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Reconstruction of Spherical Torus Equilibria

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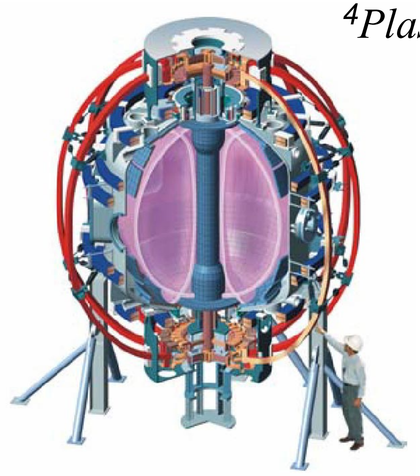

Innovative Confinement Concepts Workshop

May 27, 2004

Madison, Wisconsin

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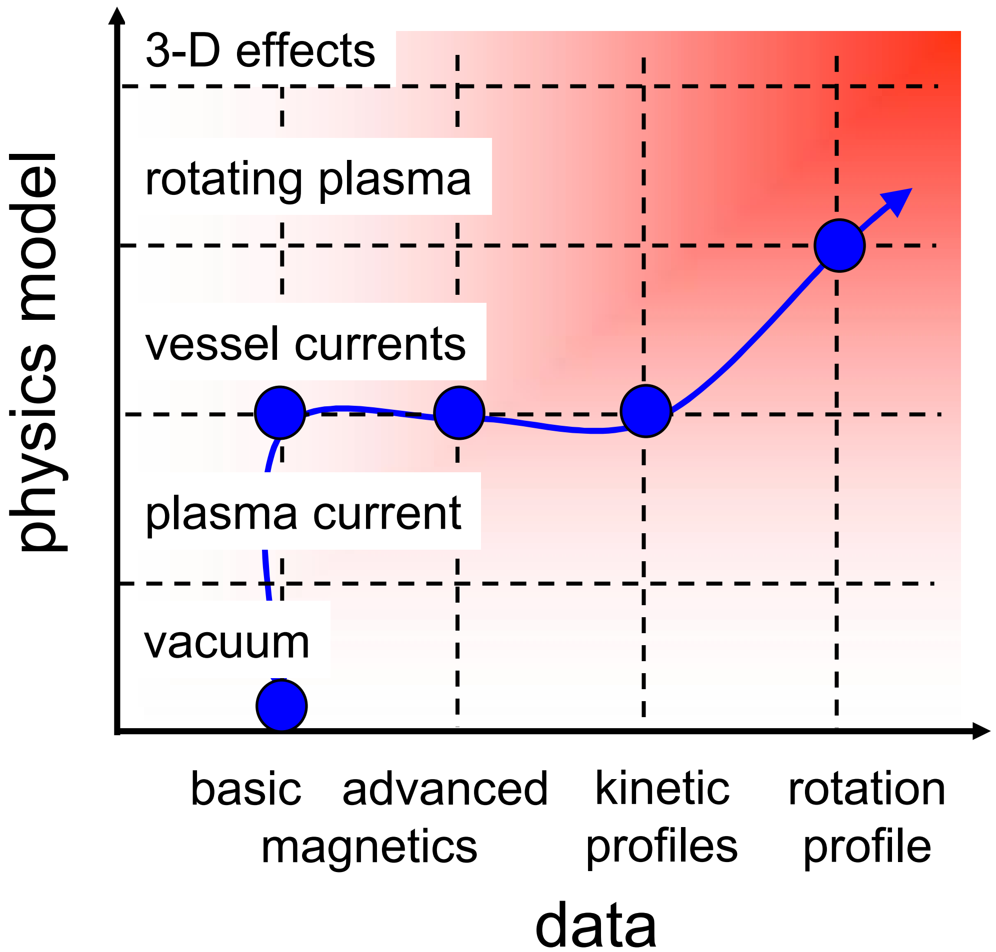
Neutral beam heated, high β ST plasmas provide opportunity for advancing equilibrium reconstruction

- Motivation
  - Equilibrium reconstruction is a basic and essential component of plasma analysis
    - Stored energy, l_i, β, β_N, boundary evolution, etc.
  - Results are needed for a myriad of subsequent analyses

- Outline
  - Philosophy
  - Summary of reconstruction technique
  - Development of improved reconstructions
  - Application to NSTX
  - Near-term and future directions

Aim to "rapidly" reconstruct "best" equilibrium

- Philosophy
  - "Best" model
    - for a given physics / data set fit all data within error
    - improved physics/data set reduces artificial constraint
  - "Rapid" reconstruction
    - between-shots
    - find one constraint set for a given (data,model) pair
  - Upgrade toward "perfect" equilibrium
    - complete physics
    - complete measurements



EFIT* provides a flexible equilibrium solution

- Solve for (1) poloidal flux, ψ, and (2) toroidal current, J_t
  - that satisfy the GS equation: $\Delta^*\psi = -\mu_0 R J_t(\psi)$, where

    $\Delta^*\psi = R^2\nabla\bullet(\nabla\psi/R^2)$; $J_t = Rp'(\psi) + \mu_0 ff'(\psi)/(4\pi^2R)$; $f(\psi) = RB_t$
  - that provide a least-squares fit to a set of constraints
- Typical constraints for fit
  - Diagnostic data - response from plasma and external coils
    - Magnetic (flux loops, I_p, coils, diamagnetic loop, stabilizing plates)
    - Pressure profile (from Thomson, CHERS, Zeff)
    - Motional Stark effect
  - Specified global / local parameters (ℓ_i, β, q_0, edge J)
  - Specified profile shapes, or boundary
    - Yields shaping coil currents, diagnostic measurements

*L. Lao, et al., Nucl. Fusion **25** (1985) 1611.

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EFIT Parameterization of Equilibrium Solution

- $\psi_t = \psi_{plasma} + \psi_{coils}$, J_t solved on rectangular grid
- For fitting, J_t modeled using various basis functions
  - splines
  - polynomial
    - $P'(\psi) = \Sigma\,(\alpha_n\ \ \psi_n\ )^n$
    - $FF'(\psi) = \Sigma(\gamma_n\ \psi_n\ )^n$
    - solution vector $\alpha\ = [\alpha_n\ ,\ \gamma_n]$
- External coil currents I_c, reference flux ψ_{ref}
- Solution vector for fit
  - $U = [I_c\ ,\ \alpha,\ \psi_{ref}]$

EFIT "Loop 1" : Determine J_t (Constraint fitting)

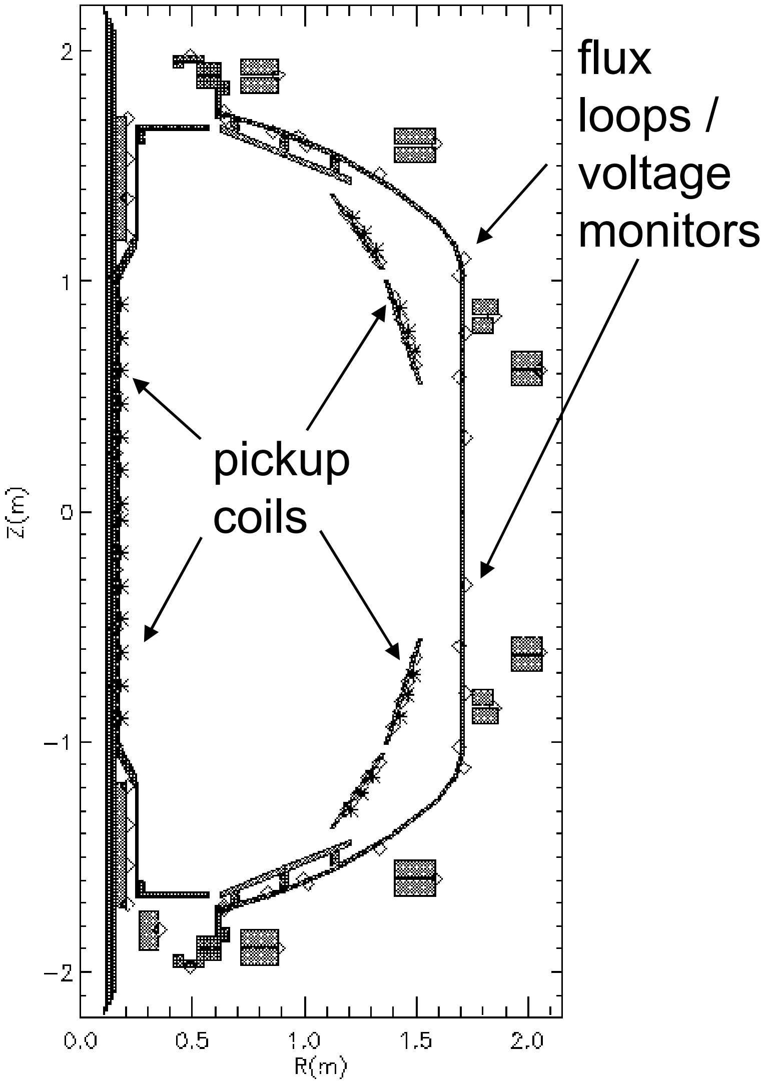
- Constraint equations
  - D(t) = R x U(t)
    (Response matrix R; D(t) = diagnostics data / constraints)
  - Submatricies of R
    - Diagnostic response to I_{coils}: Coils Green function matrix G_c
    - Diagnostic response to J_t: Plasma Green function matrix G_p
    - Relations linking MSE measurements to currents
    - Any extra constraints relating the elements of U
- Find U that minimizes $\chi^2 = \Sigma(M_i - C_i)^2/\sigma_i^2$
  - Include fitting weights F: |F • R x U - F • D| is minimized
  - Singular Value Decomposition gives least square solution:
    - $(F \cdot R)^{-1} = V \times [\mathrm{diag}(1/w_j)] \times Y^t$
    - $U(t) = (V \times [\mathrm{diag}(1/ w_j)] \times Y^t) \times (F \cdot D)$; (note: functions of ψ)

EFIT "Loop 2" : Solving for Poloidal Flux

- ψ_{coils} solved by Green's function response given I_c
- ψ_{plasma} solved by inverting Grad-Shafranov equation
  - finite difference method
  - largest change in ψ across grid < tolerance
- Boundary / ψ surfaces determined by contour routine
- Axis determined by maximum $|\psi|$

NSTX EFIT* alterations for ST geometry

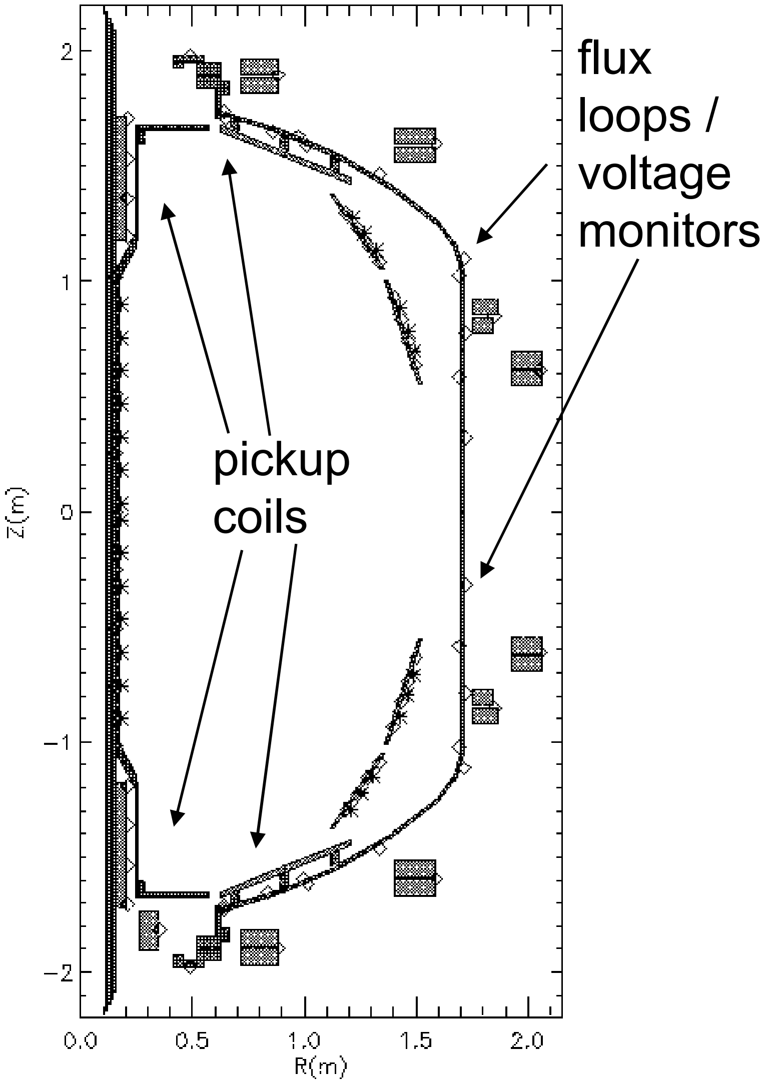
- Minor (important) changes made for NSTX geometry
  - Uniform discretization of elements at low aspect ratio
  - Vessel currents required
    - Lower A components have lower resistance
    - Total vessel currents ~ 0.3 MA; plasma current ~ 1.0 MA
    - Vessel / plates broken into 20 groups (poloidally)
    - Wall currents determined by local loop voltage data (9 loops)
    - Current distribution matched against LRDIAG models of vacuum field shots
  - Stabilizing plates / divertor plates included (~5 kA)
    - plate currents not well-diagnosed



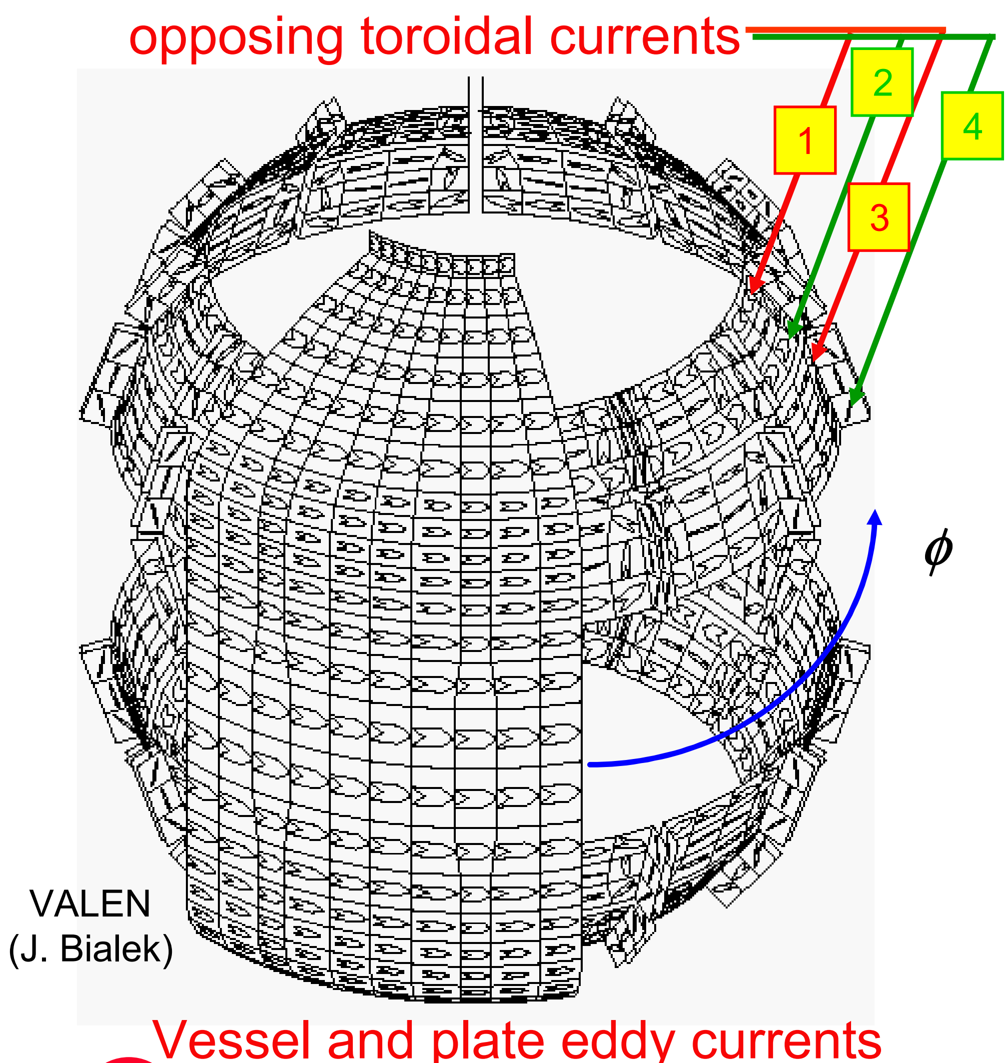
*S.A. Sabbagh, et al., Nucl. Fus. **41** (2001) 1601.

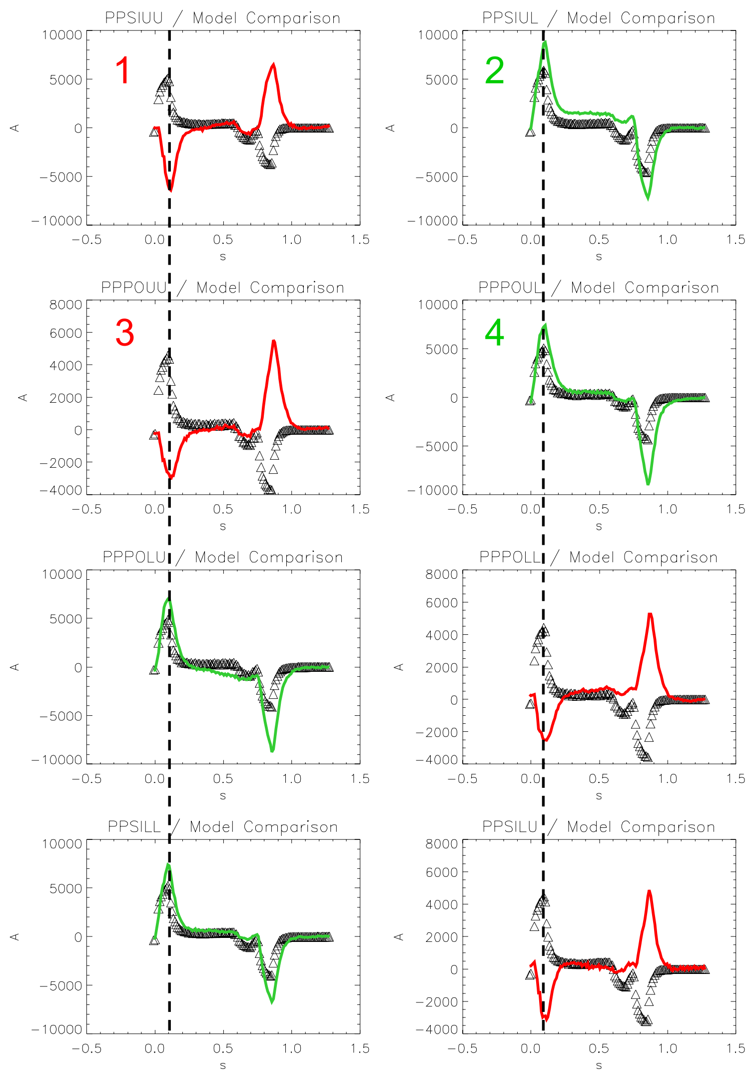
Expanded magnetics set required more accurate X-point and plate current determination

- Significant upgrade to magnetics set
  - Lower A components have lower resistance
  - ## flux loops vs. ##
  - ## Pickup coils vs. ##
  - 25 local loop voltage data vs. 9 for wall current distribution
- Modification to vessel hardware
  - Resistance of vessel elements matched against LRDIAG models of vacuum field shots
- Stabilizing plates / divertor plates currents now better resolved



Expanded magnetics set reproduces 3-D eddy currents as axisymmetric currents during OH ramp





NSTX EFIT using external magnetics data

- "External Magnetics-only" model
  - over 60 attempted model variations
  - allows finite edge current (to model current transients)
  - 9 shaping / OH coil current measurements
  - 45 flux loops; 23 Mirnov loops; 1 plasma Rogowski
  - 9 loop voltage monitors for vessel current
  - Profile constraints: p'(0) = 0, (ff')'(1) = 0
    - Results in resonable
  - 4 profile variables (1 p', 3 ff'; 2nd order polynomial in p', 3rd order in ff')

Time-evolved output here

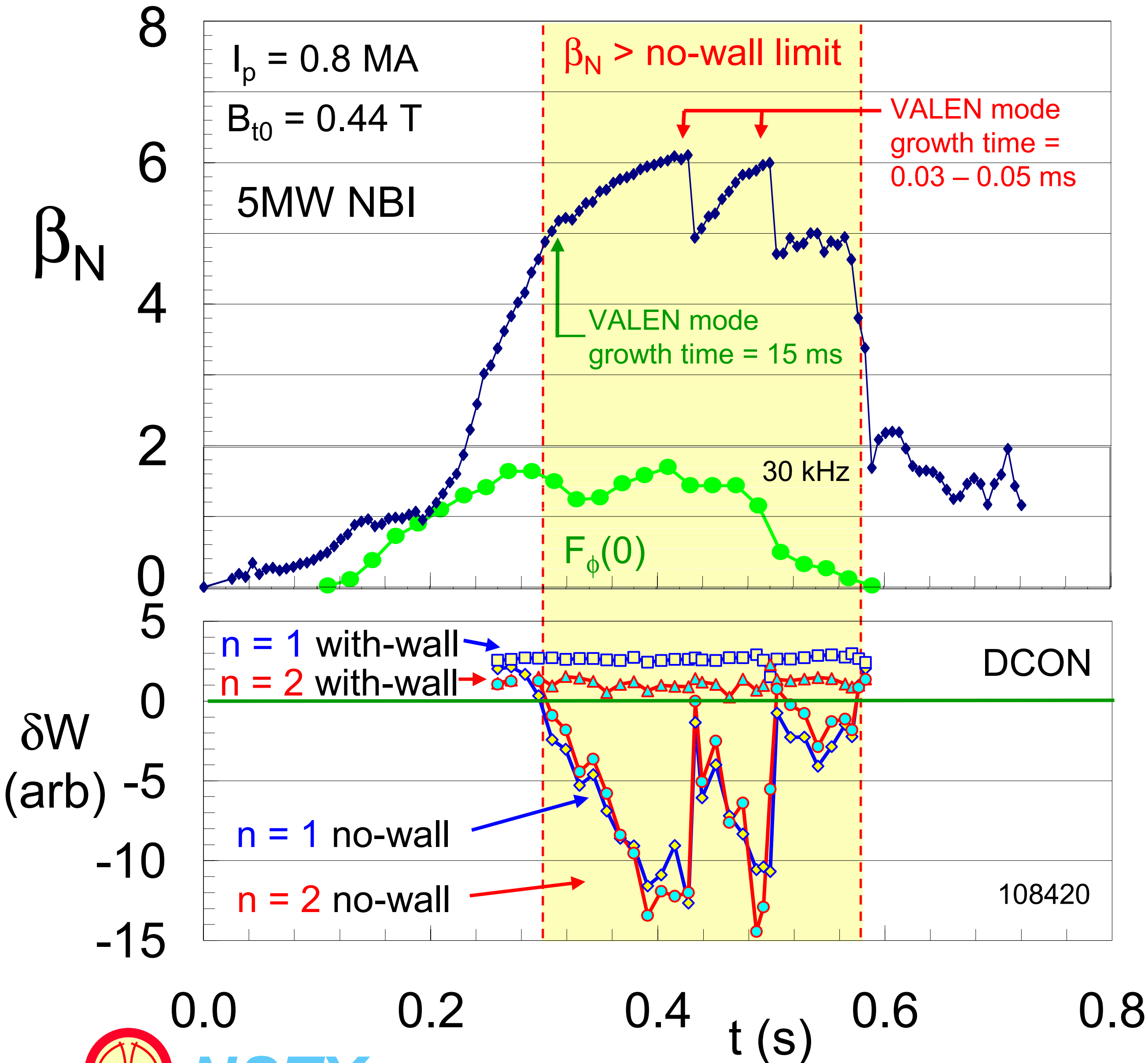
Partial kinetic prescription reduces artificial constraint

- “Partial Kinetic” model
  - over 110 attempted model variations used
  - allows finite edge current (to model current transients)
  - 9 shaping / OH coil current measurements
  - 45 flux loops; 23 Mirnov loops; 1 plasma Rogowski
  - 9 loop voltage monitors for vessel current
  - 10 Thomson scattering P_e points to constrain P profile shape
    - $P_{tot} = P_e$ + “P_i” + “P_{fast}”; errors summed in quadrature (large total error)
  - 1 diamagnetic loop to constrain stored energy
    - Required greater freedom in ff’ basis function for good fit over full discharge evolution and for various shots
  - Profile constraints: on p’(0), ff’(0) to have “reasonable” q(0)
  - 10 profile variables (5 p’, 5 ff’)

flux plot here

pressure profile here

EFIT equilibria used extensively in stability analyses



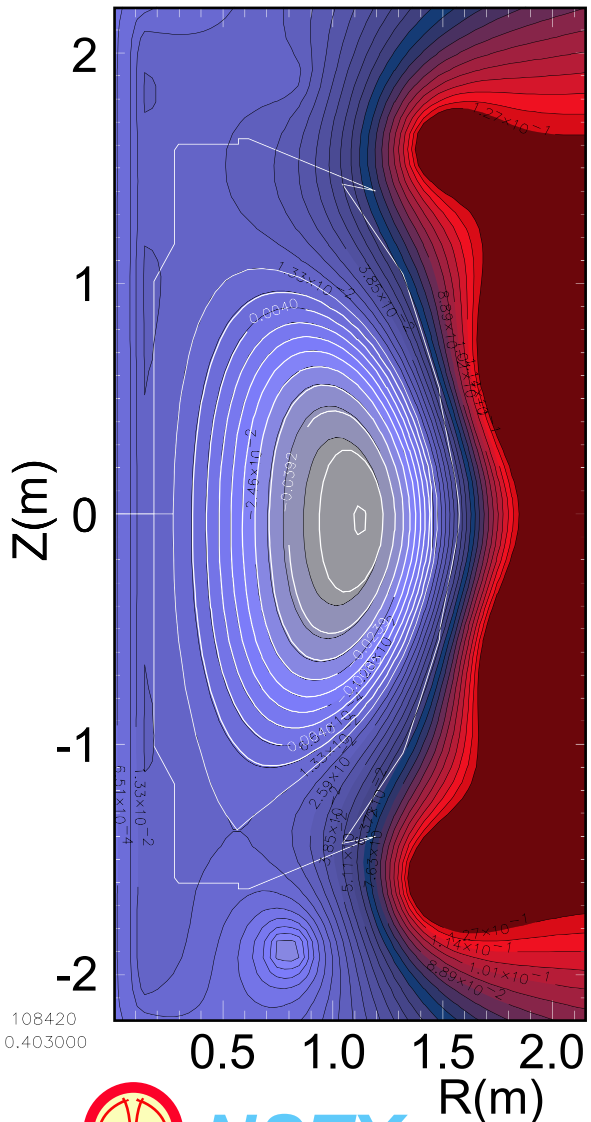
- Control room ideal stability analysis with DCON
  - time-evolved calculations
- Global mode growth rates in presence of passive stabilizers with VALEN
  - computed mode eigenfunction from DCON

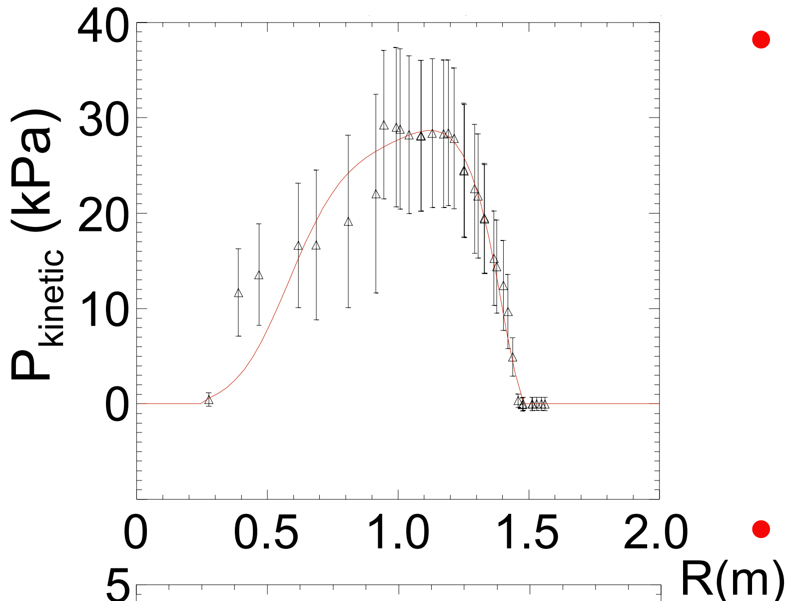
Toroidal flow allows a tractable rotation solution

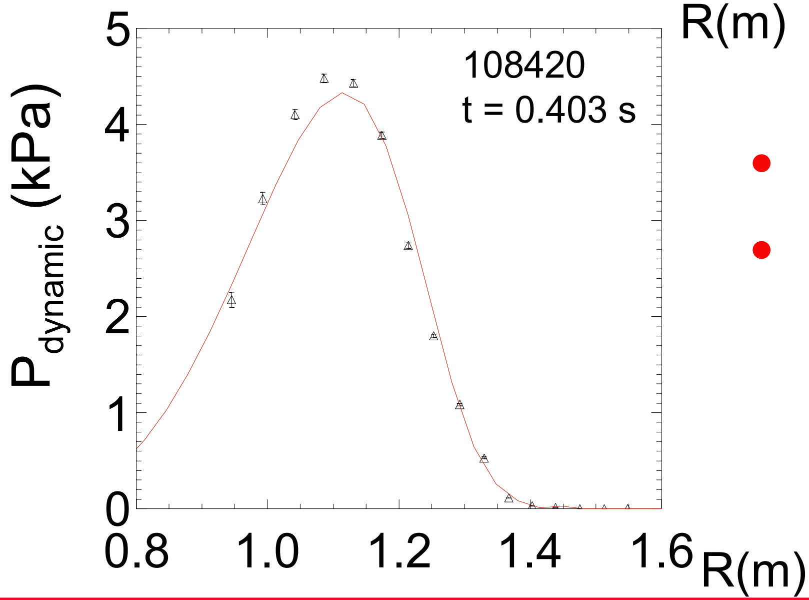
- Solve $\nabla\phi$, $\nabla\psi$, ∇R components of equilibrium equation
  - MHD: $\rho \mathbf{v} \bullet \nabla \mathbf{v} = J \times B - \nabla p$; ρ = mass density
    - $\nabla\phi$: $f(\psi) = RB_t$
    - ∇R: $2P_d(\psi,R)/R = p'(\psi,R)|_\psi$; $P_d \equiv \rho(\psi,R)\omega^2(\psi)R^2/2$ (Bernoulli eq.)
    - $\nabla\psi$: $\Delta^*\psi = -\mu_0 R^2 p'(\psi,R)|_R - \mu_0^2 ff'(\psi)/(4\pi^2)$ (G.S. analog)
  - Pure toroidal rotation and $T = T(\psi)$ yields simple solution for p
    - $p(\psi,R) = p_0(\psi) \exp\left(m_{fluid}\, \omega^2(\psi)(R^2 - R_t^2)/2T(\psi)\right)$
- Typical constraints for fit
  - EFIT reconstructs two new flux functions
    - $P_w(\psi) \equiv \rho(\psi)\, R_t^2\omega^2(\psi)/2$; $P_0(\psi)$ defined so that:
    - $p(\psi,R) = P_0(\psi) \exp\left(P_w(\psi)/P_0(\psi)\, (R^2 - R_t^2)/ R_t^2\right)$
  - Standard input: $P_w(\psi)$, $P_0(\psi)$ from approximation or transport code
  - New approach: Use direct measurement of P_d, P on midplane
    - Solve for $P_w(\psi)$, $P_0(\psi)$ in terms of measured $P(\psi,R)|_{z=0}$, $P_d(\psi,R)|_{z=0}$

NSTX EFIT reconstructions now using T_i, Z_{eff} profiles, and fit to plasma toroidal rotation

Poloidal flux and pressure







- Exact rotation solution fitting total and dynamic plasma pressure at (R, Z=0)
  - A few thousand shot*times run
  - Simple estimate for P_{fast}
- Stored energy with/without $V\phi$ = +/- 3%
- $(R_{pmax} - R_{axis})/a = 8\%$
- Significant drop in χ_{mag}^2 and χ_p^2 even though 50% more P channels and smaller error bars

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Several upgrades now being tested for June 04 run

- Data
  - 51 channel CHERS data recently made available
    - $P_d(\psi,R)|_{z=0}$
    - P_i available – will reduce error bars on "partial kinetic" $P(\psi,R)|_{z=0}$
  - Motional Stark effect
    - Internal pitch angle constraint
  - Nearly 400 measurements will be used
- Physics model
  - Flux iso-surface constraint – fit $T_e = T_e(\psi)$
    - Required to insure self-consistent solution with toroidal rotation
    - Should reduce error on q profile by fitting $(\partial\psi/\partial R)|_{z=0}$

NSTX EFIT continues to advance physics modeling and to meet group needs

- NSTX EFIT results are widely-used
  - Post-shot analysis used in various ways
  - Operations use to help guide the run
- Automated control (PHOENIX code)
  - Full discharge time evolution is computed minutes after the shot
  - About 5000 equilibria generated each run day
  - Extensive initial testing – only one constraint set used for a given data set
- Toroidal rotation now included in reconstructions
  - Large shift of core pressure contours from magnetic surfaces
  - Reconstructed stored energy essentially unchanged
- Flux iso-surface constraint being tested for June run
- High resolution profile diagnostics suggest examining magnetic island effects (3-D) in present 2-D model

Reconstruction of Spherical Torus Equilibria

S. A. Sabbagh, L.L. Lao, Z. Cheng, M.G. Bell, R.E. Bell, B.P. LeBlanc, J.E. Menard

A basic and essential component of the analysis of experimental plasmas is the accurate reconstruction of free-boundary magnetohydrodynamic equilibria based on measurements. A classic approach to this problem is the EFIT[1] algorithm, used successfully on many devices to provide toroidally symmetric representations of plasma discharges. The algorithm is reviewed and application to spherical torus (ST) geometry[2] is discussed. Special considerations for the ST include the modeling of passive conducting structure and plasma rotation. For a given set of measurements, a minimum set of modeling constraints is found that reliably produce equilibria over the entire discharge. These sets are considered for NSTX, for which about 5000 equilibria are generated during each run day. Full discharge reproduction is available minutes after the plasma is produced. Use of partial profile information to reduce modeling constraints is considered. Upgrades to the diagnostic set allow greater physics detail and reduced modeling constraint. Generalization of NSTX EFIT to utilize these new measurements are discussed, including plasma toroidal rotation and flux iso-surface constraint.

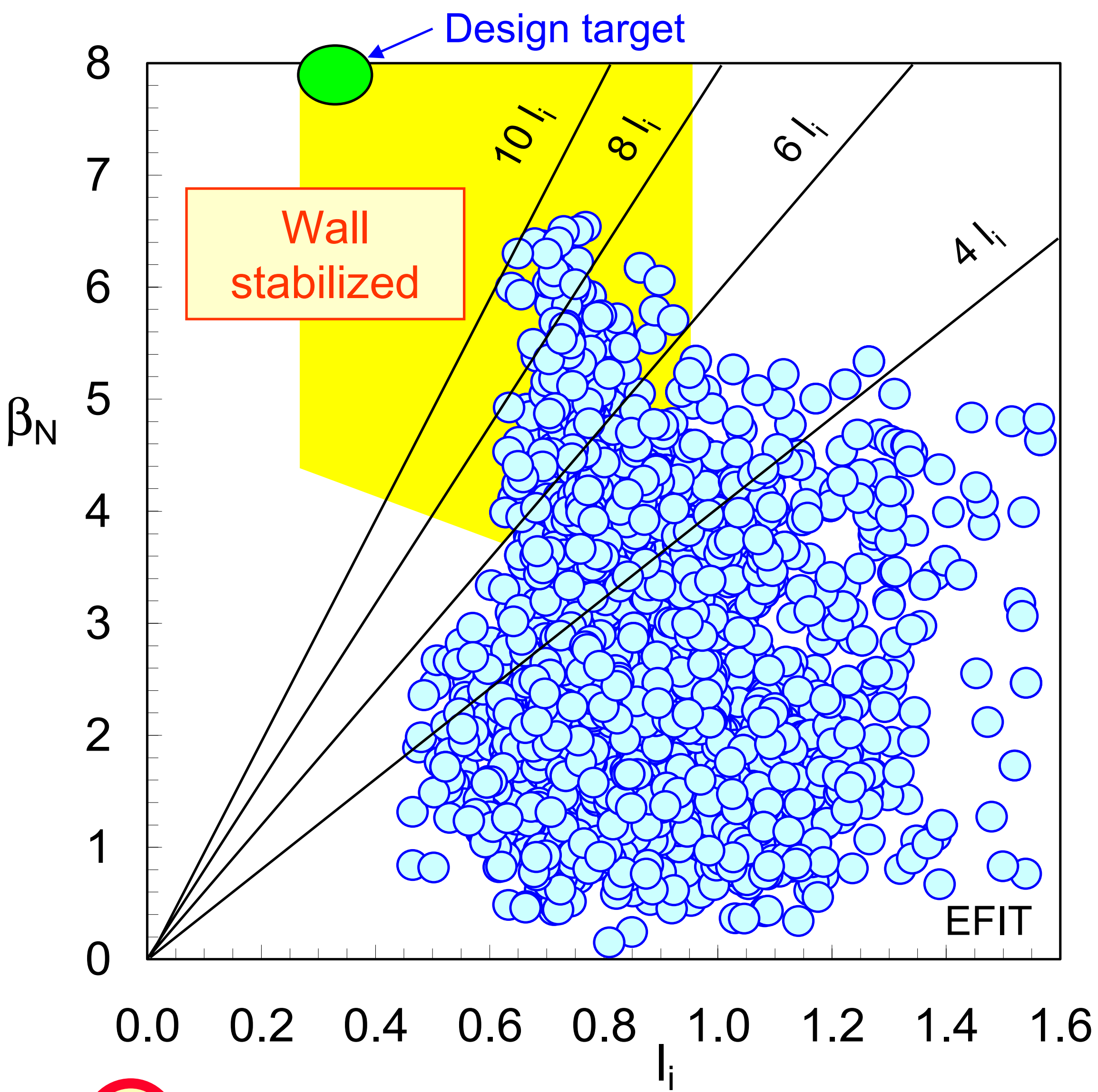
[1] L. Lao, et al., Nucl. Fusion **25** (1985) 1611.

[2] S.A. Sabbagh, et al., Nucl. Fusion **41** (2001) 1601.

Outline - 25 min talk / 5 min questions

- Talk should have sufficient review material for students
- Philosophy
  - “perfect” model has all physics and all diagnostics (for constraints)
  - Upgrade toward perfect model (eliminate artificial constraint and add physics)
  - between-shots or bust
- Quick summary of present reconstruction techniques
  - external magnetics only & “partial kinetic” as a reduction of artificial constraint
- Developing toward better reconstruction
  - upgraded external magnetics and wall model
    - result: plate currents now being resolved in vacuum shots
  - toroidal rotation – required physics upgrade for rotating ST
  - flux iso-surface constraint – required to make rotation consistent
    - reduced error in q profile
- Applications
  - High beta, handling fast current penetration, ELMs, consistent results with rotation (reduced chi^2), stability input
- Future directions (anisotropy, soft X-ray constraint, drop in SXR elongation at increased rotation).

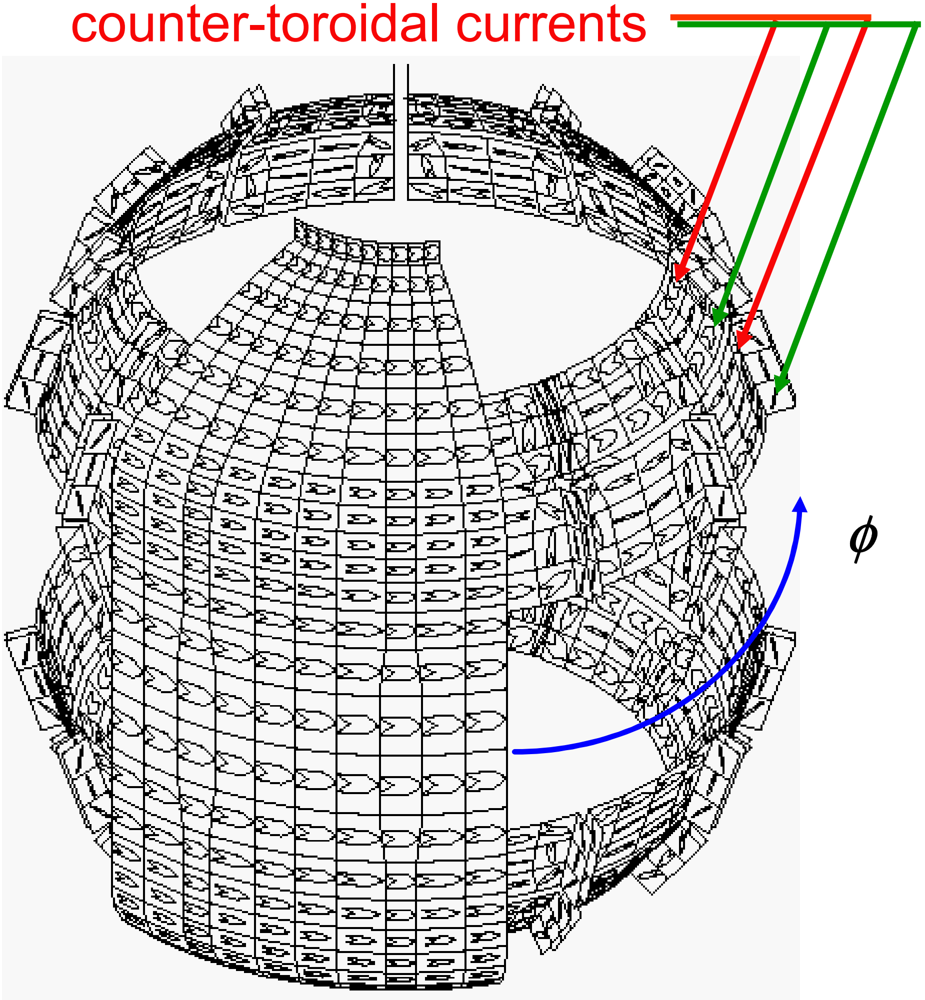
NSTX plasmas operate in wall-stabilized space



- Normalized beta, β_N = 6.5, with β_N/l_i > 9.5
- β_N up to 35% over $\beta_{N\ \text{no-wall}}$ (computed using DCON)
  - Stability limit dependent on both l_i and pressure peaking
- Toroidal beta has reached 35% ($\beta_t = 2\mu_0\langle p\rangle / B_0^2$)

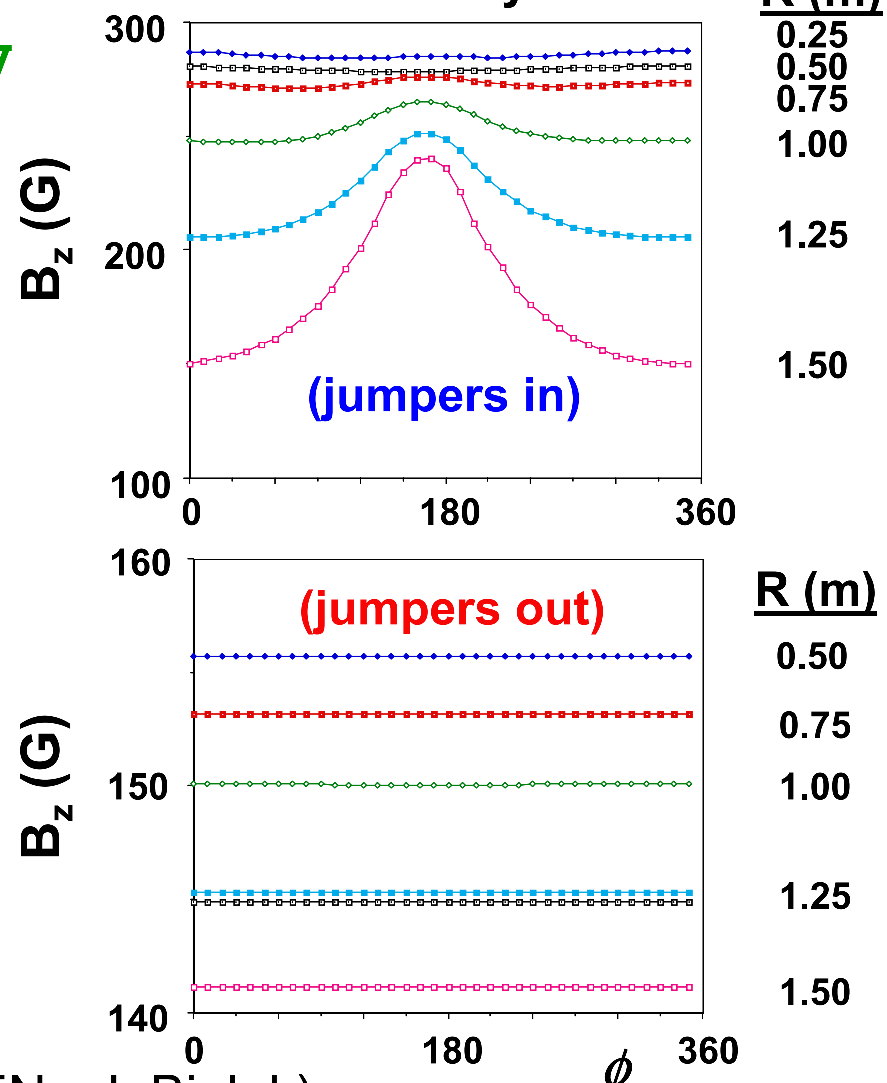
3-D eddy currents reproduced as axisymmetric currents with expanded magnetics

counter-toroidal currents



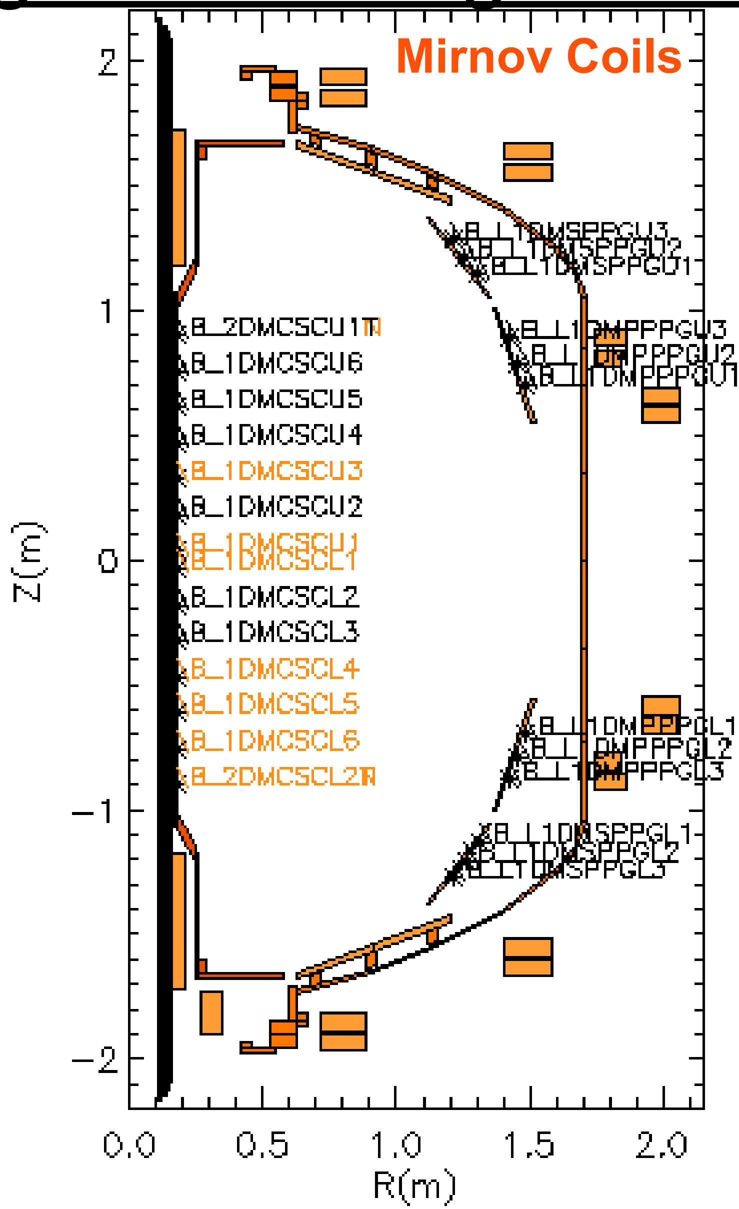
Vessel and plate eddy currents

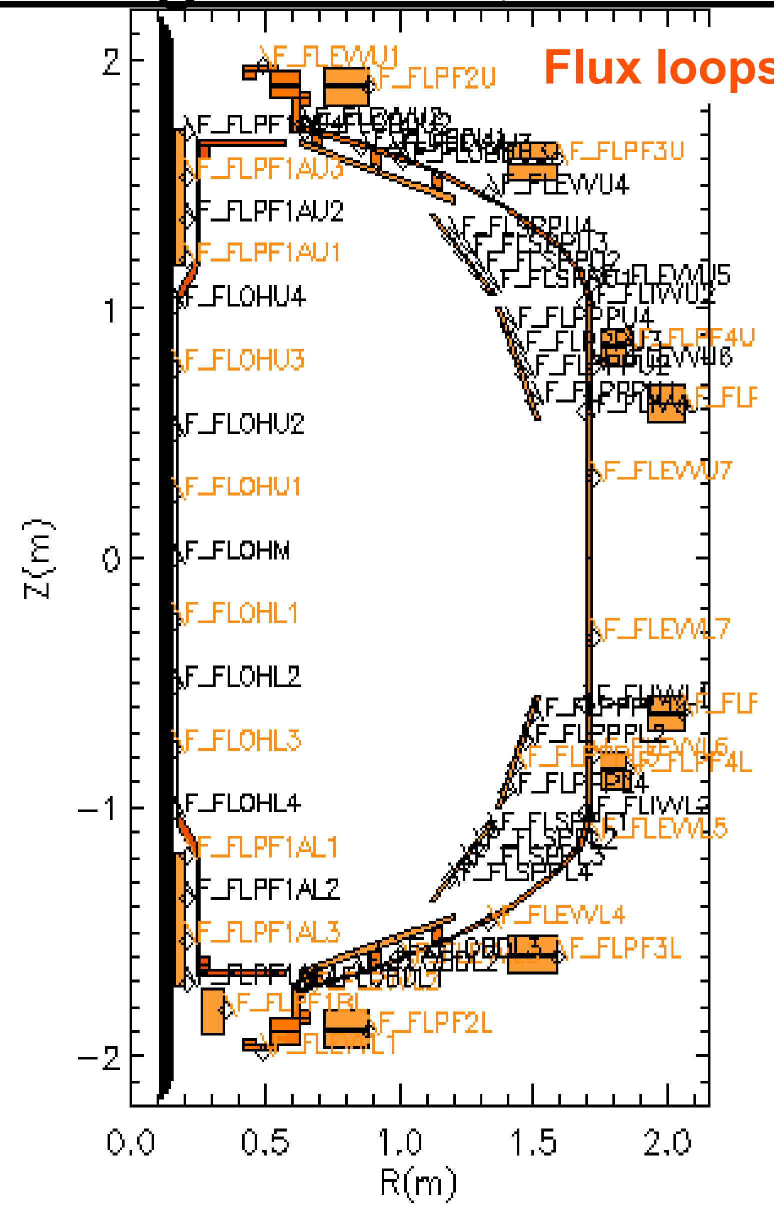
Field from eddy currents



(VALEN - J. Bialek)

Magnetics configuration / diagnostics for EFIT





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NSTX Constraint models for between-shots EFIT (I)

- Significantly different approach to constraint modeling
  - Find one constraint set that will work for all plasmas for a given set of diagnostic input
  - Eliminates "tweaking" of model that might lead to reasonable results for one shot/time, but not for other shots/times
- "Magnetics-only" model ("m1a")
  - over 60 attempted model variations used to find it
  - allows finite edge current (to model current transients)
  - 9 shaping / OH coil current measurements
  - 45 flux loops; 23 Mirnov loops; 1 plasma Rogowski
  - 9 loop voltage monitors for vessel current
  - Profile constraints: $p'(0) = 0$, $(ff')'(1) = 0$
  - 4 profile variables (1 p', 3 ff'; 2nd order polynomial in p', 3rd order in ff')