P1-20

When is it Valid to Assume that Heat Flux is Parallel to *B*?

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Abstract

It is frequently assumed that heat flow in the plasma scrape-off-layer is everywhere parallel to ***B***, due to the strong anisotropy in electron thermal conductivity. This assumption is convenient but paradoxical. Here are examined three situations where this assumption has sometimes been applied: 1) extrapolating from midplane $T_e(R)$ measurements to divertor heat flux profile, 2) determining the location of the separatrix from measured midplane $T_e(R)$, combined with total heat flux leaving the plasma, and 3) predicting the heat flux to plasma-facing components in the scrape-off-layer of diverted plasmas. Numerical solution of the anisotropic, nonlinear heat equation suggests that the first application is poor, the second well justified, and the third far from accurate. Additional plasma physics effects may mitigate these results, but the simple assumption of dominant parallel heat flow due to anisotropy in electron thermal conductivity is not supported in many important cases.

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Introduction

It is convenient to assume, for many applications, that heat flow, $\boldsymbol{q}$, in the scrape-off-layer (SOL) of a fusion plasma is everywhere parallel to the magnetic field, $\boldsymbol{B}$. With the further assumption that $\nabla \cdot \boldsymbol{q} = 0$, one has that the parallel heat flux, $q_{//} \propto B$. This set of assumptions, however useful, is fundamentally paradoxical as the SOL width itself is set by the competition between parallel and cross-field transport, both of which are generally assumed to be continuous along $\boldsymbol{B}$. Here are examined three situations where this set of assumptions has sometimes been applied: 1) extrapolating from midplane $T_e(R)$ measurements to divertor heat flux profile, 2) determining the location of the separatrix from measured midplane $T_e(R)$, combined with total heat flux leaving the plasma, and 3) predicting heat flux to plasma-facing components in the scrape-off layer of diverted plasmas.

Extrapolating from midplane $T_e(R)$ measurements to divertor heat flux profile

A previous study[1] examined the relationship between the parallel heat flux at the divertor plate and the midplane temperature profile, based on analytic and numerical solution of the anisotropic, nonlinear heat equation:

$$\frac{\partial}{\partial r}\chi_{\perp 0}T^{\alpha}\frac{\partial}{\partial r}T+\frac{\partial}{\partial z}\chi_{\parallel 0}T^{\beta}\frac{\partial}{\partial z}T=0 \qquad \text{Eq. 1}$$

in rectilinear geometry, with fixed heat flux from the main plasma and $T = 0$ boundary condition at the divertor plate. Other effects such as flux expansion, convective heat flux, spatial variation of plasma density, ion thermal transport, volumetric power loss, sheath resistance to power flow and heat flux limitation at low collisionality were in that work (and are here) neglected. Interestingly, a nonlinear eigenmode solution[2] to equation (1) was found to reproduce the well-known two-point model result that the parallel heat flux at the divertor plate, $q_{//,Div}$ is proportional to $T^{\beta}/(\beta+1)$ at the midplane. However the constant of proportionality was found to depend on both α and β. Furthermore, numerical investigation revealed that the nonlinear eigenmode, analogous to the fundamental eigenmode of a linear problem, cannot match arbitrary boundary conditions at the main-plasma / SOL interface. In the radial region close to the separatrix the two-point proportionality can be strongly violated, so the overall width of the divertor heat flux is decoupled from the midplane temperature profile, particularly as the ratio of the connection length to the divertor divided by the connection length to the x-point, $\tau \equiv L_{Div}/L_x$, varies.

Here, instead of assuming constant perpendicular heat flux across the separatrix, the perhaps more realistic assumption is made of $q_\perp \propto \cos(\pi z/2L_x)$, where z represents distance along $\boldsymbol{B}$ from the outer midplane. Spitzer-like parallel and Bohm-like perpendicular diffusivities are assumed. Characteristic values of τ are taken from C-Mod and NSTX, as shown in Figure 1. It is found for these assumptions that the divertor heat flux width is about three times greater than the $T^{7/2}$ profile width, quite inconsistent with the simple two-point model result, which is based on assuming parallel-only heat flux.

Determining the location of the separatrix from measured midplane T_e(R) combined with total heat flux leaving the plasma

Another use to which the assumption of $\boldsymbol{q} \parallel \boldsymbol{B}$ is put is to determine the location of the separatrix, based on comparing the predicted heat flux to the divertor via the two-point model with the measured heat flux crossing the plasma surface. This is less sensitive than predicting the divertor heat flux width from $T^{7/2}$, because it is both an integral quantity and also involves taking a measured quantity (the total heat flux) to the 2/7 power. Assuming a cosine dependence of deposited heat flux above the x-point, it is straightforward to generalize the two-point model to give:

$$\frac{T_0^{7/2}}{q_{\parallel,Div}} = \frac{7}{2}\frac{L_{Div}}{\chi_0}\left(1 - \frac{1-2/\pi}{\tau}\right) \qquad \text{Eq. 2}$$

Comparing this expression with the computational result gives an error of 1.251 for the C-Mod case, and 1.042 for NSTX. Assuming a fixed profile shape for *T,* this implies an error of only 6.6% and 1.2% respectively in determining T_{max}, at the midplane separatrix location.

Predicting heat flux to first-wall components in the scrape-off-layer of diverted plasmas

It is difficult, but very important, to estimate the heat flux to components in the SOL of fusion plasmas. A common and convenient approximation is to assume that components which protrude into the SOL will intercept the same total heat flow, $\int q_{\parallel}\, dR$, as would have been intercepted by the divertor plate "behind" them, in their absence. More precisely, it is assumed that a set of *N* such components, plus the divertor plates, will each absorb 1/(*N*+1) of this heat flux, with a radial profile that depends on the assumed model for how heat penetrates into the reduced connection length. This approach assumes in some respects $\boldsymbol{q} \parallel \boldsymbol{B}$ (the total heat flow) and in other respects denies it (equal sharing despite shadowing effects).

On the other hand, it is well known that there is a precise analogy between the linear, isotropic heat equation and the electrostatic equation, and that the electrostatic field at sharp points diverges, as then must heat flux. Figure 2 shows the numerical result for an infinitesimally thick scraper in a plasma SOL. This is achieved by placing a zero temperature boundary condition along a line *between* computational zones. As the spatial resolution was increased to 8000 radial zones and 4000 zones along *B*, it was found that the heat flux at the scraper tip diverged, varying with the distance from the tip as $1/\delta r^{1/2}$. This is consistent with the result from electrostatics, even in this highly anisotropic, nonlinear case. Thus there is evidently no constraint in the heat equation indicating that the heat flux profile at a scraper must reflect that which would have been accepted at a divertor plate in its shadow.

Such high resolution solution is made possible by using Jacobi iteration and an unconditionally stable Alternating Direction Implicit numerical technique, with the time step exponentially decreasing to the Courant Condition. This mimics in some regards a multi-grid solution, as large time steps correspond to coarse spatial resolution. The computational effort scales with the number of grid points as $N^2 ln(N)$, rather than N^4 as with an explicit solution. Convergence studies resulted in power balance accuracy better than 1%. Other calculations shown here employ 2000 radial zones and 1000 zones along ***B***.

In general, it is clear that a cold surface attracts heat flux across ***B***; sharp points are not required. Figure 3 shows temperature contours in the case of a shaped first-wall structure of 2 cm depth across ***B*** and 4 m extension along ***B.*** The total heat flux to this structure is 1.85% of the heat influx to the SOL, as compared with 0.742% that goes to the divertor plate behind it, in its absence. The effective parallel heat flux (which includes flux both along *and* perpendicular to ***B***) to this structure is very large, as shown in Figure 4. It greatly exceeds the local $q_{||}$ in its absence. The large peak near 0.03 m, the inner edge of the structure, is associated with cross-field heat flux to the plasma-facing surface. This effect is reminiscent of the particle "funnel" described by Stangeby *et al*[3]. It is not clear that cross-field heat flux of this sort to material components is included in 2-D and 3-D SOL and divertor codes, since the necessary sheath conditions have not been established. For a case with four of these structures evenly spaced along ***B***, the total heat flux reaches 3.75% of the SOL heat influx. The conventional model would predict 8/9 x 0.742% (taking into account the mirror symmetry of the solution) or 0.66%, equal to 18% of what is calculated.

Discussion and Conclusions

The analyses provided here are based on solution of the anisotropic, nonlinear heat equation, in rectilinear geometry, with $T = 0$ boundary condition at the divertor plate and plasma-facing components. Many important physical effects are neglected, so these results are more appropriate as motivation for questioning the even simpler assumptions being tested than as final answers. Nonetheless it is clear that the mapping from midplane $T^{7/2}$ to $q_{//,Div}$ is not simple, and depends both on the assumed profile of heat flux across the separatrix and also on $\tau \equiv L_{Div}/L_x$. The use of the total heat flux coupled with a generalized two-point model to determine T_{max} at the midplane separatrix is not strongly disturbed by cross-field transport effects. Most concerning, however, is that observation that cross-field heat diffusion to components in the SOL of high power plasmas may be significantly underestimated by the simple approximations that derive from $\boldsymbol{q} \parallel \boldsymbol{B}$. In particular more research on the cross-field heat flux to such components appears to be called for.

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Figure Captions

Figure 1. Midplane $T^{7/2}$ and divertor heat flux, normalized to their peak values, for C-Mod and NSTX-like geometries. Also shown is the comparison with nonlinear eigenmode solution for $q_{||}/T^{7/2}$. Widths are given by the integral definition, for example: $\lambda_q \equiv \int q\, dR \,/\, q_{max}$. Calc/2PM indicates total calculated heat flux divided by simple two-point model prediction.

Figure 2. Temperature contours in the SOL of a JET-size plasma, with a 2 cm deep and infinitesimally wide scraper. Contours are located at T/T_{max} = 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.02, 0.01, 0.005, 0.002, 0.001.

Figure 3. Figure 2, with a 2 cm deep and 4 m wide scraper.

Figure 4. Effective parallel heat flux (arbitrary units) to scraper of figure 3. Detailed structure (already smoothed here) is due to "staircase" discretization of scraper shape.

Figures

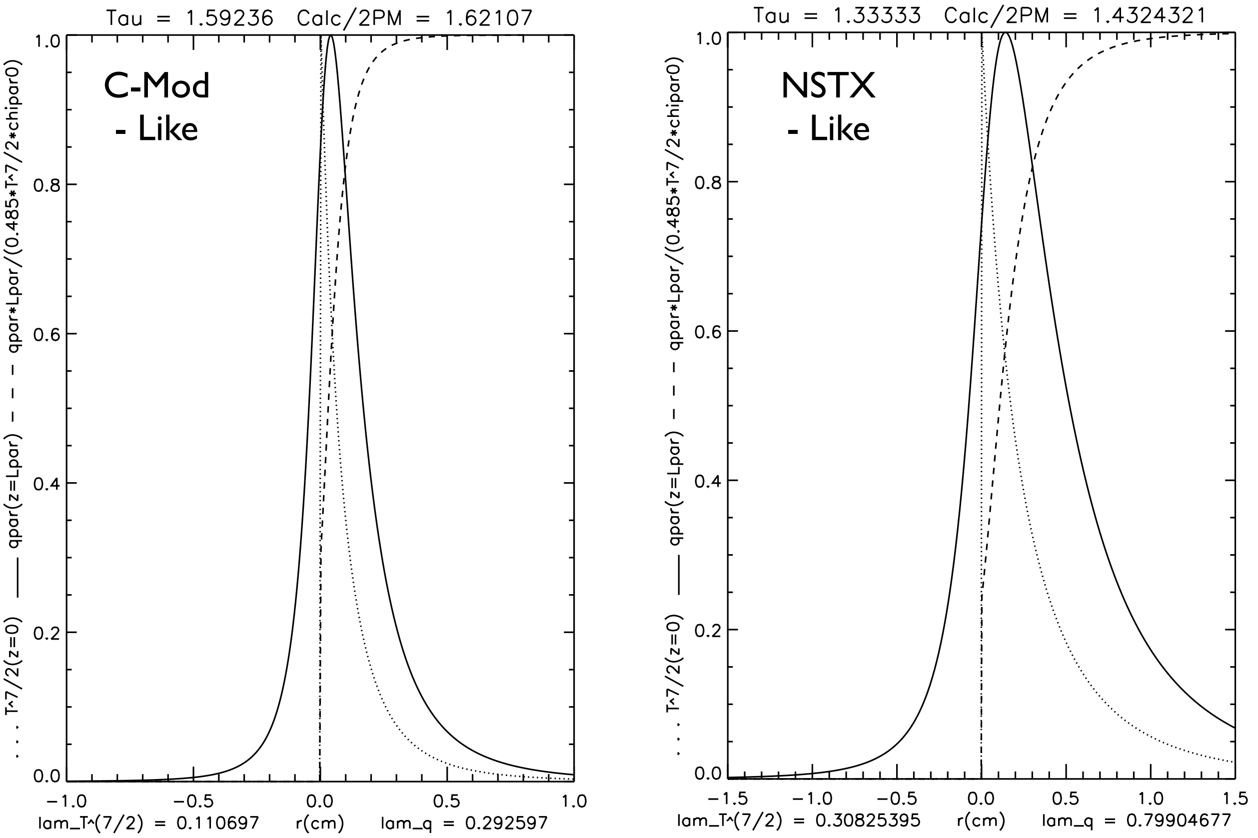

Figure 1.

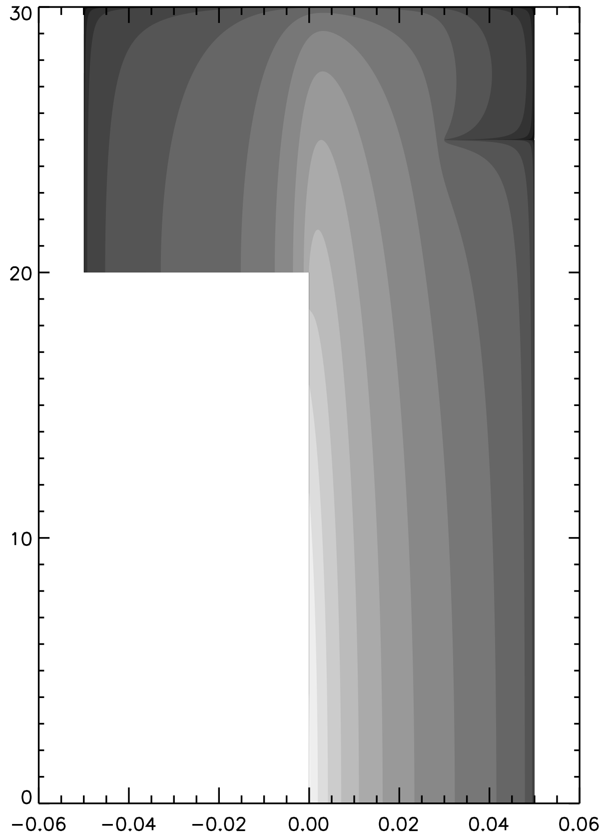

Figure 2.

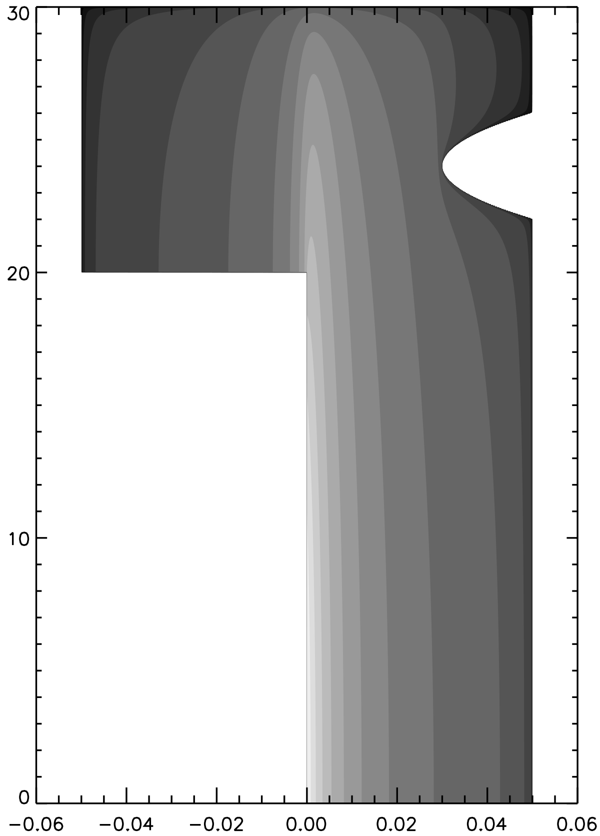

Figure 3.

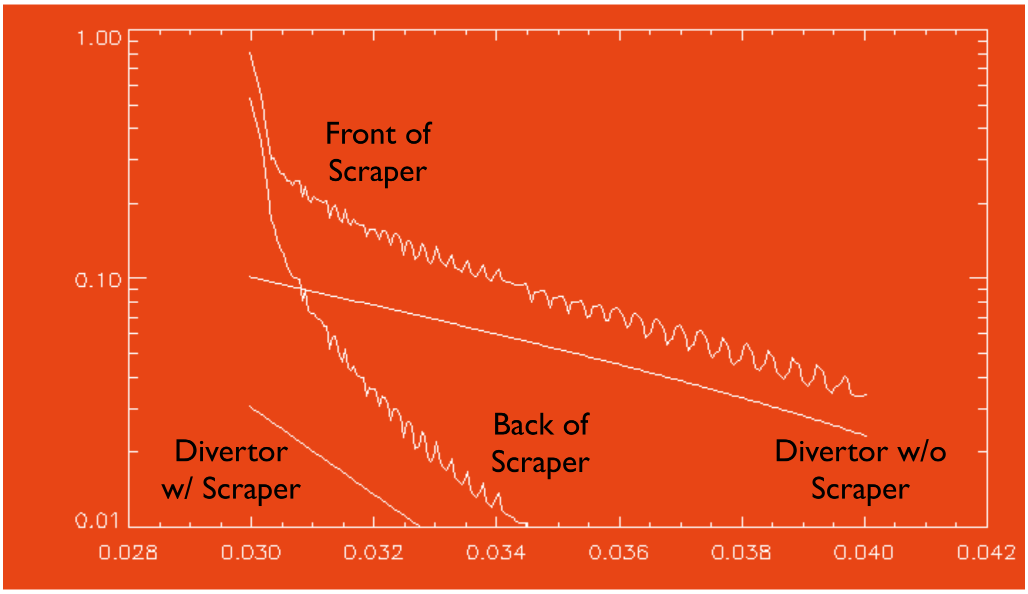
1.00
0.10
0.01
0.028
0.030
0.032
0.034
0.036
0.038
0.040
0.042
Front of Scraper
Back of Scraper
Divertor w/ Scraper
Divertor w/o Scraper

Figure 4.