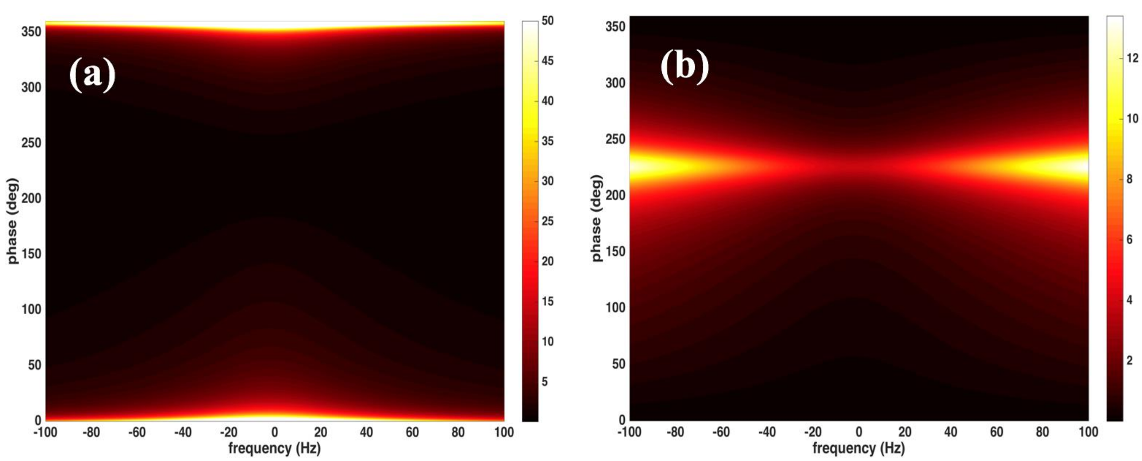

Fig.7. The phase and frequency scan of two extracted transfer functions' (a) $|\lambda_1/\lambda_2|$ *and (b)* $|\lambda_2/\lambda_1|$.

4. Summary

Nyquist analysis is a very powerful tool that can help to reveal a range of underlying physics associated with 3D fields. As discussed in this paper, Nyquist analysis shows the drift kinetic modification on the eigenmodes, which eventually results in the modification of plasma response. Damping rates of extracted transfer function can be inferred and used to quantify plasma stability. The dissipation of kinetic effects makes the plasma more stable through the stabilization of the eigenmodes. Nyquist analysis also shows increasing EXB rotation can decrease the plasma stability in kinetic-MHD approach and make kinetic plasma response multi-modal. It finds the secondary mode can be dominant in DIII-D kinetic response. Without the Nyquist analysis, it is hard to obtain a quantitative physical understanding in both experiments and time dependent MHD simulation when plasma is stable. A modified Nyquist method is developed in this work that includes both coil phase and frequency dependences. The new method reduces the experimental time required to measure the Nyquist contour of a potentially multi-mode plasma response, making it feasible to pursue in multiple plasma regimes. The new transfer functions give us the capability to optimize the coil phase and frequency for amplifying the preferred eigenmode during ELM control experiments. Future work is aimed at continuing to utilize the Nyquist contour method in order to design MHD control systems and to better predict plasma behavior in future experiments. The modified Nyquist method can also be a candidate for active MHD spectroscopy and real-time monitoring of the plasma stability in long-pulse fusion reactors.

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