M3D Simulation Studies
of
NSTX

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Outline

- M3D code
  - MHD, two-fluids, hybrid models.
- NSTX studies including flow effects
  - 2D steady states.
  - Evolutions of IRE's.
- TAE, BAE modes – G.Y. Fu, Session V-A

M3D Project

W. Park et al., Phys. Plasmas **6,** 1796 (1999)
http://w3.pppl.gov/~wpark/pop_99.pdf

Multilevel 3D Project for Plasma Simulation studies

Various physics levels are needed to understand the physics.
The best method depends on the problem at hand.

Physics

- MHD
- 2 Fluids
- Gyrokin. Hot P./MHD
- Gyrokin.Ion/Fluid Elect.
- ….

Processing

- MPP
- Serial

Meshes

- Unstructured FE
- Structured FD

State

- Equilibrium
- Linear
- Nonlinear

MHD model

- Solves MHD equations.

$$\left\{\begin{array}{l}
\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v} \\
\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}), \quad \mathbf{J} = \nabla\times\mathbf{B} \\
\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0 \\
\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\nabla(p/\rho)
\end{array}\right.$$

The fast parallel equilibration of T is modeled using wave equations;

$$\left\{\begin{array}{l}
\partial T/\partial t = s\,\mathbf{B}/\rho\cdot\nabla u \\
\partial u/\partial t = s\,\mathbf{B}\cdot\nabla\mathbf{T} + \upsilon\nabla^2 u
\end{array}\right.$$

s = wave speed/ v_A

Two-fluid MH3D-T

- Solves the two fluid equations with gyro-viscosity and neoclassical parallel viscosity terms in a torus.

- **Equations**

$$\left(\begin{array}{l}
\mathbf{v} \equiv \mathbf{v}_i - \mathbf{v}_i^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}_{\parallel}/en, \\
\mathbf{v}_e^* \equiv -\mathbf{B}\times\nabla p_e\,/(enB^2), \quad \mathbf{v}_i^* \equiv \mathbf{v}_e^* + \mathbf{J}_{\perp}/en,
\end{array}\right.$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} + \rho(\mathbf{v}_i^*\cdot\nabla)\mathbf{v}_{\perp} = -\nabla p + \mathbf{J}\times\mathbf{B} - \mathbf{b}\cdot\nabla\cdot\Pi_i,$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}) - \nabla_{\parallel}p_e/en - \mathbf{b}\cdot\nabla\cdot\Pi_e,$$
$$\mathbf{J} = \nabla\times\mathbf{B},$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}_i) = 0,$$

$$\begin{aligned}
\partial p/\partial t + \mathbf{v}\cdot\nabla p = {} & -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p/\rho) \\
& - \mathbf{v}_i^*\cdot\nabla p + (1/en)\mathbf{J}\cdot\nabla p_e \\
& - \gamma p\nabla\cdot\mathbf{v}_i^* + \gamma p_e\mathbf{J}\cdot\nabla(1/en)
\end{aligned}$$

$$\begin{aligned}
\partial p_e/\partial t + \mathbf{v}\cdot\nabla p_e = {} & -\gamma p_e\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p_e/\rho) \\
& + (1/en)\mathbf{J}_{\parallel}\cdot\nabla p_e - \gamma p_e\nabla\cdot(\mathbf{v}_e^* - \mathbf{J}_{\parallel}/en)
\end{aligned}$$

GK Hot Particle /MHD Hybrid MH3D-K

- **Fluid equations**

$$\rho \partial \mathbf{v}/\partial t + \rho \mathbf{v}\cdot\nabla \mathbf{v} = -\nabla p - (\nabla\cdot \mathbf{P}_h)_\perp + \mathbf{J}\times\mathbf{B}$$ (Pressure coupling)

or

$$\rho \partial \mathbf{v}/\partial t + \rho \mathbf{v}\cdot\nabla \mathbf{v} = -\nabla p + (\nabla\times\mathbf{B} - \mathbf{J}_h)\times\mathbf{B} + q_h \mathbf{V}\times\mathbf{B}$$

(Current coupling)

$$\partial \mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = \mathbf{v}\times\mathbf{B} - \eta(\mathbf{J} - \mathbf{J}_h), \quad \mathbf{J} = \nabla\times\mathbf{B}$$

$$\partial \rho/\partial t + \nabla\cdot(\rho \mathbf{v}) = 0$$

$$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p \nabla\cdot\mathbf{v} + \rho \nabla\cdot\kappa\cdot\nabla\,(p/\rho)$$

- **Gyrokinetic equations for energetic particles**

$$d\mathbf{R}/dt = u[\, \mathbf{b} + (u/\Omega)\mathbf{b} \times (\mathbf{b}\cdot\nabla\mathbf{b})\,] + (1/\Omega)\mathbf{b} \times (\mu\nabla\mathbf{B} - q\mathbf{E}/m),$$

$$du/dt = -[\, \mathbf{b} + (u/\Omega)\mathbf{b} \times (\mathbf{b}\cdot\nabla\mathbf{b})\,] \cdot (\mu\nabla\mathbf{B} - q\mathbf{E}/m).$$

GK Particle Ion / Fluid Electron Hybrid

- **Pressure coupling**

$$\rho \partial \mathbf{v}/\partial t + \rho \mathbf{v}\cdot\nabla \mathbf{v} = -\nabla\cdot\mathbf{P}i - \nabla Pe + \mathbf{J}\times\mathbf{B}$$

$$= -\nabla\cdot\mathbf{P}i^{CGL} - \nabla\cdot\Pi i - \nabla Pe + \mathbf{J}\times\mathbf{B}$$

$\nabla\cdot Pi^{CGL}$: from particles following GK eqns.

$\nabla\cdot\Pi i$: fluid picture as 2 fluid eqns,
or from particles.

- **Fluid electrons**

$$\mathbf{E} = -\mathbf{V}e \times \mathbf{B} + \eta\mathbf{J} + \nabla\cdot\mathbf{P}e/ne$$

$$= -\mathbf{V}e \times \mathbf{B} + \eta\mathbf{J} + \nabla Pe/ne + \mathbf{bb}\cdot\nabla\cdot\Pi e/ne$$

$$\partial \mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{J} = \nabla\times\mathbf{B}$$

Pe eqn currently, but $P_\parallel$ and $P_\perp$ eqns are planned.

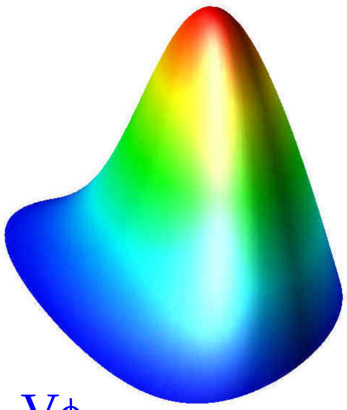
2D steady state with toroidal sheared flow

Quasi neutrality: $r\mathbf{V}\cdot\nabla\mathbf{V}+\nabla\cdot\overleftrightarrow{\mathbf{P}}-\mathbf{J}\times\mathbf{B}=0$

$$\overleftrightarrow{\mathbf{P}} = \overleftrightarrow{\mathbf{P}}^{CGL} + \overleftrightarrow{\Pi}_g$$

$$= p\overleftrightarrow{\mathrm{I}} + (P_{\parallel}-P_{\perp})\overleftrightarrow{\Pi}_{ii} + \overleftrightarrow{\Pi}_g$$

MHD Hot Particle/MHD 2-Fluids



MHD:

At the magnetic axis: $\mathbf{J}\times\mathbf{B}=0$

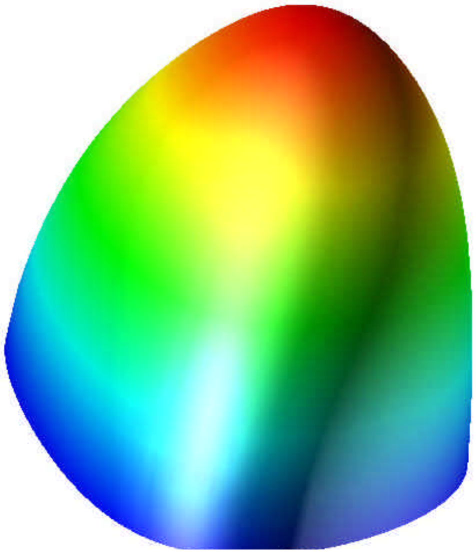
$$-\frac{r\mathrm{V}_f^{\,2}}{R}+\frac{T\partial r}{\partial R}=0$$

Relative shift of $r \equiv \frac{R\partial r}{r\partial R}=\frac{\mathrm{V}_f^{\,2}}{T}=\frac{2M_A^{\,2}}{b}$

Density profile dependence on sheared Rotation

MHD

$\varepsilon=1.3$ $q_o=0.8$ $q_b=5$

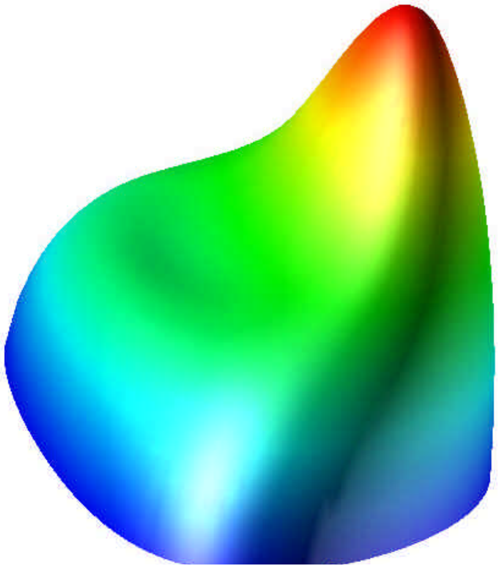


$M_A=0$

Sh=0

$\rho_{max}=1$

$\rho_{min}=0.5$

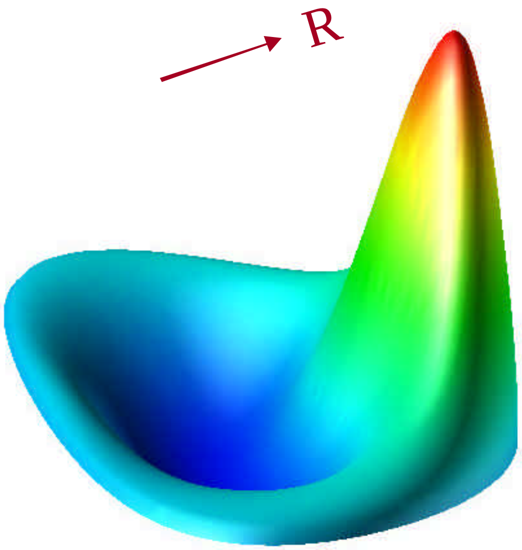


$M_A=0.2$

Sh=0.3

$\rho_{max}=1.1$

$\rho_{min}=0.5$

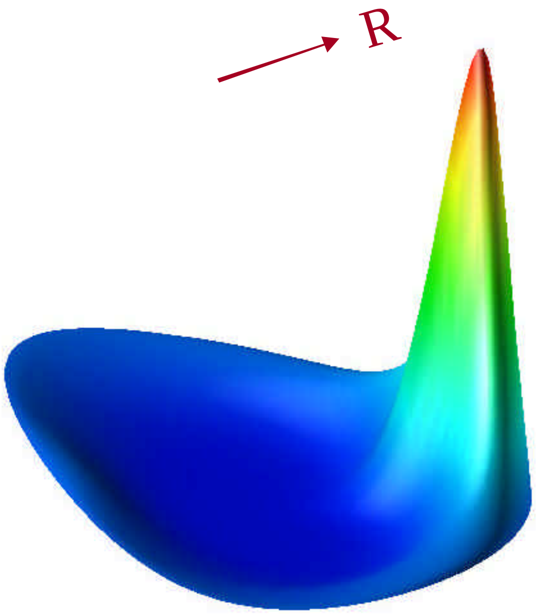


$M_A=0.5$

Sh=0.4-0.07=0.33

$\rho_{max}=1.9$

$\rho_{min}=0.2$



$M_A=0.8$

Sh=0.5-0.15=0.35

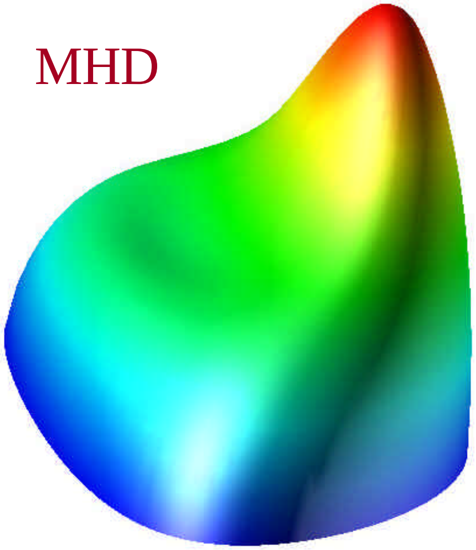
$\rho_{max}=5.2$

$\rho_{min}=0.005$

Density profile dependence on Physics model

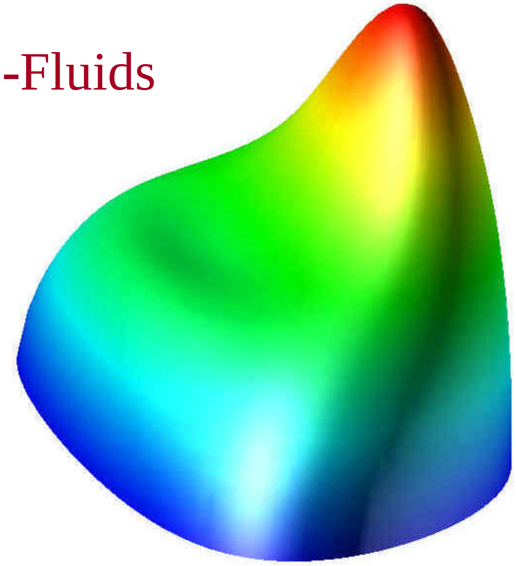
NSTX $\varepsilon=1.3$ $q_o=0.8$ $q_b=5$

MHD



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.1$
$\rho_{min}=0.5$
$RelSh=1$

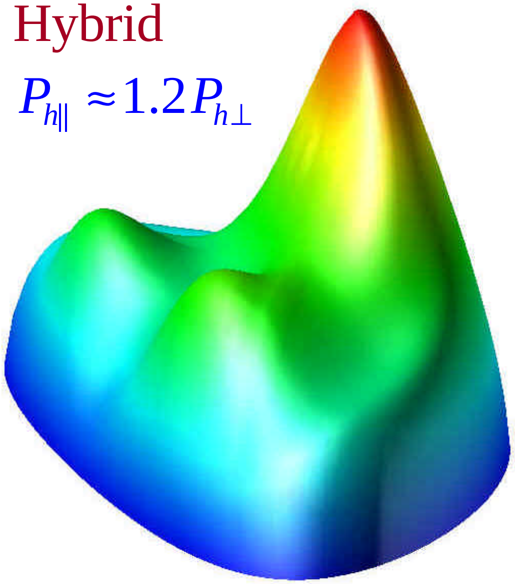
Two-Fluids



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.1$
$\rho_{min}=0.5$
$RelSh=1$

Hybrid

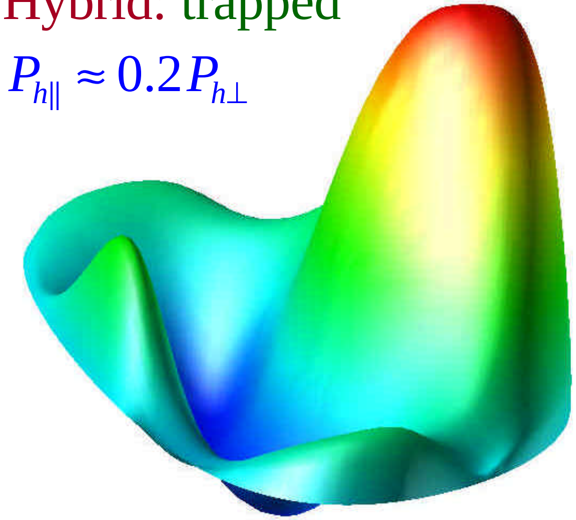
$P_{h\|} \approx 1.2 P_{h\perp}$



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.2$
$\rho_{min}=0.5$
$RelSh=0.8$

Hybrid: trapped

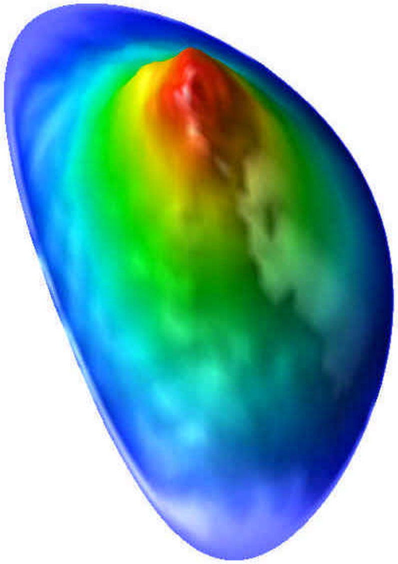
$P_{h\|} \approx 0.2 P_{h\perp}$



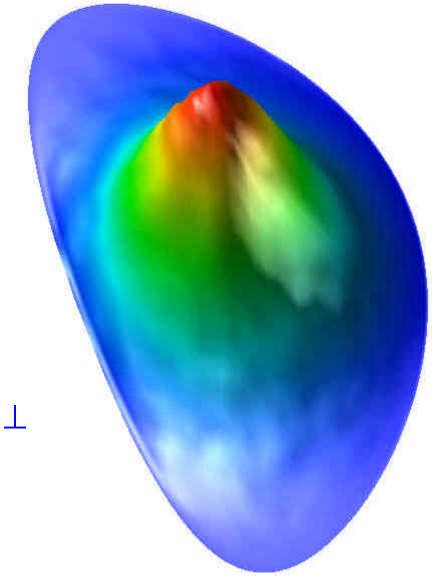
$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.8$
$\rho_{min}=0.15$
$RelSh=1.9$

Hot particle pressure $\mathbf{P}_h$ in the hybrid simulation

$P_{h\|}$



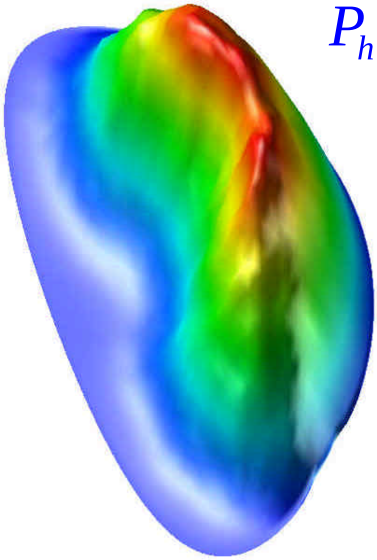
$P_{h\perp}$

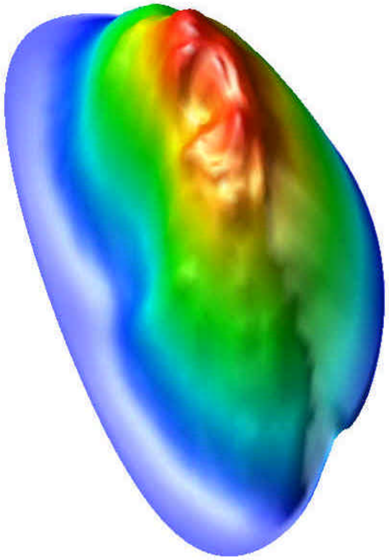


$P_{h\|} \simeq 1.2 P_{h\perp}$

Similar to
Experimental situation

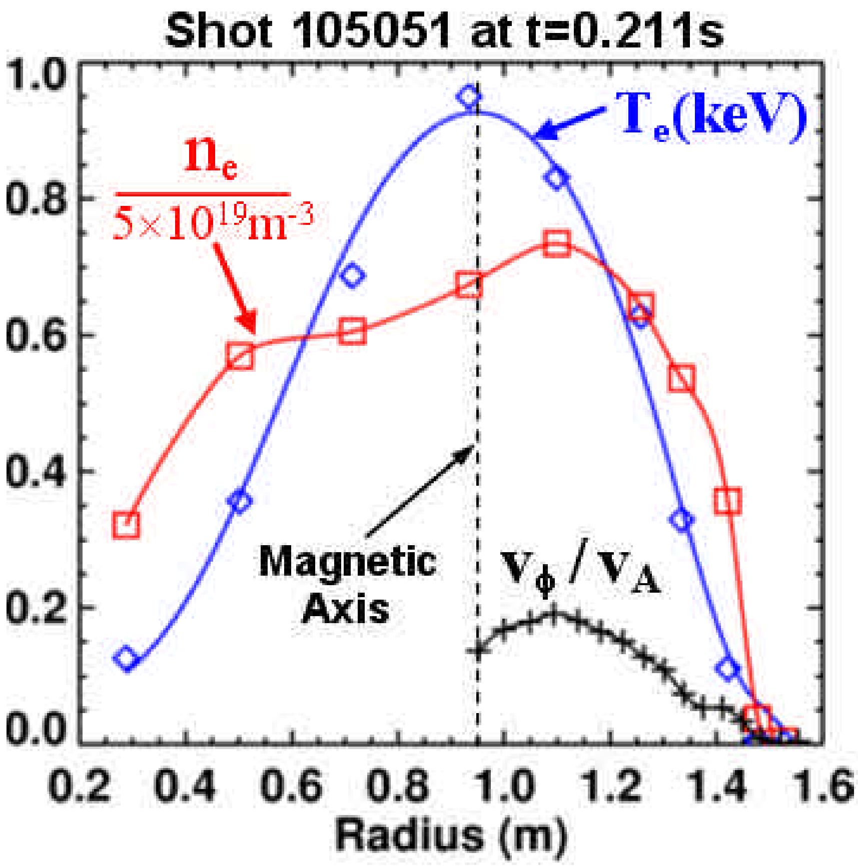
$P_{h\|} \simeq 0.2 P_{h\perp}$





Mostly trapped particles

NSTX experimental data



agrees with MHD derived

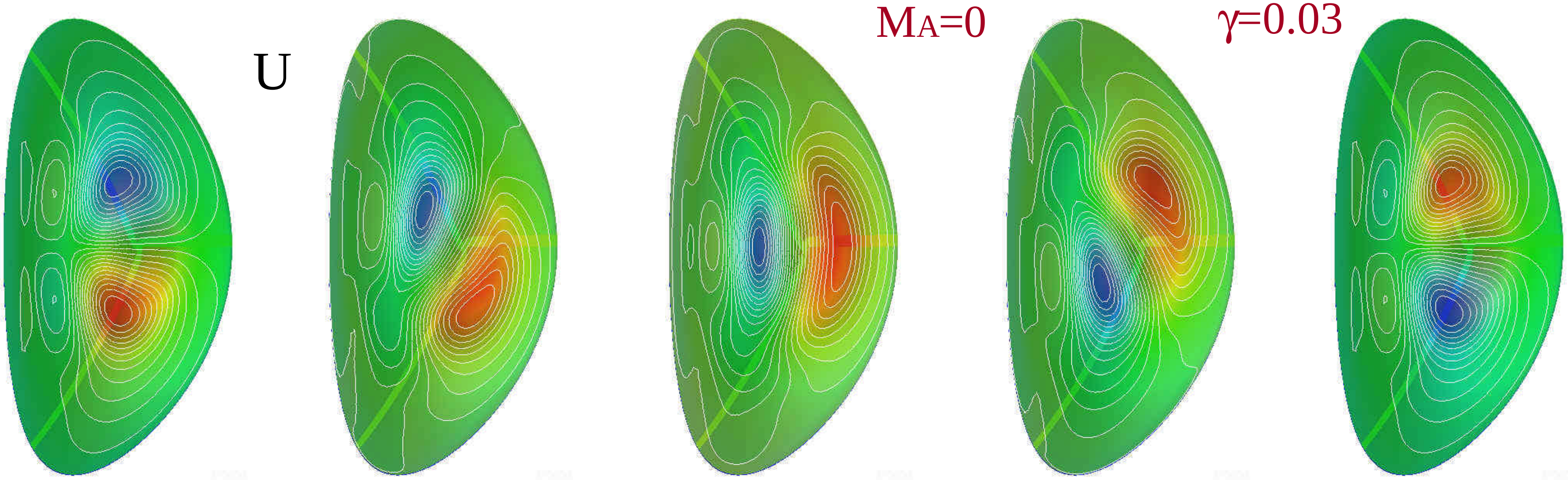
Relative shift of *r*

$$\frac{R\partial r}{r\partial R} = \frac{2M_A{}^2}{b}$$

Hot particle centrifugal force

~ Bulk plasma

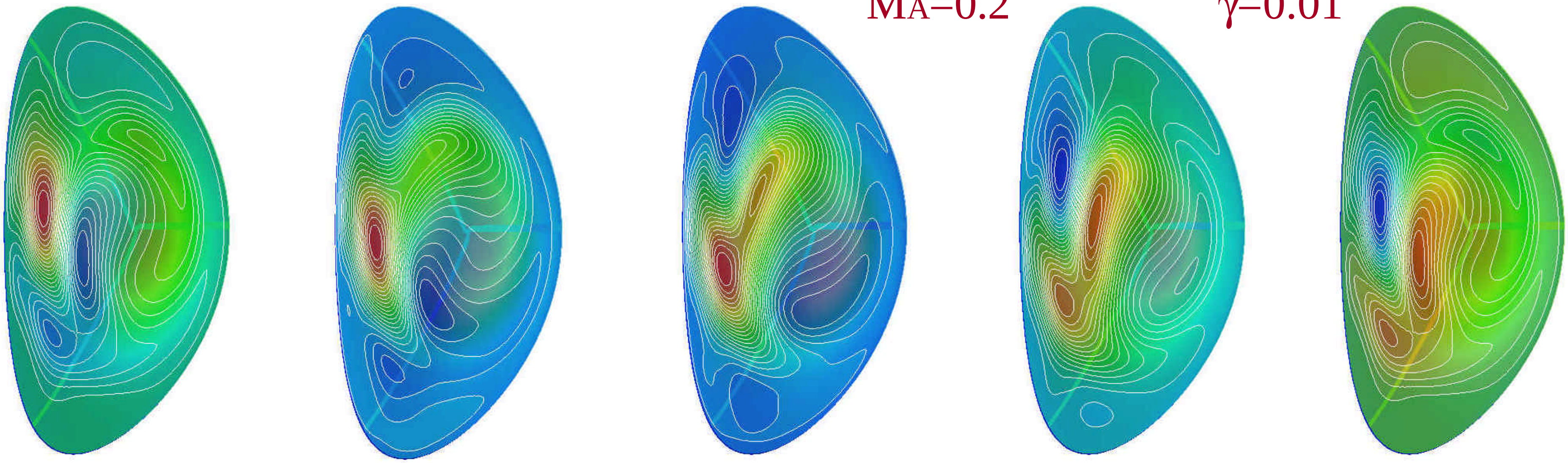
Linear Eigenmodes: shear flow reduces growth rate



U

$M_A=0$ $\gamma=0.03$

$f = 0$ $f = 0.25p$ $f = 0.5p$ $f = 0.75p$ $f = p$



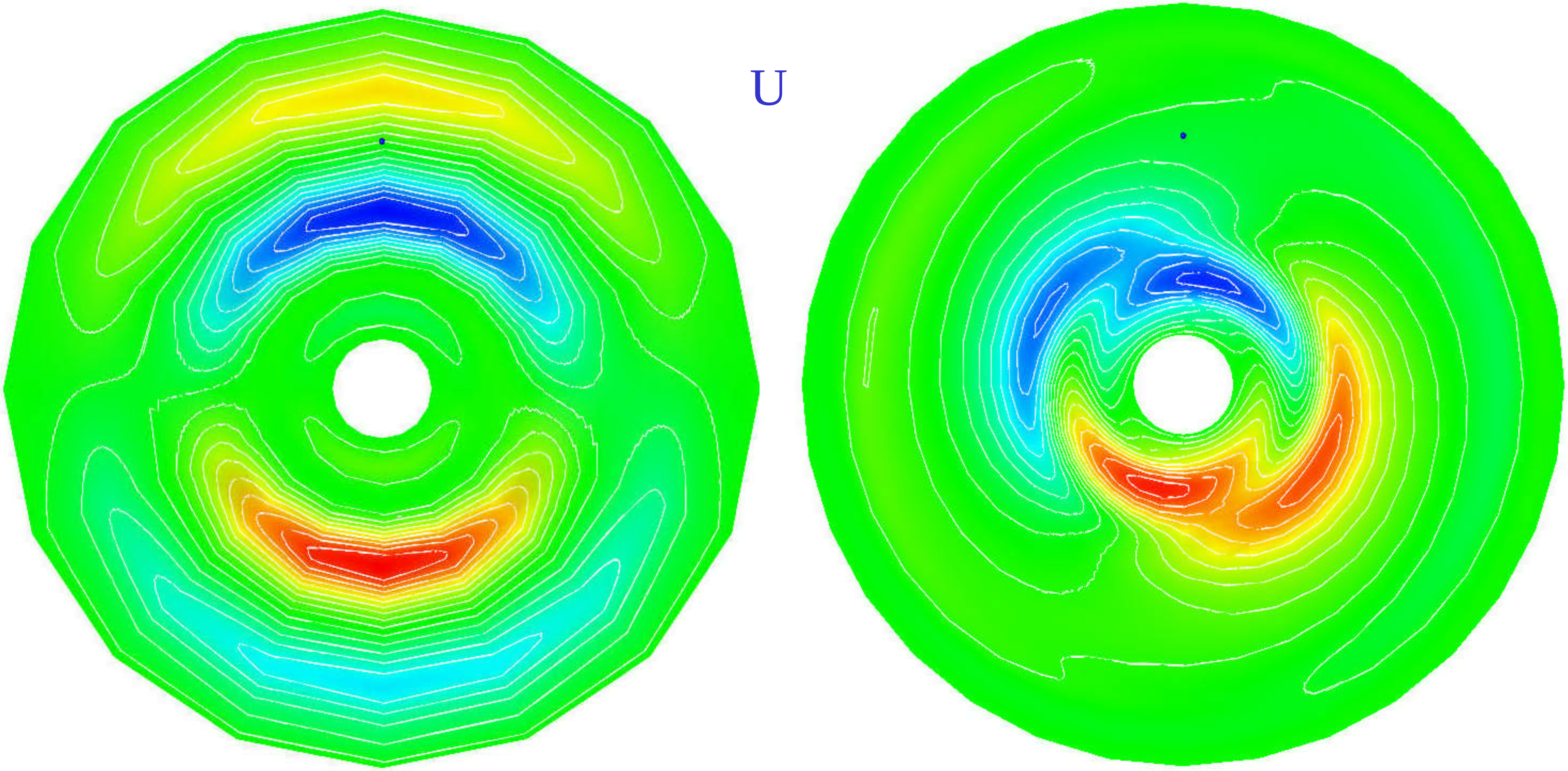
$M_A=0.2$ $\gamma=0.01$

Linear Instability Eigenmodes

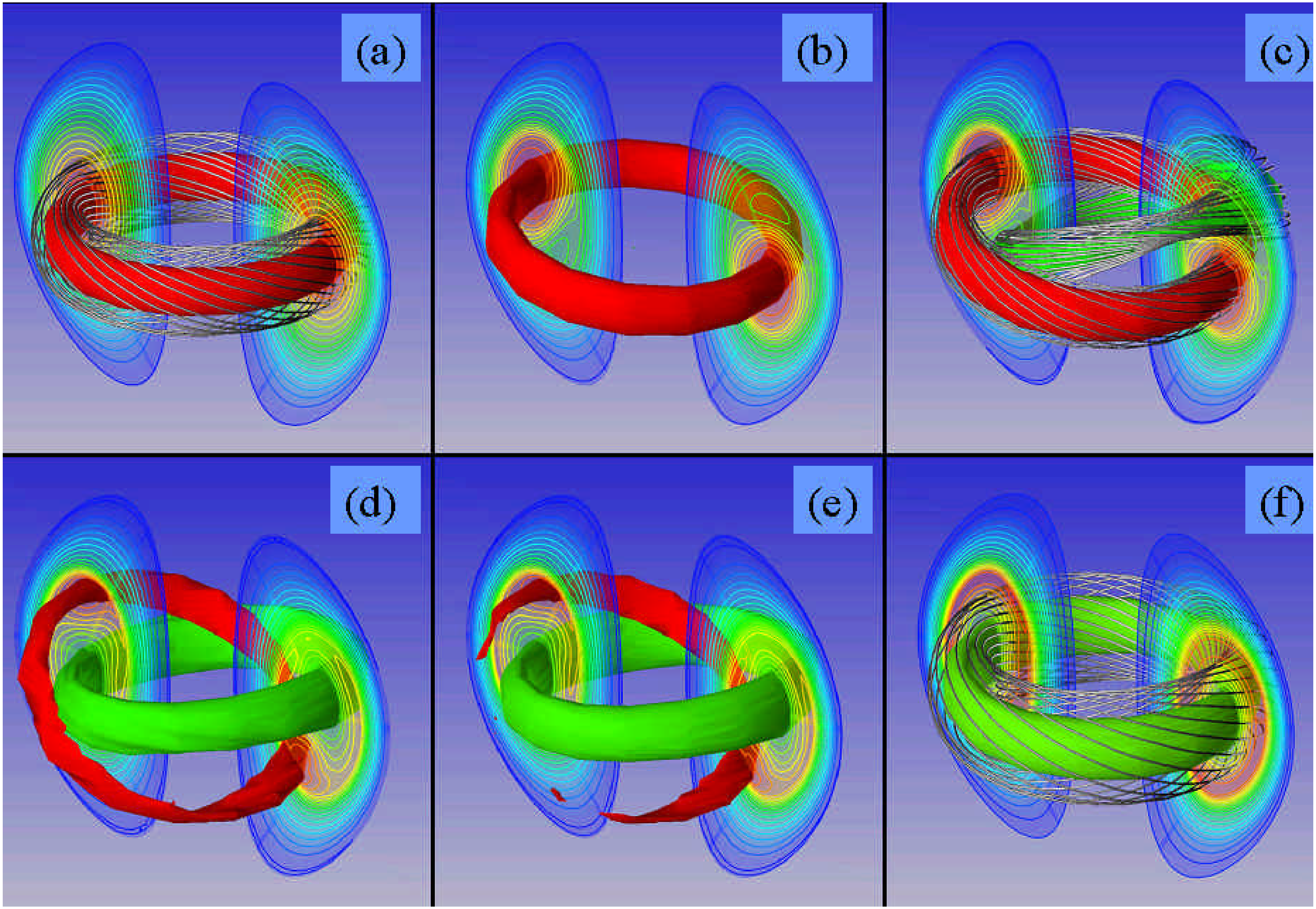
Top view on the horizontal mid-plane

$M_A=0$
$\gamma=0.03$
$\Omega m=0$

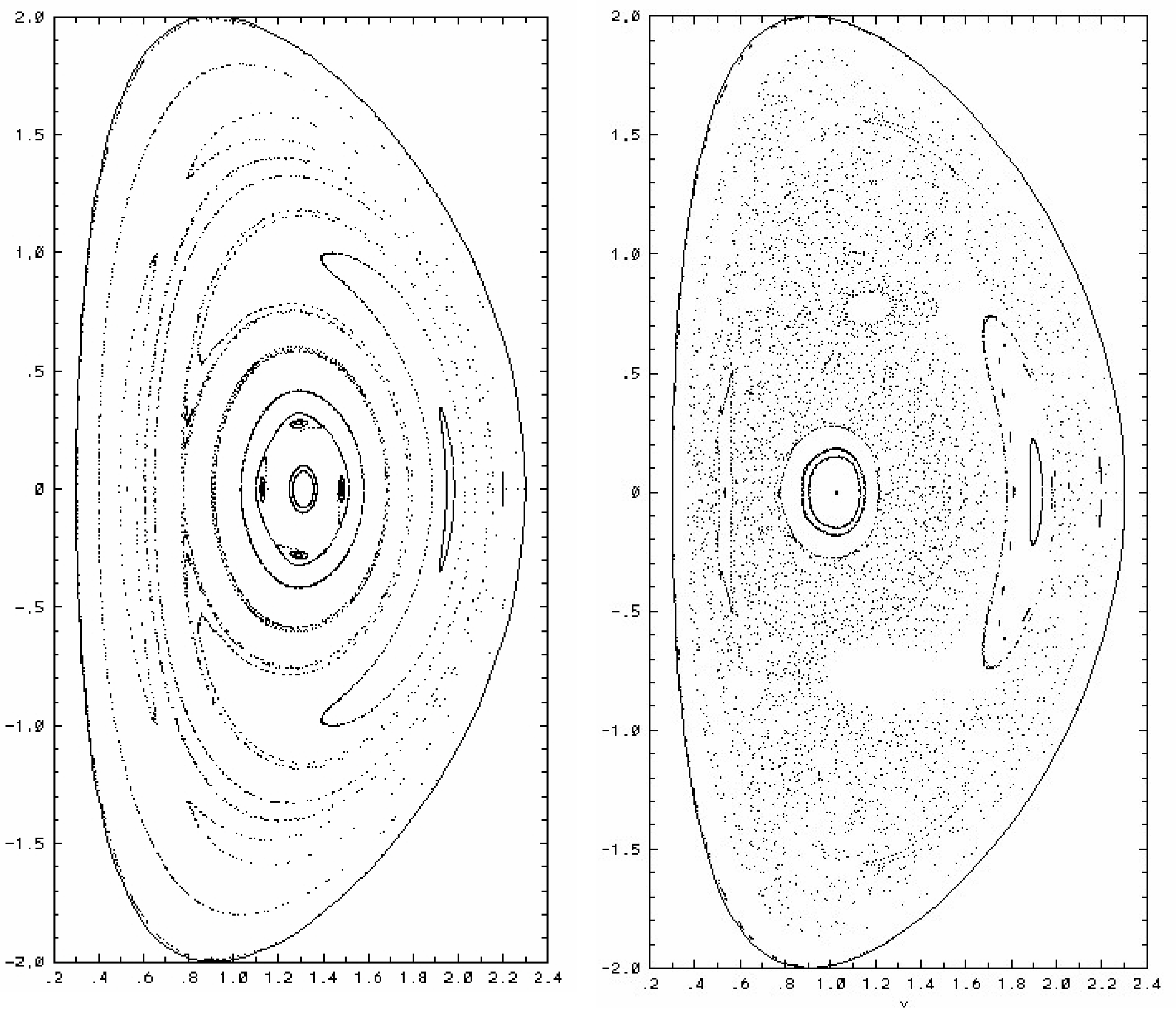
With shear flow: $M_A=0.2$
Reduced growth: $\gamma=0.01$
Rotating mode: $\Omega m=0.13$

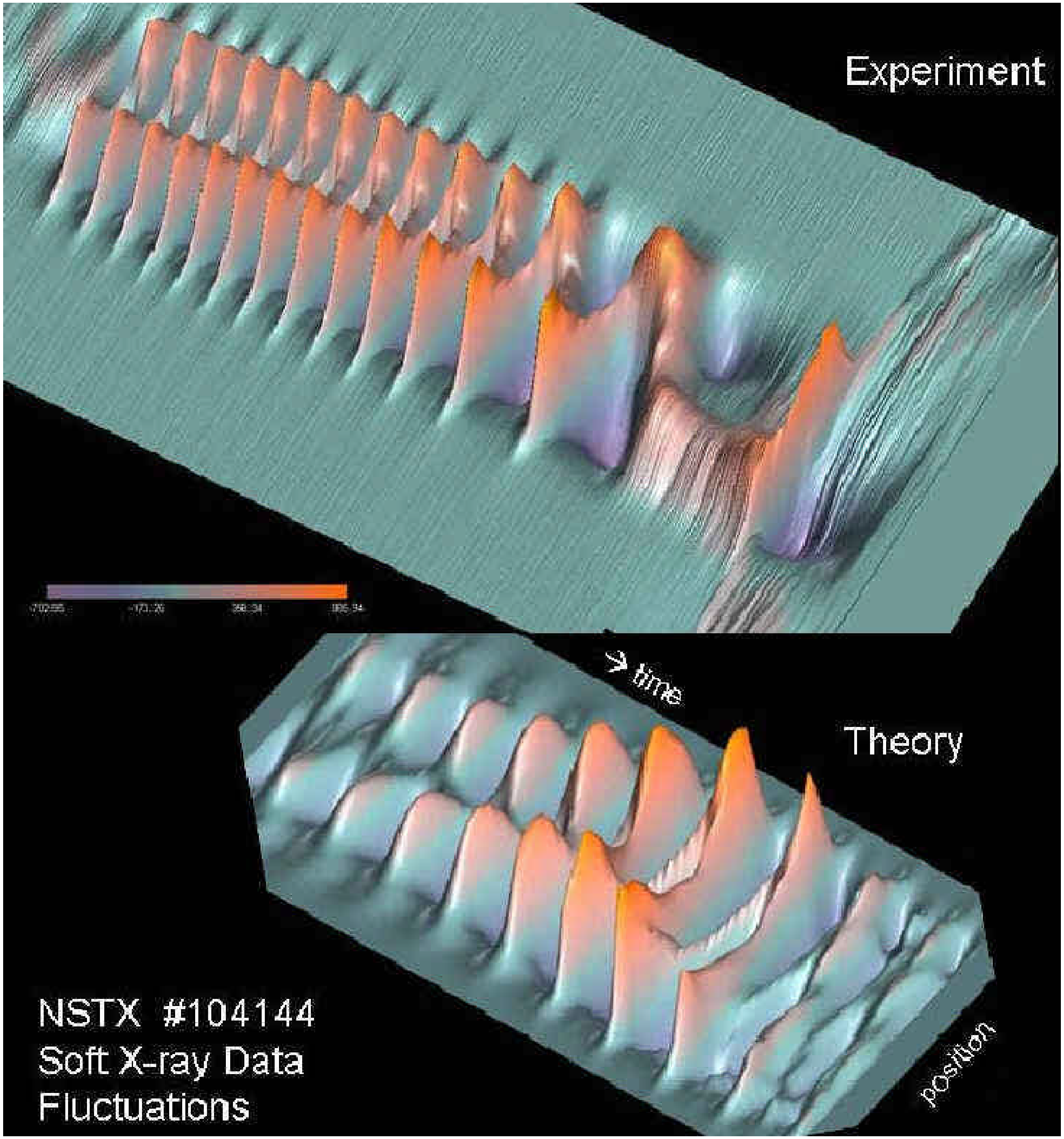


Nonlinear Evolution without strong flow: similar to a sawtooth crash



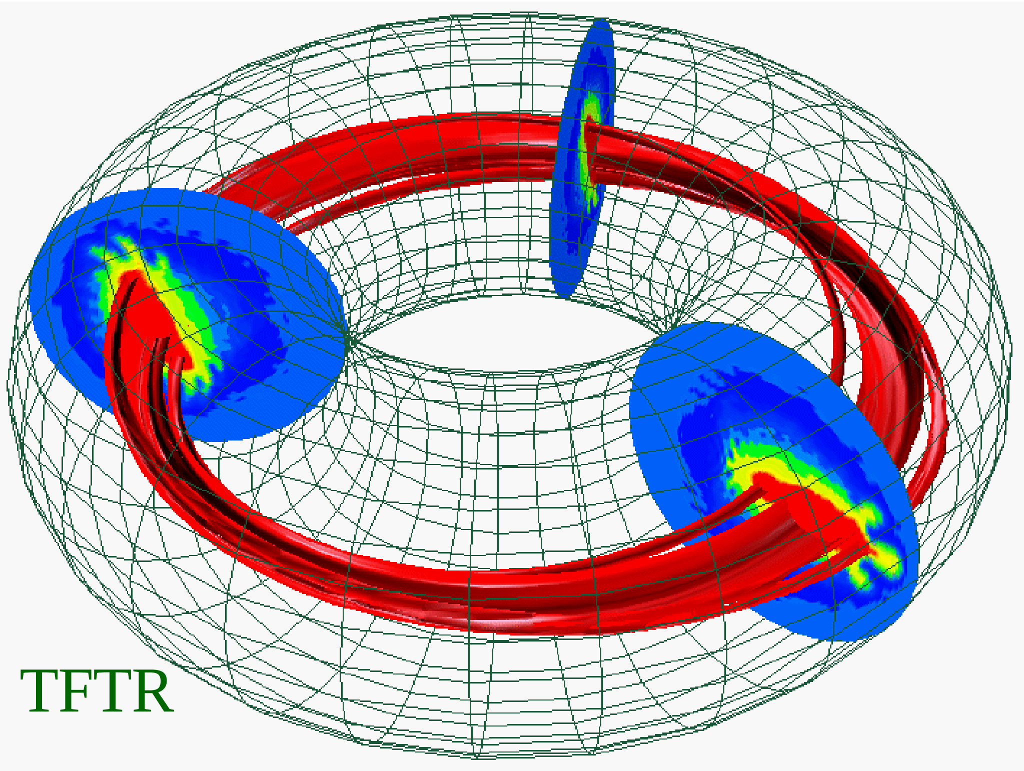
When the inversion radius is large or the plasma β is increased,
magnetic islands overlap and become stochastic.
Disruption due to field line stochasticity.





Soft X-ray signals compared:

Theory agrees with experiment on general characters, but does not have wall locking and a saturation phase.



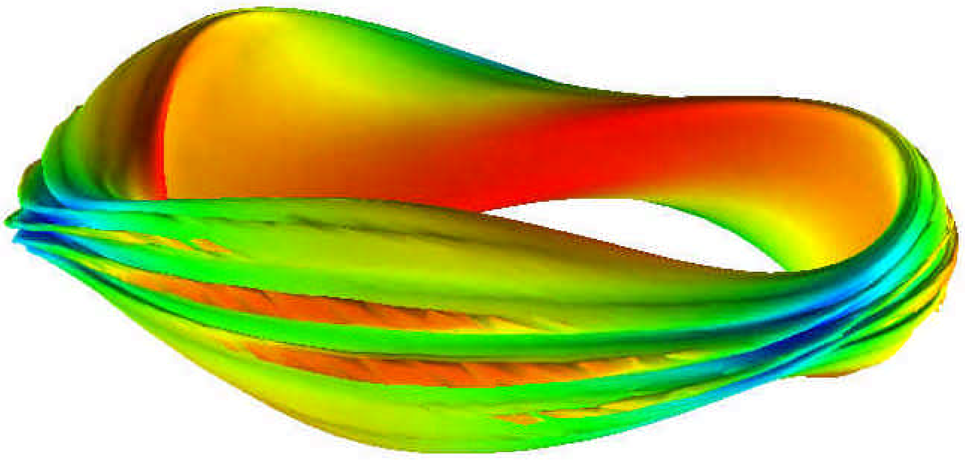
IRE

- Sawtooth
- Disruption due to stochasticity.
- Disruption due to localized steepening of *P* driven modes,

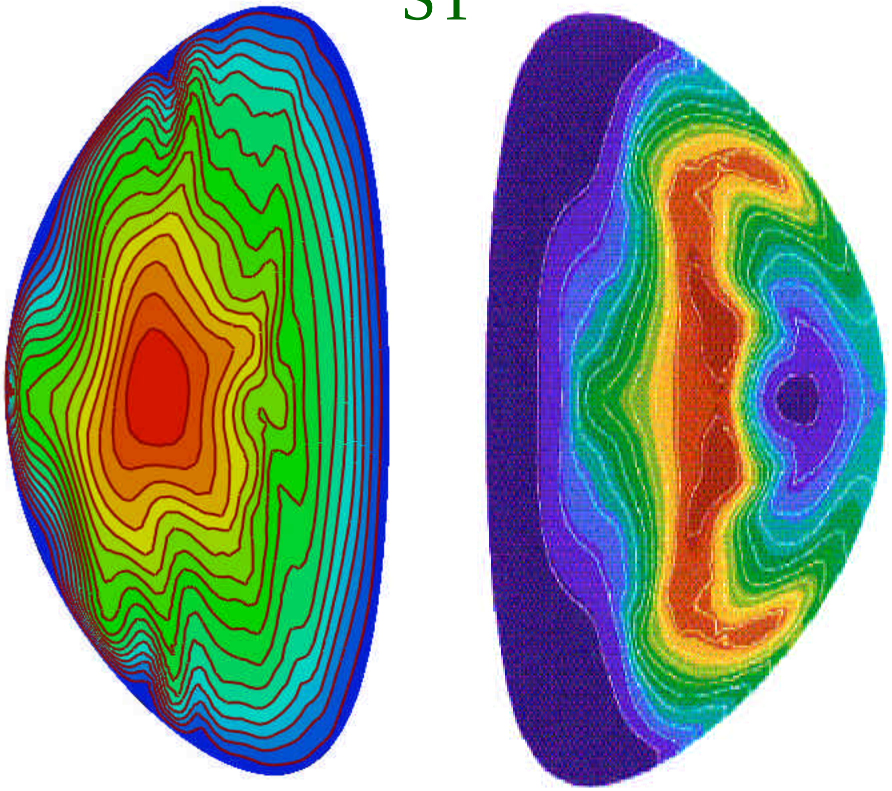
as in Tokamaks

Stellarator

P profiles



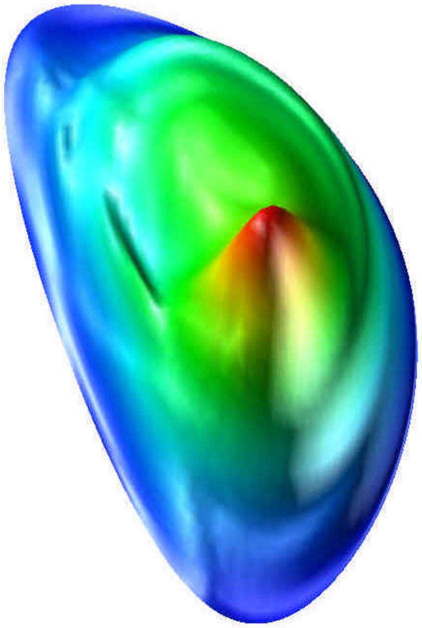
ST



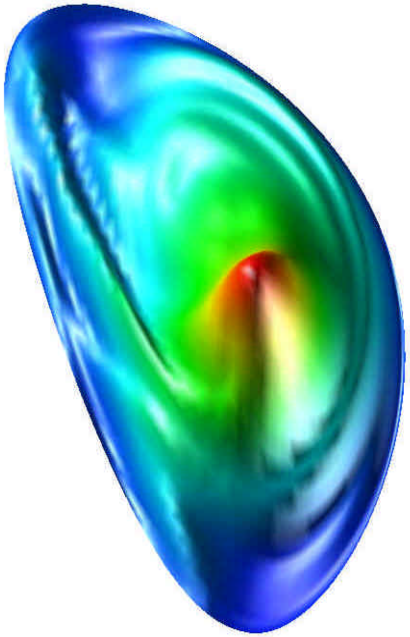
Nonlinear Evolution with peak rotation of $M_A=0.2$

Sheared rotation causes mode saturation,
if rotation profile is roughly maintained.

However, with a normal momentum source rate,
$V\phi$ profile flattens with reconnection,
and full reconnection usually occurs.



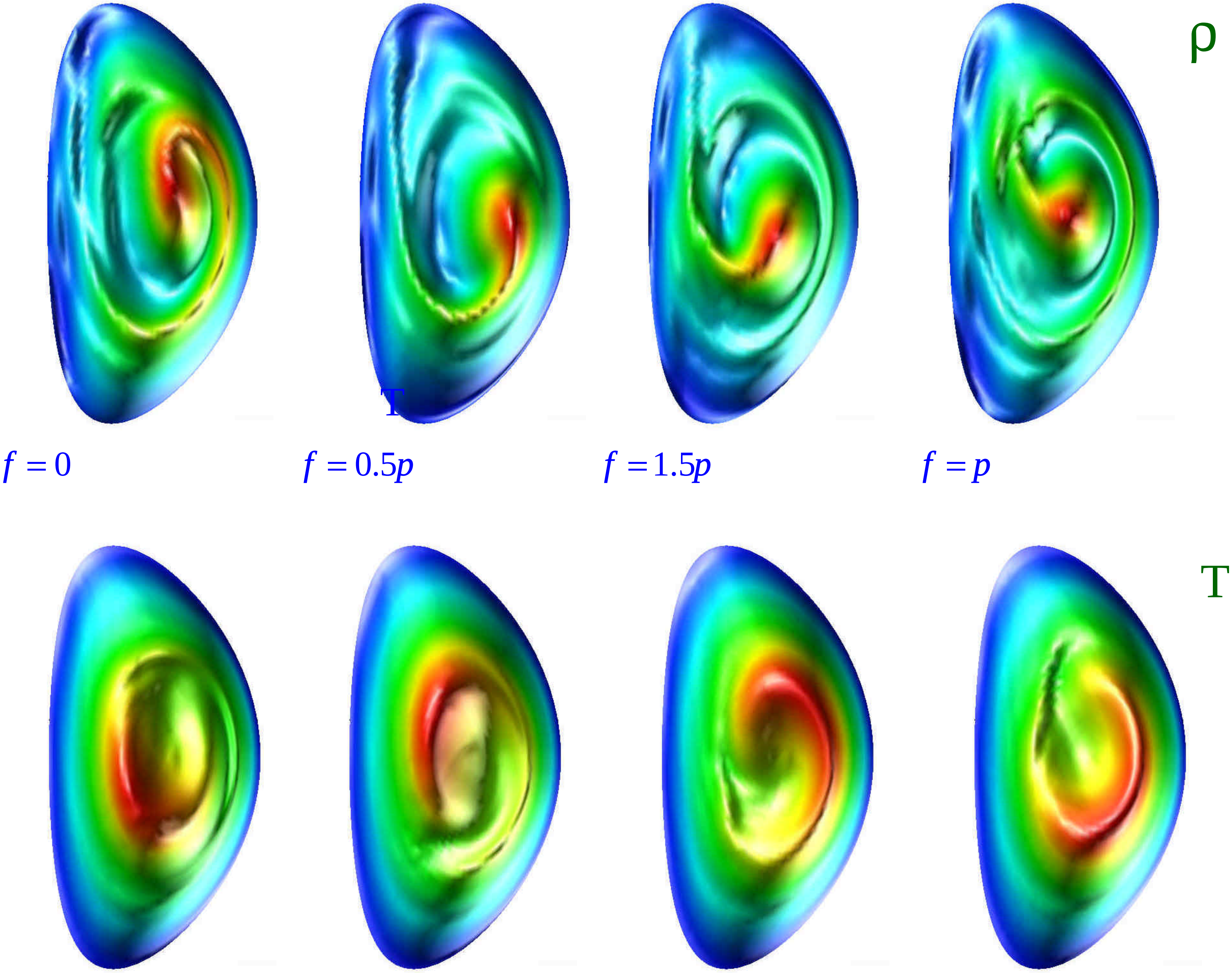
P



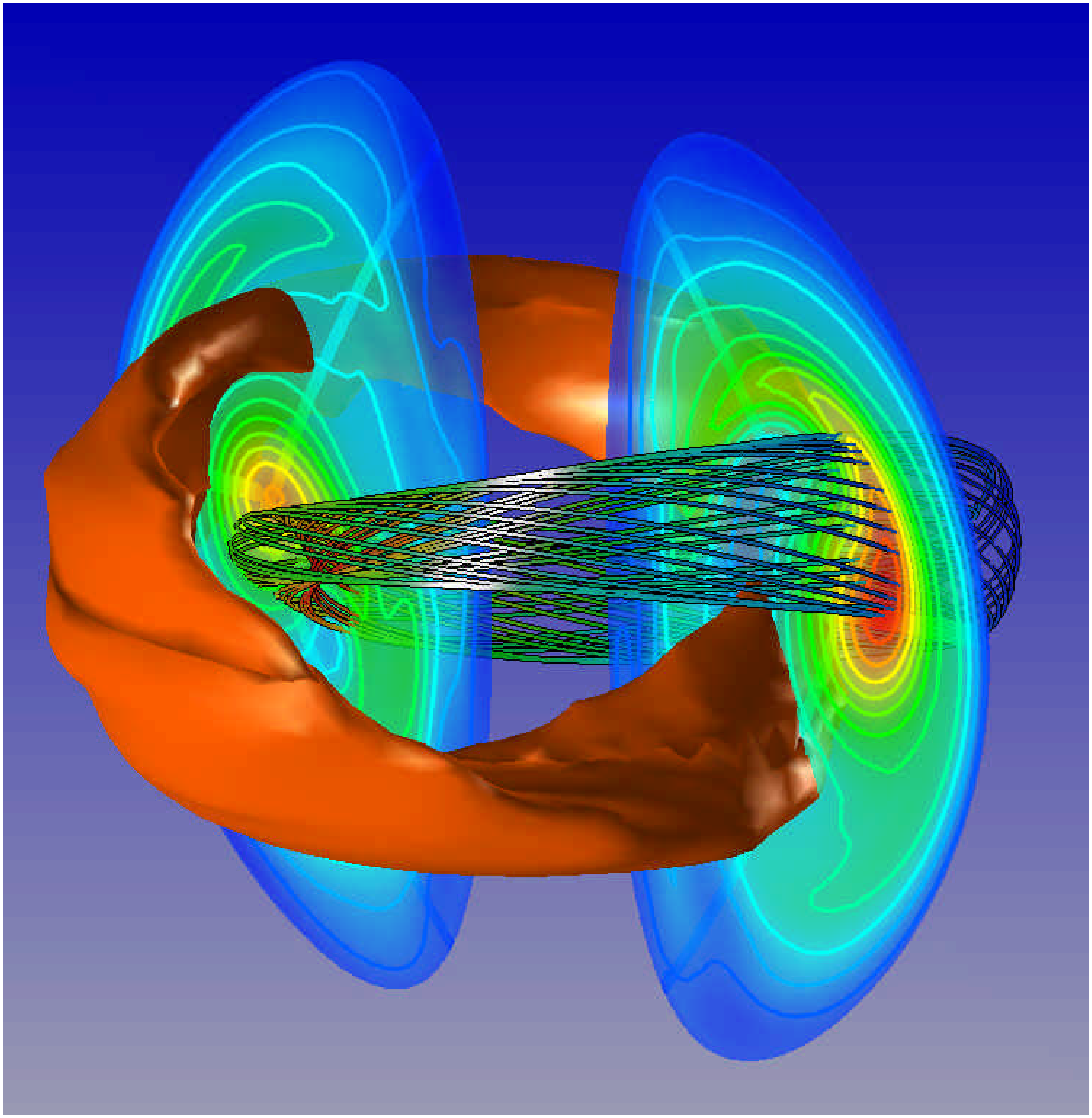
$V\phi$

Pressure and $V\phi$ profiles
are flattened inside island.
Also seen in experiment.

Sometimes, ρ and T out of phase spontaneously occur, saturate the mode



Saturated steady state with strong sheared flow



B Field line
in the island
Density (Pressure)
contours
Temperature
isosurface

Pressure peak inside the island together with shear flow causes the mode saturation.

Summary

- M3D code studies of NSTX.
- The relative density shift relation holds both in the simulation and experiment, with the centrifugal force of the hot component included.
- Toroidal sheared rotation reduces linear growth of internal kink. It is strongly stabilizing nonlinearly, but is normally flattened by reconnection. In some cases, pressure peaking in the island causes a mode saturation.
- IRE: Sawtooth, Disruption due to stochasticity, and Disruption due to nonlinear steepening of pressure driven modes, as in tokamaks.
- Resistive wall and coil currents are being added to extend the applicability regime.