INVESTIGATION OF FAST PARTICLE REDISTRIBUTION INDUCED BY SAWTOOTH INSTABILITY IN NSTX-U

D. KIM[1], M. PODESTÀ[1], D. LIU[2], G. HAO[*2] and F. M. POLI[1]

[1]Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ USA
[2]Department of Physics and Astronomy, University of California Irvine, Irvine CA USA

Corresponding Author: dkim@pppl.gov

Abstract

Analysis of the redistribution of fast ions in L-mode sawtoothing plasmas on National Spherical Torus Experiment Upgrade demonstrated that standard sawtooth models (full/partial reconnection models) in the TRANSP code were not capable to fully reproduce the behaviour of fast ion during sawtooth crashes. Some global parameters, such as neutron rate, can match the experimental values using the standard models while fast ion distribution functions were different from the measurements. In order to develop a more comprehensive model for sawtooth-induced fast ion redistributions, simulations using the ORBIT code that takes into account the characteristics of fast ions in phase space (energy, pitch, etc.) have been carried out. The simulation results confirm the experimental observation that fast ions are redistributed by sawtooth crash differently based on their orbit type and energy. The newly developed kick model has been applied to calculate the probability of the change of fast ion energy and canonical angular momentum in the presence of the sawtooth instability based on the ORBIT modeling. The calculated probability matrix has been used as an input parameter for NUBEAM module in TRANSP. TRANSP simulations using the kick model results can reproduce experimental neutron rates within 10% difference. The qualitative comparison of the measurement and FIDA simulations using the TRANSP results with kick model shows a good agreement and therefore shows that the application of phase space variables can improve describing redistribution of fast ion induced by sawtooth instability.

1 Introduction

Sawtooth-induced fast ion redistribution has been studied in the National Spherical Torus Experiment Upgrade (NSTX-U) [1] experiment. During 2016 campaign, 2s long sawtoothing L-mode discharges were obtained [2] providing well-reproducible discharges to investigate sawtooth physics in NSTX-U. From the experiments, it is observed that sawteeth have different effects on fast ions depending on the orbit types [3] and the simulation work confirms the experimental observation [4].

The analysis of sawtoothing discharges has been done using the tokamak transport code TRANSP [5] and the implemented sawtooth models: full reconnection (Kadomtsev) [6] or partial/incomplete reconnection (Porcelli) model [7]. For the interpretative simulation, the sawtooth crash times are given from the measurement and the effect of sawtooth crashes are included in the conditionally averaged measured thermal profiles. The sawtooth models are applied to evaluate current, safety factor (q) profile reconnections as well as fast ion redistributions. By choosing a model and adjusting free parameters in the model, e.g. fast ion redistribution fraction, partial reconnection fraction, TRANSP simulations can reproduce some global parameters such as neutron rate [3]. However, it is difficult to find the proper set of free parameters since it cannot be self-consistently determined. Furthermore, the agreement of global parameters does not assure the agreement of estimated features of fast ions like the fast ion distribution functions, as the sawtooth models do not take into account the characteristics of fast ions. From Fast Ion D-Alpha (FIDA) measurement [8, 9], it is seen that fast ions in the centre move outside causing the increase of fast ion population outside the inversion radius. However, the FIDA simulation (FIDASIM) [10] using the TRANSP simulation results that reproduce the experimental neutron rate shows a decrease of the population of fast ions across the whole plasma minor radius [3].

Therefore, a more reliable model that includes the effect of fast ions energy, orbit type and other phase space variables is required to evaluate the behaviour of fast ions more quantitatively in the TRANSP

*Present address: Southwestern Institute of Physics, PO Box 432 Chengdu 610041, China

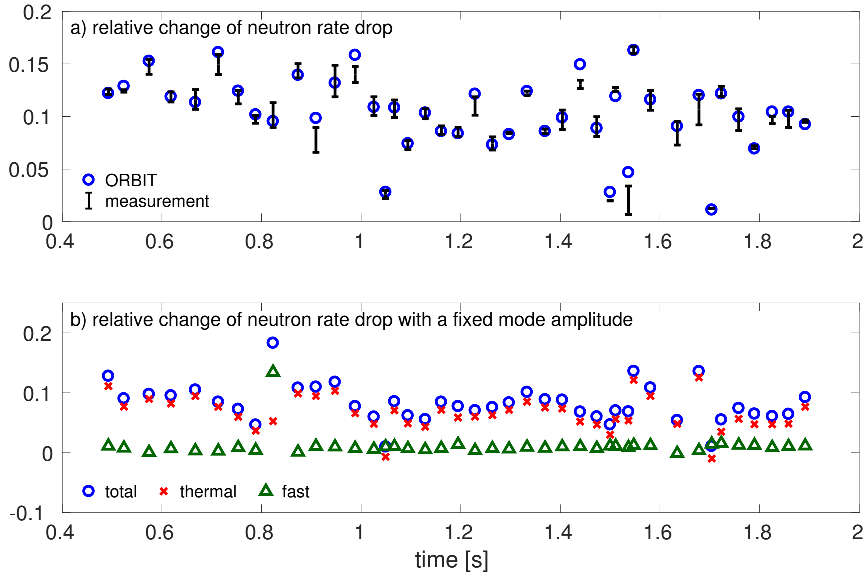

FIG. 1: a) Mode amplitude can be determined by the comparison of the simulated relative change of neutron rate before and after a crash with the measurement. b) Deuterium density accounts for most of the relative change of neutron rate. Thus the uncertainties in deuterium density estimation should be reduced to have more accurate mode amplitude calculation.

simulations. In this work, we have implemented the newly developed kick model [11, 12] into the ORBIT code [13] to take into account the effect of phase space variables on the sawtooth modeling. The simulation results from ORBIT-kick model have been applied to TRANSP simulations by updating input parameters for NUBEAM module [14, 15]. TRANSP simulation results using the kick model show features of sawtooth induced fast ion redistribution that are not found from the conventional sawtooth models. The TRANSP results are taken as input data for the FIDASIM to compare with the measured fast ion distribution functions. The FIDASIM results show that the modification of fast ion distribution from kick model application has similar features as the FIDA measurement, while that from conventional models cannot reproduce the experimental observation as already seen in Ref. [3]. The preliminary test results confirm that the features of fast ions such as energy, canonical angular momentum, pitch should be included in the modeling to properly describe the behaviour of fast ions in sawtoothing discharges.

2 Simulation methodology

Simulations have been performed based on the NSTX-U discharges #204083 and #204163 (L-mode sawtoothing discharges with 1MW NB power) using the Hamiltonian guiding-center code ORBIT [13]. Sawtooth instability is represented by a magnetic perturbation implemented in the ORBIT code (See Ref. [4] for more detail). The mode amplitude of each crash is determined by comparing the relative change of neutron rate drops [4] with measurements (Fig. 1*a*). As the calculation of the relative change of neutron rate drops is based on the reaction rate between thermal and fast particles, most of contribution of the estimated value comes from the thermal plasma density (Fig. 1*b*). Since the profile diagnostics in NSTX-U have a time resolution comparable with the sawtooth period (10∼16ms), the profile data are re-processed through conditional average to reconstruct the evolution of profiles on a finer time grid. Due to the process, thermal particle profile can have uncertainties that may cause an incorrect estimation of the relative change of neutron rate drops. For more accurate simulations, the time resolution of plasma density and temperature measurement should be increased to reduce the uncertainties.

The dependencies of fast ion redistribution during sawtooth crashes on the fast ion phase space

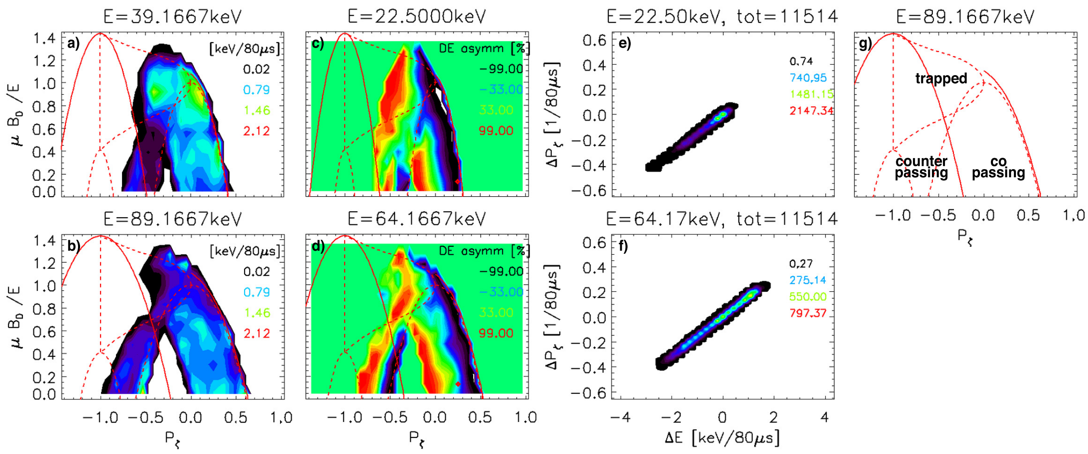

FIG. 2: a, b) The averaged root-mean-square of the particle energy change due to a sawtooth crash. c, d) The balance of the averaged positive and negative energy kicks at each location. e, f) Calculated probability matrices from the kick model for two energy cases at a certain location on the fast ion phase space (Red diamonds in c) and d)). g) The boundary of each orbit type; co and counter passing and trapped particles.

variables can be tested using the reduced kick model [11, 12] in conjunction with the ORBIT modeling results. The kick model is based on transport probability matrices to represent the fast ion transport by instabilities. Using the estimation from the ORBIT code in the presence of sawtooth instability, the kick model computes the probability matrices that represent the change of particle's energy (ΔE) and canonical angular momentum (ΔP_ζ) around a certain position in phase space (E, P_ζ, μ_0) during their orbiting over a certain time interval. The resulting probability matrix $p(\Delta E, \Delta P_\zeta | E, P_\zeta, \mu_0)$ from the kick model can be applied to time dependent transport simulations. In this work, the kick model results are used as an updated input value to the NUBEAM module [14, 15] in TRANSP [5] to include the effects of sawtooth instability. NUBEAM uses $p(\Delta E, \Delta P_\zeta | E, P_\zeta, \mu_0)$ and the given mode amplitude to update the evolution of fast ion E, P_ζ, which affects the fast ion distribution functions, density, etc.

To compare fast-ion distribution functions predicted using kick model and the standard sawtooth models, a synthetic FIDA diagnostic signal is calculated by the FIDASIM code [10] and compared with the experimental FIDA signal [8, 9]. The FIDASIM code models neutral beam deposition and predicts the active D_α emission, treating all relevant atomic physics such as charge-exchange, atomic excitation and ionization processes. The main inputs are plasma equilibrium, neutral beam parameters (geometry, power waveform), fast ion distribution and plasma profiles. The FIDASIM code integrates the D_α emission along each of the FIDA sightlines to yield spectra and profile for comparison with measurements.

3 Simulation results

Using the given magnetic perturbation, the kick probability matrix has been calculated in ORBIT for each sawtooth crash. As discussed in Sec. 2, the mode amplitude estimated by ORBIT modeling can be used for the calculation of probability matrix. However, TRANSP simulations using the kick probability matrix with the ORBIT estimated mode amplitudes show over-estimated neutron rate drops [16]. This may come from the uncertainties on density profile, the change of fast ion density evolution when the kick model is applied. Therefore in this work, a fixed mode amplitude has been used for the probability matrix calculation.

3.1 ORBIT modeling with kick model

One example of the kick model application to the ORBIT code from a time slice (1352ms) is shown in Fig. 2. Figures 2a, b represent the averaged root-mean-square of the particle energy change due to a sawtooth crash in terms of phase space variables. The boundary for the classification of the different orbit type is shown in Fig. 2g. For two different energy ranges, passing particles are clearly affected by a crash while the effect is negligible for trapped particles with high energy (∼90keV). As the critical energy for the redistribution [17] of trapped particle is about 40keV, the change in phase space is limited for higher energy. On the other hand, since the critical energy of passing particles is much higher than the neutral beam energy (∼100keV), most of passing particles are under influence of sawtooth instability and are redistributed.

From Figs. 2c,d, one can infer the redistribution of fast ions with different orbit type and energy. These figures show the balance of the averaged positive and negative energy kicks at each location. Note that from the relation between ΔE and ΔP_ζ ($\Delta P_\zeta/\Delta E \sim n/\omega$), the positive (negative) energy kicks correspond to positive (negative) P_ζ kicks. The phase space location where ΔE is mostly negative is displayed with blue color while red indicates mostly positive change of energy on average. One can picture in the real space that particles are expelled from near axis (blue region) or move inside (red) during a crash based on their initial position with reference to the inversion radius (green region between the blue and red). The detail of kick probability for the redistribution of fast ions are shown in Figs. 2e,f (red diamonds from the blue and green regions in Figs. 2c,d). The probability is calculated for a time step of 80μs in the ORBIT modeling. As the chosen phase space location is in blue region in Fig. 2c, the variations of P_ζ and E for lower energy passing particles near the magnetic axis in real space are mostly negative; this indicates that particles are expelled from the center due to a crash. For the higher energy case, the averaged positive and negative probabilities are balanced thus the redistribution is not clear. In this case as the same P_ζ and E position is shifted in phase and real space, particles are located near the inversion radius. Passing particles with ∼65keV have higher probability to move inside than lower energy case (larger red area in Fig. 2d). Unlike passing particles, the probability matrices for trapped particles are expected to be mostly positive although the change in (P_ζ, E) vanishes for higher energy particles.

3.2 Application of kick model to time dependent transport simulations

The calculated kick probability matrix has been applied to time dependent TRANSP simulations to test the improvement of the modeling for the fast ion redistribution induced by sawteeth. The sawtooth crash time identified from the measurement is given as an input parameter. The modification of thermal plasma temperature and density profiles during a crash is included in the conditional averaged input profiles. For the target discharges, MSE diagnostic was not available thus the q profile is evaluated by solving the poloidal field diffusion equation with a fixed boundary and is modified at the time of crash. The prediction of fast ion redistribution is described based on the chosen model. Using the result from ORBIT/kick modeling, NUBEAM module has an additional input of $\Delta P_\zeta, \Delta E$ to take into account the phase space variables for the sawtooth-induced redistribution. For simplicity, only one matrix (from 1352ms) is used for all the crash times. As previously mentioned, ORBIT-estimated mode amplitudes can bring over-estimated neutron rate drops due to uncertainties. In this work, the mode amplitudes for each crash time are determined using the relative change of measured neutron rate drops and are normalized by the amplitude at 1352ms. Fast ion redistributions can also be described in TRANSP using conventional sawtooth models (full [6] and partial [7] reconnection models). The simulation results using different models are compared in the following.

In Fig. 3, the time traces of neutron rate from TRANSP simulations using different models are shown. On the upper panel, blue, red and green lines indicate the measured neutron rate, TRANSP simulation results without fast ion redistribution and with kick model, respectively. On the bottom one, blue curve is the same and purple and light blue ones are full and partial reconnection cases. In this work, the fraction of fast ion redistribution is set to 20% and 50% for the full and partial reconnection cases, respectively. A partial reconnection fraction of 50% is used. The measured neutron rate is normalized to have the same averaged value as TRANSP simulations between 360 and 440ms, where no MHD activity has been detected. The neutron rate from the TRANSP simulation in which the effect of sawteeth on fast ions is suppressed (red) is comparable with the measured one (blue). This indicates that the change in the thermal plasma profile has the most contribution to the neutron rate drop. The application of kick model

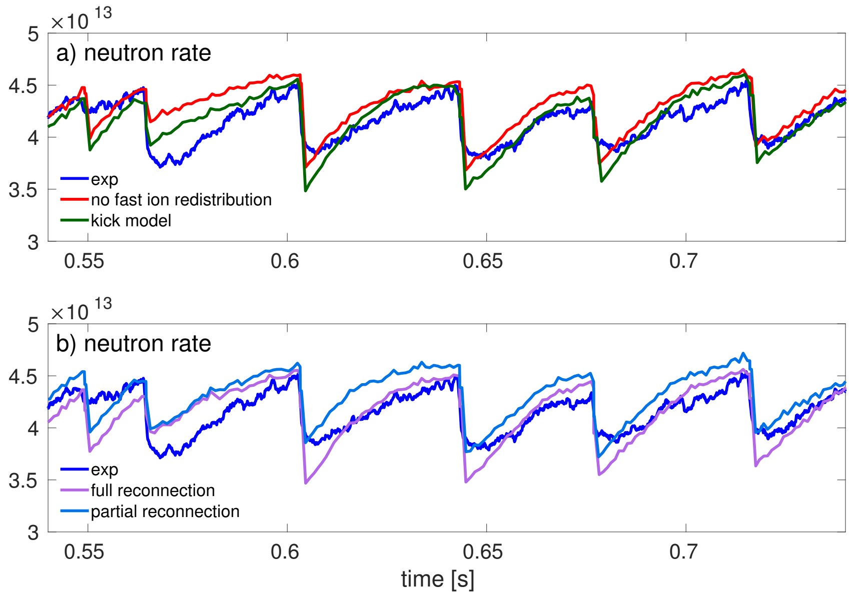

FIG. 3: a) Neutron rates from the measurement (blue), without fast ion redistribution (red) and kick model (green). It is confirmed that the main contribution to the neutron rate comes from thermal plasma. Using kick model, the neutron rate shows better match but both cases are in a good agreement with the measurement within errorbar. b) Neutron rates from the measurement (blue), full (purple) and partial (light blue) reconnection cases. By adjusting free parameters, the conventional sawtooth models can provide neutron rates close to the experimental value.

(green) brings slightly better agreement but the effect of fast ion redistribution on the neutron rate drop is not significant. Using the conventional sawtooth models, the neutron rate similar to the reference case can be produced with a proper set of free parameters; the fraction of fast ion redistribution, the partial reconnection fraction. Since these free parameters cannot be self-consistently determined, predictive modeling for sawtooth-induced fast ion redistribution is demanding using the conventional models.

From Fig. 3, it seems that the conventional models are sufficient to describe the fast ion redistribution in sawtoothing plasmas. However, although the simulated neutron rates using the different models are comparable to the measured one, detailed features are not similar. Several profiles 2ms before (solid) and after (dashed) a crash at 1353ms from kick model (blue), full (red) and partial (green) reconnection cases are compared in Fig. 4. Since the kick model does not affect the q profile, full reconnection model is applied as a default setting. As the central value of the q profile from partial reconnection stays below unity after a crash, the profile before crash is more relaxed compared to the profile from full reconnection. The estimated fast ion density profiles in Fig. 4b clearly show the difference among the models. The post-crash profile from the kick model demonstrates the effect of sawtooth on the redistribution. The central density drops while density outside inversion radius increases. Note that the inversion radius ($\psi_N \sim 0.3$) is inside the $q = 1$ surface location. Full and partial reconnection cases have similar fast ion density profiles. After a crash, density profile at the center is slightly diminished but the difference is small as the redistribution fraction is small. This may come from the assumption of the fast ion redistribution fraction. Since more than half of fast ions are not modified by the sawtooth crash, the post-crash profile keeps almost the same shape. Similar difference can be seen in the beam current density profile (Fig. 4c).

The fast ion distribution functions just outside the $q = 1$ surface ($\psi_N = 0.55$) before and after a sawtooth crash shown in Fig. 5 also illustrate the difference between applied models. Figures 5a, b are the fast ion distribution from the kick model before and after a crash while Figs. 5c, d are the partial reconnection case. Note that full and partial reconnection cases have similar distribution functions thus only partial reconnection case is shown here. The kick model results confirm that the sawtooth crash redistributes fast ions, in particular the low energy passing and trapped particles. Passing particles with

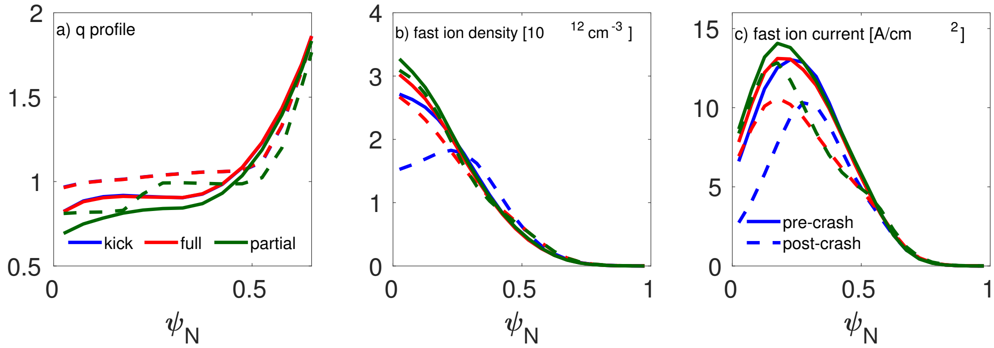

FIG. 4: a) Safety factor q b) fast ion density c) fast ion driven current profiles from each model (blue - kick model, red - full reconnection, green - partial reconnection) before (solid) and after (dashed) a crash. Using kick model, clear effect from a sawtooth crash is seen on the profiles. Since kick model does not affect q profile, full reconnection is assumed.

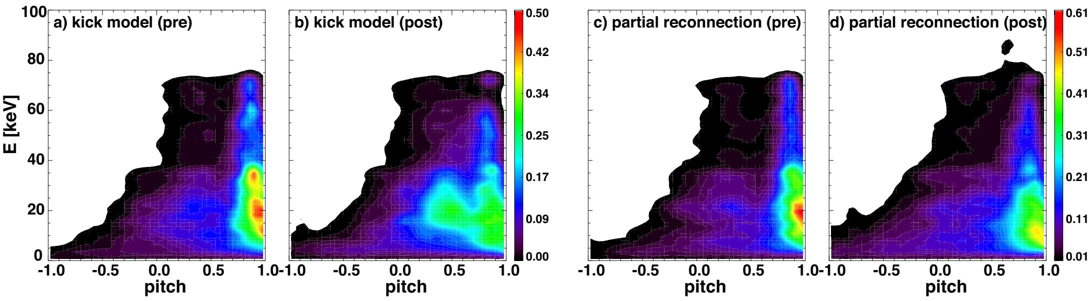

FIG. 5: The distribution of fast ion at $\psi_N \sim 0.55$ a), b) from kick model before and after a crash and c), d) from partial reconnection (50% reconnection fraction, 50% fast ion redistribution). From kick model, the population of passing particles with high pitch decreases while that of trapped particles increases. For the partial reconnection case, passing particles have the same behaviour but no clear change of trapped particles is found.

larger pitch decrease about 20-30% while trapped/passing particles with pitch around 0.5 increase about 50%. This may indicate that passing particles change their orbit type and become trapped particles. However, as seen in Fig. 2, particles move around the inversion radius and that can result in the change of the distribution of fast ions. Therefore, for the precise analysis, the tracking of particles and the change of orbit type are required. In addition to the increase of the lower energy (<40keV) in the lower pitch range (0∼0.5), the number of higher energy particles is extended as well as the increase of counter passing particles (pitch $\leq$ -0.5). On the other hand, the distributions from partial reconnection do not have a significant variation. The number of the low energy passing particles with pitch ∼1 decreases but no clear change in trapped particles. Similar to the kick model case, particles with middle pitch range become more but compared to the kick model case, the increment is much smaller. In addition, as previously discussed, trapped particles (pitch ∼ 0) with higher energy over ∼40keV are not affected by sawtooth as the distribution function does not change much for both case.

3.3 Comparison with the experimental measurements

As seen in Figs. 4, 5, the application of the kick model to TRANSP simulations shows that taking into account the phase space dependence on fast ion transport can result in different features of fast ions from the conventional sawtooth models. In order to validate the improvement from the kick model, the simulation results need to be compared with the experimental measurement [3].

The fast-ion distribution functions predicted by different models are used as inputs to the FIDASIM code [10] for the calculations of the synthetic FIDA spatial profile. The simulated FIDA spatial profiles

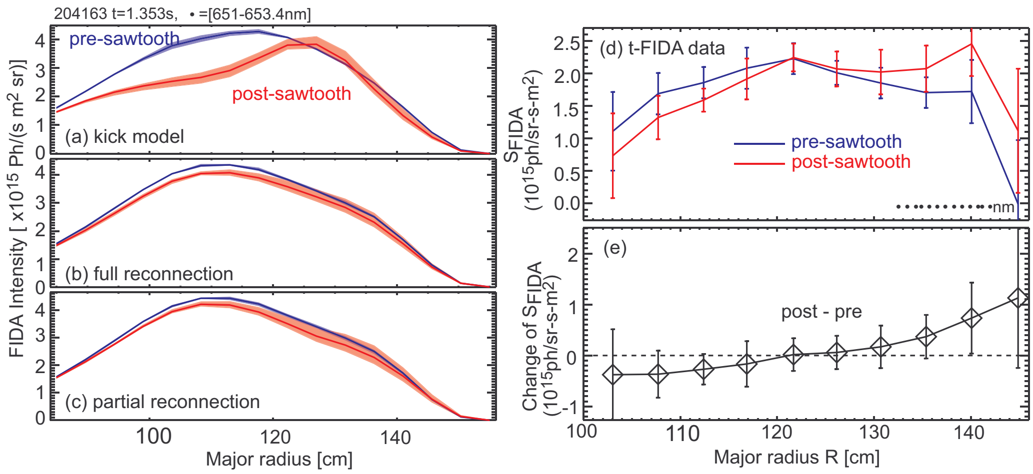

FIG. 6: The comparison of simulated t-FIDA spatial profile before and after a sawtooth crash using a) kick model b) full reconnection and c partial reconnection. All profiles are averaged over three time steps (1,3,5ms) before and after a crash. The experimental FIDA spatial profile and relative change are shown d) and e). Using kick model, qualitative features of the fast ion distribution modification due to a sawtooth crash are reproduced.

before and after a crash at 1353ms are shown in Fig. 6 together with the experimental measurements. The simulations calculate the FIDA spatial profile 1,3,5ms before (blue curve) and after (red curve) the crash and average over these three time slices since the temporal resolution of FIDA diagnostics is 10ms. The shaded regions represent the variation of the FIDASIM signal in those three time slices. Although FIDASIM provides both tangential and vertical FIDA signals (t-/v-FIDA), only t-FIDA (mostly sensitive to passing particles) is considered in this work since the v-FIDA diagnostic data is available only for the outer channels (R_{maj} ≥110cm) and the FIDA signal change is within the experimental uncertainties.

t-FIDA signals from FIDASIM are shown in Fig. 6 for three cases using kick model, full and partial reconnections for the fast ion redistribution in TRANSP. To compare the simulation results, the measured t-FIDA signal is also displayed in Figs. 6d, e [3]. The effect of sawtooth instability on the fast ion distribution function is clearly seen from the t-FIDA measurement: fast ions in the central region are expelled to outside inversion radius after a crash. Since all the three simulation cases have similar neutron rate drops and use the same thermal plasma profiles as input, one could expect that the distribution of fast ion might also be similar. Using the kick model, the t-FIDA signal after a crash decreases in the central region and the shoulder of increased signal is seen outside the inversion radius. On the other hand, the simulated t-FIDA signal from the conventional sawtooth models slightly decreases across the minor radius but no clear sign of the sawtooth-induced redistribution is found. This simulation confirms that taking the phase space variable into account can improve sawtooth modeling to describe more reliably fast ion transport. In addition, it also shows that the match of neutron rate is not a sufficient prerequisite to test models since detailed simulation results for the fast ion transport can be different. Note that the redistribution fractions of fast ion for full and partial reconnection are 20% and 50%, respectively. With higher fractions, the change of FIDA signal is similar to kick model case but the neutron rate shows larger discrepancies with respect to the measurement, with a large over-estimate of the neutron rate drops.

4 Discussion and conclusions

Modeling fast ion redistribution in sawtoothing discharges helps to understand experimental results and to set an operation scenario with prediction of behaviour of fast ions induced by sawtooth instability. Conventional sawtooth models implemented into time-dependent transport simulations such as TRANSP can be used as the models can reproduce some global parameters similar to the measurement using a proper free parameter set. However, it is difficult to self-consistently predict the parameter set and detailed fast ion properties, for instance distribution function, cannot be the same if the characteristics of fast ion transport are not taken into consideration. In this work, the reduced kick model has been applied to the ORBIT code based on NSTX-U sawtoothing discharges to find the effect of sawtooth instability and the phase space dependence on fast ion redistribution. From the ORBIT-kick modeling, the probability matrix of the change of fast ion energy and canonical momentum is calculated and this is applied as an updated input parameter to NUBEAM module in TRANSP. In this way, TRANSP simulation can have the effect of a sawtooth crash on fast ion transport taking into account the energy, orbit type of fast ion.

From the simulation results, it is shown that the application of the kick model can improve the modeling of fast ion transport in the sawtoothing discharges. In other words, for the proper modeling of sawtooth-induced fast ion redistribution, the phase space variables should be included in the modeling. Using the kick model, the neutron rate is in a good agreement with the measurement. The fast ion related profiles and distribution function from TRANSP show clear effects from sawteeth that conventional models do not show. Comparison of FIDA simulation using the TRANSP results with FIDA measurement confirms the improvement of modeling when the kick model is applied.

The simulation results show that sawteeth mostly affect low energy particles for trapped particles. The critical energy for the redistribution of fast ions can be estimated using the criteria in Ref. [17]. The zero-dimensional prediction can help to understand the different behaviour of fast ions, in particular trapped particles, with their energy. However, more quantitative criteria to describe the level of redistribution of fast ions after a sawtooth crash are required. ORBIT modeling is expected to find more general and quantitative criteria and it can improve kick model or help to develop a new model for fast ion transport.

Acknowledgements

This work is supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences under contract number DE-AC02-09CH11466.

References

[1] MENARD, J. et al., Nucl. Fusion **52** (2012) 083015.
[2] BATTAGLIA, D. et al., Nucl. Fusion **58** (2018) 046010.
[3] LIU, D. et al., Nucl. Fusion **58** (2018) 082028.
[4] KIM, D. et al., Nucl. Fusion **58** (2018) 082029.
[5] HAWRYLUK, R., An empirical approach to tokamak transport Physics Close to Thermonuclear Conditions vol 1 ed B. Coppi et al (Brussels: Commission of the European Communities) p 19, 1980, For more details on the TRANSP code, please refer to the TRANSP webpage at http://w3.pppl.gov/~pshare/help/transp.htm.
[6] KADOMTSEV, B., Soviet Journal of Plasma Physics **1** (1975) 710.
[7] PORCELLI, F. et al., Plasma Phys. Control. Fusion **38** (1996) 2163.
[8] BORTOLON, A. et al., Rev. Sci. Instrum. **81** (2010) 10D728.
[9] PODESTà, M. et al., Rev. Sci. Instrum. **79** (2008) 10E521.
[10] HEIDBRINK, W. W. et al., Commun. Comput. Phys. **10** (2011) 716.
[11] PODESTÀ, M. et al., Plasma Phys. Control. Fusion **56** (2014) 055003.
[12] PODESTÀ, M. et al., Plasma Phys. Control. Fusion **59** (2017) 095008.
[13] WHITE, R. et al., Phys. Fluids **27** (1984) 2455.
[14] GOLDSTON, R. et al., J. Comput. Phys. **43** (1981) 61.
[15] PANKIN, A. et al., Comput. Phys. Commun. **159** (2004) 157.
[16] KIM, D. et al., Modeling of sawtooth-induced fast particle redistribution in NSTX-U, in Proc. 45th EPS Conference on Plasma Physics (Prague, Czech Republic, July 2018).
[17] KOLESNICHENKO, Y. I. et al., Phys. Plasmas **4** (1997) 2544.