Self-consistent optimization of neoclassical toroidal torque with anisotropic perturbed equilibrium in tokamaks

J.-K. Park[1], N. C. Logan[1], A. H. Glasser[2], Z. R. Wang[1], E. Kolemen[3], and A. H. Boozer[4]
(email: jpark@pppl.gov)

[1]Princeton Plasma Physics Laboratory, Princeton, NJ08543, USA
[2]Fusion Theory and Computation Inc., USA
[3]Princeton University, Princeton, NJ08543, USA
[4]Columbia University, New York, NY10027, USA

Control of toroidal rotation is an important issue for tokamaks and ITER since the rotation and its shear can significantly modify plasma stability from microscopic to macroscopic scales. One of the promising actuators is the non-axisymmetric (3D) magnetic perturbation, which is under consideration in ITER for other purposes such as error field or ELM control, since neoclassical toroidal momentum transport can be substantially increased in the presence of non-axisymmetry. This so-called neoclassical toroidal viscosity (NTV) process is quadratically non-linear to the magnitude of the applied field, and furthermore has complex dependency on the 3D field distribution. Therefore, the optimization of the 3D field distribution for NTV and ultimately rotation control is non-trivial, requiring a number of numerical simulations up to a thousand code runs even with advanced optimizers. In this paper we present a new method that entirely redefines the optimizing process, by solving 3D equilibrium and NTV consistent with each other and constructing the so-called torque response matrix function. As shown in Figure 1, the NTV profile optimization can be achieved by a single code run, with much better efficiency and accuracy than the previous methods with stellarator optimizers [1].

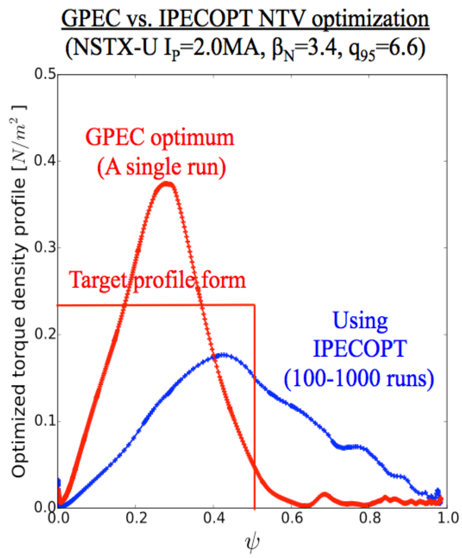

Figure 1. Optimized torque profile to maximize torque in ψ<0.5 while minimizing torque in ψ>0.5 for an NSTX-U target, using GPEC vs. IPEC-coupled stellarator optimizers.

The new, general perturbed equilibrium code (GPEC) solves the single-fluid quasi-neutral anisotropic pressure perturbed equilibrium in the first gyro-radius ordering; $F[\xi] = \delta j \times B + j \times \delta B + \nabla\cdot(\xi\cdot\nabla p) - \nabla\cdot\delta\Pi = 0$ from a Maxwellian plasma in axisymmetry. NTV torque comes from the toroidal component of the anisotropic pressure force $\nabla\cdot\delta\Pi$, and the net torque arises at the second order in perturbation from the surface average on perturbed flux surfaces. The anisotropic pressure force $\nabla\cdot\delta\Pi$ that appears in the force balance has the same kinetic origin as that in the NTV, but is the first order change locally inducing torque distribution and non-ambipolar currents that can modify the field penetration. If NTV is calculated based on the δB established by this anisotropic perturbed equilibrium, both transport and equilibrium calculations will be consistent with each other. This force operator, however, is no longer self-adjoint due to the finite toroidal torque, and therefore must be solved directly for each of three components, rather than using the variational method with δW that is popular in equilibrium calculations. Nevertheless, the direct treatment yields the modified kinetic Euler-Lagrange equation $\left(F\Xi_\psi{}' + K_R\Xi_\psi\right)' - \left(K_L^{+}\Xi_\psi' + G\Xi_\psi\right) = 0$, similarly to self-adjoint case. Here Ξ_ψ is the vector containing Fourier components of radial displacements and F, K, G are matrices containing fluid and kinetic contributions. For IPEC and DCON, F, G matrices become Hermitian and $K_R = K_L$, and in cylindrical geometry it becomes the Newcomb equation with the matrices being scalar.

Integrating energy and torque, one can show $\delta W + \frac{i\tau_\varphi}{n} = -\int \xi \cdot F[\xi] = \Xi_\psi^+ \cdot W_P \cdot \Xi_\psi$, in which only the surface term remains since the volumetric term vanishes by the force balance. The anti-Hermitian part of the plasma response matrix function $W_P(\psi)$ is the torque response matrix function $\mathrm{T}(\psi)$, which provides the self-consistently calculated NTV in any point of radius as the quadratic form involving the external field Φ on the control surface, i.e. $\Phi^+ \cdot \mathrm{T} \cdot \Phi$. Given $\mathrm{T}(\psi)$, one can immediately answer various questions for optimization. An important example is the maximum (or minimum) torque possible for any arbitrary interval (ψ_1, ψ_2), and corresponding 3D field distribution, with the integrated total torque up to the boundary ψ_b fixed. The answer is the maximum (or minimum) eigenvalue and eigenvector of the composite matrix $\mathrm{T}^{-1}(\psi_b)[\mathrm{T}(\psi_2) - \mathrm{T}(\psi_1)]$. The example shown in Figure 1 is obtained by simply calculating the first eigenvector of this matrix for $0< \psi <0.5$, and applying it to plasma as an external field on the boundary. Furthermore, the eigenvectors provide a way to properly order and decompose 3D fields with respect to local torque. In many cases, it has been shown that negative m modes (backward helicity modes) play an important role in balancing the positive m modes for local torque optimization.

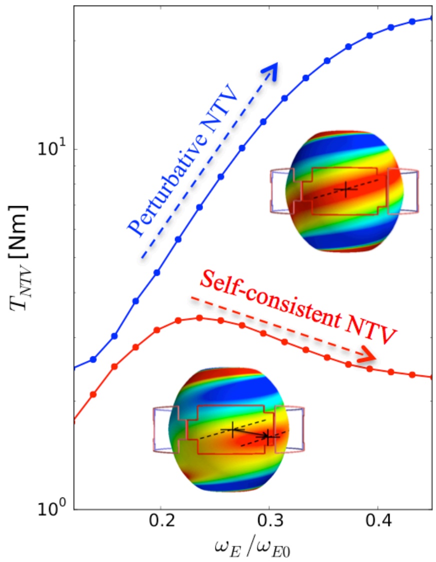

Figure 2. Perturbative NTV with IPEC vs. self-consistent NTV with GPEC for an NSTX-U target with scaled E×B. Subfigures in 3D shows the NSTX-U plasma response to applied 3D fields from midplane coils.

NTV torque profiles obtained by this method are self-consistent across equilibrium and transport, which can be very important whenever local or global torque is substantial, due to the strong toroidal phase shift in response and consequent inefficiency in coupling between plasma and external field. This is called the self-shieling process [2] as illustrated in Figure 2, which can significantly change NTV predictions. In fact, the importance of self-consistent calculations with $\nabla \cdot \delta\Pi$ in 3D response has been highlighted by recent MARS-K applications to DIII-D [3] and NSTX. In principle, MARS-K and GPEC should provide the same results in the zero-frequency limit, and this has been successfully verified in this work. The unique feature of GPEC is the full eigenmode structure of 3D plasma response consistent with $\nabla \cdot \delta\Pi$ and neoclassical toroidal torque, which thereby provides a new and systematic way of optimization for NTV and non-resonant fields.

[1] S. Lazerson, J.-K. Park et al., Plasma Phys. Control. Fusion **57**, 104001 (2015)
[2] A. H. Boozer, Phys. Rev. Lett. **86**, 5059 (2001)
[3] Z. R. Wang, M. Lanctot, Y. Liu, J.-K. Park et al., Phys. Rev. Lett. **114**, 145005 (2015)

*This research was supported by U.S. DOE contracts #DE-AC02-09CH11466.