

# Equilibrium and Stability

## Characterization of $A < 1.3$ Plasmas in the PEGASUS Toroidal Experiment

Aaron Sontag for the PEGASUS Team

University of Wisconsin-Madison

STW 2002  
November 20, 2002

*Work supported by U.S. DoE Grant No. DE-FG02-96ER54375*



## PEGASUS is an ultra-low aspect ratio ST

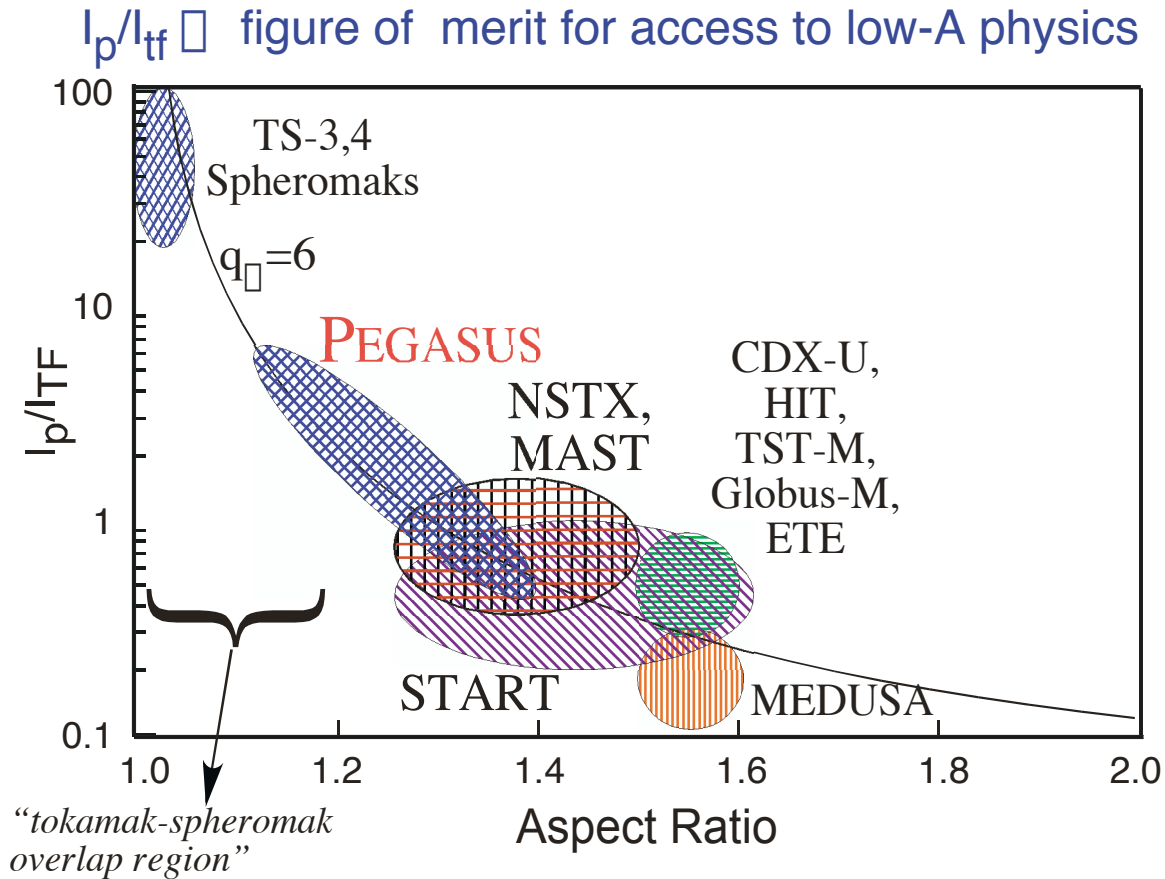
---

- Test limits of beta and safety factor as  $A \ll 1$
- Access high toroidal beta and high toroidal utilization factor ( $I_p/I_{tf}$ )
- A “soft limit” occurs at  $I_p/I_{tf} \approx 1$ 
  - *due to MHD activity and some reduction in V-s*
- Beginning to explore the edge kink stability boundary
  - *$q_{95}=5$  is found unstable*
- Upgrades will allow further challenges to stability limits
  - *tools for increased plasma control*





# PEGASUS is exploring the $A < 1.3$ operating regime



## Achieved Parameters:

- $A = 1.12-1.3$
- $R = 0.2-0.45$  m
- $I_p = 0.16$  MA
- $RB_t \approx 0.03$  T-m
- $\beta = 1.4 - 3.7$
- $\tau_{\text{pulse}} = 0.01-0.03$  s
- $\langle n_e \rangle = 1-5 \times 10^{19} \text{m}^{-3}$
- $\tau_t \approx 20\%$   
( $\tau_t \equiv 2\beta_0 \langle p \rangle / B_{t0, \text{vac}}^2$ )





# New Equilibrium Reconstruction Code Developed

## • Motivation:

- robustness
- easy incorporation of new diagnostics
- portability

## • Description:

- full solution of Grad-Shafranov equation at each iteration  
*G-S solver uses multi-grid Gauss-Seidel PDE solver*
- minimize  $\chi^2$  of fit to measurements  
 $\chi^2$  minimization via *Levenberg-Marquardt method*
- has been validated against TokaMac

## • Profile parameterization:

- GG' as 3 term polynomial
- p' as 2 term polynomial

$$F(x) = F_0 + (F_1 + F_2 \chi_N + F_3 \chi_N^2 + \dots + F_n \chi_N^{n-1}) \chi_N^n (F_1 + F_2 + \dots + F_n)$$

$$\chi_N = \frac{\chi - \chi_0}{\chi_{lim} - \chi_0}$$

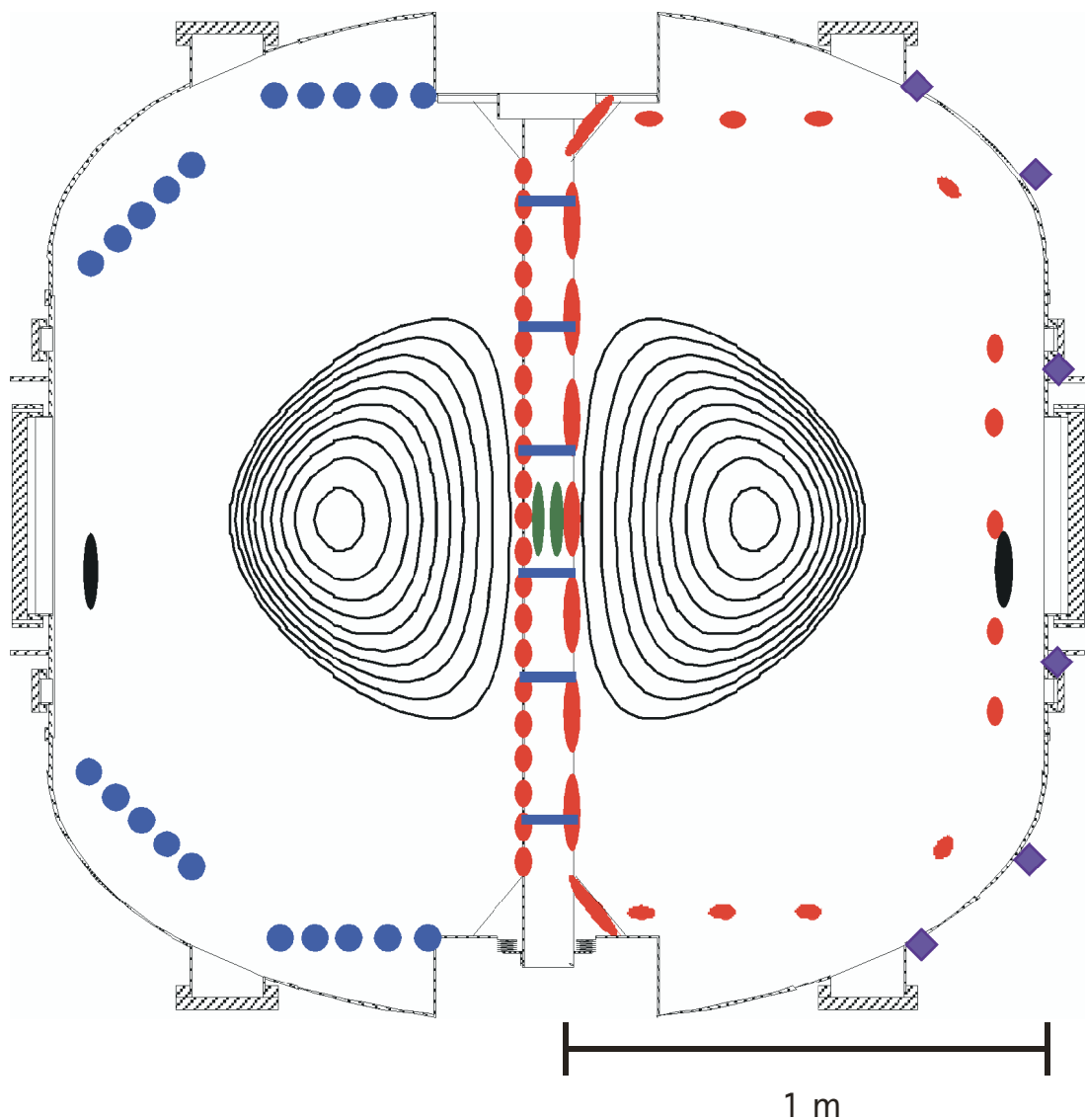
## • Drawbacks:

- computationally intensive  $\chi$  slow
- average fit takes approximately 1.5 minutes with 1.3 GHz Athlon





# A full set of magnetics diagnostics provides equilibrium and stability data



## Installed Magnetics

- Flux Loops (26)
- Poloidal Mirnov Coils (22 + 21)
- LFS Toroidal Mirnov Coils (6)
- HFS Toroidal Mirnov Coils (7)
- ◆ External Wall Loops (6)

Not shown:

- Internal Plasma Rogowski Coils (2)
- Internal Diamagnetic Loops (2)
- Diamagnetic Compensation Loop





# Internal view of PEGASUS shows narrow centerstack

Flux loops

Centerstack armor

Outboard limiter

HHFW antenna



Mirnov coils

10 cm

Segmented divertor plates





# Resistive vacuum vessel wall modeled as set of axisymmetric current filaments

- Induced wall currents calculated by numerically integrating resulting set of differential circuit equations

- coupled current filaments described by matrix equation

$$\overline{M} \cdot \frac{d\overline{I}}{dt} + \overline{R} \cdot \overline{I} = \overline{V}$$

- inductance matrix ( $M$ ) determined by coil set self-inductances and mutual-inductances

*inductance of individual filament (wall)*

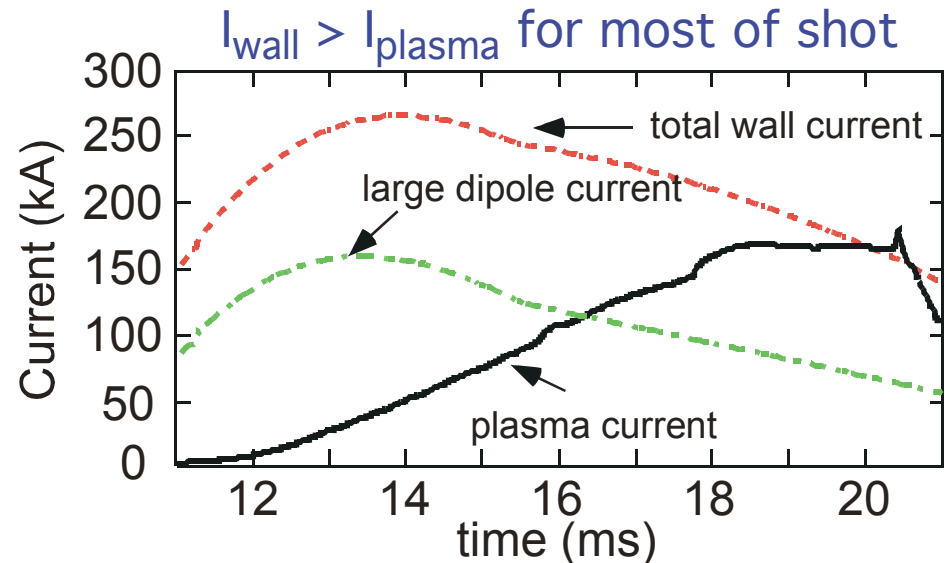
$$L_i = \mu_0 R \ln \left[ \frac{8\sqrt{R}}{\sqrt{A}} \right] \frac{7}{4}$$

*self-inductance of coil set  $i$*

$$L_i I_i = \sum_{k=1}^{N_i} \sum_{l=1}^{N_i} \mu_{ij}^{k,l}$$

*mutual inductance of coil set  $i$  with coil set  $j$*

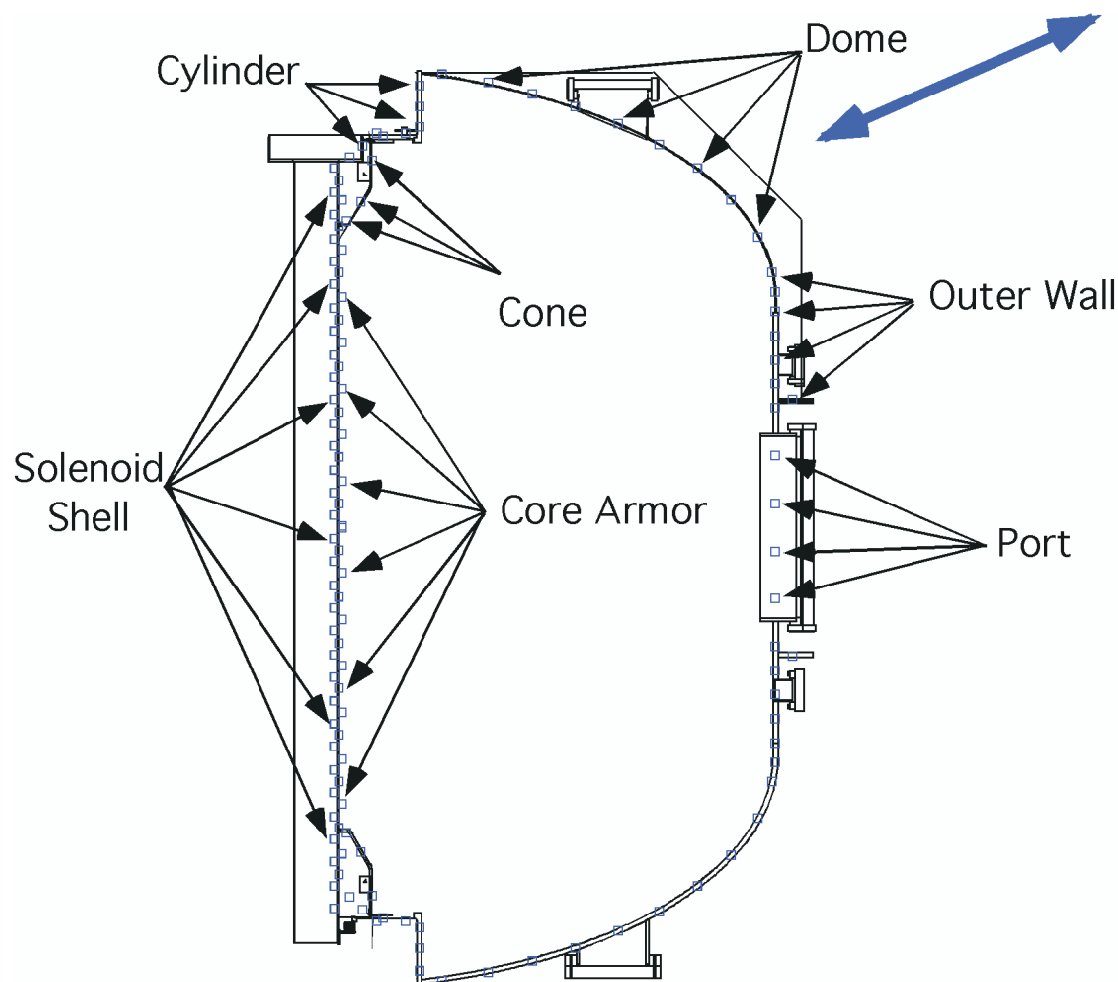
$$M_{ij} I_j = \sum_{k=1}^{N_i} \sum_{l=1}^{N_j} \mu_{ij}^{k,l}$$



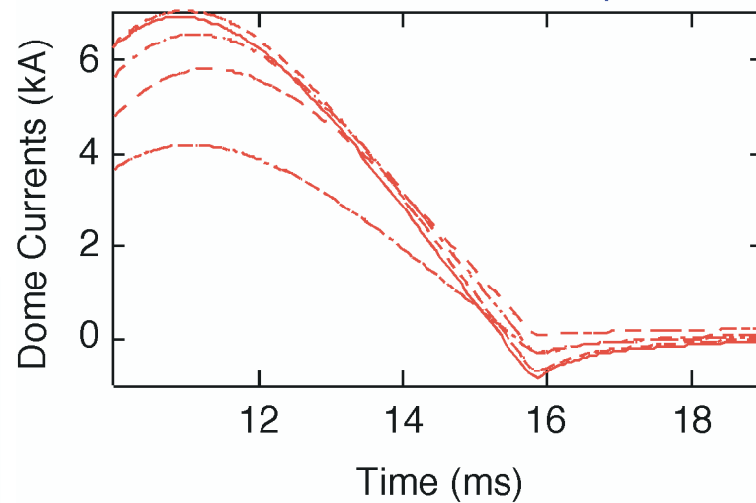


# Wall filaments are grouped into coil packs

- Wall modeled as 91 current filaments
- Filaments grouped into coil packs
  - coil pack currents are fit by equilibrium code



Coil Pack Currents Exhibit Similar Temporal Behavior



**pack currents constrained via mounted flux loops**

Dome and outer wall most significant  
2 loops on dome, 1 on outer wall







# Monte Carlo analysis gives uncertainty in fit parameters

## • Uncertainty estimation technique:

- single time-slice of discharge reconstructed 100 times
- Gaussian noise added to measurement data  
*Gaussian width from diagnostic uncertainty starting  $\chi^2 \sim 8 \times$  final  $\chi^2$*
- $\chi^2$  of fit parameter distributions gives uncertainty

## • Variety of discharges analyzed

- wide range of fit parameters covered:

$$75 \text{ kA} < I_p < 150 \text{ kA}$$

$$8\% < \chi_t < 18\%$$

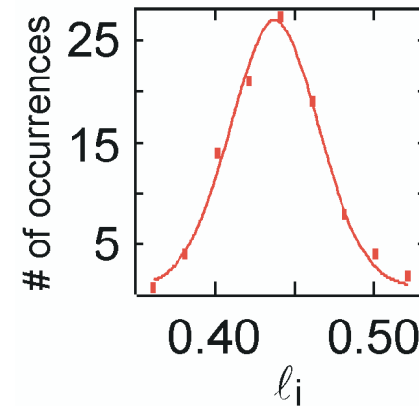
$$0.2 < l_i < 0.4$$

$$0.23 \text{ m} < R_0 < 0.33 \text{ m}$$

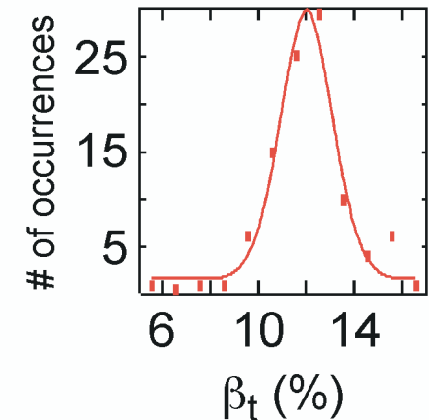
- no significant variation in relative uncertainty

$\chi^2$  uncertainty determined by diagnostic uncertainties

Parameter	Rel. Uncertainty
$I_p$	$\pm 2\%$
$R_0$	$\pm 4\%$
$l_i$	$\pm 9\%$
$\beta_t$	$\pm 15\%$
$\beta_p$	$\pm 15\%$
$q_{95}$	$\pm 6\%$
$q_0$	$\pm 20\%$



$$\frac{\sigma_{l_i}}{l_i} = 9\%$$

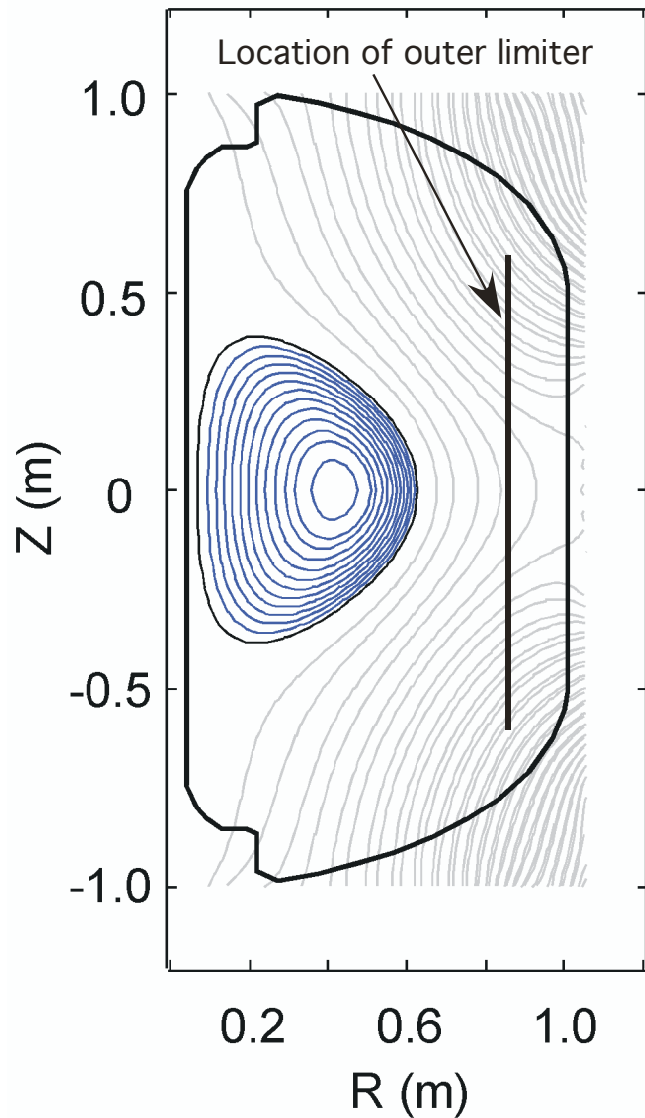


$$\frac{\sigma_{\beta_t}}{\beta_t} = 15\%$$



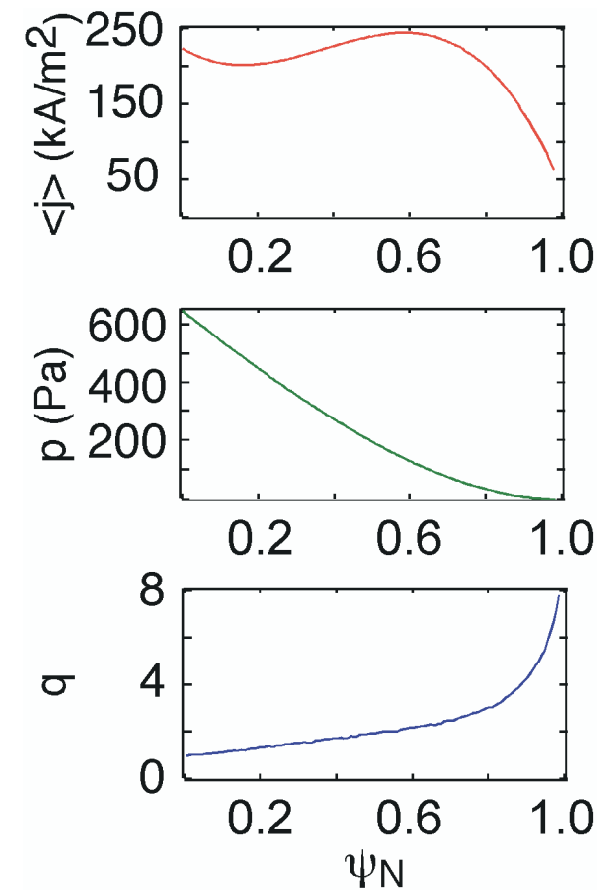


# Sample equilibrium results



Shot  
12445

$I_p$	78.3 kA
$R_0$	0.337 m
$a$	0.274 m
$A$	1.22
$\beta_t$	1.4
$B_t$ (axis)	0.048 T
$\beta_t$	18%
$\ell_i$	0.40
$q_0$	0.98
$q_{98}$	7.8



• **Small plasma size due to crude vertical field control**





# Equilibrium reconstructions show low-A characteristics

---

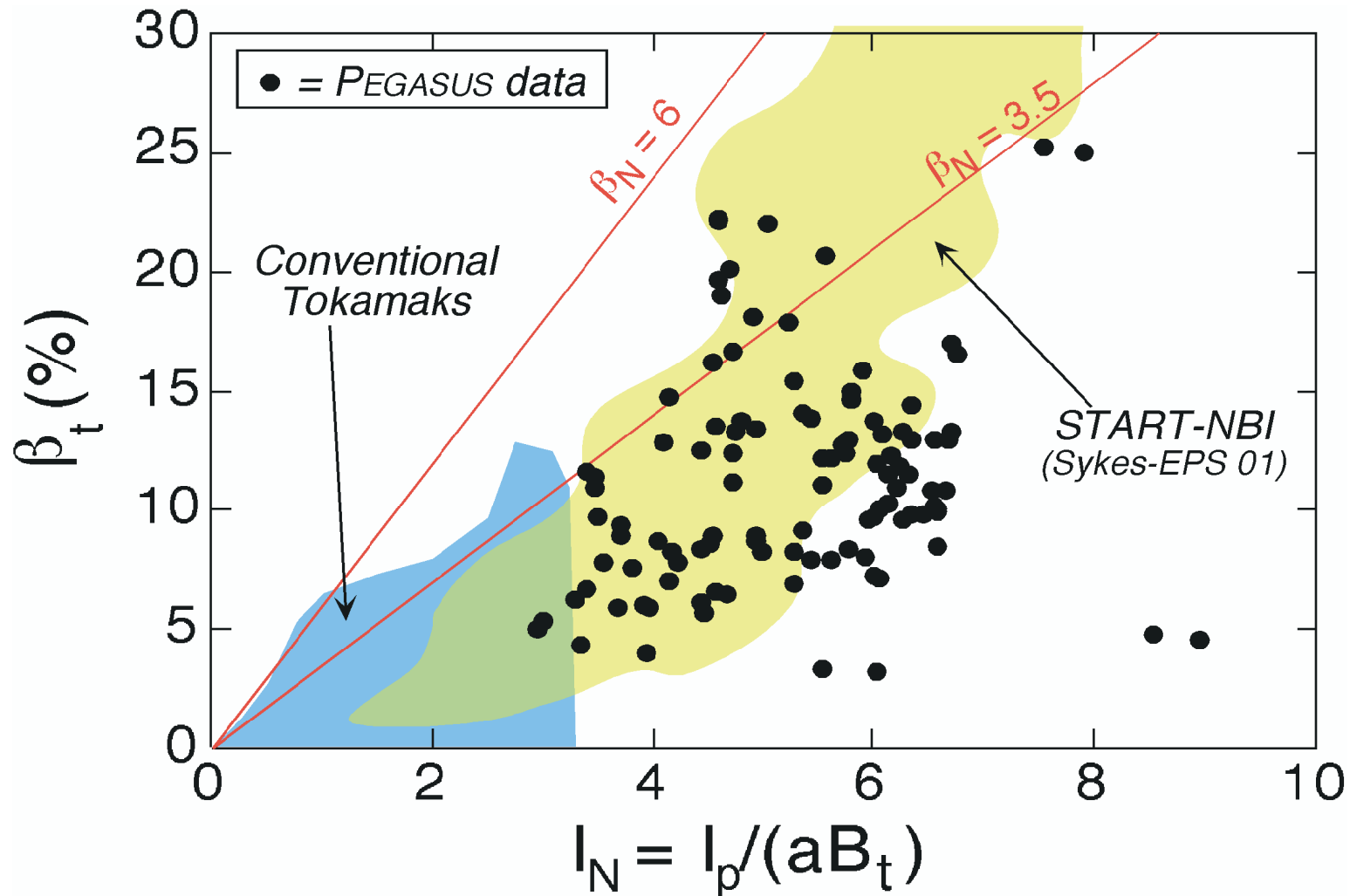
- High- $\beta_t$  (Ohmic):  $\beta_t \sim 20\%$
- High- $\beta_N$  (Ohmic):  $\beta_N \sim 5$
- High- $I_N$  (Ohmic):  $I_N \sim 6.5$
- High  $I_p/I_{TF}$ :  $I_p/I_{TF} \sim 1$
- High- $\beta$  (natural):  $\beta > 2$
- High field windup: high  $q_a$  at low TF
- Paramagnetic:  $\beta_p = 0.3$  at  $\beta = 0.83$ ;  $F/F_{vac} \sim 1.5$  on axis





$A < 1.3$  ready ohmic access to high  $\beta_t$

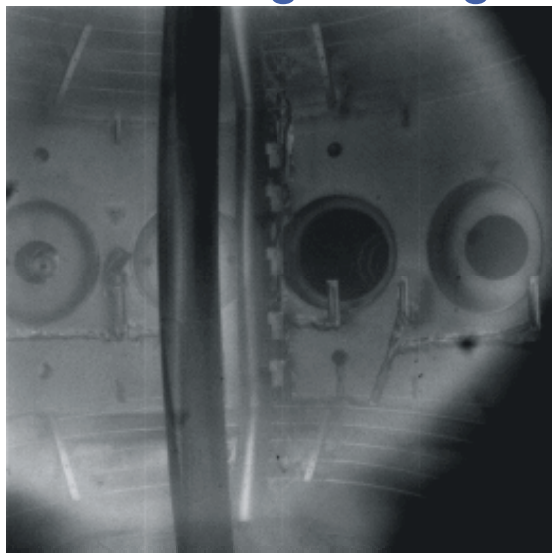
- $\beta_t \sim 20\%$  and  $I_N > 6$  achieved ohmically
- Low field  $\beta$  high  $I_N$  and  $\beta_t$



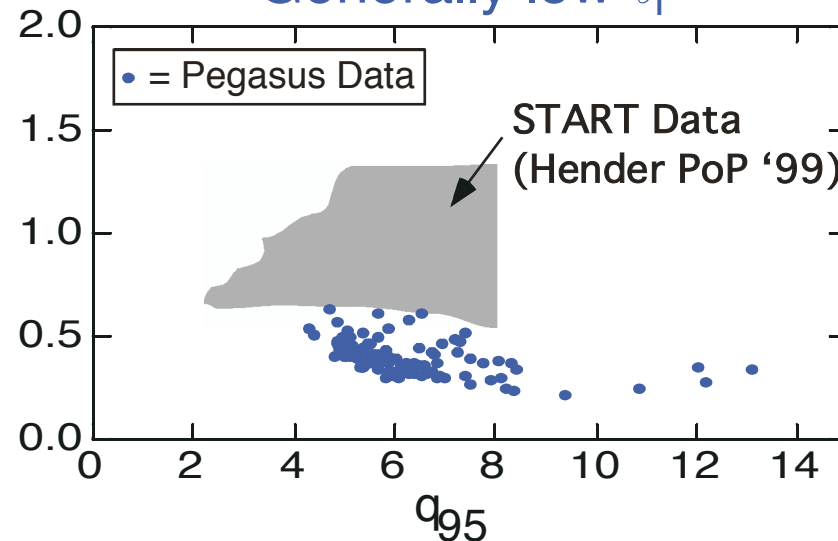


# PEGASUS operates in low- $\ell_i$ , high-density space

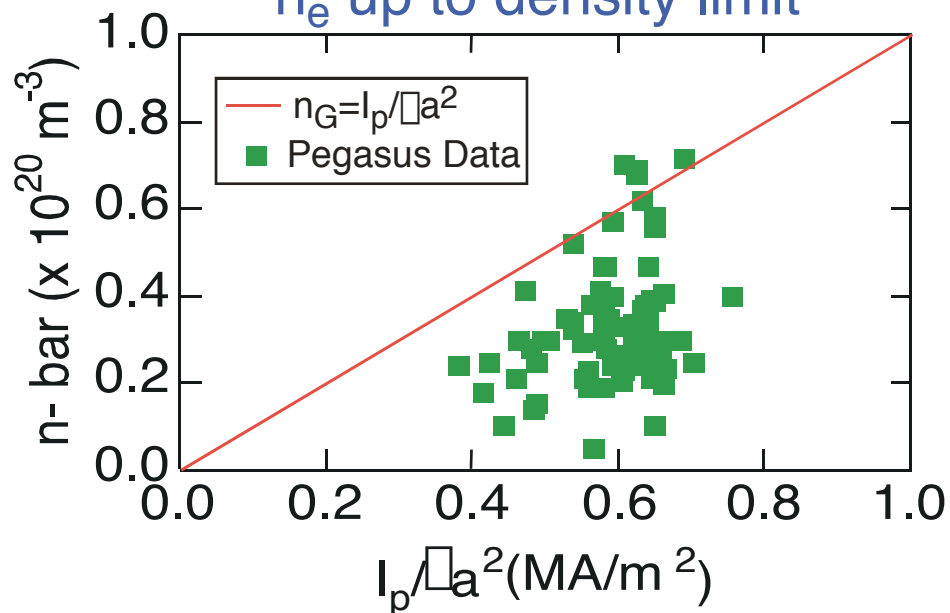
## Visible light image



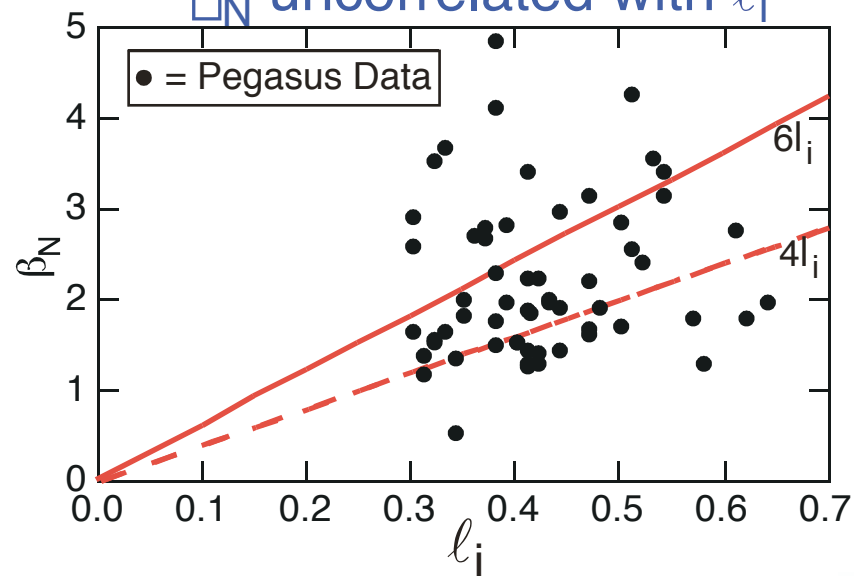
## Generally low $\ell_i$



## $\bar{n}_e$ up to density limit



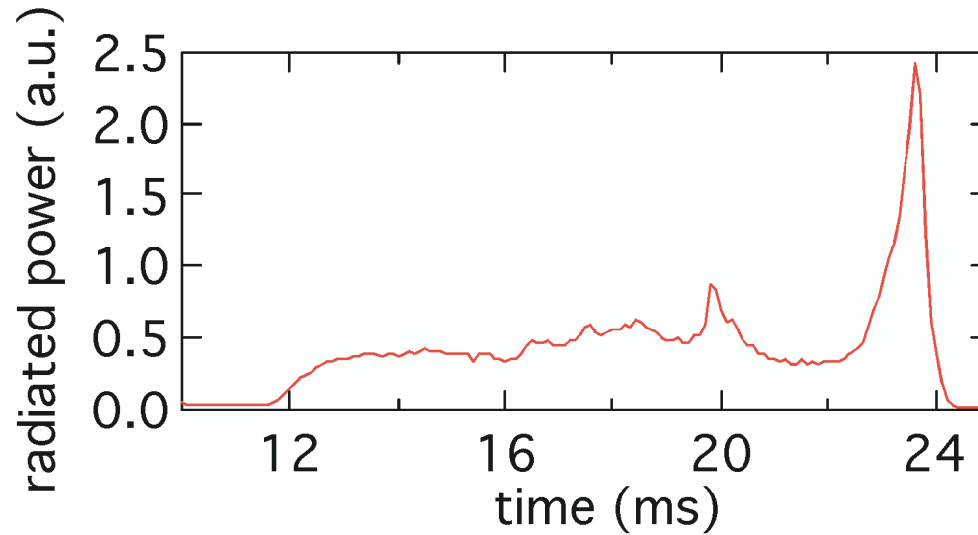
## $\beta_N$ uncorrelated with $\ell_i$



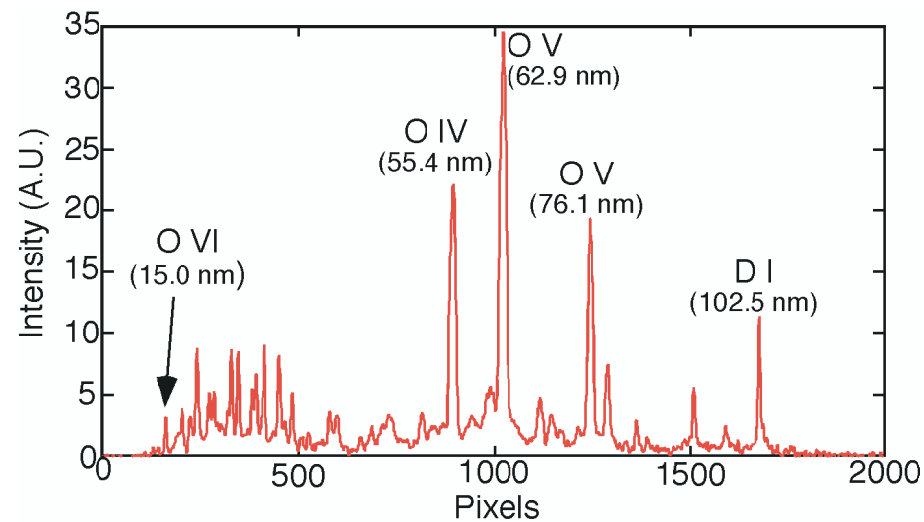


# General discharge characteristics

- Bolometry shows no indications of impurity build-up



- SPRED indicates oxygen is dominant impurity species

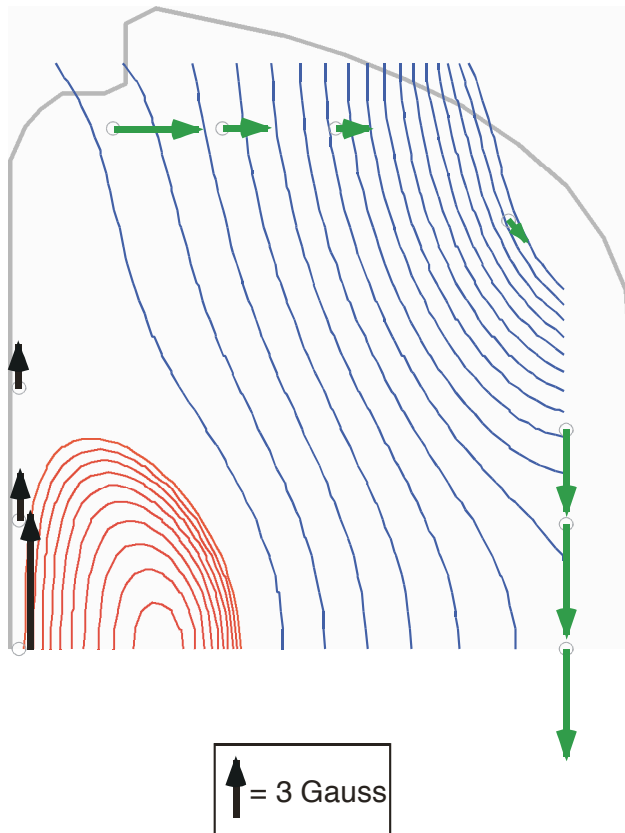




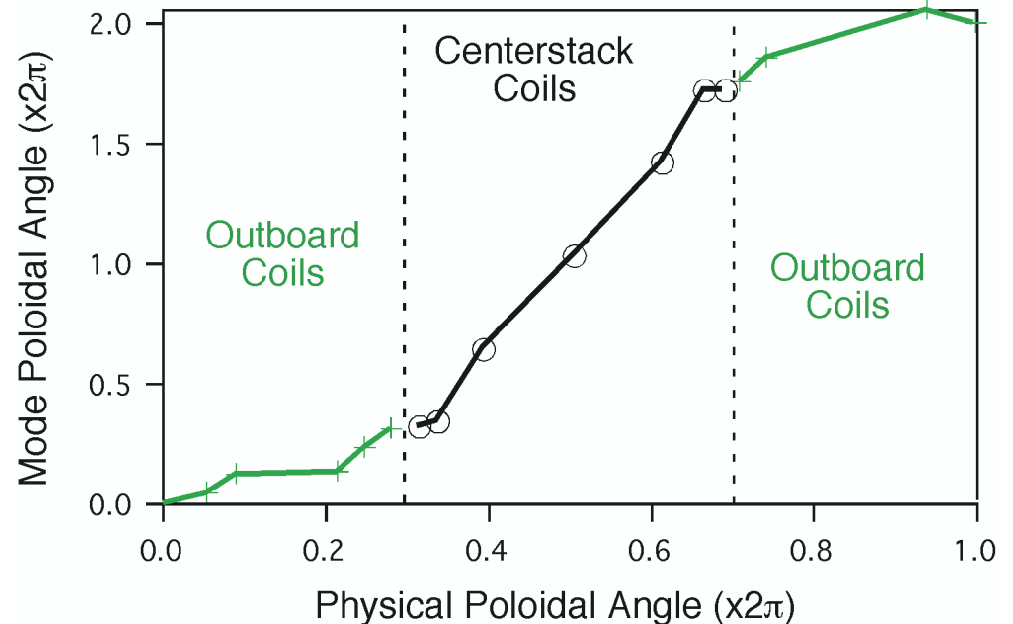
# Ultra-low A □ strong poloidal asymmetry to tearing mode

- Large phase shifts observed along centerstack for  $m/n=2/1$ 
  - 1.5 wavelengths observed across  $120^\circ$  poloidally
  - similar structure observed for  $3/2$  and higher  $m/n$
- Mode is strongest on the low-field side
  - LFS coils  $\sim a$  from edge
  - HFS coils  $\sim a/10$  from edge

Perturbed Field Magnitude at the Wall



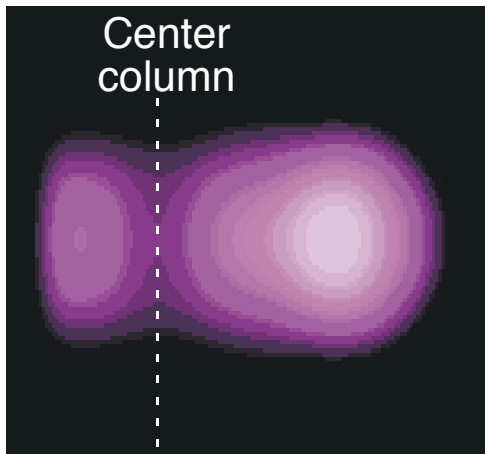
2/1 Poloidal Phase at the Wall



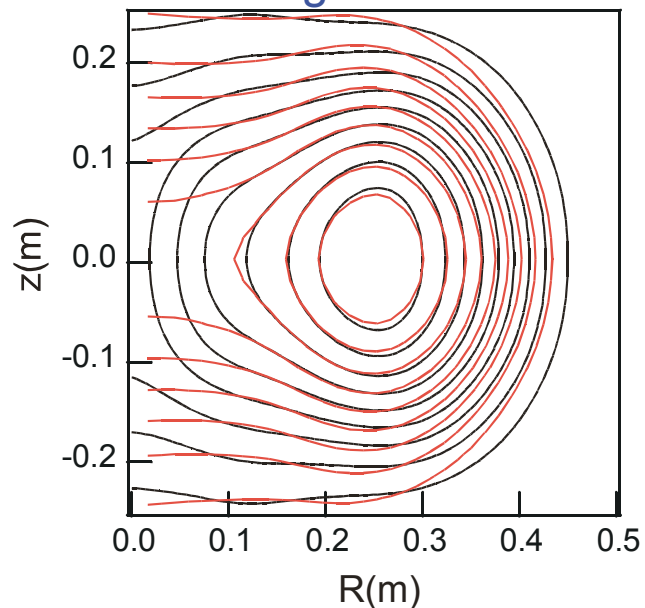


# Measured q-profile indicates low central shear

Tangential PHC SXR image

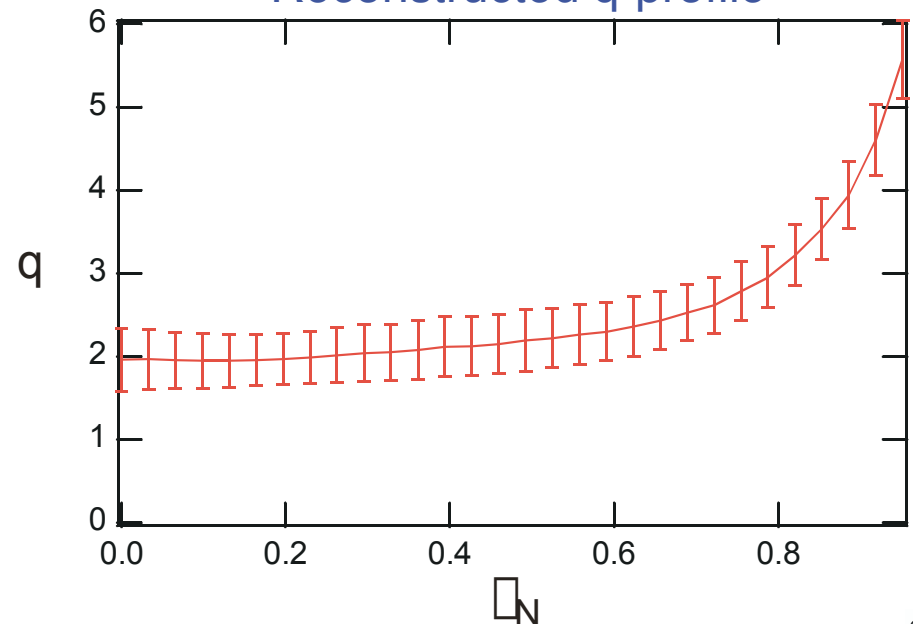


Measured and reconstructed image contours



- 2D soft x-ray camera gives q-profile
  - Measures constant-intensity surfaces
  - Used as internal constraint on equilibrium
  - Useful as q-profile diagnostic
- Measured q-profile  $\square$  zero central shear
  - Typical of low-A
  - Confirms shape predicted by external magnetics

Reconstructed q-profile

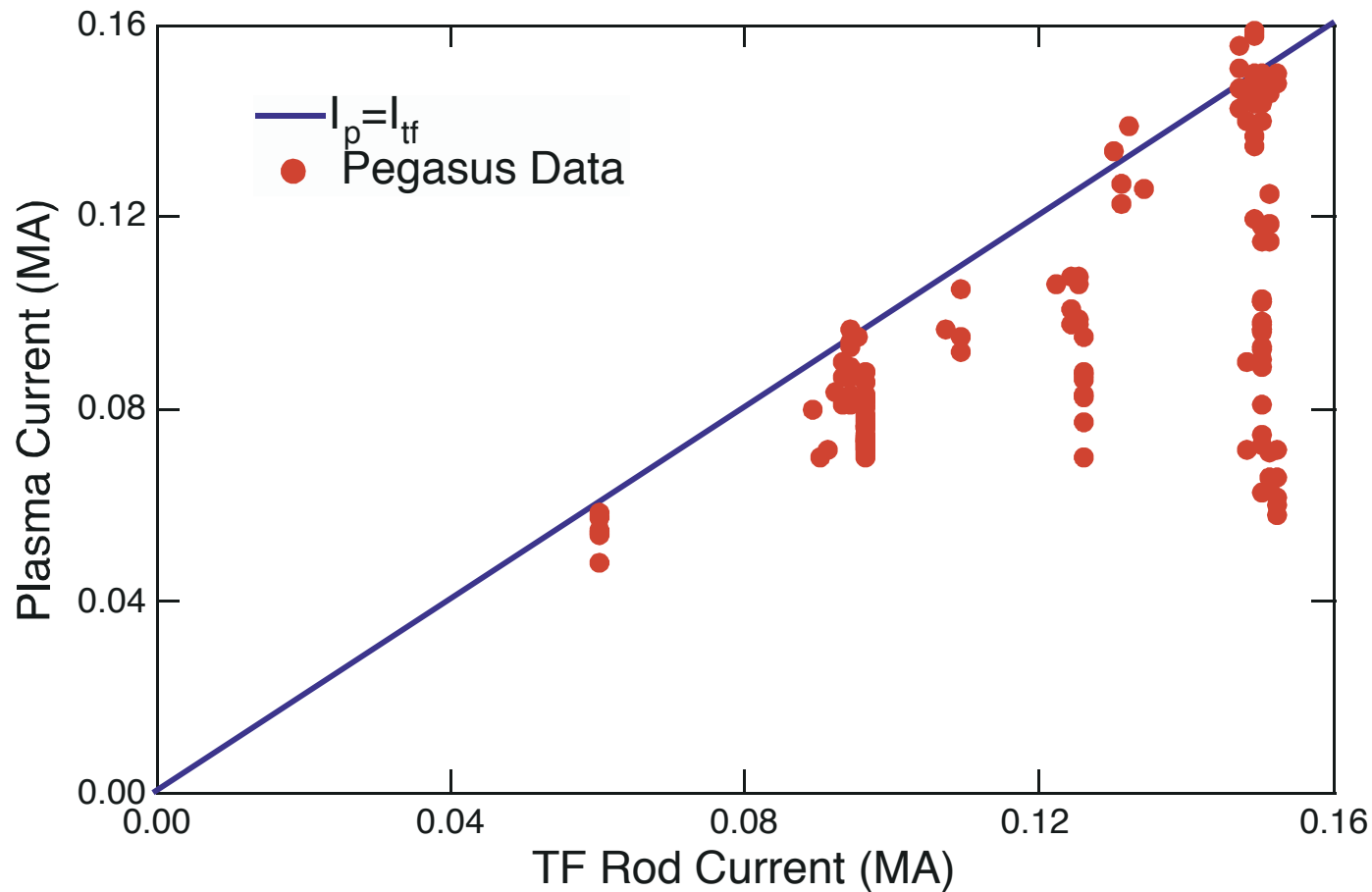






# Toroidal field utilization exhibits a “soft limit” around unity

- Maximum  $I_p \approx I_{tf}$  in almost all cases
- Limit is not disruptive or abrupt
  - $I_p$  saturates or rolls over





## Two factors contribute to the $I_p/I_{tf} \sim 1$ soft limit

### Large resistive MHD instabilities degrade plasma as TF $\square$

- low  $B_t$  and fast  $dI_p/dt$   $\square$   $q =$  low-order  $m/n$  early in discharge
- high resistivity early in plasma evolution
- ultra-low  $A$   $\square$  low central shear
- in the Rutherford regime:

$$\frac{dw}{dt} \sim \square \quad w_{\text{sat}} \sim q \left( \frac{dq}{dr} \right)^{-1}$$

- $\square$  Result is early rapid growth of tearing modes and large saturated island widths

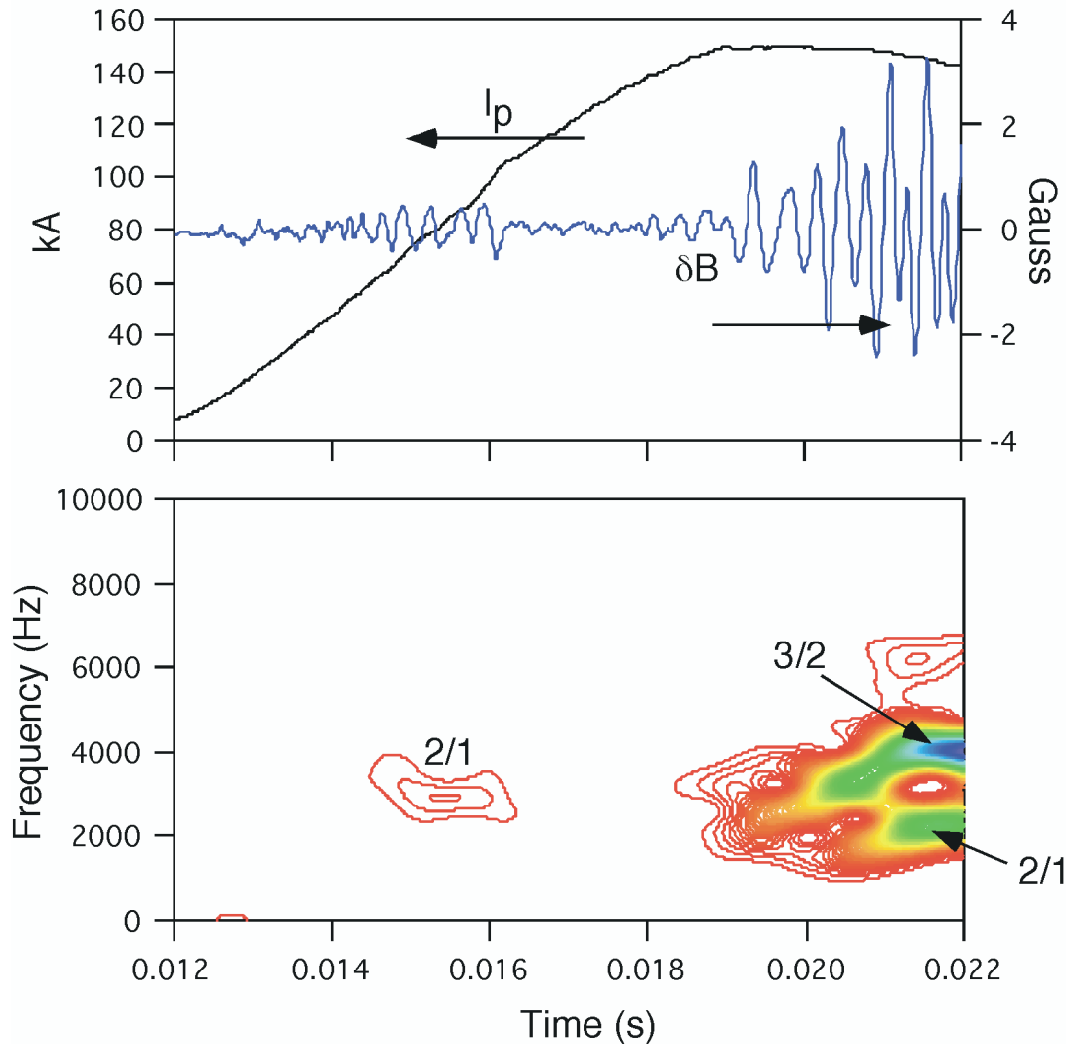
### Reduced available Volt-seconds as TF $\square$

- reduction in toroidal field  $\square$  delayed startup
- delayed startup  $\square$  reduction in available volt-seconds
- only partially explains drop in  $I_p$  with reduced  $I_{tf}$





# Significant tearing activity is observed in most discharges



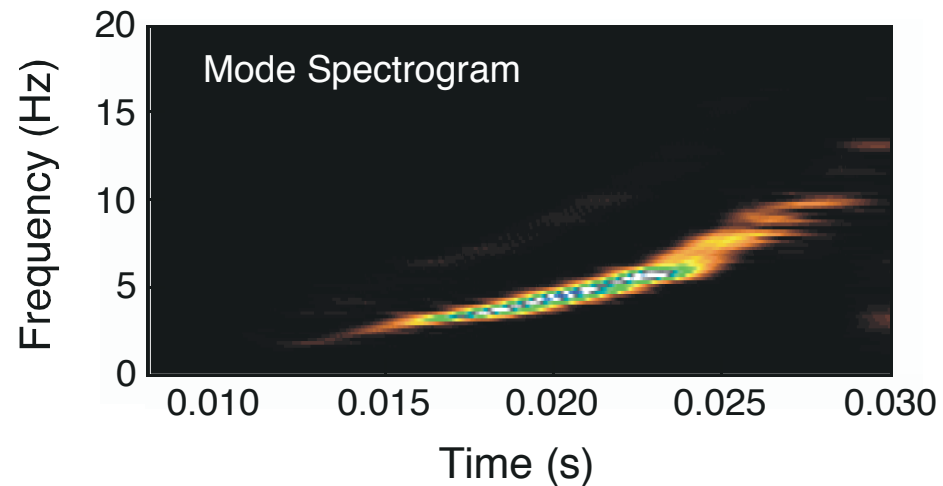
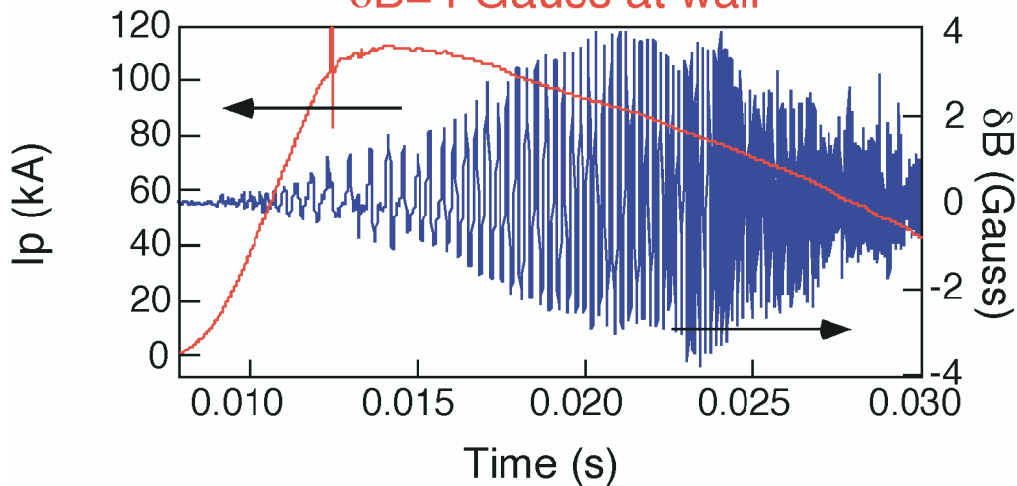
- Most common mode  $m/n=2/1$ 
  - other  $m/n$  also observed
  - evidence of 2/1-3/2 coupling
- Leads to increased  $C_E$ , decreased  $I_p$ 
  - Less efficient flux consumption in presence of internal MHD
  - Degradation of  $\langle \epsilon_E \rangle$
  - Decrease in  $dI_p/dt$  and  $I_p$
  - Large radial extent
    - $\square$  affects entire plasma





# Island widths are on the order of plasma minor radius

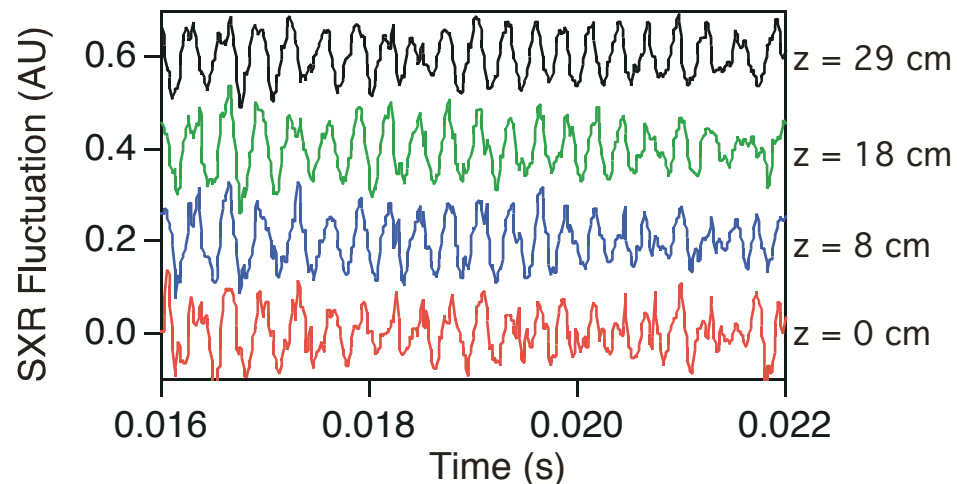
•  $\delta B = 4$  Gauss at wall



• Island width estimates give  $w > 10$  cm

$$w \approx 4 \sqrt{\frac{\Delta B}{B_t} \frac{qR}{n \frac{dq}{dr}}} \sim a$$

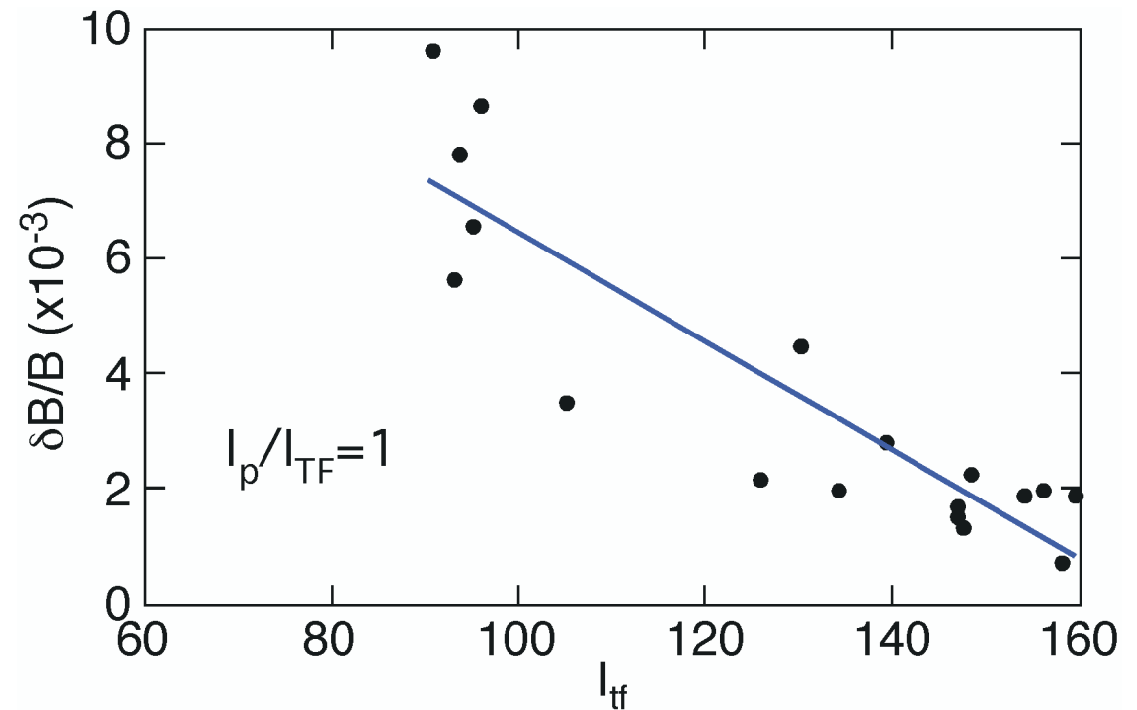
• SXR  $\square$  large radial extent of mode





# Mode amplitude decreases as $I_{TF}$ is increased

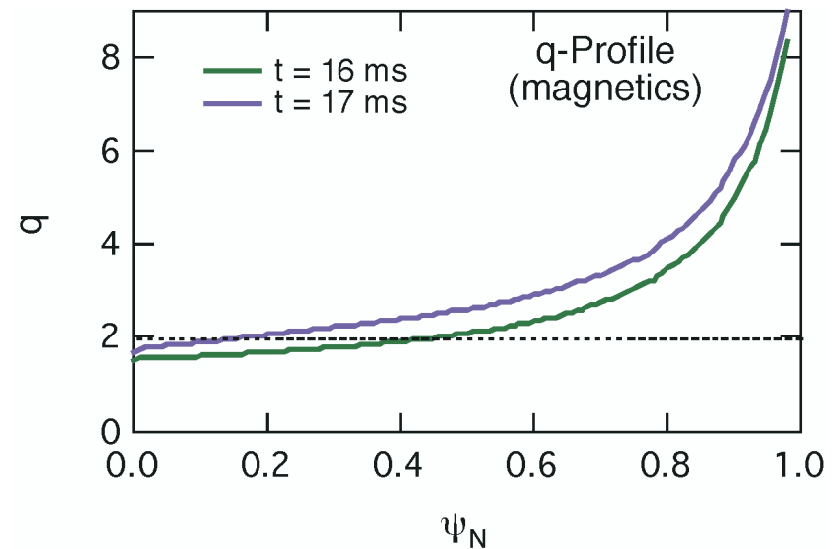
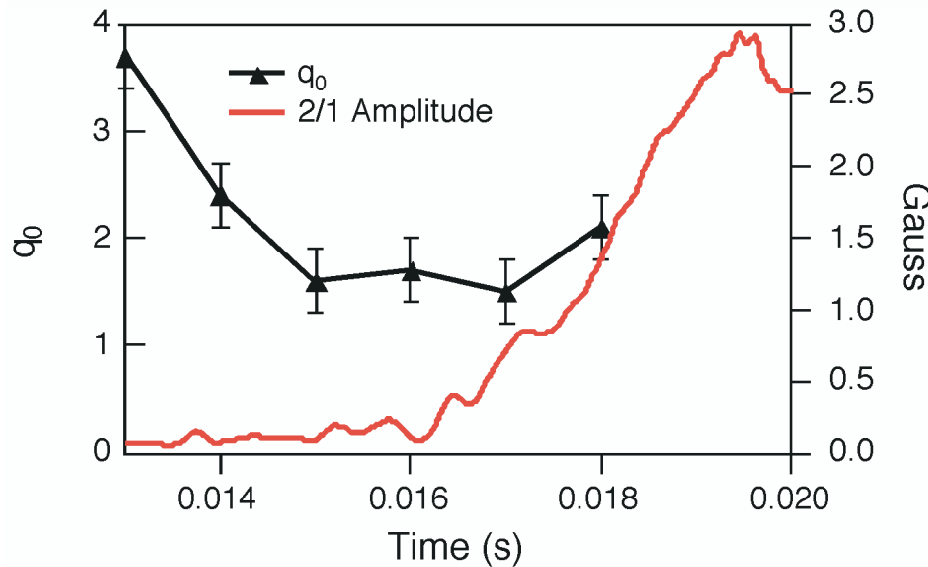
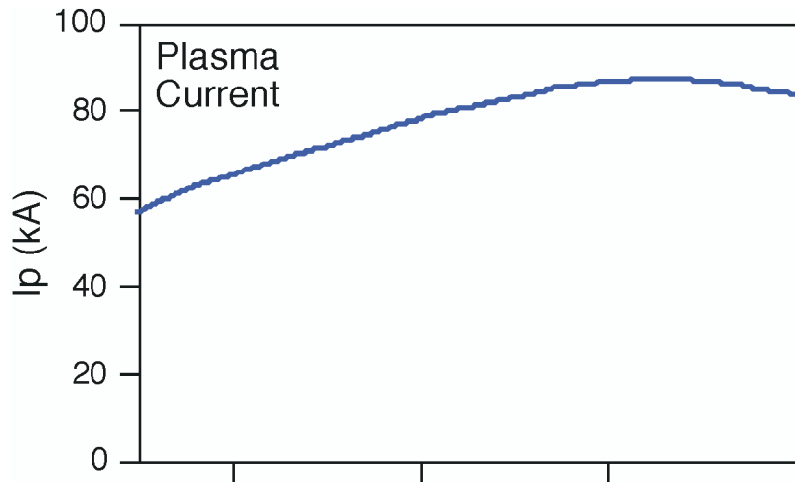
- Along  $I_p \sim I_{TF}$  contour:  $\delta B \uparrow$  as TF  $\square$
- At high TF effect of MHD minimal
  - $C_E = 0.4$
- At lower TF MHD amplitude increases
  - $C_E$  increases
  - Stored energy decreases





# Tearing modes correlated with appearance of low $q=m/n$ in broad low-shear region

- Low-A and low toroidal field
  - appearance of  $q=2$  surface early in discharge
- □ high early in shot
- Broad low-shear region gives large radial extent of mode
  - 2D SXR imaging shows low central shear





$I_p/I_{TF} \sim 1$  implies low-order  $q_0$

- Cylindrical approximation OK for central flux surfaces:

$$q(r) = \frac{2\mu r^2 B_t}{\mu_0 R I(r)} \frac{1 + \mu^2}{2}$$

- Assuming flat  $j(r)$  implies:

$$q_0 \sim \frac{1}{A^2} \frac{I_{TF}}{I_p} \frac{1 + \mu^2}{2}$$

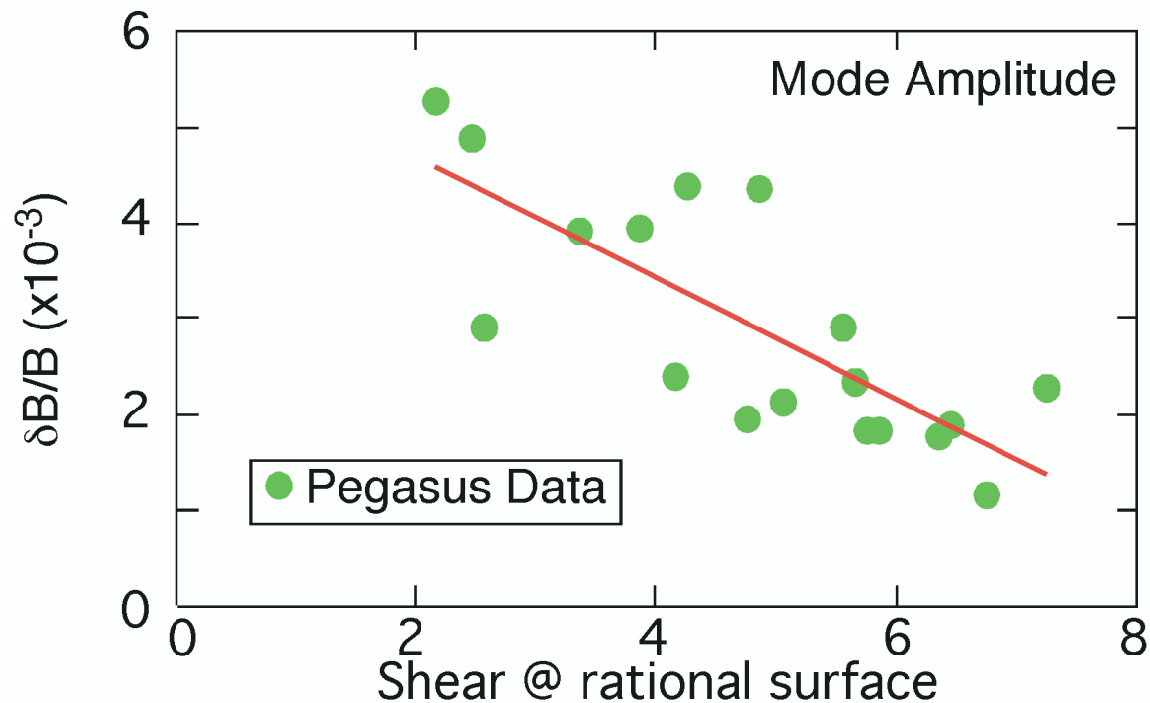
- For PEGASUS at  $I_p \sim I_{TF}$   $\mu$   $q_0 = 1.5 - 2$

$\square$  Low-order rationals in low-shear region for  $I_p = I_{TF}$



# Mode amplitude reduced by manipulation of shear and $q_0$

- Improved wall conditions & EF control  $\square$  plasmas with reduced MHD activity
    - Increased  $W$ ,  $I_p$
  - Increased shear, increased  $q_0$   $\square$  delay tearing onset
  - MHD amplitude decreases with increasing shear @ mode rational surface
- $\square$  Access higher toroidal field utilization via higher  $q_0$ ,  $T_e$ , shear







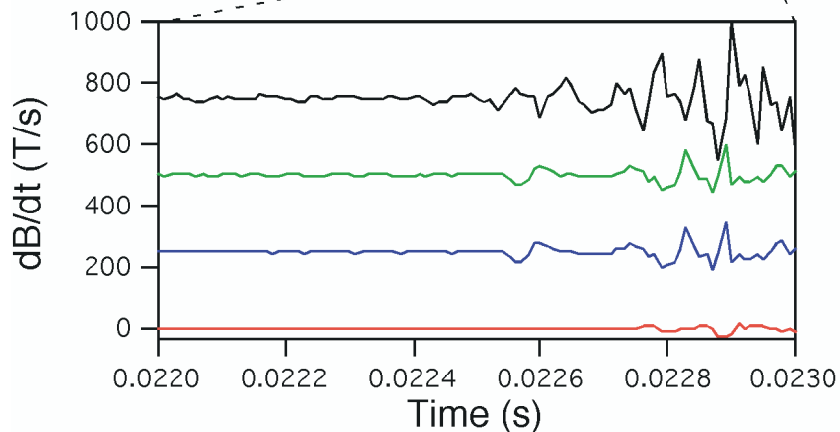
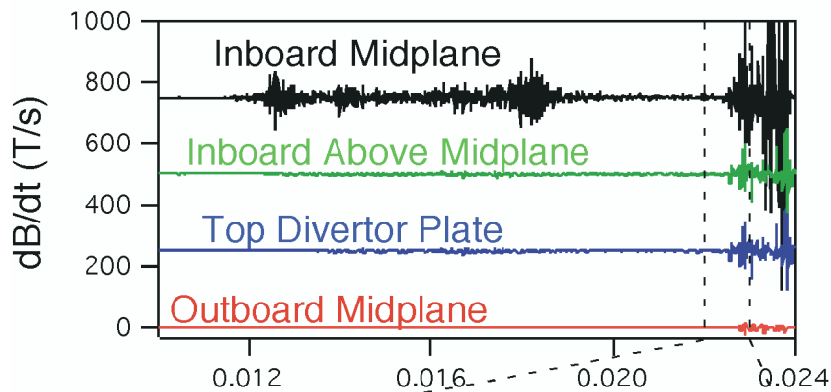
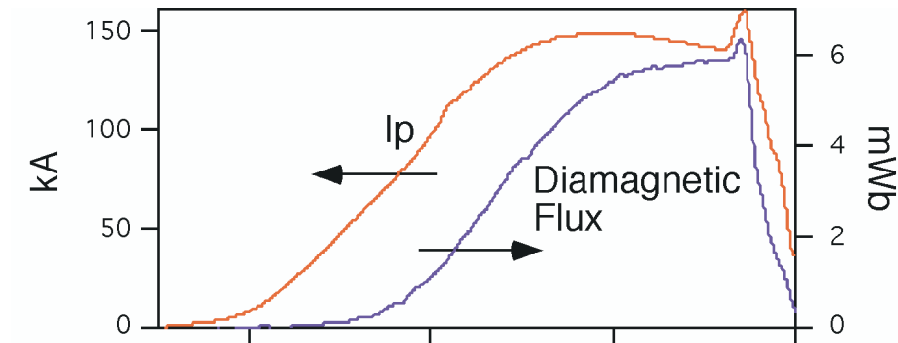
# Summary of internal MHD effects

- **Large scale tearing modes observed in almost all plasmas**
  - low  $B_t$  and fast  $dI_p/dt$   $\square$  low  $q$  early in discharge
  - high resistivity early in plasma evolution  $\square$  fast island growth
  - ultra-low  $A$   $\square$  large island widths
- **MHD activity contributes to  $I_p \sim I_{tf}$  soft limit**
  - large tearing modes dissipate input flux
  - mode onset is related to appearance of low-order rational  $q$  surfaces  
*onset at lower  $I_p$  for lower TF*
  - MHD amplitude increases as TF decreased
  - mitigated by lower  $\square$ , increased  $q_0$ , increased shear
- **At highest  $I_p$  ideal external kink is observed ...**





# Disruptive instabilities end discharge in some cases

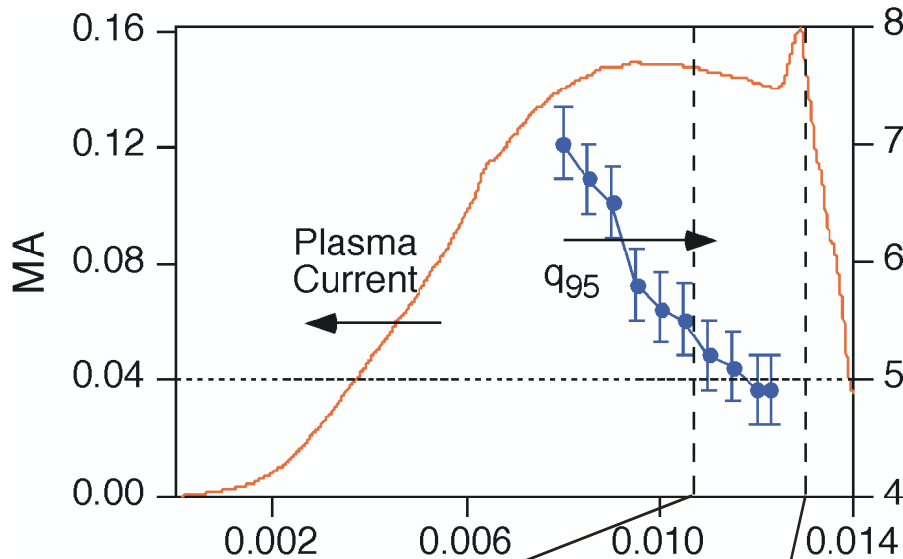


- Higher-current plasmas (150 kA class) often terminate in abrupt disruptions
- $n=1$  fluctuations are observed on core Mirnov coils immediately prior to disruption
  - Dominant frequency is order of 10 kHz
  - Mode is observed a few 100  $\mu$ s before IRE
- These fluctuations are not observed in lower-current shots

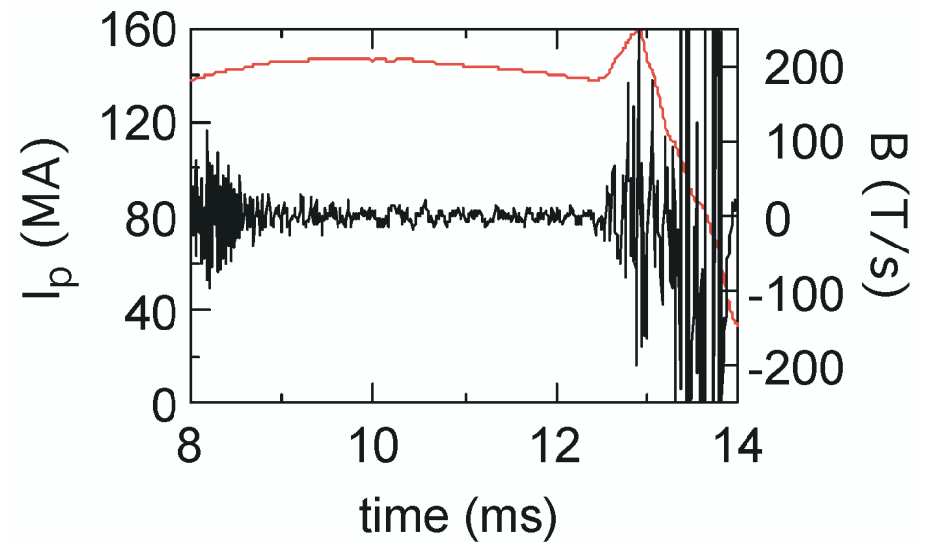
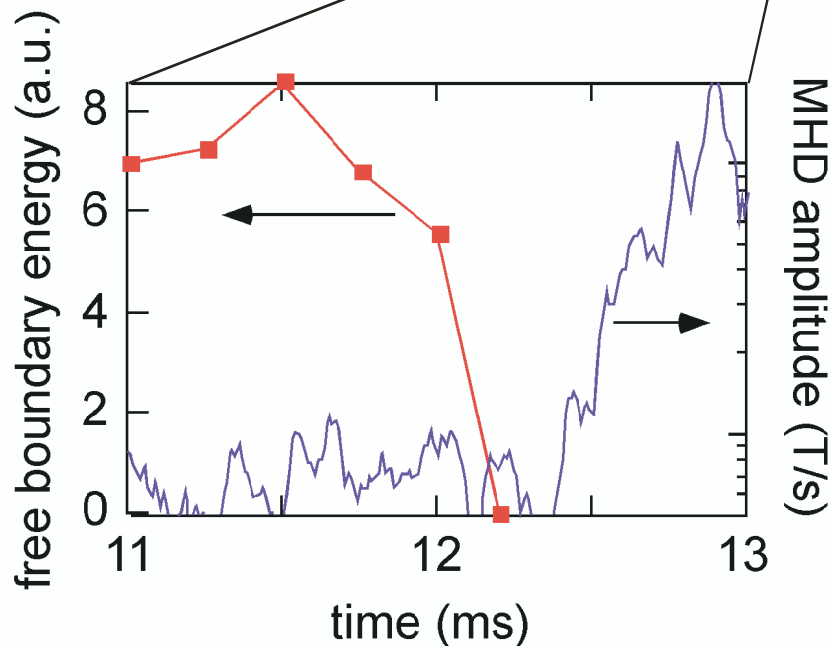




# External kink observed at $q_{95} \sim 5$



- tearing modes suppressed
- free boundary energy  $\approx 0$  as  $q_{95} \approx 5$
- disruption immediately follows
- mode grows on a hybrid time scale between  $\tau_A$  and  $q(dq/dt)^{-1}$ 
  - Roughly as expected for a plasma slowly crossing instability boundary





# Upgrades will allow access to high $I_p/I_{tf}$ , $\beta_t$ , low-q operation

---

## Goals:

- Manipulate q-profile: suppression of large internal modes
- Lower  $\beta$  during plasma formation: suppression of large internal modes
- Manipulate edge conditions: Expand access to external kink modes
- Access to very high  $\beta_t$  regime for stability analysis

## Additional tools being deployed:

- Programmable waveform power systems
  - Increase V-sec,  $B_t$ ; position and shape control
- Fast-response  $B_t(t)$  system
- Separatrix operation
- Increased HHFW power





# Flexible power systems in fabrication

## Ohmic Upgrade:

- Programmable waveform being implemented
  - will increase V-s capability
  - eliminates overdrive after breakdown

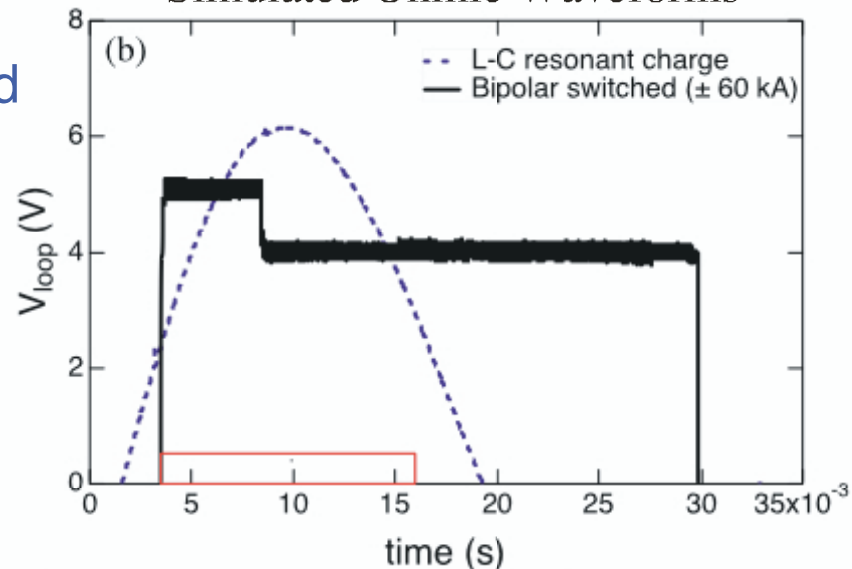
## TF Upgrade:

- Allows operation at  $> 3X$  present  $I_{TF}$ 
  - hold off  $q = 2$  surface until  $\beta$  is decreased
- Fast ramp-down capability

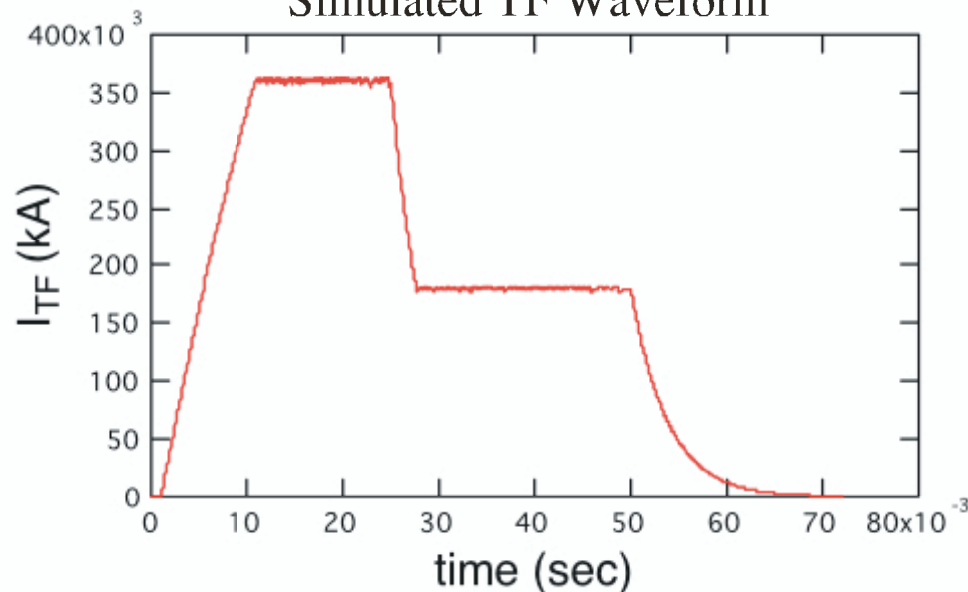
## EF Upgrade:

- Flexibility added to coil set
  - pre-programmable shape control
  - position control for improved HHFW coupling
  - energize divertor for separatrix operation

Simulated Ohmic Waveforms



Simulated TF Waveform





## Summary of PEGASUS ultralow-A results

---

- Mission: Study characteristics of plasmas as  $A \ll 1$
- Ready access to low-A physics with ohmic heating:
  - $\beta_t = 20\%$ ,  $\beta_N > 4$ ,  $n_e \approx n_{GW}$ , low central shear, paramagnetic:  $F/F_{vac} = 1.5$
- Resistive MHD activity and some Volt-second reduction result in a “soft limit” of  $I_p \sim I_{TF}$ 
  - Associated with central  $q(\rho) = 1.5-2$
- Beginning to explore the edge kink stability boundary
  - external kink observed at  $q_{95} = 5$
- Upgrades now underway will provide improved plasma control and allow access to high- $\beta_t$ , high  $I_p/I_{TF}$  regime

