Effect of thermal particle collisionality on resistive wall mode in tokamak

G.Z. Hao[1], Y.Q.Liu[2,4] S.X.Yang[3], S.Wang[4], D.Y.Liu[1], Z.X.Wang[3], A.K. Wang[4]

[1] *UC Irvine, CA,USA*

[2] *CCFE,Abingdon, UK*

[3] *DUT, Dalian,China*

[4] *SWIP, Chengdu, China*

UCIrvine
University of California, Irvine

Outline

- Motivation
- Theory model
  - Obtain the dispersion relation from an energy principle
  - Collisionality model used in theory model
- Analytical&Numerical results
  - Mode-particle resonance destabilizes a new branch
  - Collisionality can be either stabilizing or destabilizing
  - Numerical results from MARS-K consistent with analytical results
- Summary

UCIrvine
University of California, Irvine

Motivation

- RWM is a kind of external kink instability, which needs to be suppressed in advanced tokamak, such as ITER.

- The devices, such as ITER, NSTX-U, will have low collisionality operation, with different rotation levels.

- The role of thermal particle collisionality on RWM requires further investigation, based on the self-consistent computation.

UCIrvine
University of California, Irvine

Obtain the dispersion relation from an energy principle

Dispersion relation:

$$D = -i\omega\tau_w^* + \frac{\delta W^{\infty} + \delta W_k}{\delta W^b + \delta W_k} = 0$$

Y.Q.Liu,et.al. PoP,2008,
S.X.Yang,et.al. PoP,2015

Perturbed potential energy without wall :

$$\delta W^{\infty} = \frac{-4\pi}{ma^2}\frac{(m-nq)^2}{q^2}\left(\frac{v}{m-nq}-1\right)$$

Perturbed potential energy with ideal wall localized at b:

$$\delta W^{b} = -\frac{-4\pi}{ma^2}\frac{(m-nq)^2}{q^2}\left(\frac{v}{m-nq}-\frac{1}{1-b^{-2m}}\right)$$

Kinetic contribution from thermal particles:

$$\delta W_k = 2\pi^{1/2}\frac{\mu_0}{a^2 B_0^2}\frac{\mu_0 J_0^2}{4}\sum_{e,i}\int F r^{2\mu-1}\left(1-r^2\right)dr$$

$$F = \int d\lambda\left[\left(1-k_t^2\right)^2 + 4k_t^2 K^{-2}\left(k_t\right)\right]\frac{(2-\lambda)^2}{\sqrt{2\lambda \in / \hat{K}\left(k_t\right)}} I$$

$$I = \int d\hat{\varepsilon}_k\, \hat{\varepsilon}_k^{5/2}\, e^{-\hat{\varepsilon}_k}\frac{n\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_0 - \omega}{\omega_d + \omega_0 - i\nu_{eff} - \omega}$$

Resonance factor Q

Drift freq. Rotation collisionality Mode eigenvalue

UCIrvine
University of California, Irvine

Obtain the dispersion relation from an energy principle

Dispersion relation can be rewritten as :

$$\omega_r \tau_w = \frac{\left(\delta W_b - \delta W_\infty\right) \mathrm{Im}\left(\delta W_K\right)}{\left[\delta W_b + \mathrm{Re}\left(\delta W_k\right)\right]^2 + \mathrm{Im}\left(\delta W_K\right)^2} \quad .$$

$$\gamma \tau_w = \frac{\left(\delta W_b - \delta W_\infty\right)\left[\delta W_b + \mathrm{Re}\left(\delta W_K\right)\right]}{\left[\delta W_b + \mathrm{Re}\left(\delta W_k\right)\right]^2 + \mathrm{Im}\left(\delta W_K\right)^2} - 1 \quad ,$$

$\mathrm{Im}\left(\delta W_K\right)$ Always have stabilization on the instability

We assume a sample cylindrical to calculate perturbed energies,

Minor radius: $a = 1m$, Major radius: $R_0 = 3m$, Magnetic field at axis: $B_0 = 3T$,

Wall position: $b = 1.2a$ Mode number: m/n=2/1, Safety factor: $q0 = 1.42$

UCIrvine
University of California, Irvine

Collisionality model used in theory model

Two simple collisionality models:

- Energy-independent model

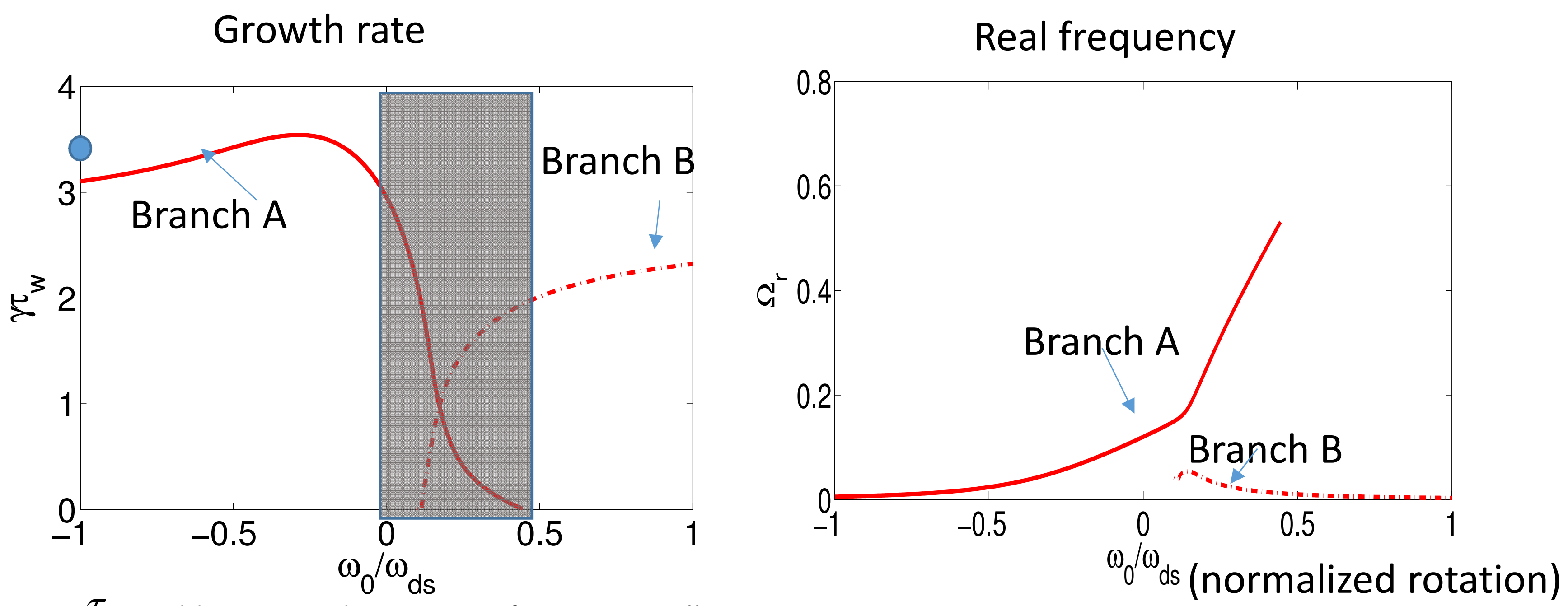
$$\nu_{eff} = \nu \equiv \frac{\sqrt{2} n_i m_{ij}^{1/2} Z_i^2 Z_j^2 e^4}{12\pi^{3/2}\epsilon_0^2 m_j T_j^{3/2}} \ln\Lambda$$

- Energy-dependent model

$$\nu_{eff} = \nu \hat{\varepsilon}_k^{-3/2} / \epsilon_r$$

UCIrvine
University of California, Irvine

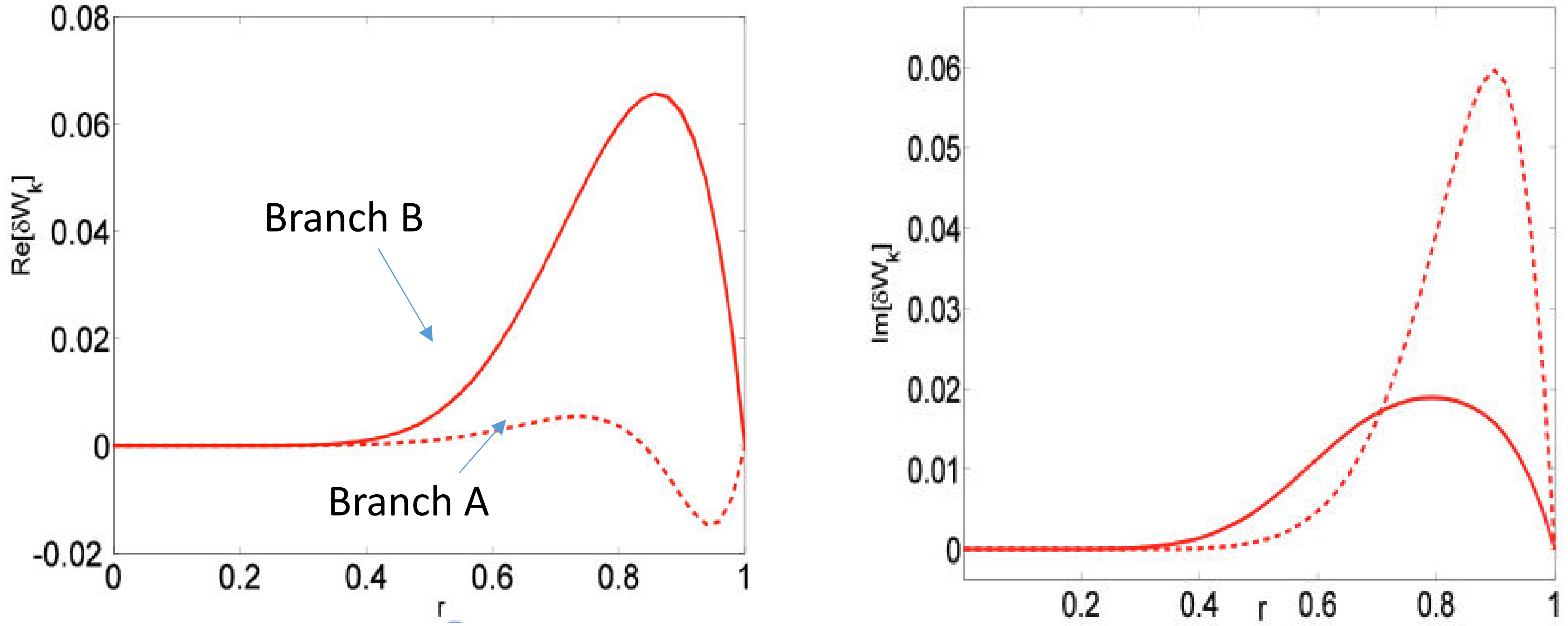
Mode-particle resonance destabilizes a new branch



τ_w *Eddy current decay time of resistive wall*

- W/O kinetic effect, normalized growth rate of RWM ~3.5, labeled by blue solid circle
- Growth rate is sensitivity to the plasma rotation in the gray region, since the resonance between the mode and precession drift motion of trapped thermal ions
- The mode-particle resonance can stabilize branch A, however, also trigger branch B as rotation exceeds a critical value.
- Here, no collisionality is included. We assume flat plasma rotation.
- Branch A with higher real frequency which is close to plasma rotation frequency
- Branch B with lower real frequency which decreases as increasing plasma rotation

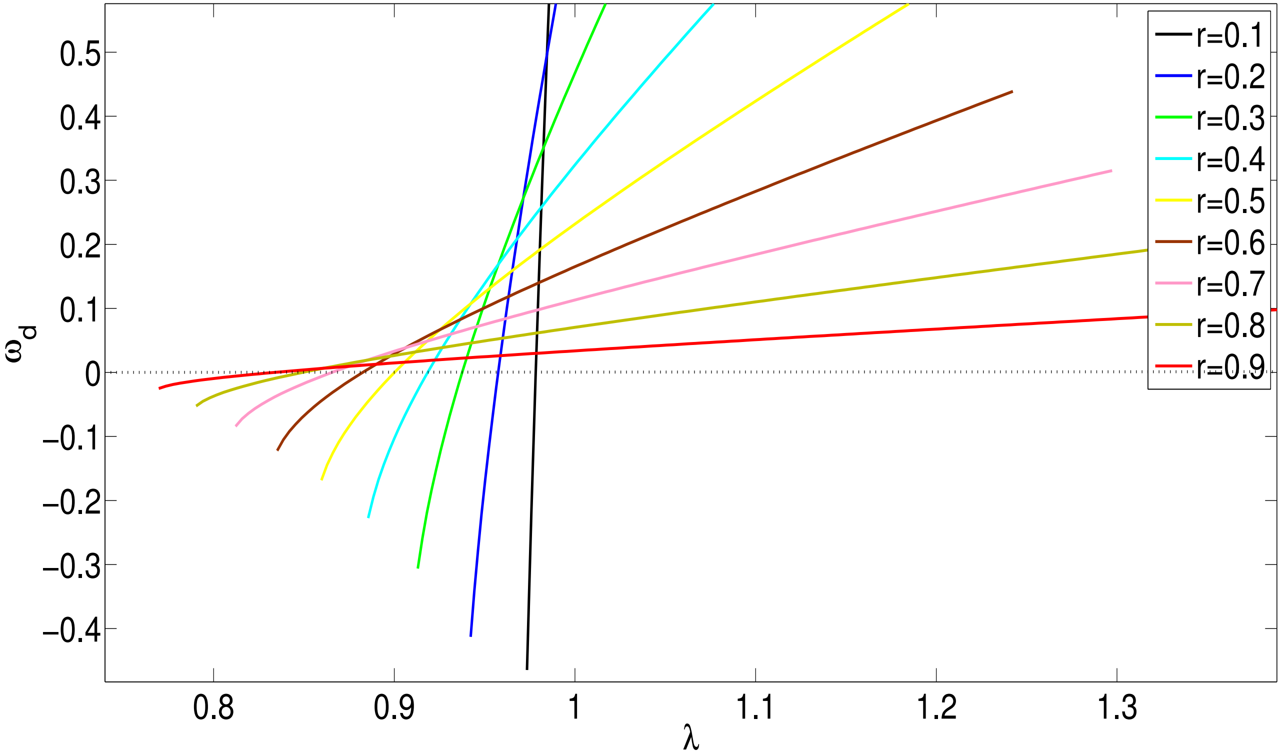
Eigenvalue of two branches depends on the mode-particle resonance



Choose rotation : $\omega_0 = 0.2\omega_{ds} = 1.2\times10^{-3}\omega_A$

- δW_k profile of branch A is very different with that of branch B, which is related to the mode-particle resonance.
- Branch A has larger $\mathrm{Im}(\delta W_k)$ and smaller $\mathrm{Re}(\delta W_k)$, which in turn determines the lower growth rate and larger real frequency of Branch A, though the RWM dispersion relation. As a analogy, Branch B has larger growth rate and smaller real frequency.
- For Branch A, profile of precession drift frequency of particles in pitch angle space has a dominant effect on profile of δW_k

precession drift frequency profile in pitch angle space



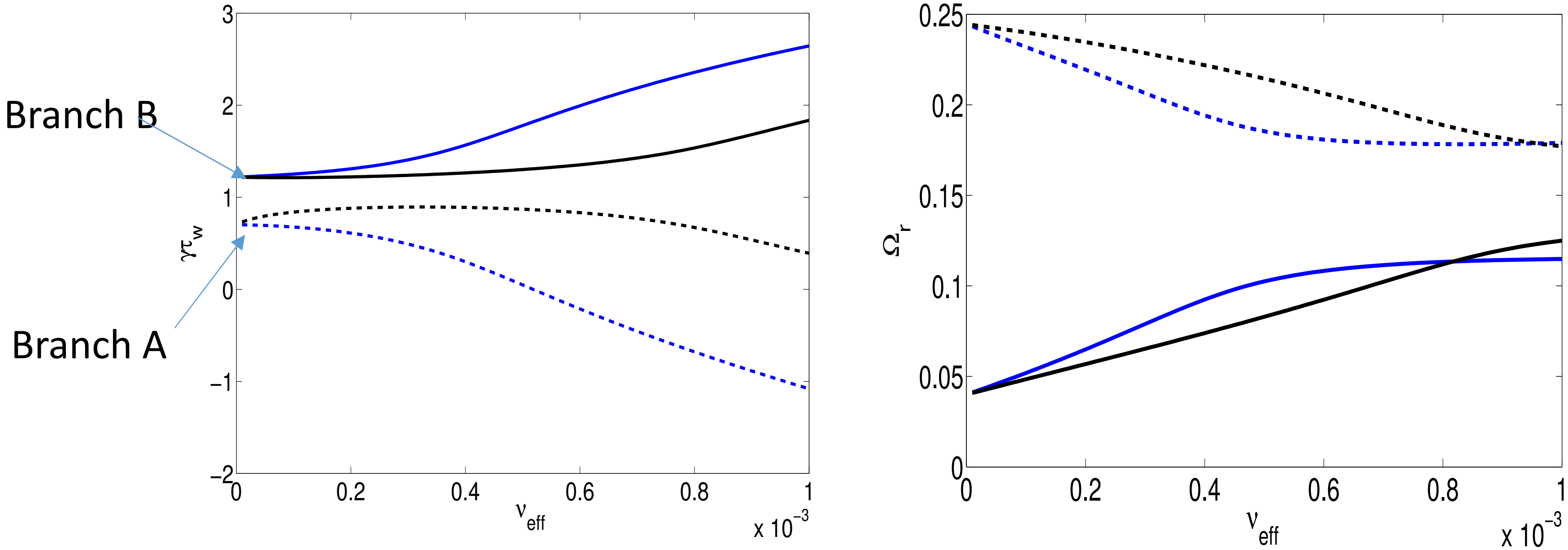
At given minor radius, precession drift frequency changes its sign at a certain value of pitch angle. For the branch with higher frequency(Branch A), this profile of drift frequency strongly enhances $\mathrm{Im}(\delta W_k)$, and reduces $\mathrm{Re}(\delta W_k)$, which in turn contributes damping on the mode.

$\lambda = \frac{\mu B_0}{\varepsilon_k}$: pitch angle. The ratio of the magnetic momentum to particle energy.

UCIrvine
University of California, Irvine

Collisionality can be either stabilizing or destabilizing

Choose rotation $\omega_0 = 1.2\times10^{-3}\omega_A$



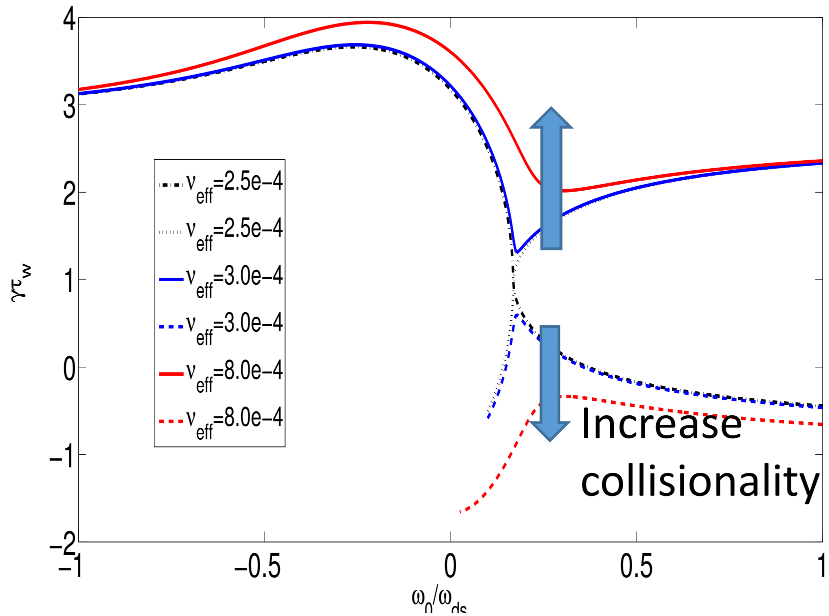
black curve: energy-dependent collisional model
Blue curve: energy-independent collisional mode

- For branch A: collisionality has stabilization effect on the mode
- For branch B: collisonality has destabilization influence on the mode
- The above effects are insensitive to the choice of the collisionality model
- Collisionality can be either stabilizing or destabilizing, depending on the value of mode frequency and plasma rotation.

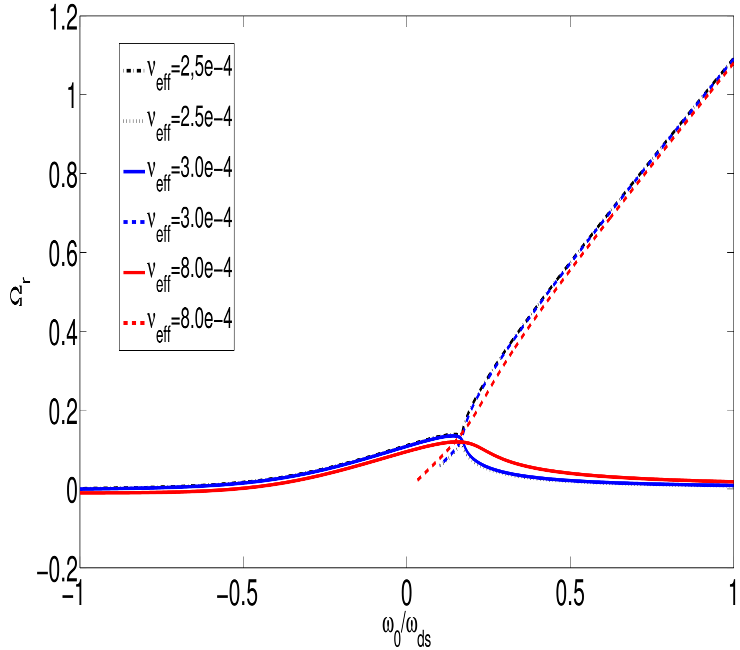
UCIrvine
University of California, Irvine

Collisionality can be either stabilizing or destabilizing

Growth rate



Real frequency

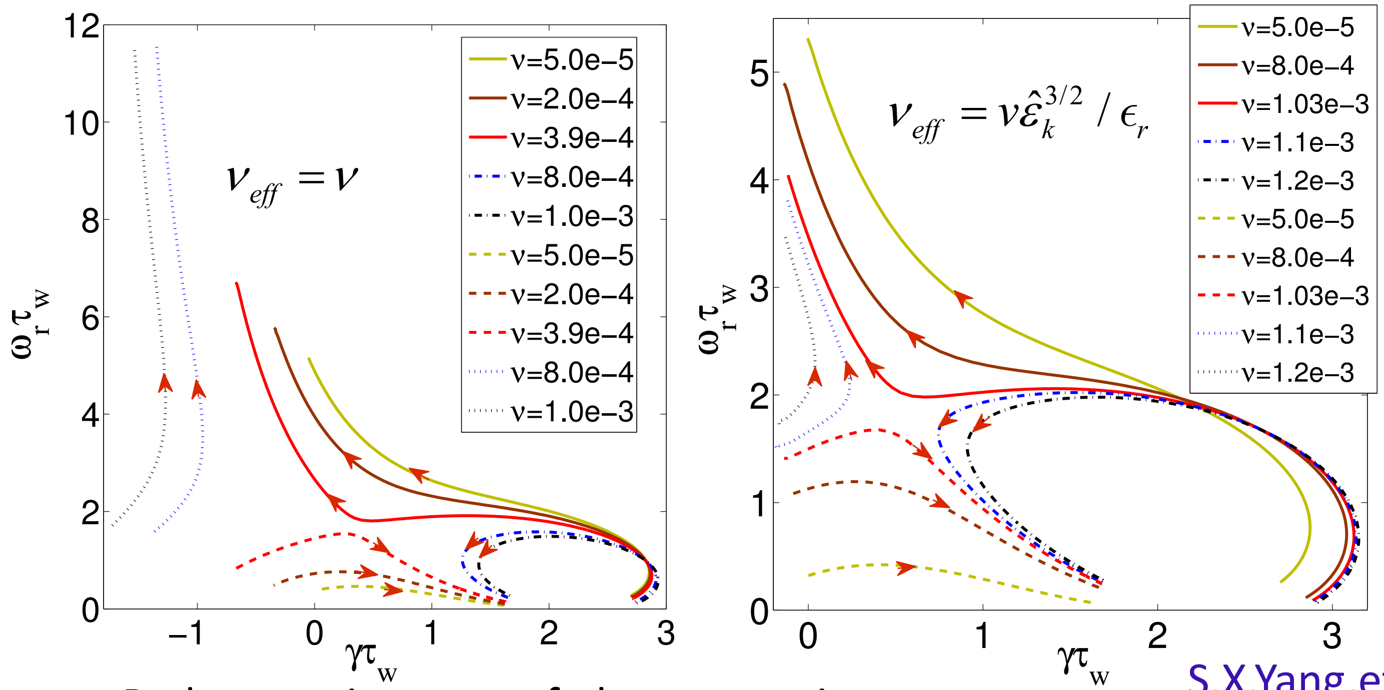


- With increasing collision frequency, the two branches shown in slide 7 merge and form two new branches.
- Collison frequency contributes destabilization effect on upper branch(blue curve); however, has stabilization effect on the lower branch

UCIrvine
University of California, Irvine

Collisionality can be either stabilizing or destabilizing

Plot the branches in (growth rate, real-freq) domain.



S.X.Yang,et.al.POP, 2015

Red arrow: increase of plasma rotation

- The merge and re-form of branches are insensitive to the collisional model
- The behavior of RWM , with varying plasma rotation, is rather different depending on the collisional regime
- In the low collisional regime, the plasma flow significantly stabilizes one branch, while destabilizes another one .
- At high collisionality, the plasma rotation does not generally change the stability of either of the branches.

Numerical results from MARS-K code

Basic formulations:

Single MHD:

$$(\gamma + in\Omega)\xi = \mathbf{v} + (\xi \cdot \nabla\Omega)R^2\nabla\phi,$$

$$\rho(\gamma + in\Omega)\mathbf{v} = -\nabla \cdot \mathrm{p} + \mathbf{j} \times \mathbf{B} + \mathbf{J} \times \mathbf{Q} - \rho\left[2\Omega\hat{\mathbf{Z}} \times \mathbf{v} + (\mathbf{v} \cdot \nabla\Omega)R^2\nabla\phi\right],$$

$$(\gamma + in\Omega)\mathbf{Q} = \nabla \times (\mathbf{v} \times \mathbf{B}) + (\mathbf{Q} \cdot \nabla\Omega)R^2\nabla\phi,$$

$$(\gamma + in\Omega)p = -\mathbf{v} \cdot \nabla P,$$

$$\mathbf{j} = \nabla \times \mathbf{Q},$$

$$\mathrm{p} = p\mathbf{I} + p_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}} + p_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}),$$

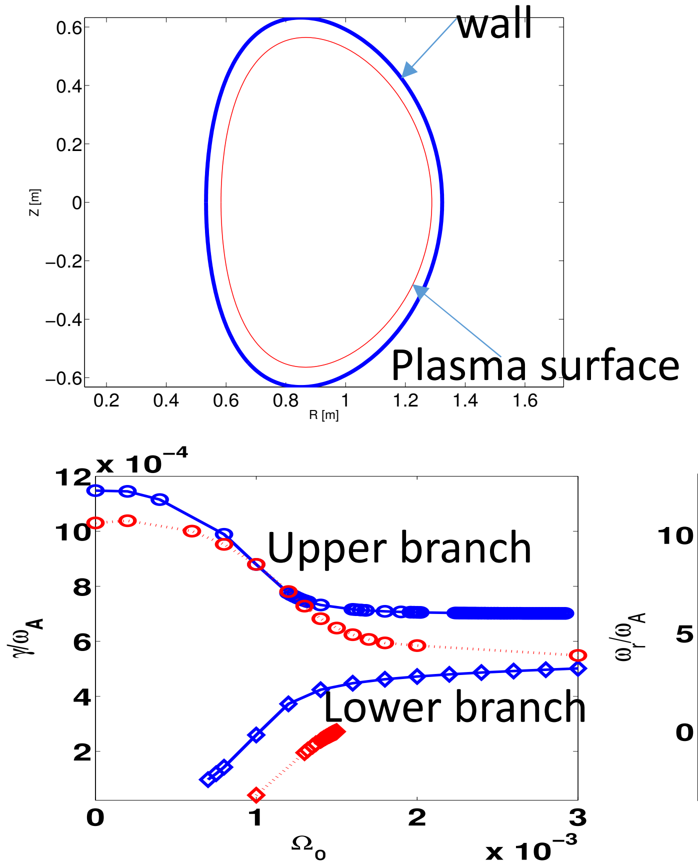
Kinetic contribution from particles:

$$p_{\|}e^{-i\omega t + in\phi} = \sum_{e,i}\int d\Gamma M v_{\|}^2 f_{th}^1(\xi_{\perp}) + \int d\Gamma M v_{\|}^2 f_h^1(\xi_{\perp}),$$

$$p_{\perp}e^{-i\omega t + in\phi} = \sum_{e,i}\int d\Gamma \frac{1}{2} M v_{\perp}^2 f_{th}^1(\xi_{\perp}) + \int d\Gamma \frac{1}{2} M v_{\perp}^2 f_h^1(\xi_{\perp})$$

MARS-K: full toroidal geometry, self-consistent computation of the kinetic effect of the particles on MHD instability

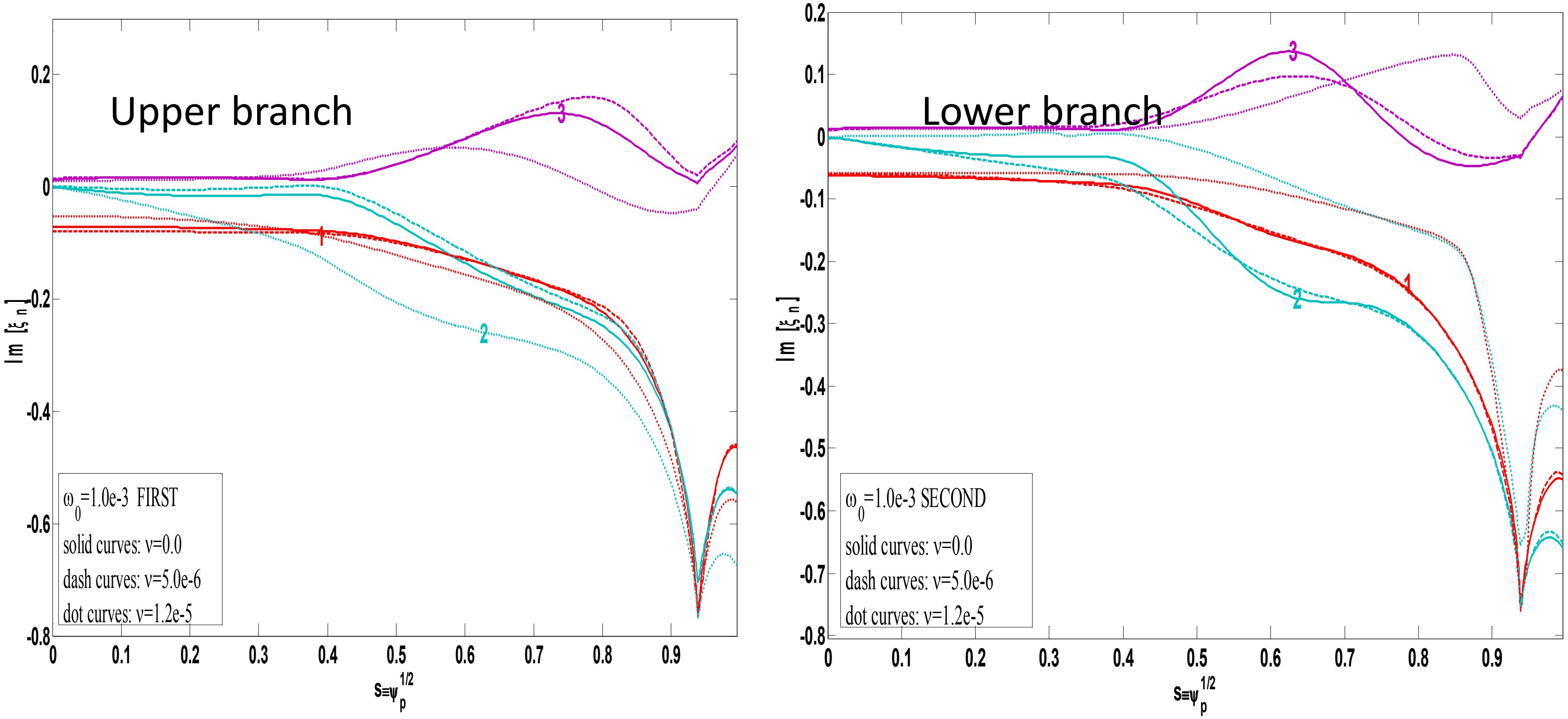
Numerical results consistent with analytical results



Red curves: ion collision frequency = 3E-4; Blue curves: no collisionality

- w/o collisionality, there exist two branches when plasma flow exceeds a critical value
- Collisionality stabilizes the lower branch, but destabilizes the upper one

Collisionality globaly affect the mode structure



For upper/lower branch, collisionality slightly pushes the mode perturbation to the core/edge plasma region

UCIrvine
University of California, Irvine

Summary

- The resonance between the mode and precession drift frequency of trapped thermal ions is stabilizing; however, the resonance destabilizes a new branch when the rotation exceeds a critical value

- Collisionality can be either stabilizing or destabilizing, depending on the value of mode frequency and plasma rotation.

- In the low collisionality regime, plasma flow stabilizes one branch but destabilizes another

- The numerical results of full toroidal, self-consistent computation using MARS-K confirms the main conclusion of analytical model

- The main conclusion is still kept, when electron kinetic effect is included.

- The results expand the zoology of RWM behavior in low collisionality tokamaks such as NSTX-U

UCIrvine
University of California, Irvine