Plasma Confinement in the Vicinity of a Magnetic Island in Toroidal Plasmas

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Outline

- **Magnitude of the symmetry breaking in |*B*| in a tokamak with an island and magnetic field model in the vicinity of the island.**

- **Enhanced transport fluxes in the collisionless regime.**

- **Radial electric field, bifurcation, and plasma confinement.**

- **Extend the theory to a rotating island.**
- **Conclusions.**

Symmetry Breaking in $|B|$

- $|B|$ in a large aspect ratio tokamak:

$$B/B_0 = 1 - \varepsilon \cos\theta$$

B_0 : B on axis;
$\varepsilon = r/R$;
θ: Poloidal angle.

- In presence of an island, toroidal symmetry is broken.

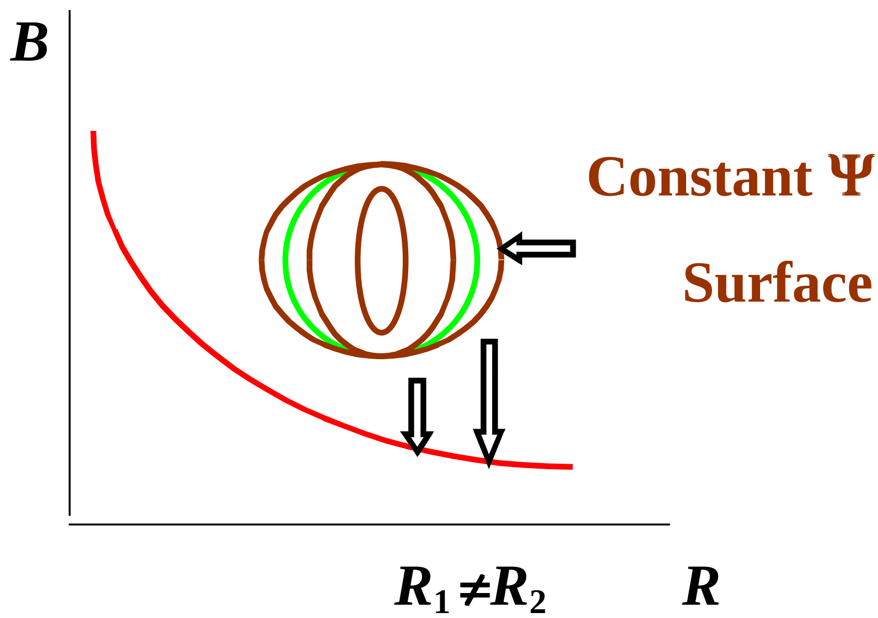
- However, the magnitude of the broken symmetry is thought to be small:

$$B + \text{island } \delta B_r \Rightarrow [B^2 + (\delta B_r)^2]^{1/2}$$

$\Rightarrow$ Modification on B:

$$\sim (\delta B_r)^2 / B^2 << 1$$

Variation of |*B*| on Island Magnetic Surface

- 

- |*B*| on the island surface:

$$B/B_0 = 1 - \left[\frac{r_s}{R} \pm \frac{r_w}{R}\left(\overline{\Psi} + \cos\xi\right)^{1/2}\right]\cos\theta$$

$\overline{\Psi}$: Normalized helical flux function,
ξ : $m(\theta - \zeta / q_S)$, helical angle,
m : Poloidal mode number,
q_S : Safety factor at rational surface,
ζ : Toroidal angle,
r_w : A measure of the width of the island,
r_s : r at the rational surface.

Magnitude of |*B*| Variation on Island Surface

- **Magnitude of the broken symmetry:**

$$\varepsilon\,(r_w / r_s) \sim (\delta B_r)^{1/2}$$

- **For** $(r_w / r_s) \sim 10\%$, $\varepsilon\,(r_w / r_s) \sim 1\%$.

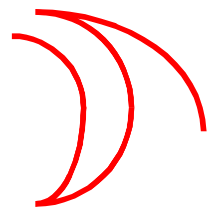
- **Similar to a rippled tokamak except this occurs in the hot plasma core.**

- **This leads to enhanced plasma transport and momentum dissipation over those of the standard neoclassical theory.**

- **In general, if *B* is not spatially uniform, *e.g.*, $B=B(x)$, with x the radial variable, the symmetry breaking effect in *B* is of the order of $B'(x_0)(\Delta x)$, Here, x_0 is the position of the singular layer, prime denotes d/dx, and Δx is the width of the island.**

Enhanced Plasma Transport

- **Banana particles are no longer close on themselves in a poloidal plane:**



Fraction of the trapped particles: $\varepsilon^{1/2}$

- **The radial drift speed v_{dr} is due to the $|B|$ variation on island surface:**

Island	Neoclassical Theory
$v_{dr} \sim m\delta_w$	$v_{dr} \sim \varepsilon$
$\delta_w = \frac{r_w}{R}$	
$\Delta r \sim v_{dr}/(\nu/\varepsilon)$	$\Delta r \sim v_{dr}/(\varepsilon^{1/2} v_t/Rq)$

- **Step size:**

ν: Collision frequency

- **Heat conductivity relative to Ψ:**

$$\chi \sim (m\delta_w)^2\, \varepsilon^{3/2}/\nu$$

$$\chi \sim \nu\varepsilon^{1/2}/(v_t/Rq)^2$$

- **Ratio:**

$$\chi\, /\, \chi \sim (m\delta_w)^2/(\varepsilon\nu_*)^2$$

ν_* : collisionality parameter

- **In the banana regime: $\nu_* < 1$**

- **Island induced heat flux can be larger than that of the standard neoclassical theory and may be comparable to the anomalous flux.**

- **Radial particle flux:**

$$\Gamma_r \sim 0.5N(cT/eBr)^2 (m^2\, \delta_w^{\,2}\, \varepsilon^{3/2}/\nu) H(\bar{\Psi})$$

$$\times [(dP/dr)/P + e(d\Phi/dr)/T]$$

P: Pressure,
T: Temperature,
Φ: Potential,
N: Density,
$H(\bar{\Psi})$: Form factor,
e: Charge.
c: Speed of light

- **In low aspect tokamaks, the effect is more pronounced because there are more bananas.**

- **The $1/\nu$ scaling cannot persist indefinitely. Eventually, finite drift orbit width limits the transport.**

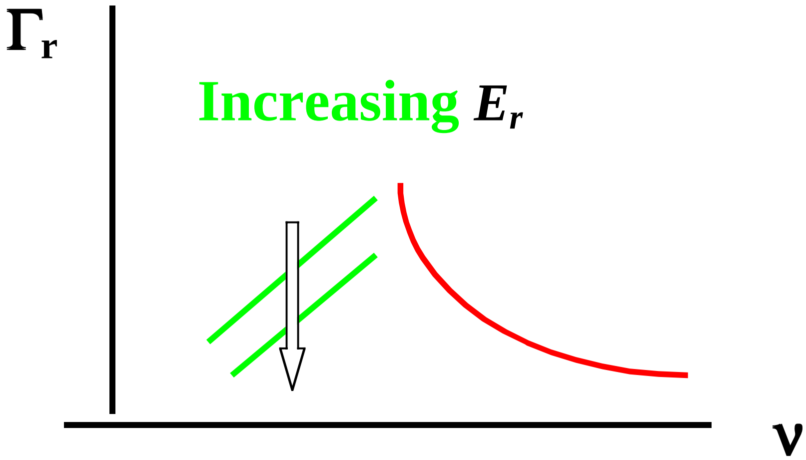
- **When $(\nu/\varepsilon) < \omega_E (RB_P)(q_s' r_W / q_s)$, the particle flux becomes**

$$\Gamma_r = -\,0.22\, N\, \nu\, (cT/eBr)^2 (\delta_W/\omega_E)^2 \varepsilon^{-1/2} G(\overline{\Psi})$$

$$\times [(dP/dr)/P + e(d\Phi/dr)/T]$$

$\omega_E = cE_r/(Br)$: $E\times B$ angular speed,
$E_r = -\,d\Phi/dr$: Radial electric field,
$G(\overline{\Psi})$: A form factor.

- **The flux depends nonlinearly on the radial electric field. It decreases when E_r increases.**



- When $\omega_E = 0$, we need to include super-bananas to remove the singularity.

Radial Electric Field

- **The radial electric field can be determined by the quasi-neutrality condition: $\Gamma_i = \Gamma_e$.**

  Γ_i: Ion particle flux,
  Γ_e: Electron particle flux.

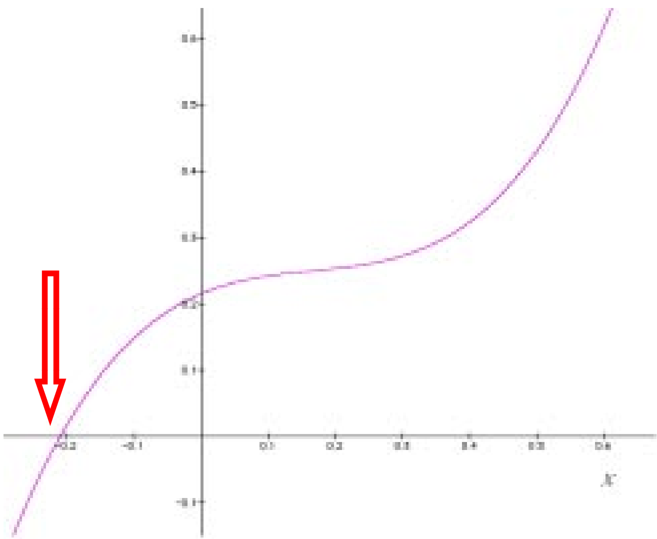
- **Combining $1/\nu$ and $1/(\omega_E)^2$ fluxes, we obtain a nonlinear equation for the electric field:**

$$m^2 (X/C)^3 + m^2 (X/C)^2 + [(M_i/M_e)^{1/2}(\nu_i/\varepsilon)^2 + (\nu_i/\varepsilon)^2](X/C^3) - [(M_i/M_e)^{1/2}(\nu_i/\varepsilon)^2 - (\nu_i/\varepsilon)^2]/C^2 = 0,$$

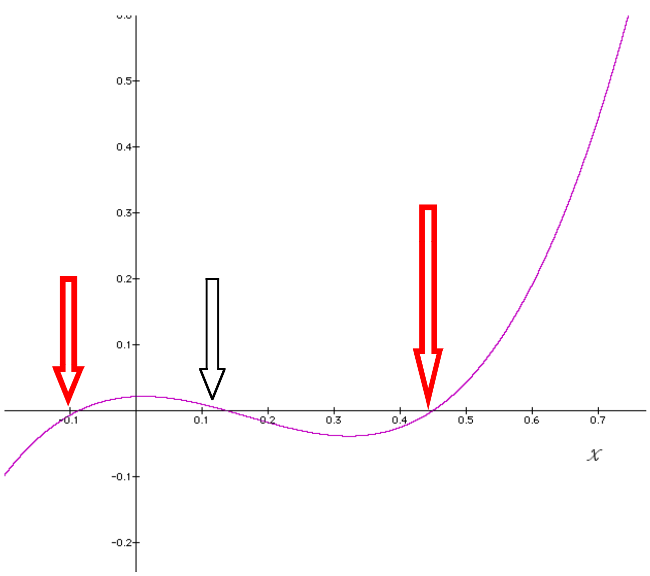
$X = \omega_E (RB_P)(q_S' r_W / q_s)$,
$C = (cT/|e|Br)(RB_P)(q_S' r_W / q_s)(N'/N)$.
M_i: Ion mass, M_e: Electron mass

- **This equation can have multiple equilibrium solutions.**

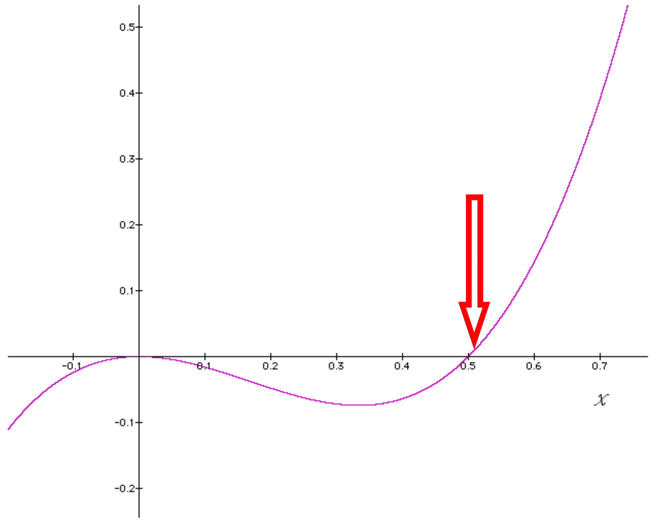
- **Examples:** $C = -0.5$, $m = 2$, $(M_i / M_e)^{1/2} = 43$



$\nu_i/\varepsilon = 0.1$. There is one equilibrium solution.



$\nu_i/\varepsilon = 0.0316$. There are three equilibrium solutions. The one in the middle is unstable. The one on the right is the new solution.



ν_i/ε = 0.0001. The two solutions on the left almost merge into one.

- **It is possible that E_r can bifurcate to a large value.**

- **This in turn will suppress the turbulent fluctuations and improve the overall plasma confinement in the vicinity of a magnetic island.**

- **This mechanism for the confinement improvement is the same as the one used in the H -mode theory.**

- **It has been observed in tokamak and stellarator experiments that plasma confinement improves in the vicinity of low order rational surfaces.**

- **The theory presented here may play a role in understanding this phenomenon because there can be magnetic islands centered on the lower order rational surfaces in tokamaks.**

Island forms in low order rational surface

⇩

Enhanced non-ambipolar flux

⇩

E_r bifurcation

⇩

Turbulence suppression

⇩

Confinement improvement

- **The theory may be also applicable for stellarators.**

- **To check the theory, E_r needs to be measured.**

Rotating Island

- **Island rotation frequency ω is assumed to vanish (in the laboratory frame) or be small.**

- **However, some islands do rotate.**

- **The theory is extended to a rotating island.**

- **ξ is now defined as $\xi = m\,(\theta - \zeta/q_S) + \omega t$.**

- **The electric field parallel to B, $E_{||} = 0$.**

- **The electrostatic potential Φ has the form**

$$\Phi = -(\omega q/mc)\,(\psi - \psi_S) + F(\Psi),$$

$F(\Psi)$: An integration constant.

- **The non-rotating theory remains valid for a rotating island if**

$$d\Phi/d\Psi \quad \Longrightarrow \quad dF(\Psi)/d\Psi$$

- **The bifurcation is in $dF(\Psi)/d\Psi$ when the island rotates.**

- **$F(\Psi)$ does not have to be the same as:**

  $N(\Psi)$:Density profile

  or $T(\Psi)$: Temperature profile.

- **The theory becomes related to island rotation theory.**

- **There are 3 unknowns that need to be determined:**

  ω,

  r_W (Island width),

  $F(\Psi)$.

- **ω : $sin\xi$ component of the Ampere's law,**

- **Island width: $cos\xi$ component of the Ampere's law.**

- **In the past, $F(\Psi)$ has been determined using:**

A transport equation
or
a vorticity equation + Braginskii viscosity.

- **Here, it is determined from:**

$(\boldsymbol{B}\times\nabla\Psi)/B^2 \bullet$ total momentum equation:

$$\Longrightarrow \quad \langle \boldsymbol{J}\bullet\nabla\Psi\rangle = \langle \boldsymbol{B}\times\nabla\Psi\bullet\nabla\bullet\Sigma_j \boldsymbol{P}_j \,/\, B^2\rangle,$$

⇑

Island induced viscosity

J **: Plasma current density,**
$\boldsymbol{P}_j$: Pressure tensor for the species j.

- **The equilibrium quasineutrality condition:**

$$\langle \boldsymbol{J}\bullet\nabla\Psi\rangle = 0.$$

Conclusions

- **A theory for the transport processes in the vicinity of a magnetic island in tokamaks where the toroidal symmetry in |*B*| is broken.**

- **This leads to enhanced transport fluxes that can be comparable to the anomalous ion transport flux.**

- **The radial electric field can now be determined from the quasineutrality condition, or equivalently the momentum equation on the island magnetic surface.**

- **The equation that governs the radial electric field is nonlinear. It can have bifurcated solutions. After electric field bifurcation, turbulence can be suppressed and confinement can be improved.**

- **This mechanism may play a role in the confinement improvement in the vicinity of the low order rational surfaces observed in tokamak and stellarator experiments.**

- **The theory is extended to a rotating island. We find the non-rotating theory remains valid for a rotating island if we replace $d\Phi/d\Psi$ in the non-rotating theory by $dF(\Psi)/d\Psi$.**

- **The theory for a rotating island is also related to the island rotation theory. We determine $F(\Psi)$ from the island-induced nonlinear viscosity instead of Braginskii viscosity.**

- **The theory will be incorporated in NCLASS to simulate plasma and momentum transport processes in tokamaks with islands.**