Global Alfvén eigenmode stability dependence on fast-ion distribution

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Abstract. During the initial operation of NSTX-U [Ono, *et al*., Nucl. Fusion **55** (2015) 073007] it was discovered that injection of nearly tangential neutral beams (creating a population of high pitch fast-ions) would strongly suppress an otherwise ubiquitous fast-ion driven instability on NSTX [Ono et al 2000 Nucl. Fusion **40** 557], the Global Alfvén eigenmode (GAE). This observation was shown to be consistent with an analytical theoretical model of the Doppler-shifted ion-cyclotron resonant drive (DCR) for Global Alfvén eigenmodes (GAE) [N.N. Gorelenkov, *et al.,* Nucl. Fusion **43** (2003) 228]. In this paper we combine the prediction of the DCR theory with a simple analytic dispersion relation for GAE to create an algorithm for predicting the mode numbers and frequencies of potentially unstable GAE. This model has been applied to NSTX data to qualitatively reproduce the observed scaling of GAE frequencies and mode numbers with toroidal field on NSTX.

1. Introduction

Control of fast ion populations (e.g., fusion-α's, beam ions) is important to fusion reactors like ITER as fast ions are used to heat the thermal plasma, drive current and their loss can damage plasma facing components. Instabilities excited by the free energy in non-thermal fast ion populations change the fast ion distributions and can thus affect the current drive and heating profiles, and can cause losses or channel energy from the fast ion population into the thermal plasma. Unlike typical operational regimes in present tokamaks, the fast ions in ITER (fusion-α's and 1 MeV beam ions) will be super-Alfvénic. No present tokamaks can operate in the fast-ion parameter regime expected for ITER and theoretical models are needed to predict the effects of fast ion driven instabilities and provide guidance on how to ameliorate the potentially adverse behaviors. Spherical tokamaks like NSTX [1] typically operate with super-Alfvénic beam ions, and here we demonstrate that the theoretical model of the Doppler-shifted ion-cyclotron instability drive for Global Alfvén eigenmodes (GAE) is qualitatively consistent with observations of GAE on NSTX and NSTX-U [2]. The GAE propagate counter to the tangentially injected beam ions and are excited through an ion cyclotron resonance with the co-moving beam ions [3]. The GAE are seen with a range of toroidal mode numbers and frequencies [4]. While excited through an ion-cyclotron resonance, the GAE frequencies are down-shifted from the ion-cyclotron frequency by the motion of the beam ions, that is, in the moving frame of the beam ions the GAE frequency is up-shifted to the ion-cyclotron frequency.

In the initial operation of NSTX-U it has been experimentally demonstrated that the otherwise ubiquitous GAE activity can be completely suppressed with the judicious injection of a relatively small amount of fast ions with trajectories nearly parallel to the magnetic field, that is, $V_{//}/V \approx 1$. The experiments were possible as NSTX-U was outfitted with three additional beam sources with tangency radii outside the magnetic axis. Neutral beam ions injected from these sources have trajectories nearly parallel to the equilibrium magnetic field. Fig. 1 shows an example where the addition of a source injecting high-pitch fast ions (red curve, Fig. 1c) suppresses the GAE (Figs. 1a and 1b) [5]. The black, red, blue and green contours in Fig. 1a show the presence of GAE bursts with toroidal mode numbers from n = -8 to n = -11. The suppression occurs quickly, showing that it is a fast-ion effect rather than due to an equilibrium change. The analytic Doppler-shifted ion cyclotron resonance (DCR) drive model describing fast-ion drive of the GAE [3] was used to qualitatively explain the suppression. Additional numerical modeling with the HYM code [6] very successfully reproduces the observed mode suppression.

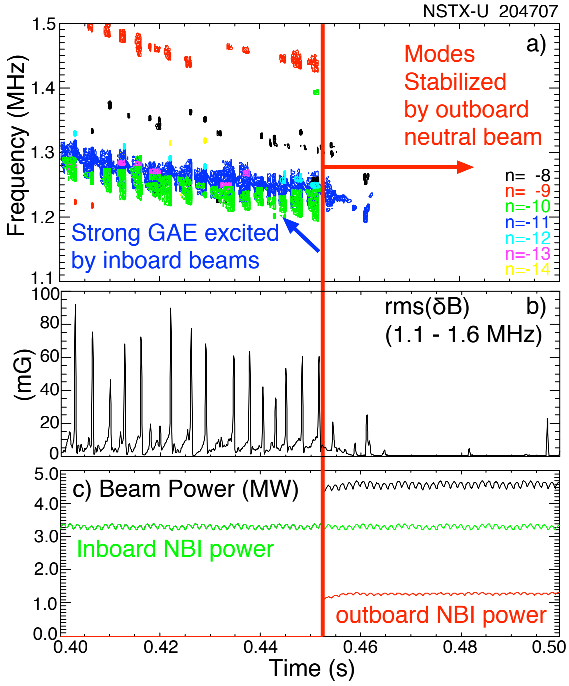

Fig. 1. a) color-coded spectrogram showing GAE activity. Dominant modes are n=-10 (green) and n=-11 (blue). b) RMS magnetic fluctuation amplitude over the frequency range 1.1-1.6 MHz, c) green curve is inboard beam power, red curve is off-axis beam power (source 2c).

Motivated by the strong qualitative agreement between the experimental observations of GAE suppression and the predictions of the analytic DCR theory, a simple model was developed to predict the toroidal mode numbers and frequencies for unstable GAE in NSTX plasmas. In this paper we apply this simple analytic DCR model of the GAE stability to explain the experimentally observed scaling of the GAE frequencies and toroidal mode numbers with toroidal field. We find good agreement, considering the simplicity of the model, which provides further validation for the qualitative value of the DCR model.

2. Application of the DCR model to predict unstable GAE

The DCR is a local model which provides qualitative guidance as to what changes to the fast-ion distribution will tend to increase or decrease the stability of the GAE. The energy transfer between fast-ions and a GAE mode was calculated analytically and the result was the prediction that *resonant* fast-ions which satisfied the condition that $1.9 < k_{\perp} \rho_L < 3.9$ would transfer energy to the mode, destabilizing it and resonant fast-ions with $k_{\perp} \rho_L < 1.9$ would take energy from the mode, damping it. Here, $k_{\perp}$ is the perpendicular wavenumber and ρ_L is the fast-ion larmor radius.

There are two steps in predicting the range of toroidal mode numbers and frequencies of GAE which might be unstable. First, we calculate the local GAE dispersion relation for each pair of poloidal and toroidal mode numbers, (m,n),

$$\omega_{mode} = k_{//} V_{Alvén} + /n/\omega_{rotation},$$

where $\omega_{rotation}$ is the bulk plasma rotation which will provide a Doppler shift of the GAE mode frequency (not to be confused with the Doppler shift due to fast ion motion in the resonance condition). A simple expression for the parallel wavenumber is used,

$$k_{//} \approx [m - n\, q(r)] / [R\, q(r)],$$

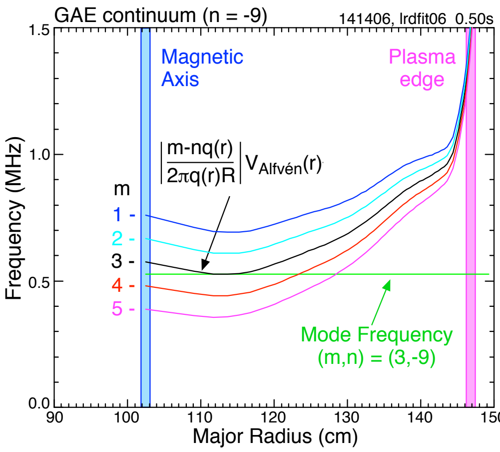

Fig. 2. Calculation of the GAE continuum for the n=-11 for m=4-8 modes (ω=$/k_{//}V_{Alfvén}$-$n\omega_{rot}/$). Indicated in green is the observed laboratory mode frequency. Overlaid are the reflectometer data (black rectangles), and modeled GAE displacement (red), simulated reflectometer response (blue).

The mode is assumed to be spatially localized near the minimum of the dispersion relation and the mode frequency is evaluated there. This is illustrated for the n = 9 GAE in Fig. 2 for a range of poloidal mode numbers. The minimum for all of the poloidal mode numbers is near the major radius of R ≈ 1.2m.

The parallel wavenumber and mode frequency determined from the dispersion relation are then used in the Doppler-shifted ion-cyclotron resonance condition,

$$\omega_{mode} + (k_{//} \pm 1/qR)V_{b//} = \omega_{ci}$$

to estimate the parallel velocity of fast-ions which would be resonant with the mode (here $V_{b//}$ is the parallel beam-ion velocity). This approach is very simplified and recent theoretical modeling has suggested that the dispersion relation itself also depends on the fast-ion distribution function [7]. The DCR theory predicts which fast-ions along this resonance curve are stabilizing or destabilizing. The application of this model for the rough prediction of GAE stability is illustrated in Fig. 3 with a fast-ion distribution calculated in TRANSP with NUBEAM, for the shot used in Fig. 2. There are two solutions of $V_{b//}$ for the parallel resonance equation and they are shown in Fig. 3 in purple for the (m,n) = (3, -9) GAE. The dashed black curve which crosses it, the 'marginal stability curve', shows the fast-ions for which $k_\perp \rho_L = 1.9$. Fast-ions on the resonance curve below the black curve will damp the GAE, those above the black curve will drive the mode. The brown (n=-8), purple (n=-9), green (n=-10) and blue (-11) dots indicate the crossing points of the (lower) resonance and marginal stability curves for the indicated poloidal mode numbers. (For simplicity, it is assumed that the lower velocity resonance would dominate the stability calculation due to the higher density of fast ions at lower energy.)

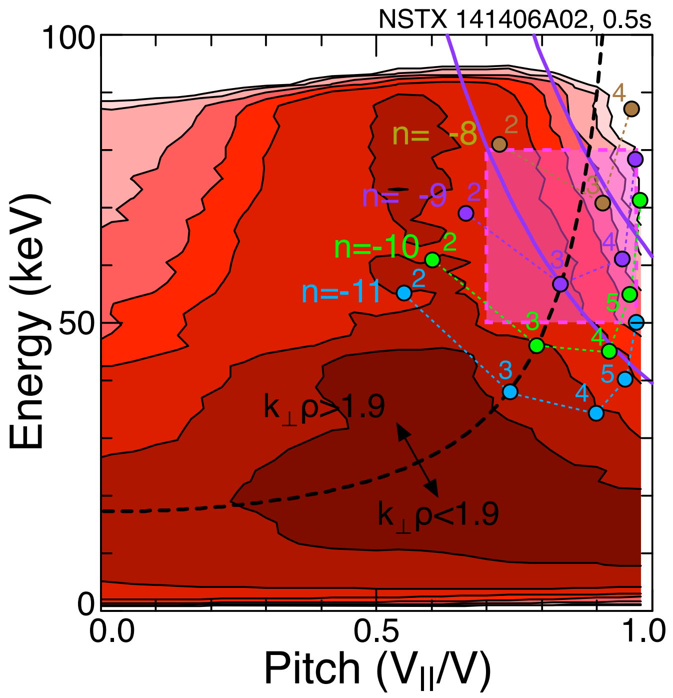

Fig. 3. TRANSP fast-ion distributions for before and after the outboard beam injection. The dashed oval indicates fast ions believed responsible for GAE suppression. Distribution functions are average over whole plasma volume. Contour spacing is linear, beginning at $1x10^7$ up to $1.1x10^8$ /cm^3/eV/dA.

Unstable GAEs are assumed to have an intersection of the resonance curve with the marginal stability curve within a specific area of the fast-ion distribution as indicated by the magenta shaded region in Fig. 3. The shots used to construct the database all had 90 kV neutral beams with similar distribution functions.

Many GAE will have resonance curves that are well above the most energetic beam ions. Similarly, for other modes the marginal stability curve will be at such a low pitch that most of the fast ions on the corresponding resonance curve would be damping. The location of the resonance and marginal stability curves was used to identify potentially unstable GAE. If the intersection is at too high an energy, there won't be enough fast ions on the resonance curve above the marginal stability point. Likewise, if the energy is too low, the pitch-angle scattering will make the distribution function too isotropic and the higher density on the lower part of the resonance curve will dominate. At too low of a pitch, there will be too many stabilizing fast-ions on the resonance curve. As this model is local and it can't quantitatively predict stability, thus is very approximate, the exact shape of the boundaries used aren't too important. The range chosen was between 50 keV and 80 keV and a pitch greater than 0.7. This range is found to qualitatively predict scaling of toroidal mode numbers and frequencies for the typical NSTX H-mode shot with 90 kV neutral beams. Obviously a different range would need to be chosen for a machine with, for example, lower energy beams or beams with different tangency radii.

3. Comparison of experimental with predicted scaling

A database of GAE frequencies and toroidal mode numbers was constructed from NSTX data spanning the toroidal field range from 2.53 kG up to 4.64 kG (here the toroidal field is defined as the vacuum field at a major radius of 1 m). To minimize variations in the fast ion distribution, the NSTX shots were chosen from moderate density H-mode plasmas with all neutral beam sources injected at 90 kV. Fourier transforms of the Mirnov coil data were done for each 0.256 ms interval and the three largest frequency peaks in the GAE frequency range

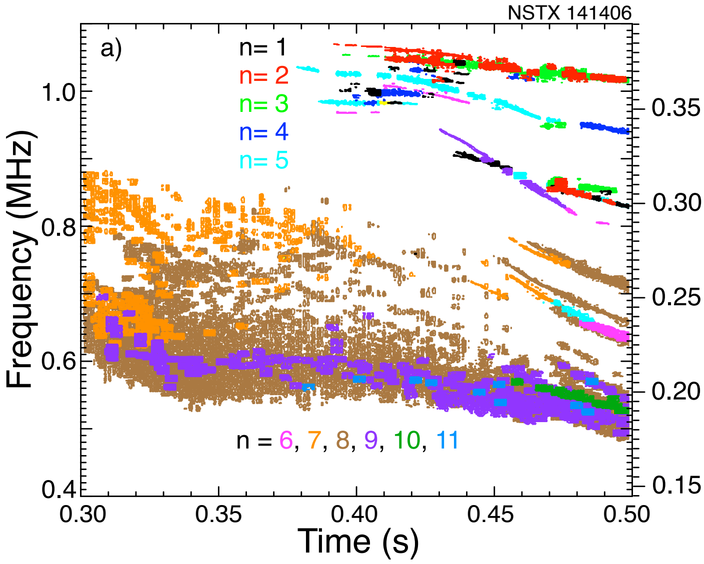

Fig. 4. a) color-coded spectrogram showing GAE activity. Dominant modes are n=-7 (orange), n=-8 (brown), n=-9 (purple), and n=-10 (green), right hand axis is frequency normalized to the ion-cyclotron frequency.

were found. The GAE were identified as the band of mostly chirping and bursting modes seen in the example spectrogram shown in Fig. 4. The band spans roughly 0.6 MHz to 0.9 MHz at 0.3s and narrows to roughly 0.48 MHz to 0.6 MHz at 0.5 s in Fig. 4. The discrete higher frequency modes present later are believed to be counter-propagating Compressional Alfvén eigenmodes. Each frequency peak was analyzed to find the best fit toroidal mode number, similar to the approach used to create the spectrogram in Fig. 4. The best fit toroidal mode numbers and corresponding frequencies were calculated at and averaged over approximately 60 time points around each Thomson scattering time. The average toroidal mode number (n_{eff}) was defined by calculating the average n_{tor} weighted by the square of the mode amplitude (as measured with the Mirnov coil array). A similarly weighted average GAE frequency was calculated and a database containing more than 1200 time points from multiple shots was created.

For each of the shots analyzed for the experimental database which had MSE-constrained equilibrium reconstructions, the DCR model was used to find the poloidal and toroidal mode numbers, and the frequencies of GAE which might satisfy the above instability criterion. The calculations were done at each Thomson scattering time point during the beam injection period, as that was when the density profile, needed to calculate the $V_{Alfvén}$ profile, was measured. As was done for the experimental data, an average toroidal mode number and frequency was calculated at each time point, although as there was no estimate for mode strength, the averages were not weighted.

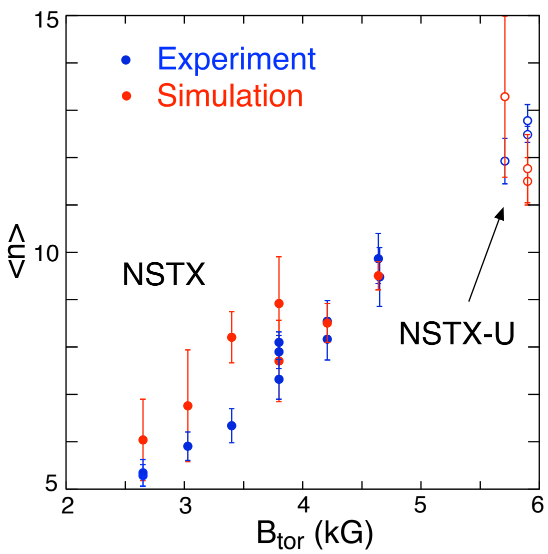

Fig. 5. Experimental (blue) and simulated (red) average GAE toroidal mode number scaling with toroidal field. (Open points are NSTX-U shots without MSE-constrained equilibrium reconstructions and include lower voltage beams.)

To compare the simulated scaling with the experimental scaling the data was grouped to similar plasma current and toroidal field values and then data in each (Ip - Btor) group was further averaged. For this averaging, data from earlier in the discharge where the q-profile, plasma density and equilibrium rotation were still strongly varying were avoided. Future analysis will attempt to separate the dependence of frequency and mode number on q_{min}, plasma rotation and density, as well as beam injection voltage.

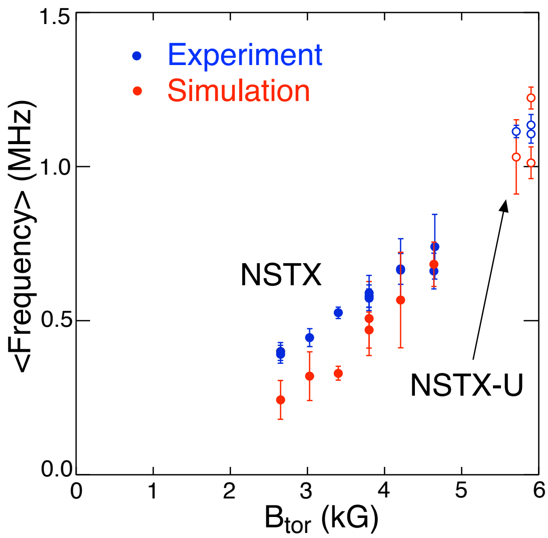

Fig. 6. Experimental (blue) and simulated (red) average GAE frequency scaling with toroidal field. (Open points are NSTX-U shots without MSE-constrained equilibrium reconstructions and include lower voltage beams.)

The results for the toroidal mode number scaling are shown in Fig. 5. Three additional points (open circles) from NSTX-U at higher field were added, although these shots included a mix of 90 kV beams and beams at lower voltage. Also, for the theoretical analysis, they did not have MSE-constrained equilibrium reconstructions so the q-profiles were less reliable. Nevertheless, these points seem to agree well with the scaling found in the NSTX data. A similar analysis was done for the GAE frequencies, shown in Fig. 6. The experimental and predicted GAE frequency and the average toroidal mode number increase roughly linearly with toroidal field. The DCR model also roughly quantitatively predicts the absolute frequencies and mode numbers.

4. Summary

In initial operation of NSTX-U it was discovered that injection of nearly tangential neutral beams (creating a population of high pitch fast-ions) would strongly suppress Global Alfvén eigenmodes (GAE). This observation was shown to be consistent with the theoretical model of the Doppler-shifted ion-cyclotron resonant drive (DCR) for Global Alfvén eigenmodes (GAE). In this paper we have combined the predictions of the DCR theory with a simple analytic dispersion relation for GAE and used that to predict the mode numbers and frequencies of potentially unstable GAE. The model qualitatively, and roughly quantitatively, predicts the observed scaling of GAE frequencies and mode numbers with toroidal field in a select set of NSTX H-mode plasmas with toroidal fields between 2.65 kG and 4.64 kG. The model assumes fast ions are stabilizing when $k_\perp \rho_L < 1.9$, and destabilizing for $1.9 < k_\perp \rho_L < 3.9$ [3]. The agreement between theory and experiment in the scaling of unstable mode frequencies and toroidal mode numbers with the experimental measurements provides additional strong validation of the DCR theory.

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Appendix:

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