

Experimental Study of Density Gradient Stabilization Effects on High-k Turbulence

J. Ruiz Ruiz¹

Y. Ren², W. Guttenfelder², A. E. White¹,

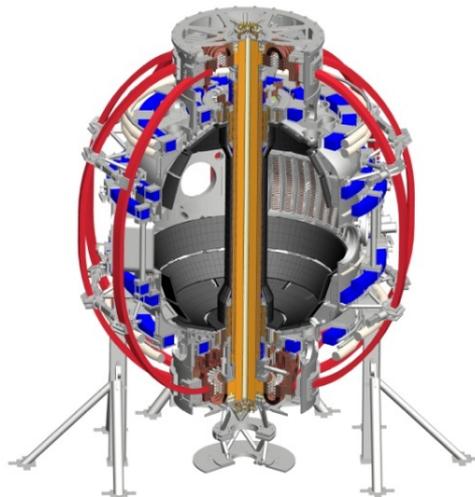
S.M. Kaye², B. P. LeBlanc², E. Mazzucato², K.C. Lee³,

C.W. Domier⁴, D. R. Smith⁵, H. Yuh⁶

1. MIT 2. PPPL 3. NFRI 4. UC Davis 5. U Wisconsin 6. Nova Photonics, Inc.

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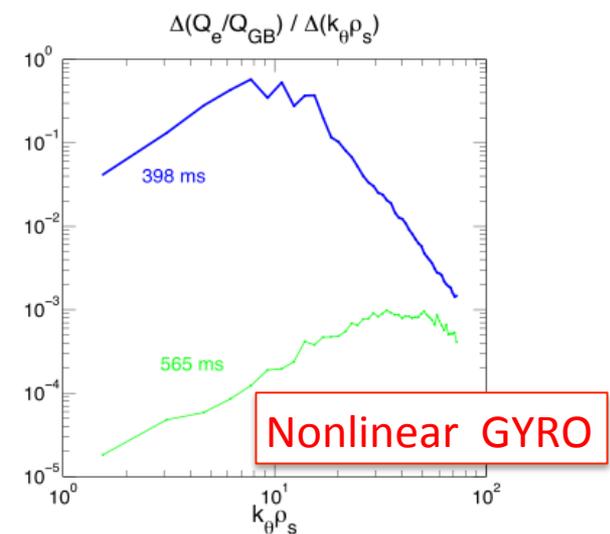
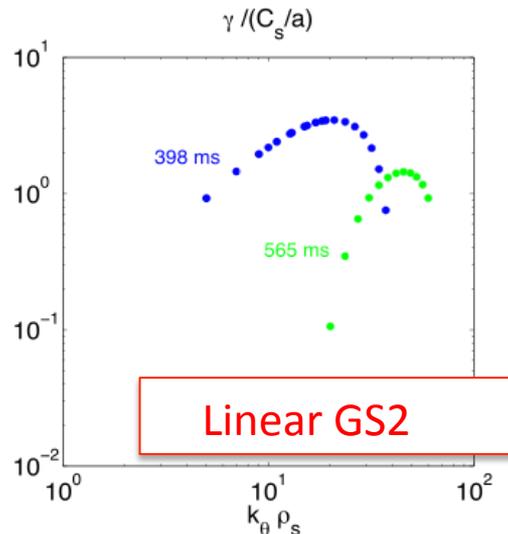
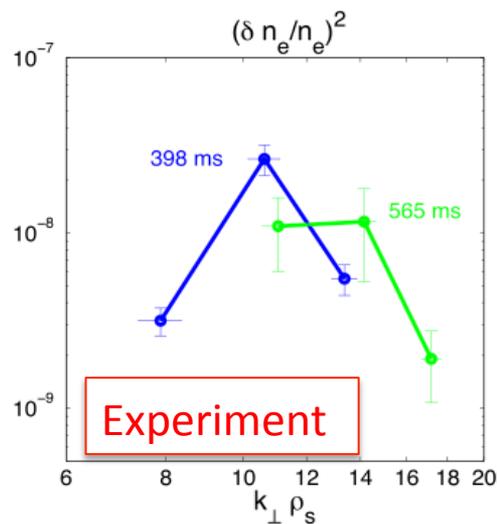


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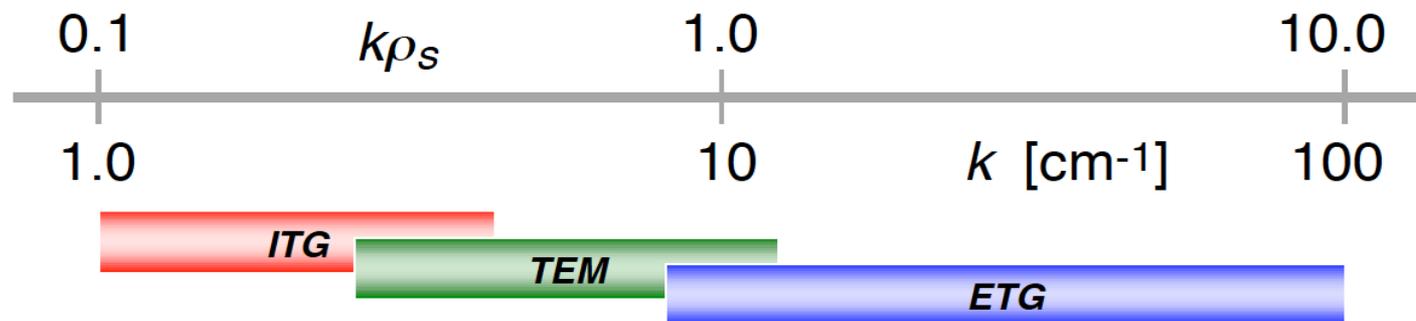
Linear and Nonlinear Gyrokinetic Simulations are Compared with Measured Trends in High-k Turbulence at NSTX

- $(R/L_{Te}^{\text{exp}}) - (R/L_{Te})_{\text{crit}}$ determines linear threshold for instability and is correlated with the presence of observed fluctuations.
- Increasing R/L_{ne} shifts high-k fluctuations to higher k values (stabilizing) and decreases real frequency ω_r , consistent with Doppler subtracted plasma frame frequency of detected fluctuations.
- Electron density gradient increases the ETG nonlinear threshold, consistent with experimental observations of reduced fluctuation amplitude, and reduces electron heat flux and stiffness.



Electron Thermal Transport is Anomalous in All NSTX Confinement Regimes

- NSTX H-mode plasmas exhibit ion thermal transport close to neoclassical levels due to low- k turbulence suppression by ExB shear [*cf. Kaye NF 2007*]. **Electron thermal transport is always found anomalous.**
- **ETG** turbulence is a candidate for anomalous electron thermal transport in some NSTX and NSTX-U operating regimes.
- A **microwave scattering diagnostic** is used at NSTX to measure electron-scale density fluctuations indicative of **high- k turbulence** ($k_{\perp}\rho_s > 1$).
- Linear and nonlinear gyrokinetic simulations are used to study high- k turbulence and electron thermal transport in an NBI-heated H-mode plasma.
- We observe a **stabilizing effect of the electron density gradient** on experimental high- k fluctuation levels and predicted electron heat flux (article submitted to PoP).



From Mazzucato PPPL presentation

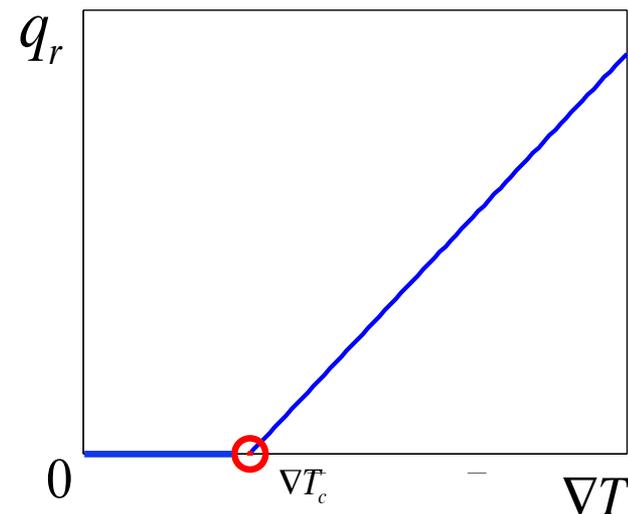
Critical Gradient and Critical ETG Formula

- Normalized gradient of quantity X

$$R / L_X = -R(\nabla X / X)$$

- Critical gradient

$$q_r^{turb} = \underbrace{\chi_{GB} f(\hat{s}, q, \nabla n_e, \dots)}_{\text{Nonlinear dependence}} \underbrace{\left(R / L_{T_e} - (R / L_{T_e})_c \right)}_{\text{Linear threshold}}$$



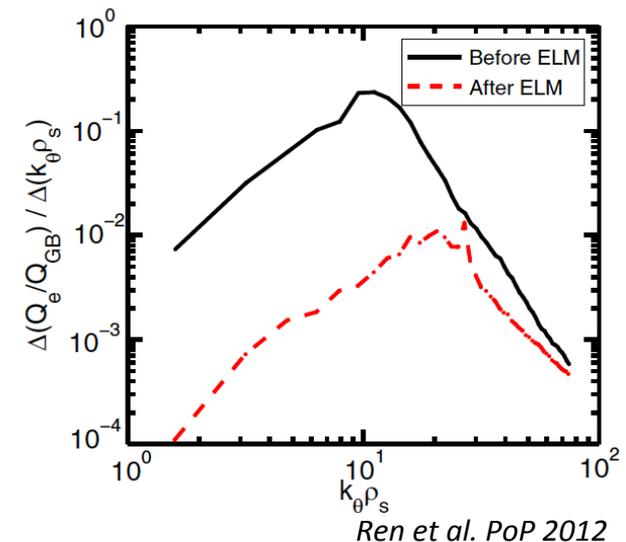
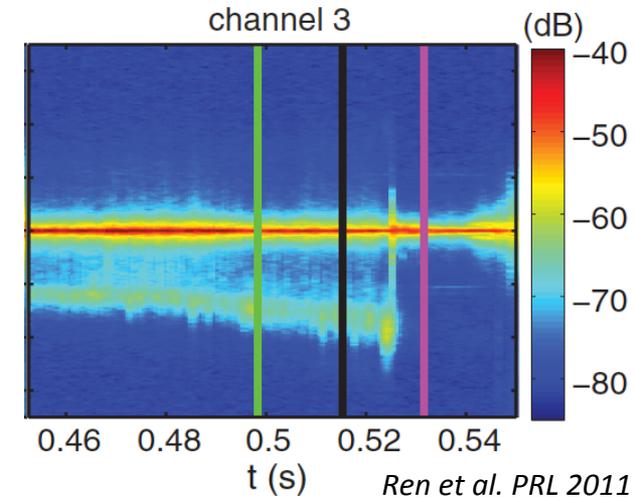
- Jenko critical temperature gradient [*cf. Jenko Phys. Plasmas 2001*].

$$(R / L_{T_e})_{crit} = \max \left\{ \begin{array}{l} 0.8R / L_{ne} \\ (1 + \tau)(1.33 + 1.91\hat{s} / q)(1 - 1.5\varepsilon)(1 + 0.3\varepsilon d\kappa / d\varepsilon) \end{array} \right. \quad \text{with} \quad \tau = Z_{eff} T_e / T_i$$

Applicability: low- β , positive \hat{s} and large aspect ratio with local Miller equilibrium (Miller *et al* PoP 1998).

Previous Work Suggested Density Gradient Stabilized Turbulence after ELM Event

- First direct experimental demonstration of density gradient stabilization of e^- -scale turbulence (*Ren et al. PRL 2011*). Shot 140620.
 - **ELM event** at $t \sim 525$ ms \rightarrow change in density gradient.
 - Stabilization of lower- k e^- -scale fluctuations ($k_{\perp} \rho_s < 10$).
- Nonlinear gyrokinetic simulations show the effect of density gradient on transport (*Ren et al. PoP 2012*). Shot 140620.
- New work presents a detailed analysis of the linear and nonlinear stabilizing effect of density gradient on ETG fluctuations *focusing on changes in critical gradient and stiffness* during controlled current ramp down experiment (Shot 141767. Article submitted to PoP 2015).

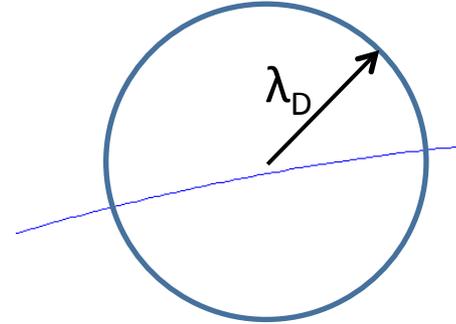


Experimental Set-Up

Collective Scattering is Used to Measure High-k Turbulence

- Collective/coherent scattering

$$k\lambda_D \leq 1$$

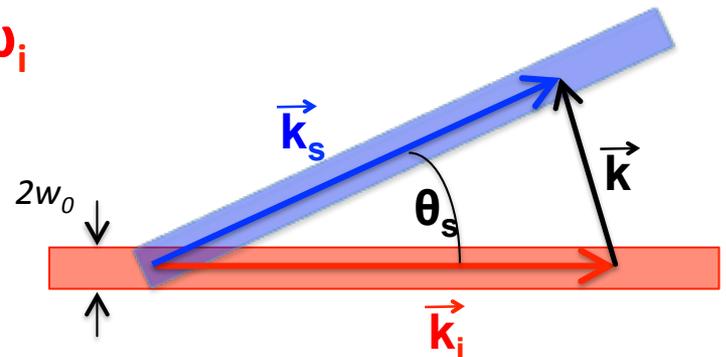


- Scattered power density $\frac{d^2 P}{d\Omega d\nu} = P_i r_e^2 L_z |\Pi \cdot \hat{e}|^2 \frac{|\tilde{n}_e(k, \omega)|^2}{VT}$
- Three wave-coupling** between incident beam (\mathbf{k}_i, ω_i) and plasma (\mathbf{k}, ω)

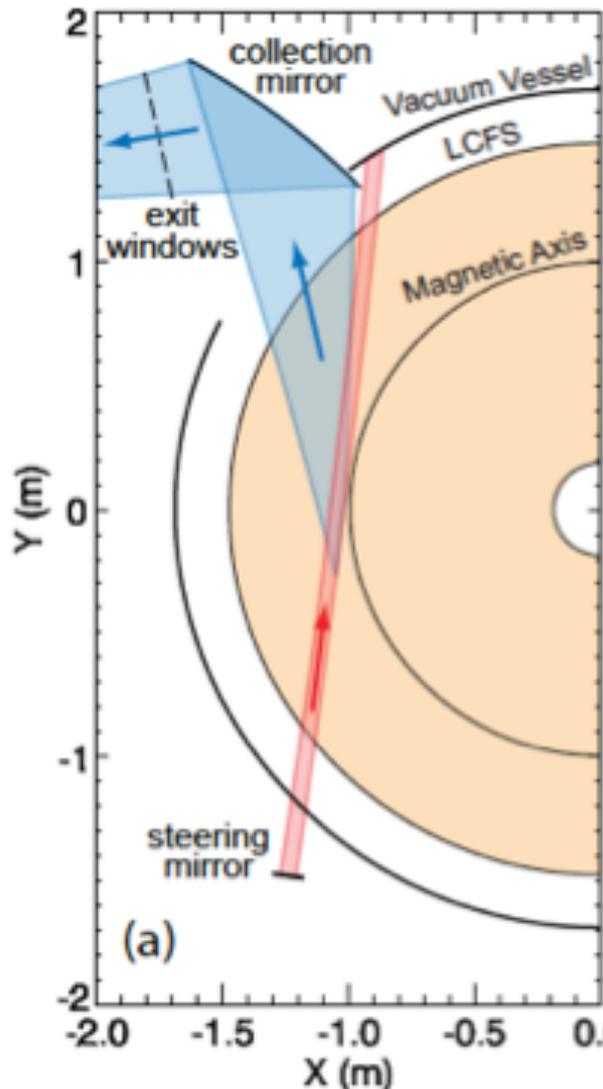
$$\vec{\mathbf{k}}_s = \vec{\mathbf{k}} + \vec{\mathbf{k}}_i \quad \omega_s = \omega + \omega_i$$

- $\omega_i, \omega_s \gg \omega$ imposes Bragg condition

$$k = 2k_i \sin(\theta_s/2)$$



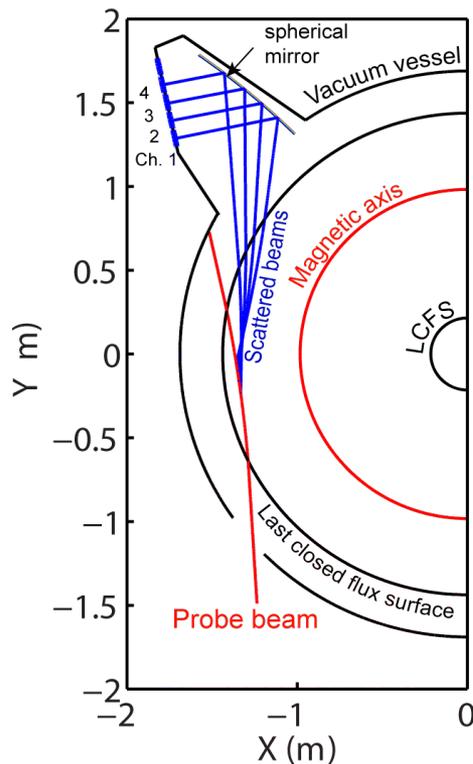
High-k Microwave Scattering Diagnostic at NSTX



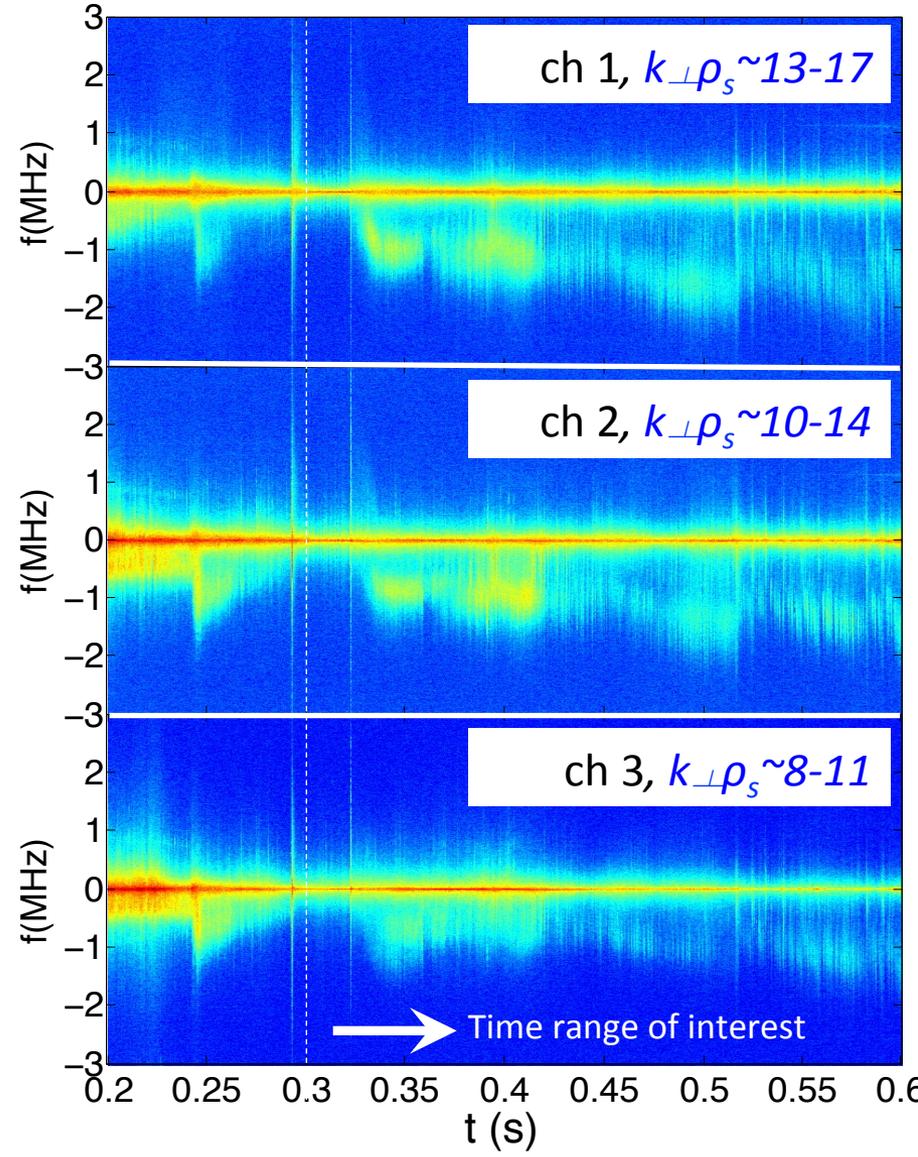
- Gaussian Probe beam: 15 mW, 280 GHz, $\lambda_i \sim 1.07$ mm, $a = 3$ cm ($1/e^2$ radius).
- Propagation close to midplane $\Rightarrow k_r$ spectrum.
- 5 detection channels \Rightarrow range $k_r \sim 5$ -30 cm^{-1} (*high-k*).
- Wavenumber resolution $\Delta k = \pm 0.7$ cm^{-1} .
- Radial coverage: $R = 106$ -144 cm.
- Radial resolution: $\Delta R = \pm 2$ cm (unique feature).

Each Channel of the NSTX High-k Scattering System Detects a Fluctuation Wavenumber k

- Channel 1 detects highest k_{\perp} and k_t , Doppler shift is greatest ($f_D = k_t v_t / 2\pi$).
- High peak at $f \sim 0$ corresponds to stray radiation.
- Scattering region $R \sim 135\text{-}136$ cm, $r/a \sim 0.7$. (major radius 0.85 m, minor radius 0.68 m).

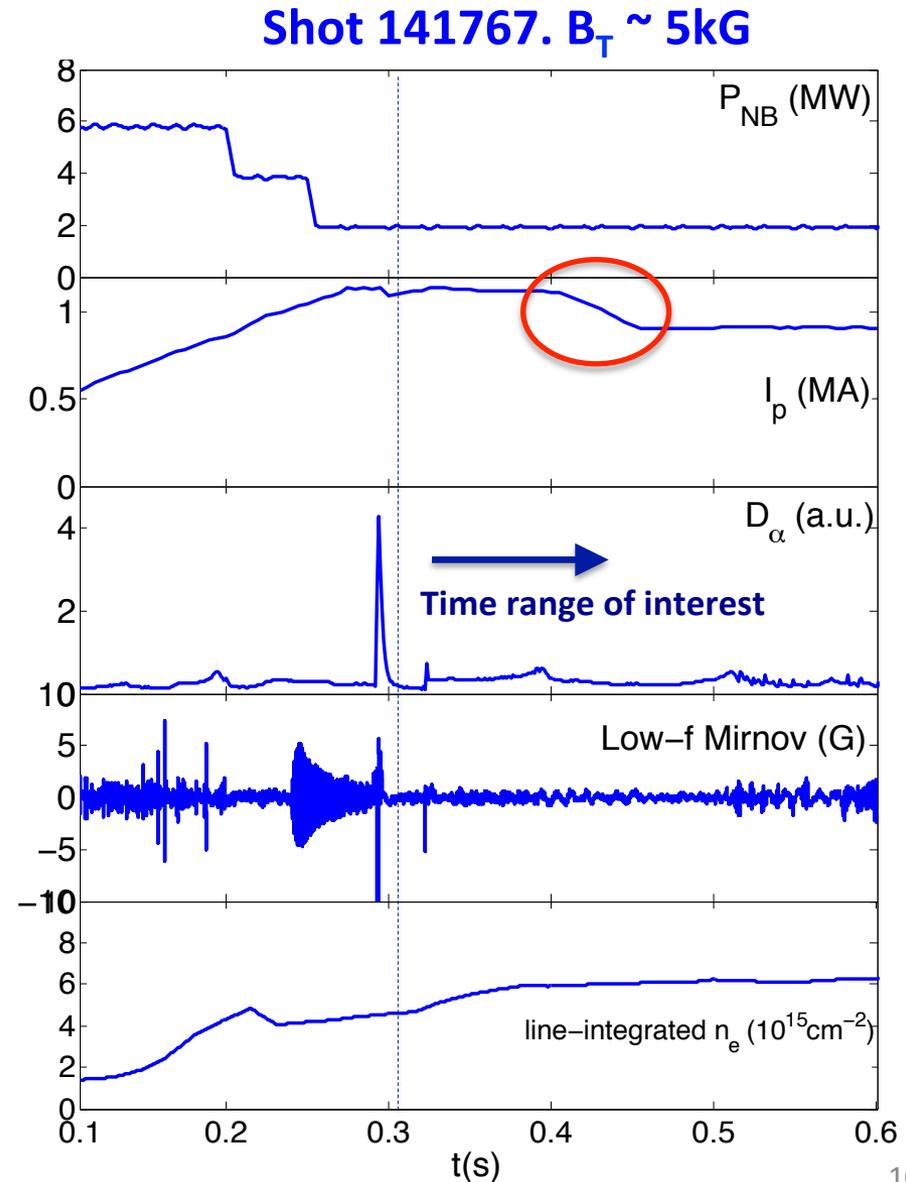


Shot 141767



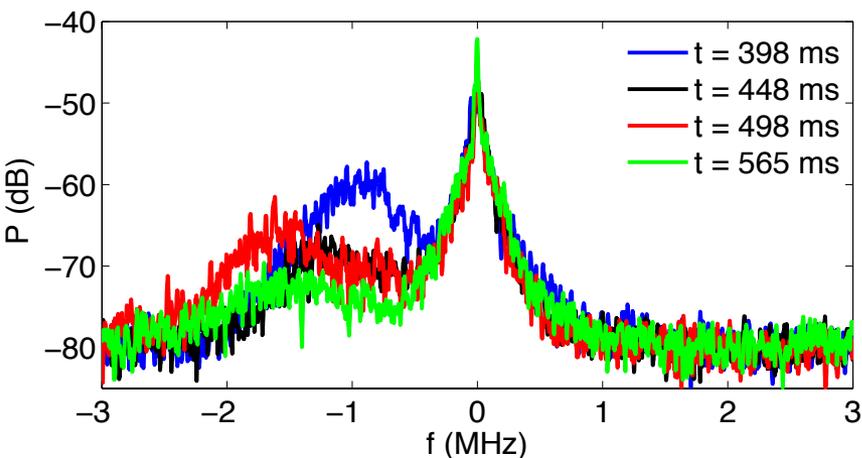
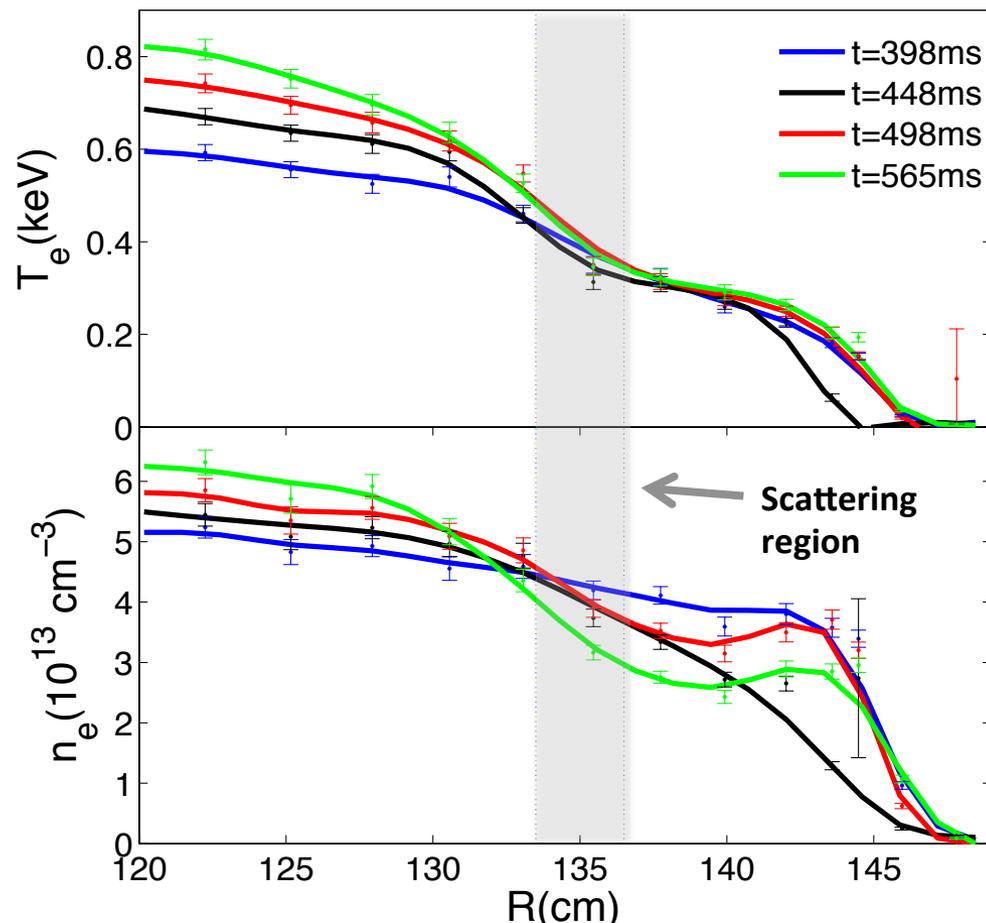
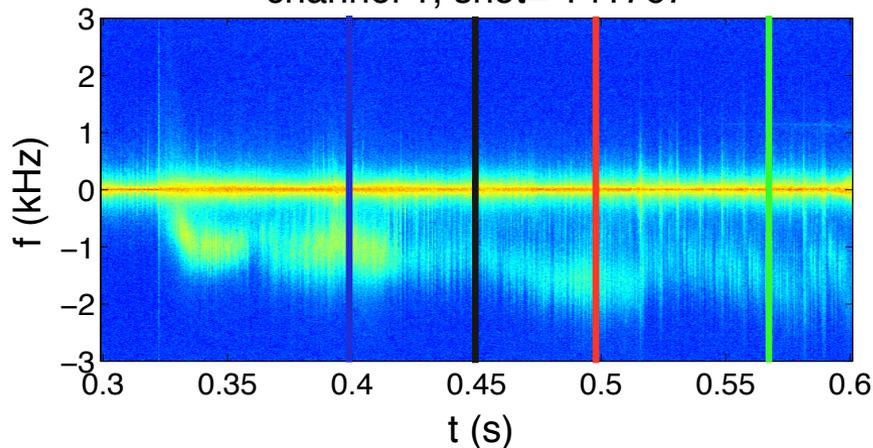
A Set of NBI-heated H-mode Plasmas is Used to Study High-k Turbulence during Current Ramp-down

- **NBI heated**, HHFW heating is absent during the run.
- **Controlled Current ramp down** between $t = 400$ ms and $t = 450$ ms (from LRDFIT).
- Time range of interest is $t > \sim 300$ ms, covering current ramp-down phase, and after ELM event at $t \sim 290$ ms.
- **MHD activity is quiet during time range of interest.** (*cf.* low-f Mirnov signal).
- Line integrated density is fairly constant during the time range of interest.



Observed High-k Fluctuations Correlate to Local Electron Density Gradient

channel 1, shot= 141767

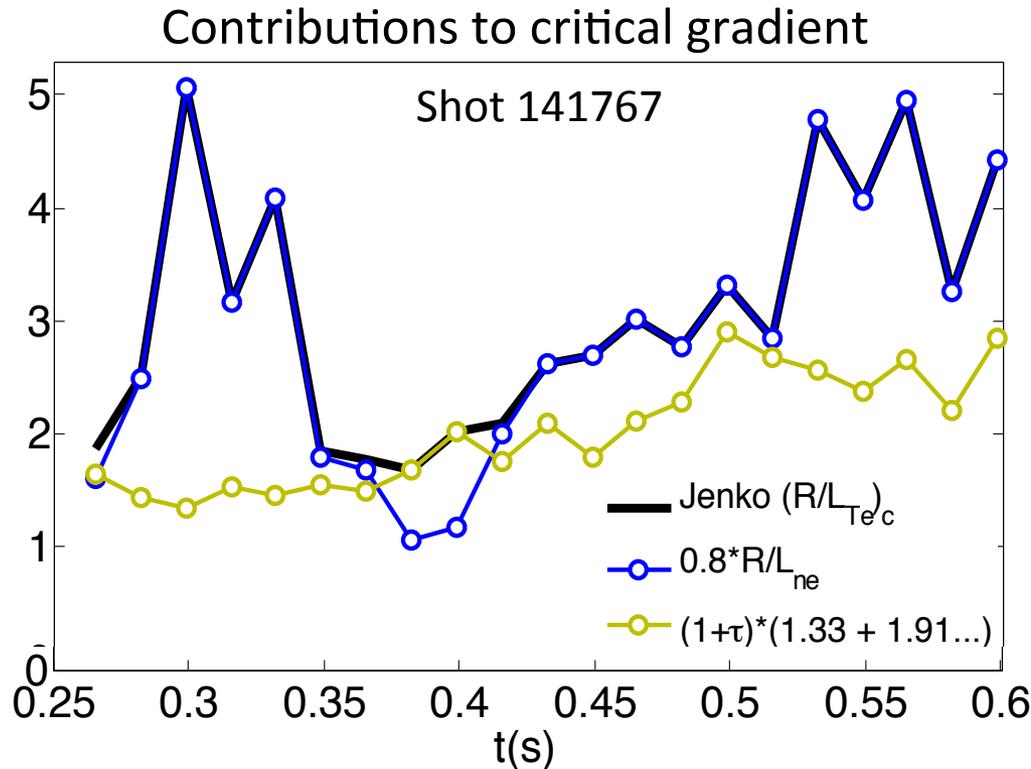


$\nabla T_e \rightarrow \text{Drives ETG}$
 $\nabla n_e \rightarrow \text{Stabilizes ETG}$

Two competing effects: ∇n_e is dominant effect.

Comparisons with Linear Critical Gradient Formula (Jenko)

Electron Density Gradient Can *Linearly* Stabilize ETG Turbulence by Increasing Critical Gradient



- Jenko critical gradient is a maximum of a R/L_{ne} term and an s/q term.

$$(R/L_{Te})_{crit} = \max \begin{cases} 0.8R/L_{ne} \\ (1+\tau)(1.33+1.91\hat{s}/q)(1-1.5\epsilon)(1+0.3\epsilon d\kappa/d\epsilon) \end{cases} \quad \text{with} \quad \tau = Z_{eff}T_e/T_i$$

- Higher values of R/L_{ne} raise the critical gradient for ETG (possibly above the experimental gradient value). This *should* have a **stabilizing** effect on turbulence.

High-k Density Fluctuations are Observed during ETG Unstable Time Periods

- Total scattered power (integrated in *freq*).

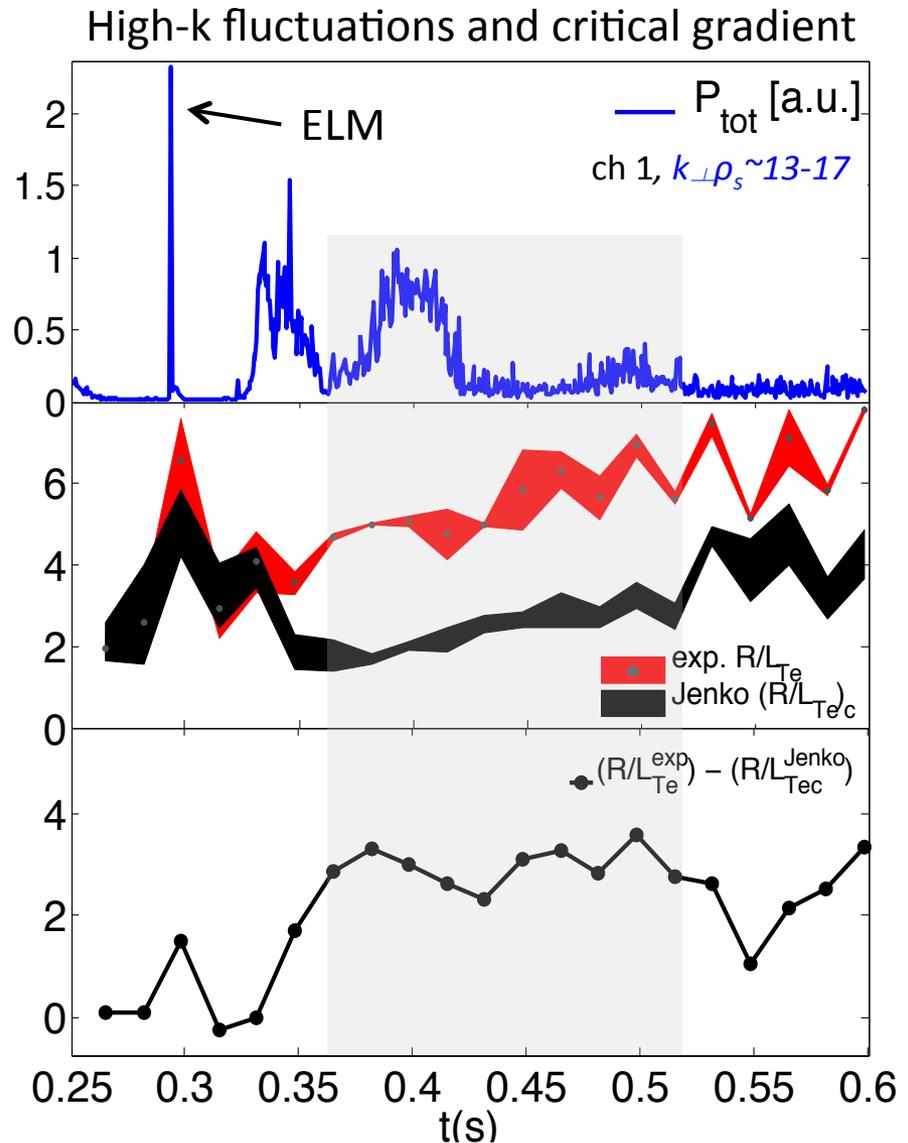
$$P_{tot} \propto (\delta n_e / n_e)^2$$

- $(R/L_{Te}^{exp}) - (R/L_{Te})_{crit}$ determines linear threshold for instability.

- $t < 320 \text{ ms}$ $(R/L_{Te}^{exp}) \sim (R/L_{Te})_{crit}$
 → ETG marginally stable, no fluctuations.

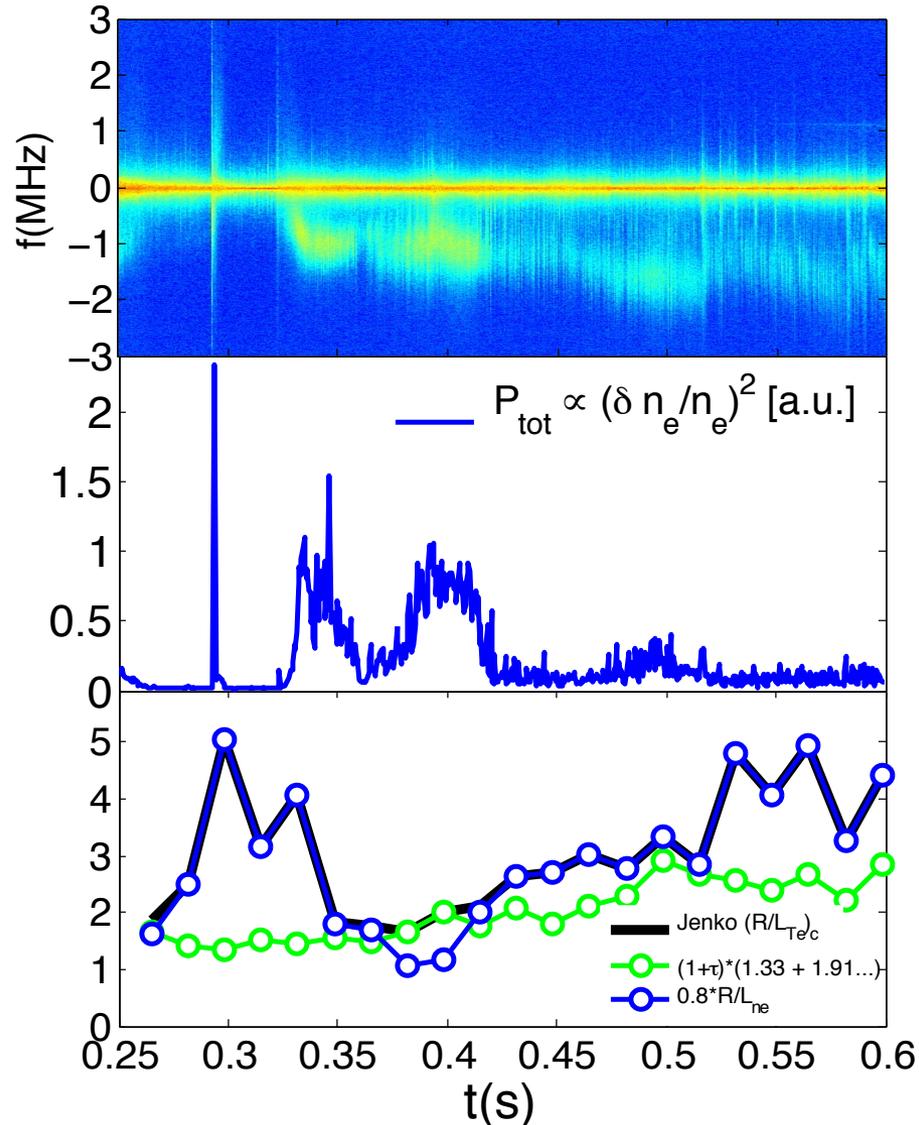
- $t > 320 \text{ ms}$ $(R/L_{Te}^{exp}) > (R/L_{Te})_{crit}$
 → fluctuations develop.

- $360 \text{ ms} < t < \sim 520 \text{ ms}$ (gray shading)
 Similar $(R/L_{Te}^{exp}) - (R/L_{Te})_{crit}$ produces **VERY** different P_{tot} . Nonlinear evolution of turbulence motivates the use of nonlinear gyro-kinetic simulations (future work).



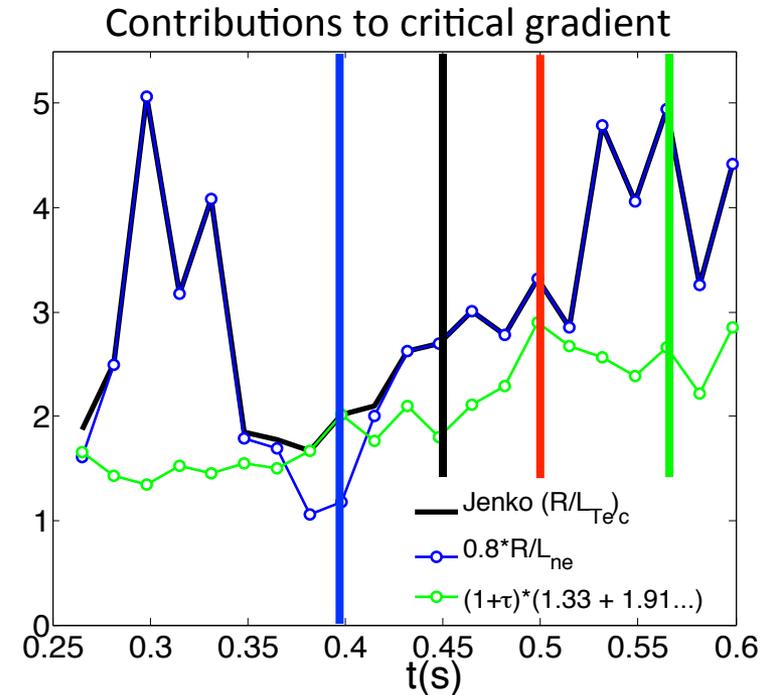
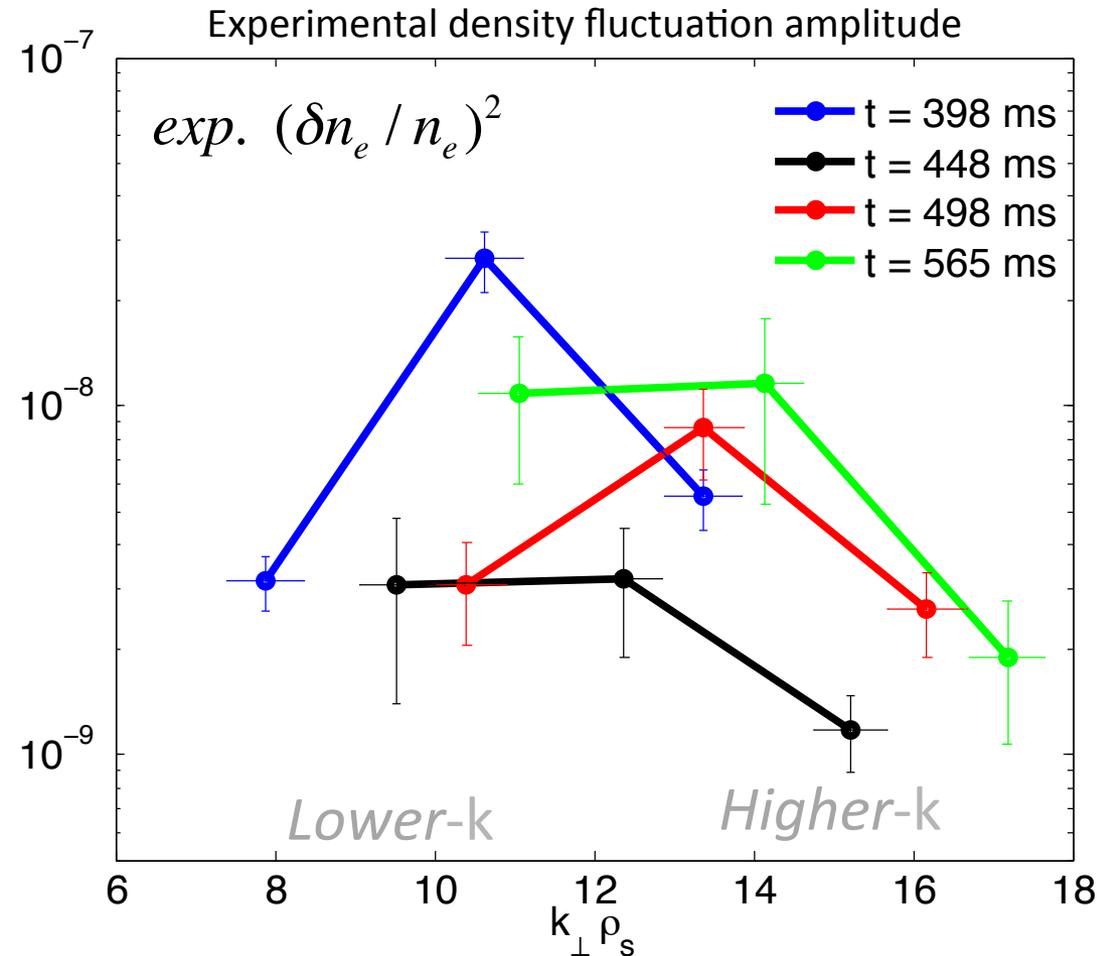
High-k Density Fluctuations are Linearly Stabilized by Density Gradient through the Critical Gradient

shot 141767, channel 1



- As R/L_{ne} increases, it dominates in Jenko's formula $(R/L_{Te})_{crit}$ ($t < 340$ ms, $t > 410$ ms & $t > 515$ ms). Fluctuations decrease during that time.
- Previous to $t \sim 320$ ms ETG is marginally stable with respect to Jenko critical gradient. No fluctuations are observed.
- R/L_{ne} is a *linear stabilizing* mechanism when it dominates the Jenko critical gradient.
- Electron density gradient stabilization of ETG turbulence first observed by Y. Ren *et al*, Phys. Rev. Letters 2011.

Wavenumber Spectrum of Fluctuation Amplitude and Electron Density Gradient

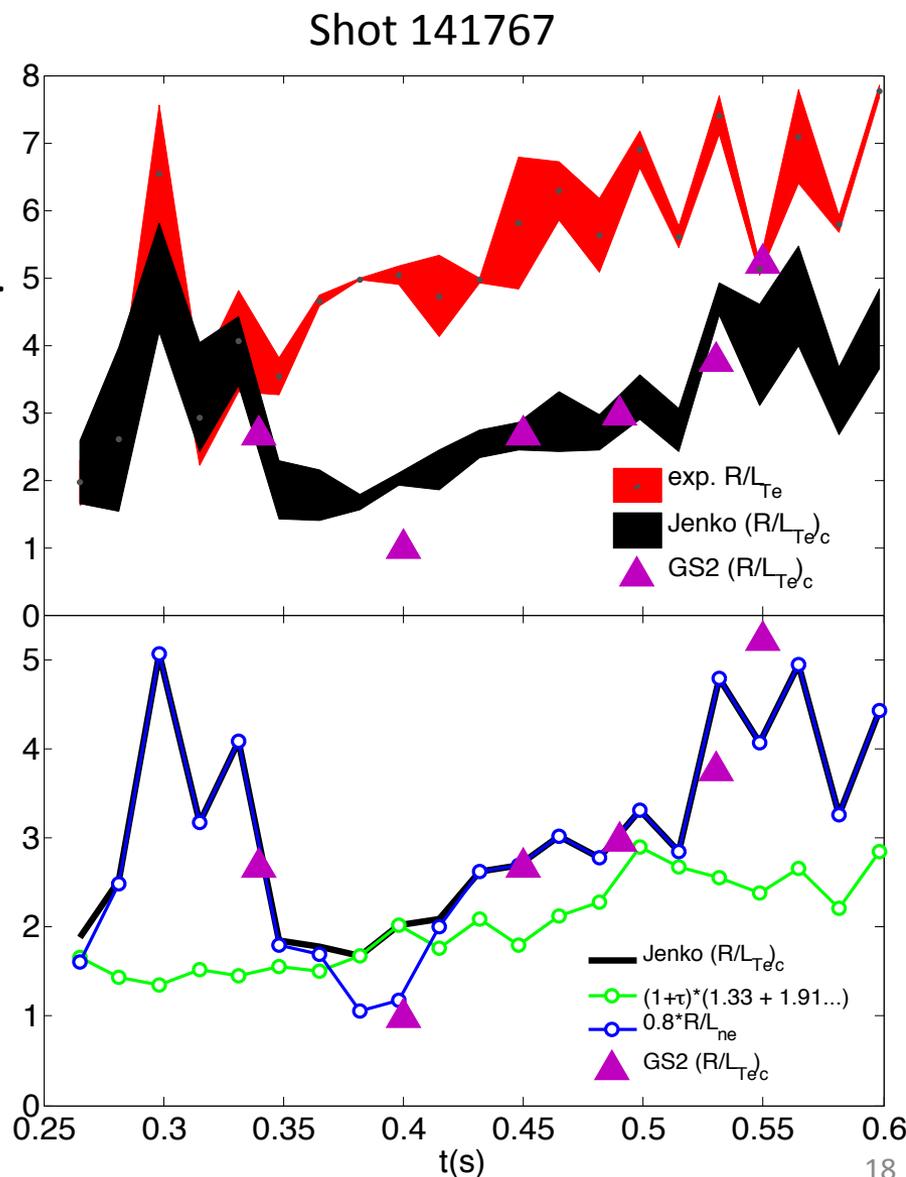


- Lower-k ($k_{\perp} \rho_s < 10$) $(\delta n_e / n_e)^2$ decreases for $398 < t < 498$ ms.
- After $t \sim 448$ ms, higher k ($k_{\perp} \rho_s \sim 12-16$) fluctuation levels increase. During that time, R/L_{ne} increases.

Comparisons between Experiment and Linear GS2 simulations

Critical Gradient Computed with GS2 Linear Runs Agrees with Jenko's Critical Gradient

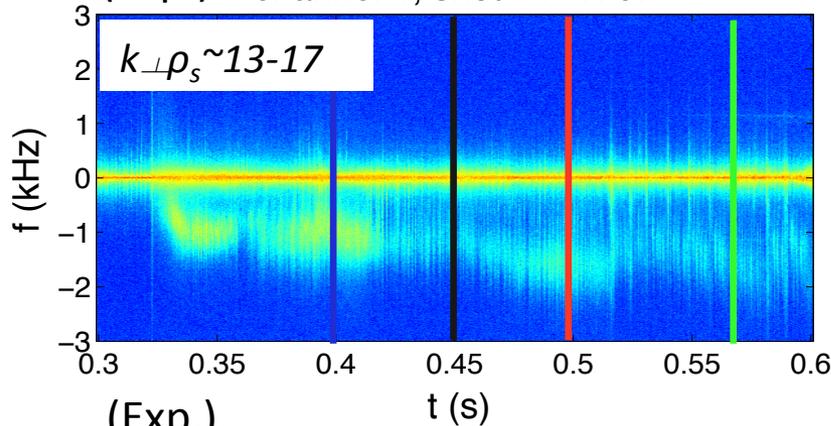
- Regime of validity of $(R/L_{Te})_{crit}$:
 - low- β
 - positive $\hat{s} > 0.2$
 - large aspect ratio
 } not NSTX parameters.
- $(R/L_{Te})_{crit}$ is explicitly calculated with GS2.
- Good agreement between **GS2 $(R/L_{Te})_{crit}$** calculations and **Jenko $(R/L_{Te})_{crit}$** :
 - Jenko's critical ETG formula is assumed valid in these NSTX plasmas.
- $GS2 (R/L_{Te})_{crit}$ seems to follow R/L_{ne} .



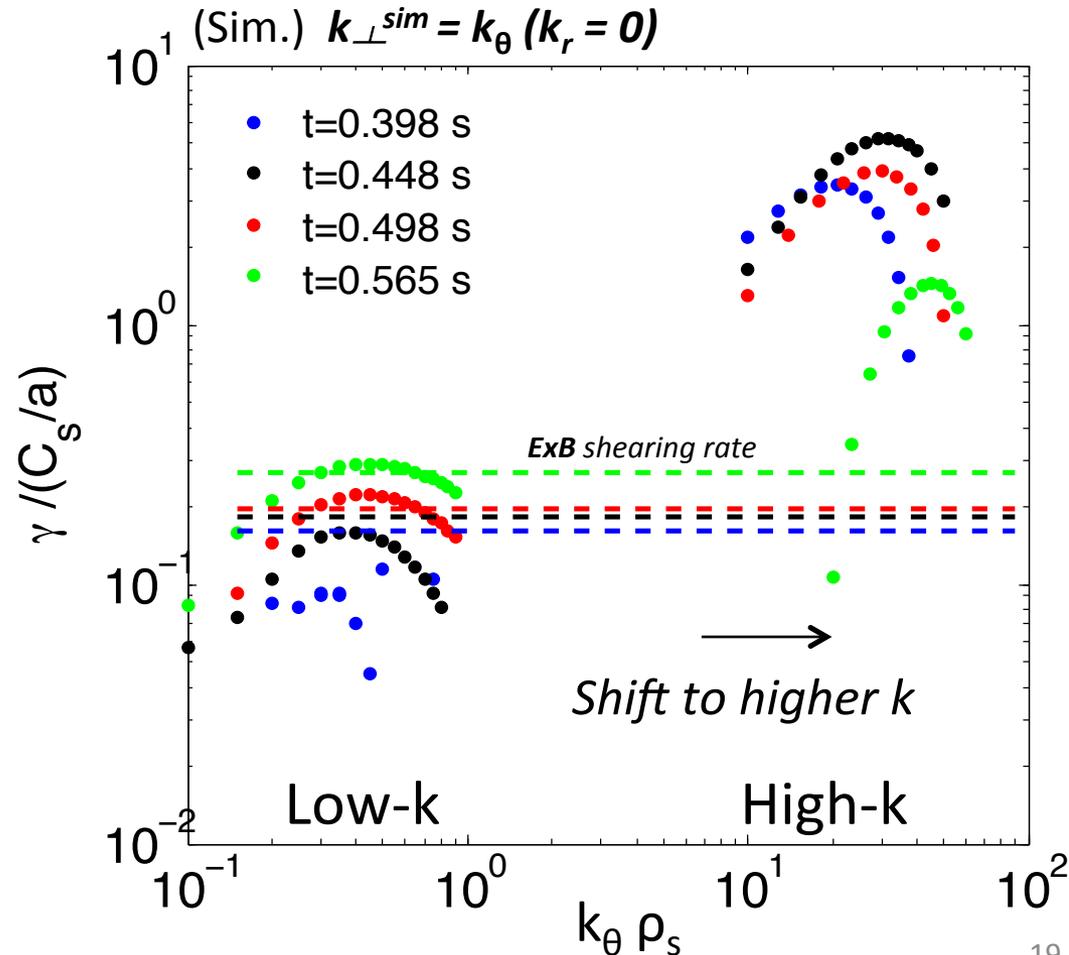
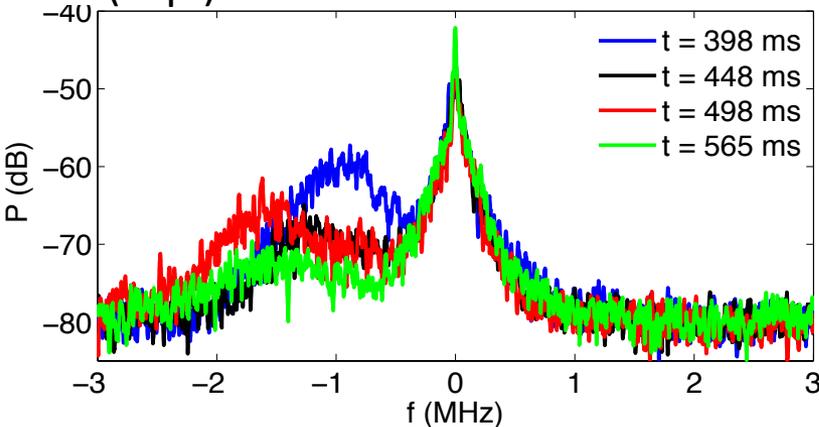
GS2 Linear Simulations Show the Wavenumbers at Maximum Growth Rate Shift to Higher k in Time

- Linear simulations compute most unstable mode ($k_r = 0$). Experimental k is found to be linearly stable.
- Low- k linear growth rates ($k_\theta \rho_s \leq 1$) are comparable to **ExB shearing rate** levels (Waltz, Miller PoP 1999).
- High- k wavenumbers corresponding to maximum linear growth rate shift towards higher- k .
- Observed fluctuations decrease as $k_b \rho_s (\gamma_{\max})$ increases.

(Exp.) channel 1, shot= 141767

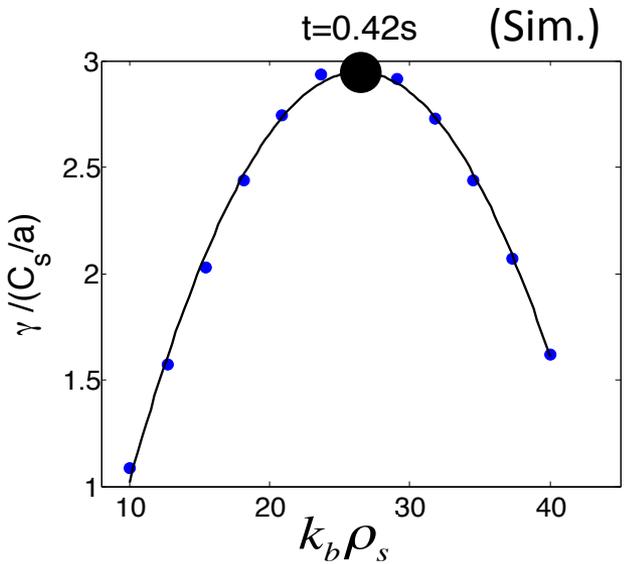


(Exp.)

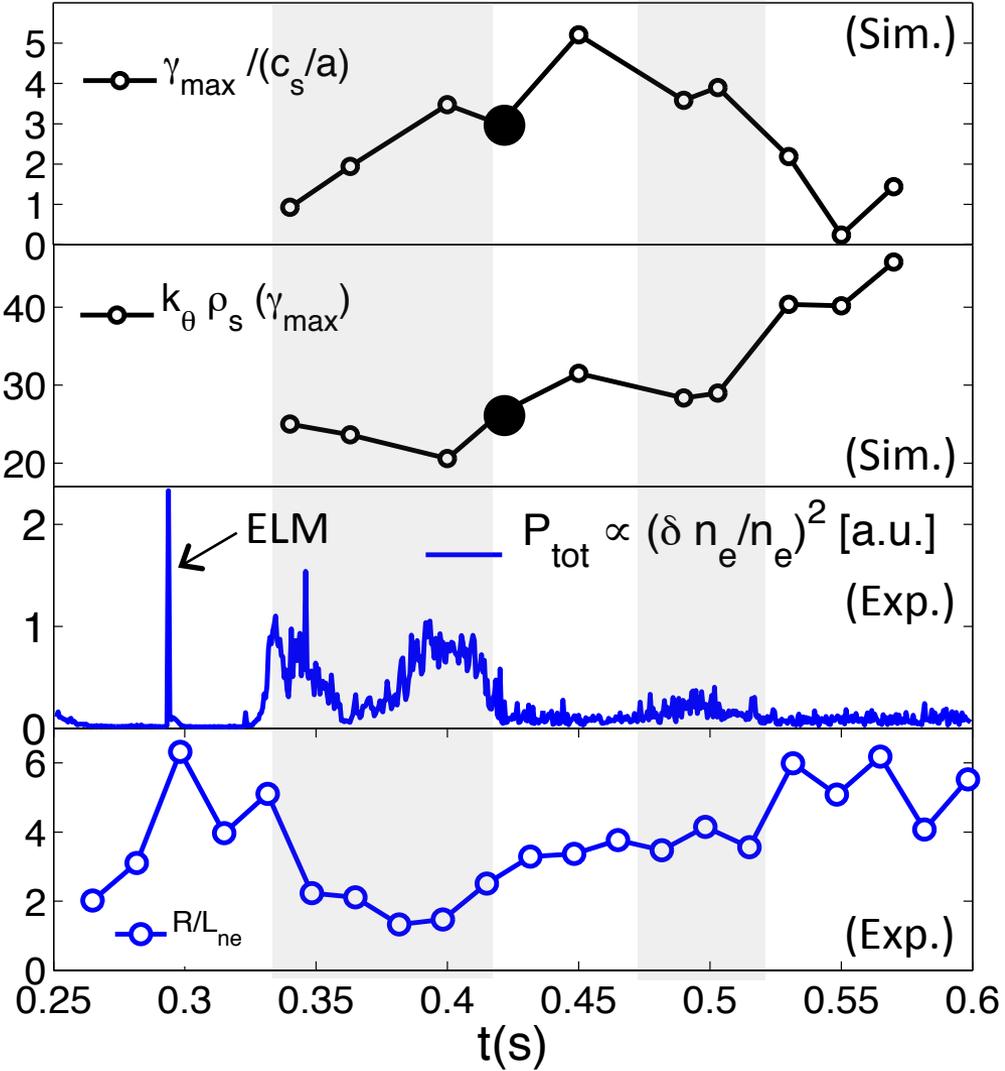


Wavenumber at Maximum Linear Growth Rate Correlates to Electron Density Gradient and Fluctuation Amplitude

- Linear growth rates are calculated at each time: determine $\gamma_{max}/(c_s/a)$ and $k_{\theta}\rho_s(\gamma_{max})$ (black dot).



- $\gamma_{max}/(c_s/a)$ not correlated with and P_{tot} or R/L_{ne} .
- $k_{\theta}\rho_s(\gamma_{max})$ correlates to total scattered power P_{tot} and R/L_{ne} at the scattering location (cf. evolution within time panels).**



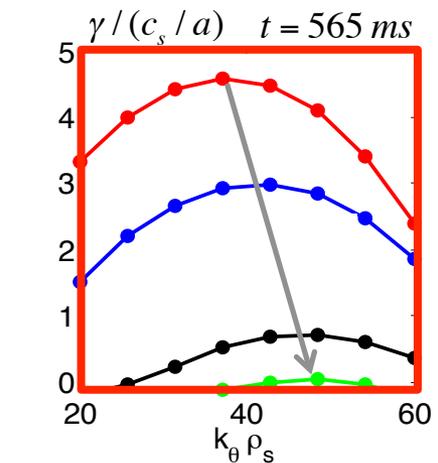
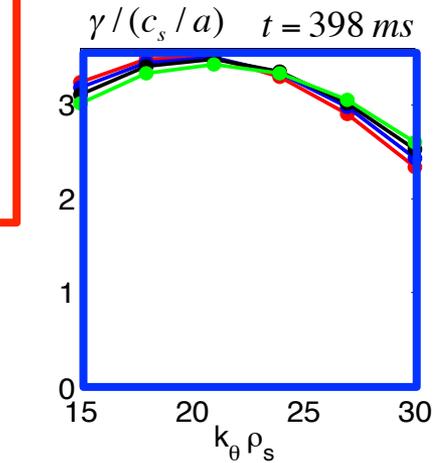
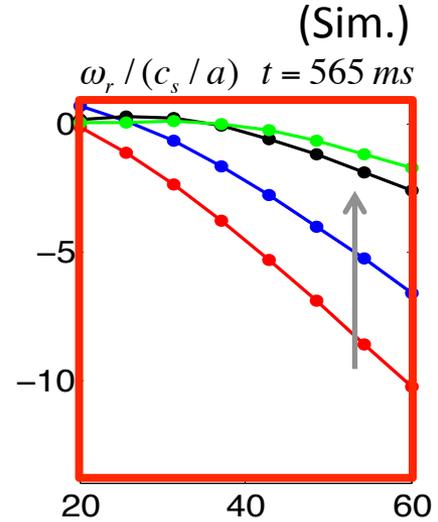
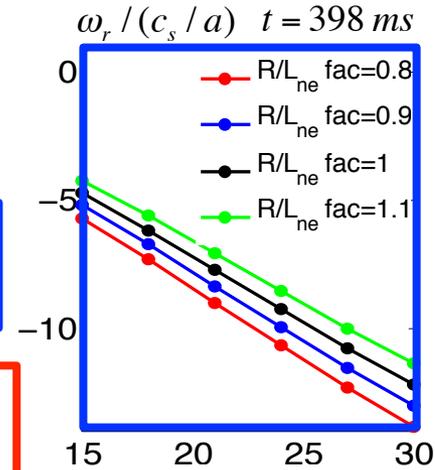
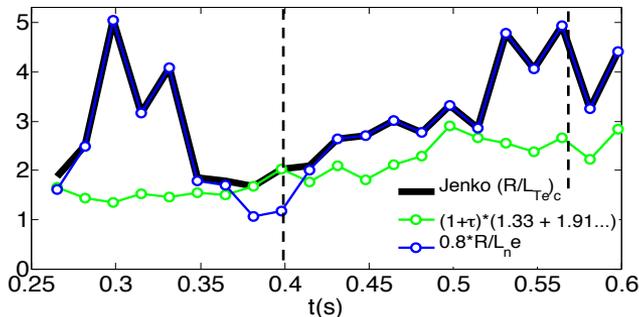
Scan with GS2 is Performed to Confirm Effect of Electron Density Gradient on High-k Turbulence

- Real frequency ω_r and linear growth rate γ are sensitive when $0.8 \cdot R/L_{ne}$ dominates Jenko's critical gradient for ETG.

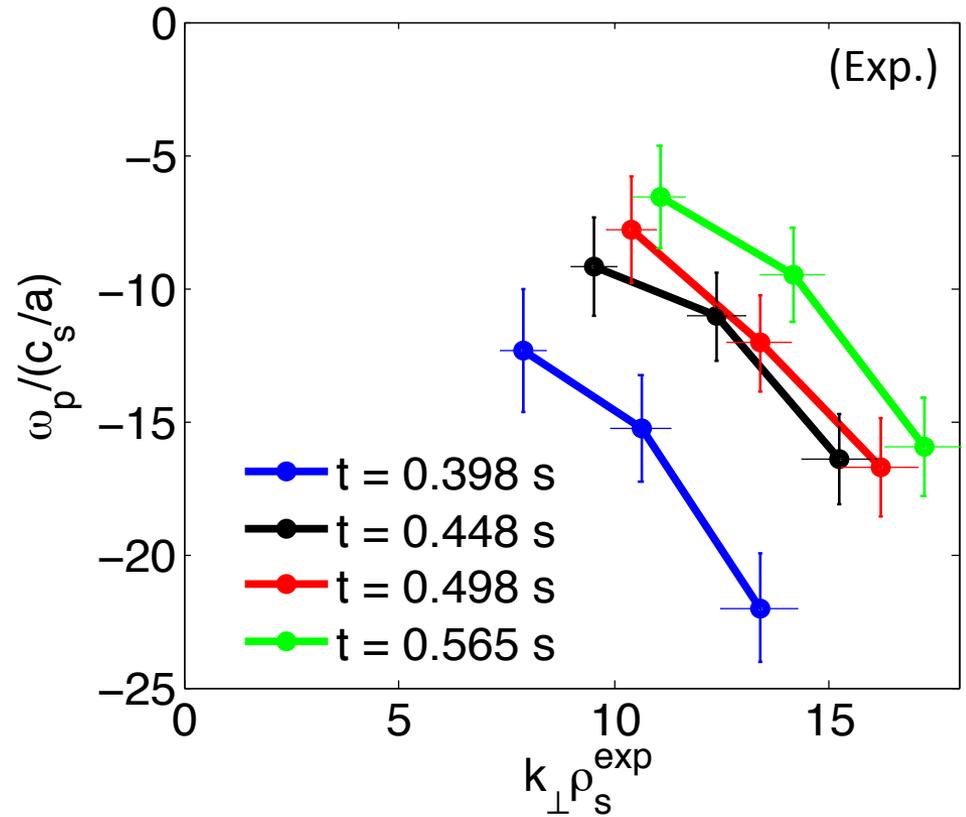
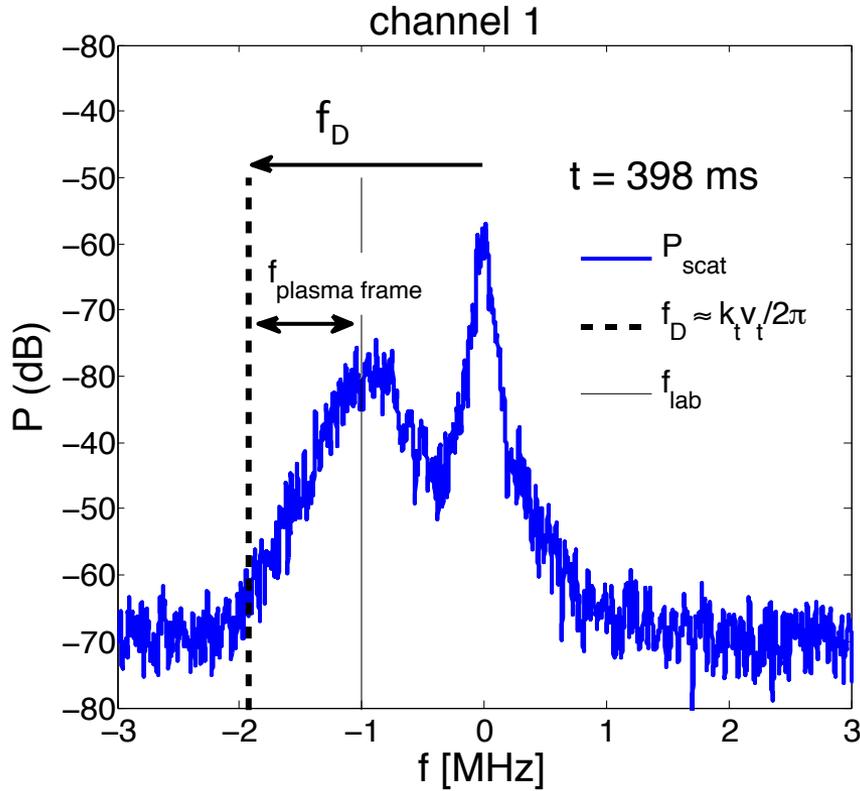
$t = 398$ ms, $0.8 \cdot R/L_{ne}$ term not dominant
 $\rightarrow \omega_r$ and γ insensitive to R/L_{ne} .

At $t = 565$ ms R/L_{ne} dominates, and R/L_{ne} decreases γ and shifts $k_{\theta} \rho_s (\gamma_{max})$ to higher-k
 \rightarrow stabilizing effect.

$|\omega_r|$ decreases with R/L_{ne} .



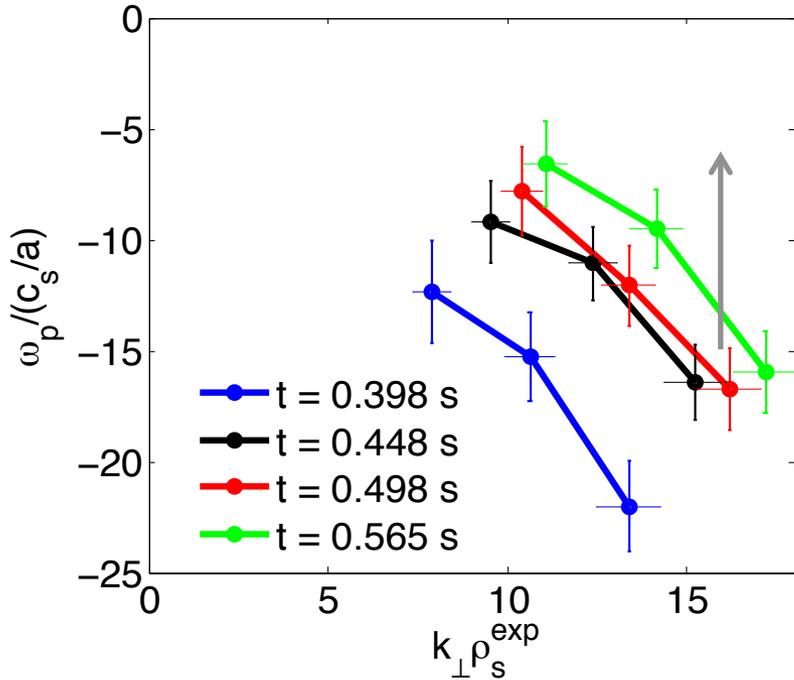
Experimental Real Frequency of High-k Turbulence is Calculated by Subtracting Doppler Shifted Frequency



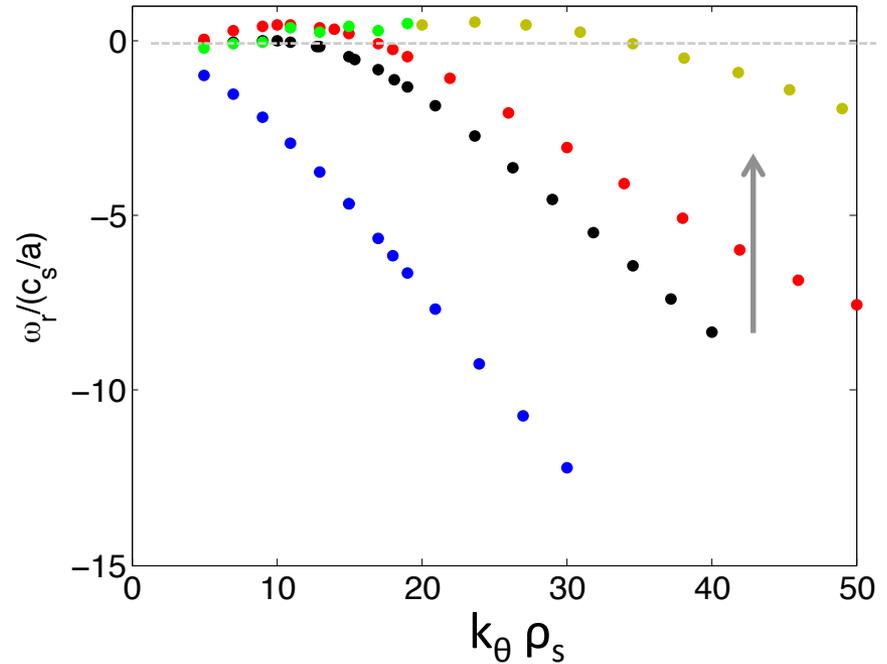
- Lab frame frequencies detected f_{lab} are Doppler shifted from plasma frame frequencies by $f_D = k_t v_t / 2\pi$, and $\omega_p / 2\pi = f_{lab} - f_D$.
- Obtain k_t from ray tracing calculations, v_t from CHERS measurement and TRANSP calculations.

Experimental Real Frequency and GS2 Real Frequency Exhibit Similar Behavior

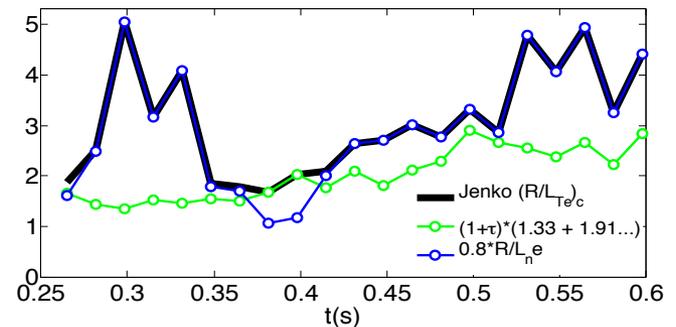
Exp: $k_{\perp}^{exp} = \sqrt{k_r^2 + k_b^2}$, $k_r/k_b \gg 1$



Sim: $k_{\perp}^{sim} = k_{\theta}$ ($k_r = 0$) (Most unstable mode)

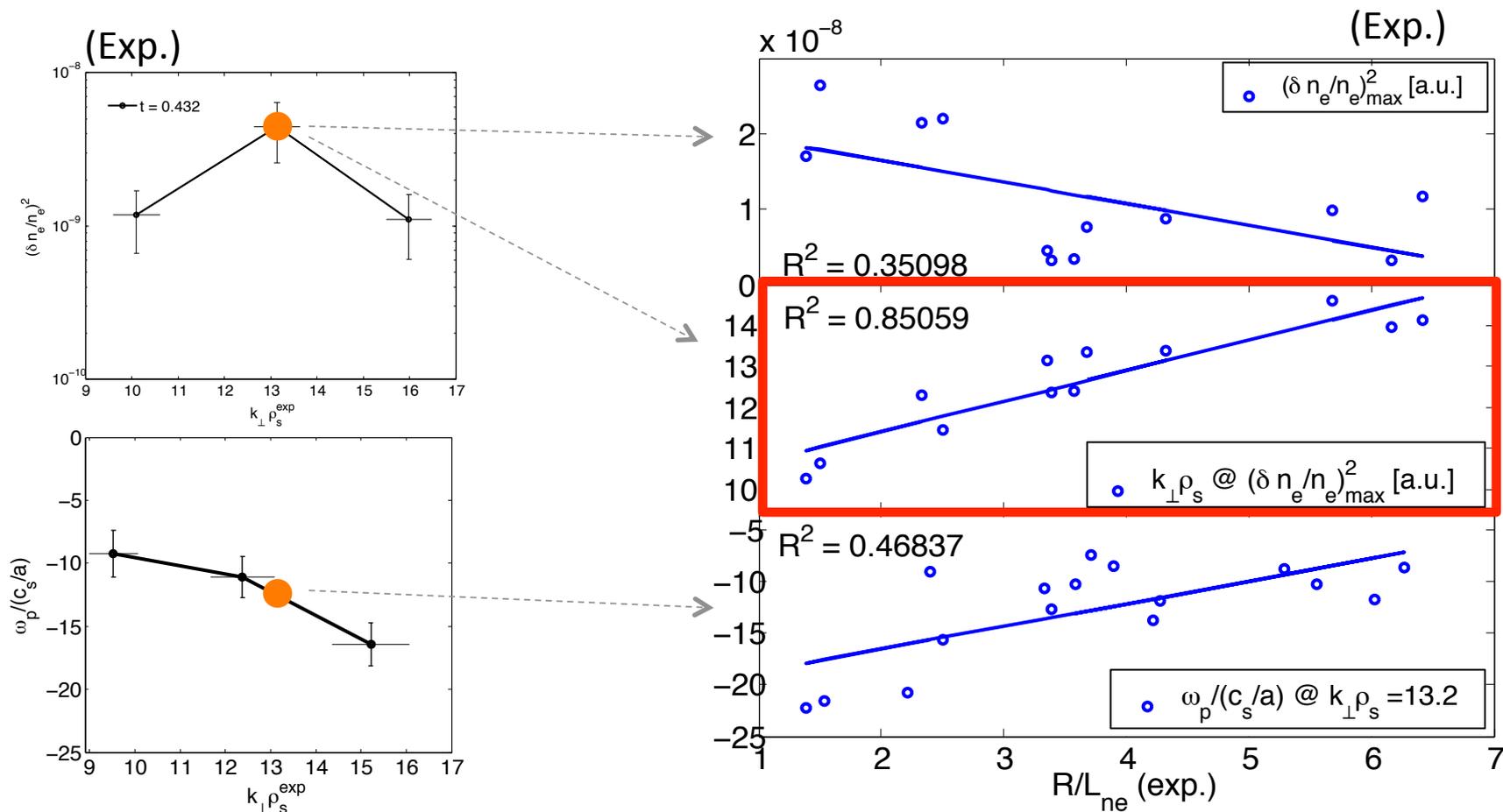


- Experimental k is linearly stable in GS2.
- $|\omega_p^{exp}|$ and $|\omega_r^{sim}|$ decrease in time.
- Only 15% change in c_s/a (normalization).
- Note R/L_{ne} increases in time.



Correlation Between Experimental Wavenumber at Maximum Fluctuation Amplitude and Density Gradient

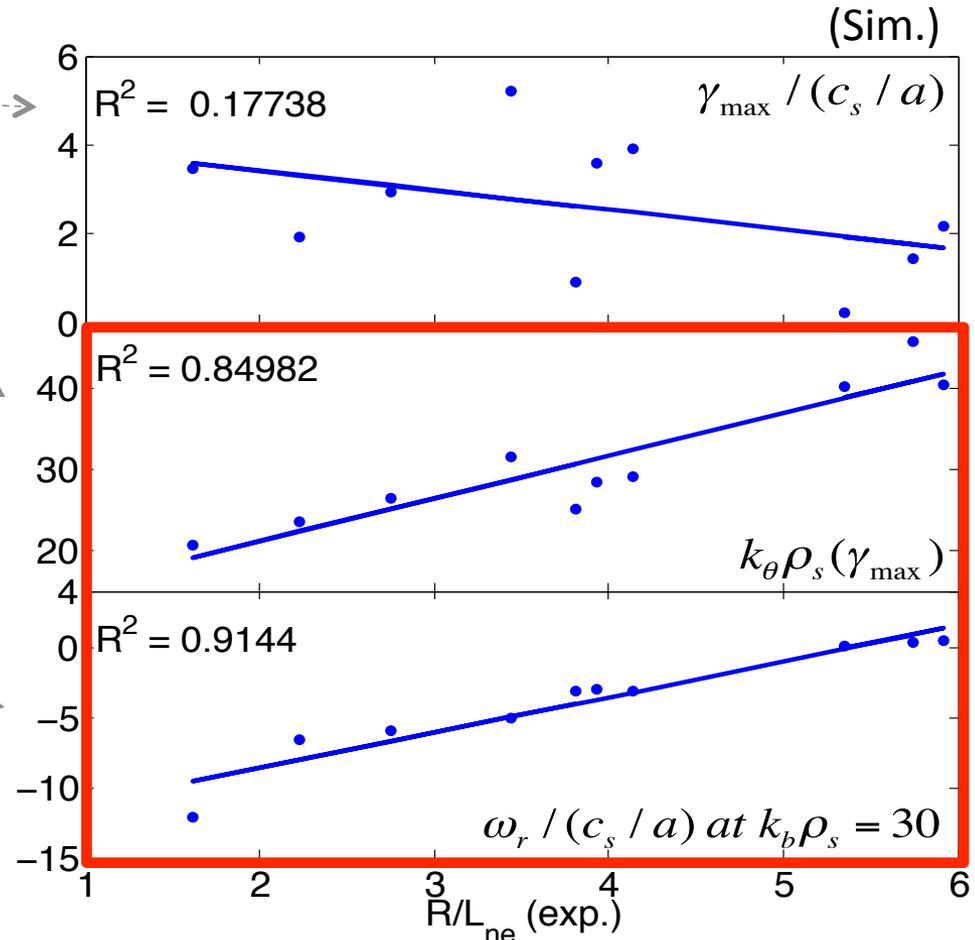
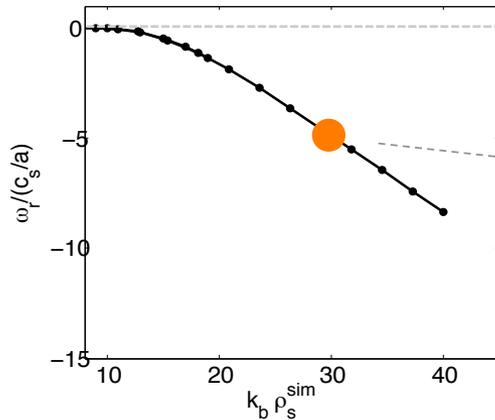
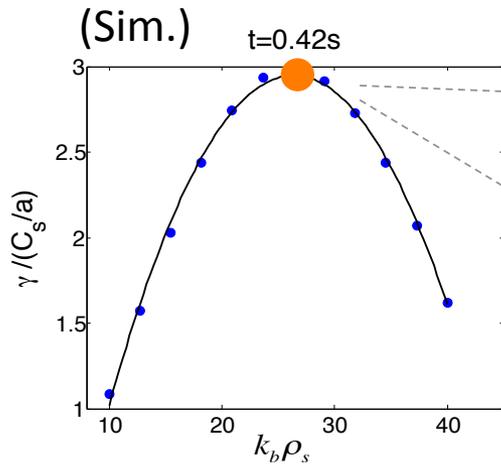
Compute $(\delta n_e/n_e)^2_{\max}$, $k_{\perp}\rho_s$ @ $(\delta n_e/n_e)^2_{\max}$, and $\omega_r/(c_s/a)$ @ $(k_{\perp}\rho_s=13.2)$ and compare to R/L_{ne} .



- Low correlation between $(\delta n_e/n_e)^2_{\max}$, $\omega_r/(c_s/a)$ and experimental R/L_{ne} , but note **similar trend** as found from GS2 linear simulations (slide 20).
- **Correlation between $k_{\perp}\rho_s$ @ $(\delta n_e/n_e)^2_{\max}$ and R/L_{ne} (possible beam refraction effects on k_{\perp}).**

Correlation Between GS2 Wavenumbers at Maximum Growth Rate, Real Frequency and Electron Density Gradient

Compute γ_{max} , $k_b \rho_s(\gamma_{max})$ and $\omega_r/(c_s/a)$ @ ($k_b \rho_s=30$) in time, compare to R/L_{ne} .

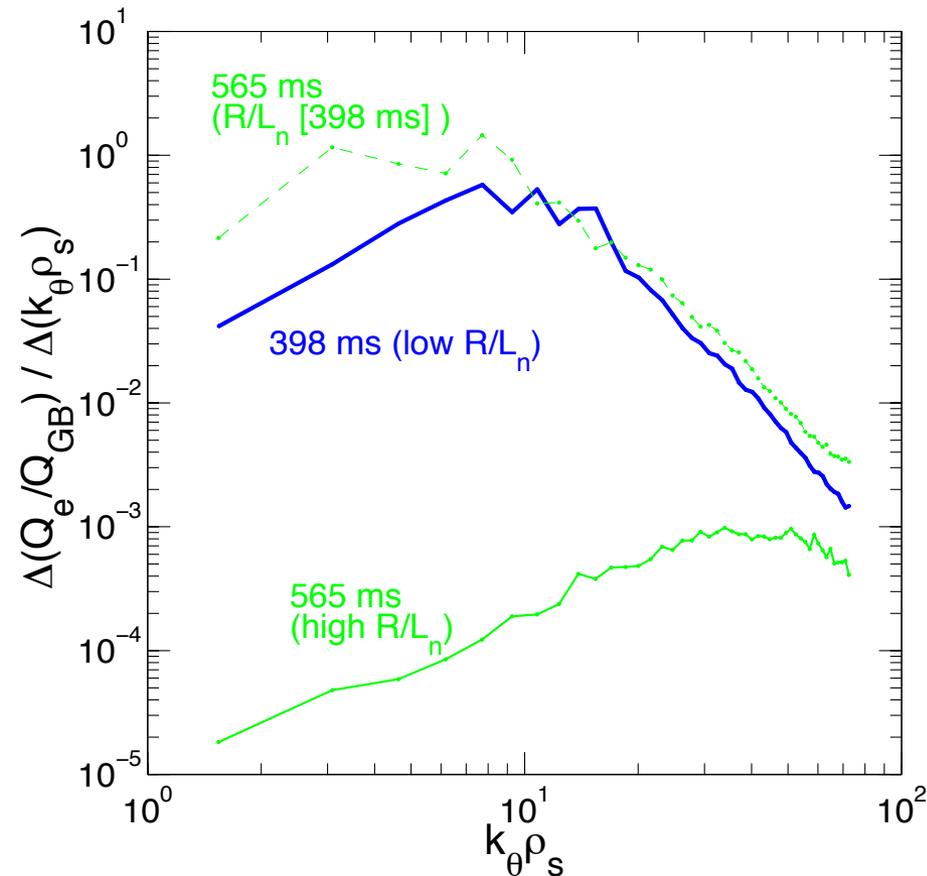
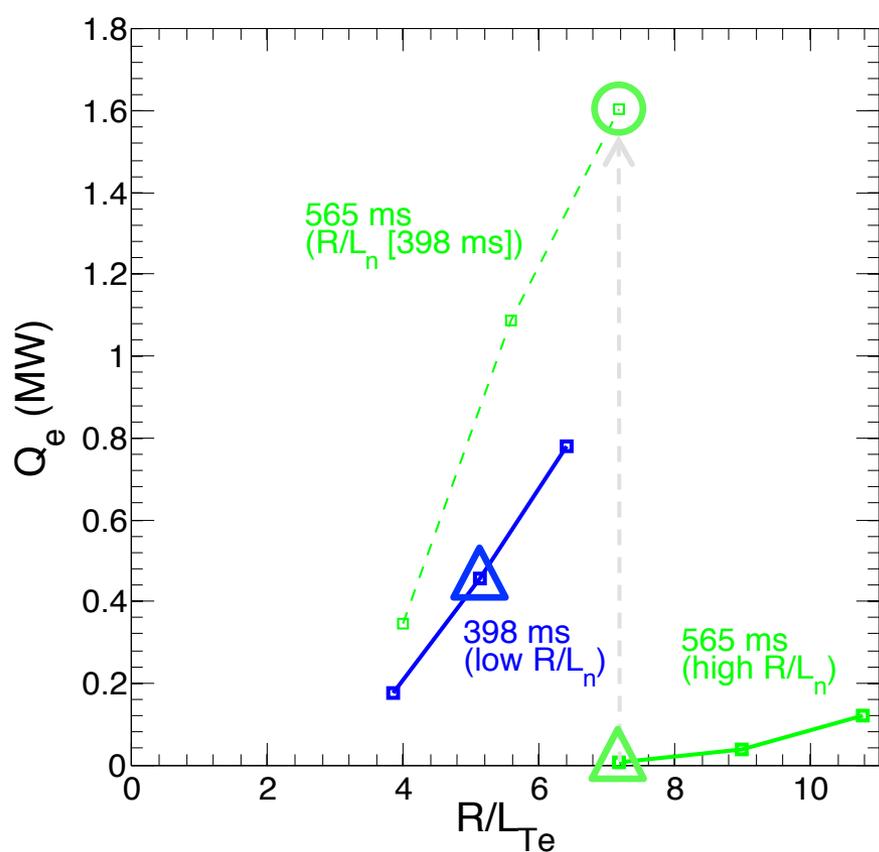


- Low correlation between γ_{max} and experimental R/L_{ne} .

- Correlation between $k_b \rho_s(\gamma_{max})$, $\omega_r/(c_s/a)$ @ ($k_b \rho_s=30$) and R/L_{ne} .

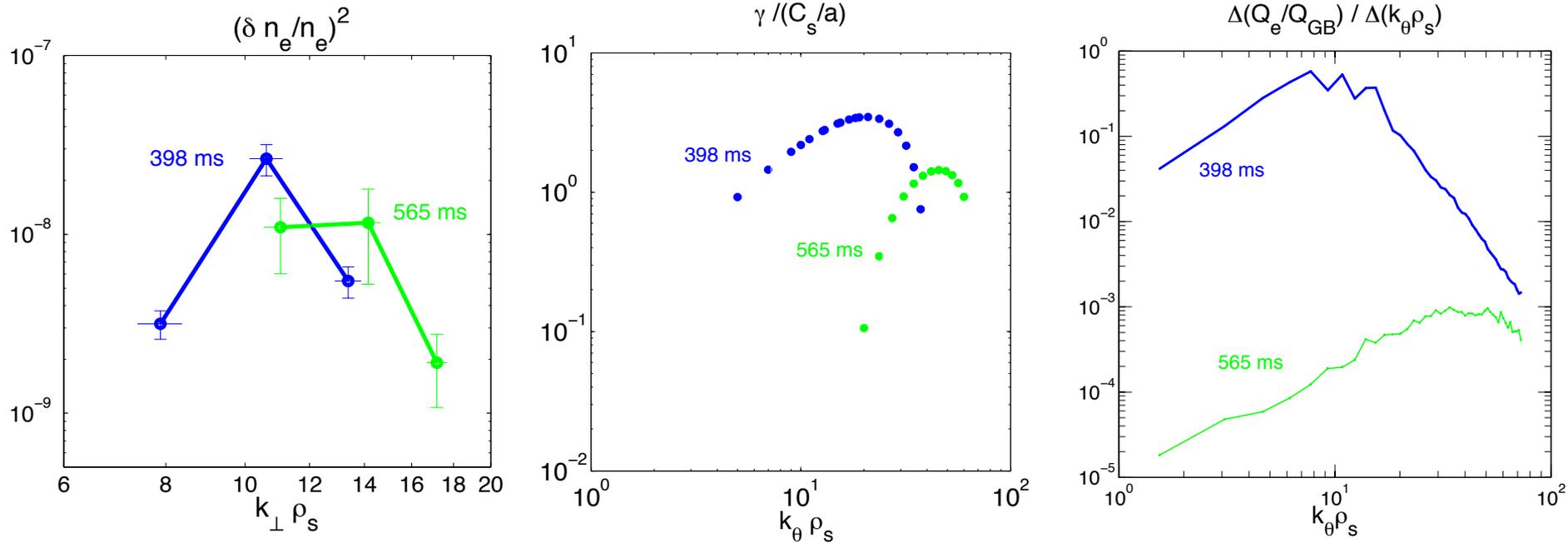
Comparisons with nonlinear GYRO simulations

Nonlinear High-k Gyrokinetic Simulations Show High Impact of Density Gradient on Predicted Electron Heat Flux



- An R/L_{Te} scan is carried out about experimental level (triangles).
- Green circle corresponds to $t = 565$ ms (high R/L_n) but using density gradient from $t = 398$ ms (low R/L_n). Dashed line is R/L_{Te} scan about green circle.
- High density gradient increases *nonlinear* critical gradient.
- High density gradient reduces electron heat flux and stiffness.
- $Q_e(t = 565 \text{ ms}) \sim 0 \rightarrow$ unlikely to account for experimental $Q_e \rightarrow$ study low-k contrib. to Q_e . 27

Experimental Density Fluctuations, Linear Growth Rates and Electron Heat Flux Spectrum Behave Similarly with R/L_n



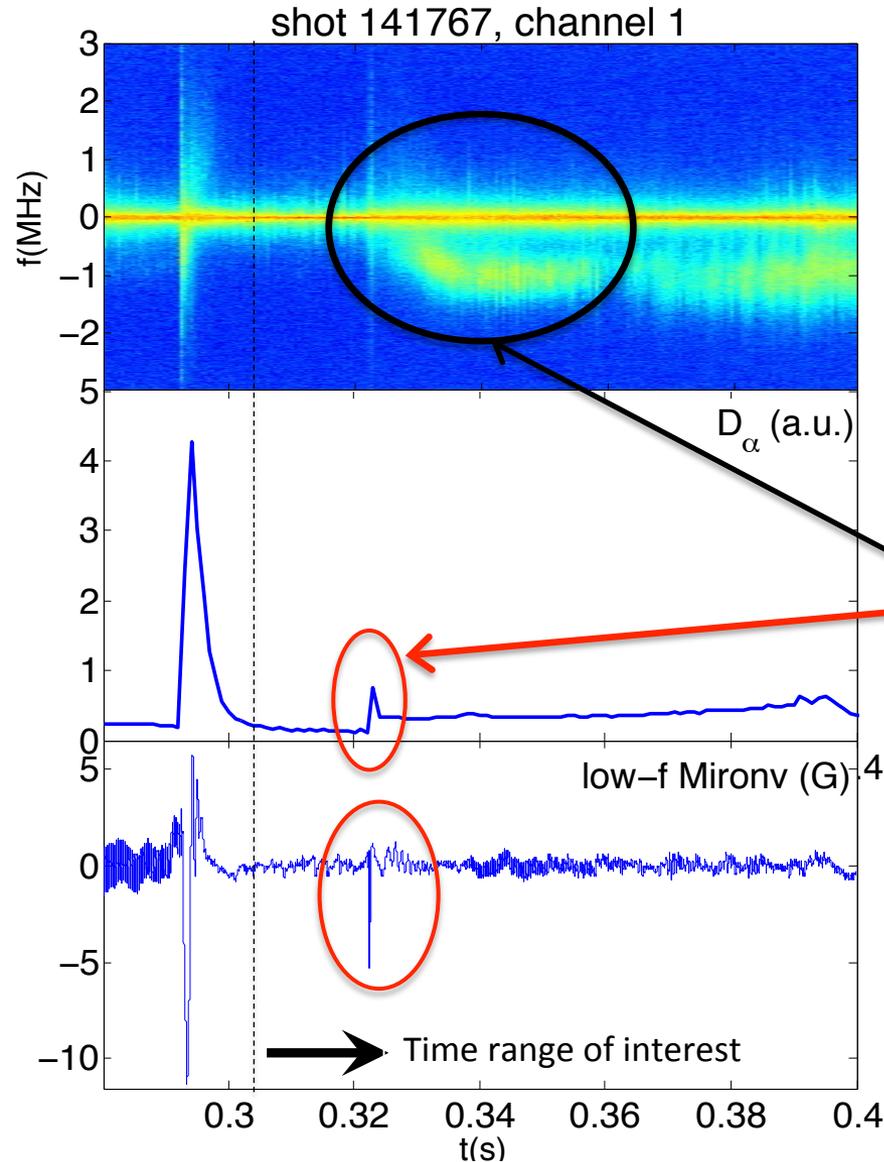
- $t = 398$ ms is low density gradient time, $t = 565$ ms is high density gradient time.
- $(\delta n_e/n_e)^2$, $\gamma/(c_s/a)$ and $\Delta(Q_e/Q_{GB})$ are greatest at $t = 398$ ms.
- The lower- k part of the spectrum of $(\delta n_e/n_e)^2$, $\gamma/(c_s/a)$ and $\Delta(Q_e/Q_{GB})$ is most reduced at high density gradient.

Future Work

- Study low-k and high-k turbulence using nonlinear GYRO in this discharge to determine if they account for experimental electron heat flux levels (TRANSP).
- Develop a synthetic high-k scattering diagnostic for GYRO to perform quantitative comparisons between experiment and gyrokinetic simulations.
- Perform global nonlinear GYRO simulations to understand profile effects in NSTX plasmas ($\rho_* \sim 1/100$).
- Use reduced models (TGLF).

Back-up slides

High-k Fluctuations Start after Small Spike in D_α and Mirnov Signal



- Before $t \sim 290$ ms, MHD activity is high. At ~ 290 ms, an ELM event takes place and MHD activity quiets.
- Between $t \sim 290$ ms and $t \sim 320$ ms, high-k fluctuations are absent and MHD activity is quiet.
- **High-k fluctuations** start at $t \sim 320$ ms, after small ELM event, detected in **D_α and Mirnov signal**.

Typical quantities in these NSTX D plasmas

- Measured fluctuation wavenumbers $k_{\perp} \sim 20 \text{ cm}^{-1} \sim 2000 \text{ m}^{-1}$
- $\omega_{pe} = 2\pi \cdot 90 \text{ GHz} \cdot \sqrt{n_e(10^{20}[\text{m}^{-3}])} \sim 3.6 \cdot 10^{11} \text{ s}^{-1}$
- $f_{pe} \sim 57 \text{ GHz}$
- $\omega_{pD} = \omega_{pe} / \sqrt{m_i/m_e} \sim 5.9 \cdot 10^9 \text{ s}^{-1}$
- $f_{pD} \sim 0.94 \text{ GHz}$
- $\Omega_{ce} = 2\pi \cdot (28 \text{ GHz/Tesla}) \sim 8.8 \cdot 10^{10} \text{ s}^{-1}$
- $f_{ce} \sim 14 \text{ GHz}$
- $\omega_{pe} / \Omega_{ce} \sim 4 \gg 1$ (no ECH)
- $\omega_{cD} = 2\pi \cdot (7.6 \text{ MHz/Tesla}) \sim 2.4 \cdot 10^7 \text{ s}^{-1}$
- $f_{cD} \sim 3.8 \text{ MHz} \gg$ drift wave fluct (low-f)
- $v_{te} = \sqrt{2} \cdot 4.2 \cdot 10^5 [\text{m/s}] \cdot \sqrt{T_e[\text{eV}]} \sim 1.3 \cdot 10^7 \text{ m/s}$
- $c_s = \sqrt{2} \cdot v_{te} / \sqrt{m_i/m_e} \sim 3.03 \cdot 10^5 \text{ m/s}$
- Debye length $\lambda_{de} = v_{te} / (\sqrt{2} \cdot \omega_{pe}) \sim 2.6 \cdot 10^{-5} \text{ m}$
- e- collisionless skin depth $\delta_e = c / \omega_{pe} \sim 8.8 \cdot 10^{-4} \text{ m}$
- Alfvén velocity $v_A = c \cdot (f_{ci} / f_{pi}) / \sqrt{1 + (f_{ci} / f_{pi})^2} \sim c \cdot f_{ci} / f_{pi} \sim 1.21 \cdot 10^6 \text{ m/s}$
- Tor. Rotation vel. v_t (CHERS) $\sim 70 \text{ km/s}$
- $\beta = c_s^2 / v_A^2 \sim 0.06$
- $\rho_e = v_{te} / (\sqrt{2} \cdot \Omega_{ce}) \sim 0.1 \text{ mm}$
- $\rho_s = c_s / (\sqrt{2} \cdot \Omega_{ci}) \sim \rho_e \cdot \sqrt{m_i/m_e} \sim 0.6\text{-}0.7 \text{ cm}$

Spatial Localization and Wavenumber Resolution

- Volume overlap of incident and scattered beams leads to poor spatial localization.
- Theory [cf. Horton *Rev. Mod. Phys.* 1999] predicts $k_{||} \sim 1/qR \ll k_{\perp} \Rightarrow \vec{k} \cdot \vec{B} \approx 0$
- Plasma fluctuations must satisfy:

$k \cdot B \approx 0$	(1) Perpendicular fluctuations.
$k = 2k_i \sin(\theta_s / 2)$	(2) Bragg Condition
- When incident beam forms a small angle with \vec{B} , (1) and (2) become highly dependent on **toroidal curvature** of magnetic field (cf. scattered beams at P_1 and P_2 in the figure). **Oblique propagation** (outside the midplane) of incident beam exploits this phenomenon and enhances **longitudinal localization** of fluctuations [cf. Mazucatto *Phys. Plasmas* 2003].

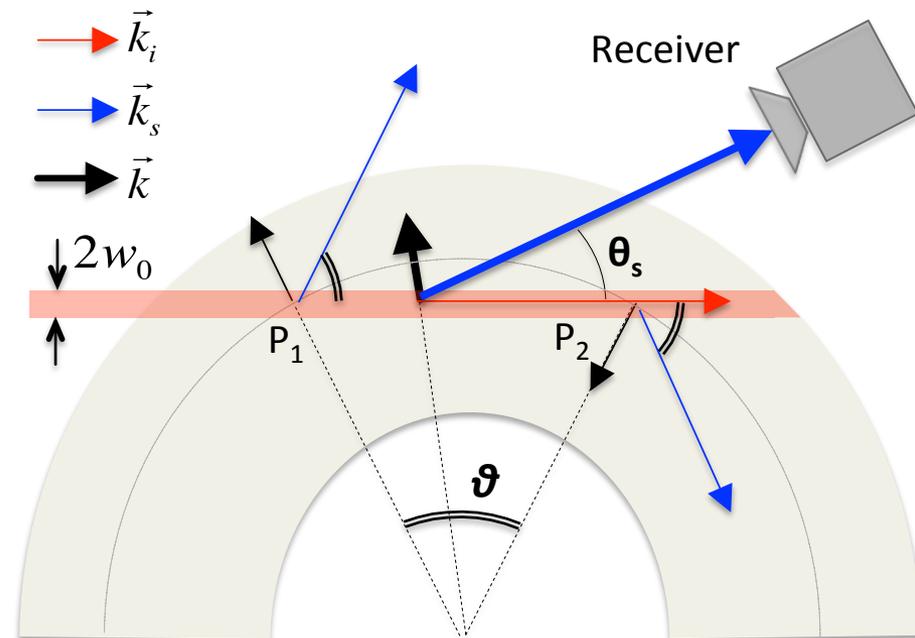
- For midplane propagation, (1) and (2) are only satisfied at P_1 and P_2 and fluctuation wavenumber is purely in the **radial direction**.
- In practice, beam propagation is out of midplane, but oblique angle is small ($\sim 5^\circ$). k is *mostly* radial.

- Gaussian beam width dictates k and R-resolution

$$A(r_{\perp}) = \exp(-r_{\perp}^2 / w_0^2)$$

$$G(k_{\perp}) = \exp(-k_{\perp}^2 / \Delta k^2)$$

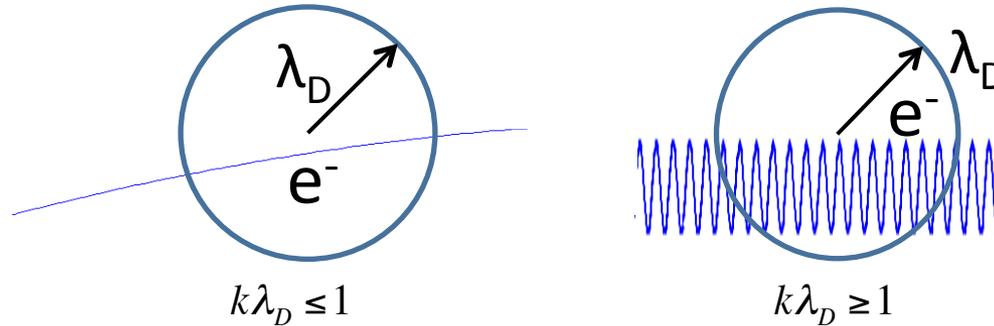
$$\Delta k = 2 / w_0$$



View from top (not to scale)

Collective Thomson Scattering Theory is used to measure ETG-scale turbulence

- Collective/coherent and incoherent scattering



- Typical values (NSTX) $\lambda_D \sim 10^{-5} \text{ m}$, $k \sim k_{\perp} < 10^4 \text{ m}^{-1}$ (*high-k*)
➔ $k\lambda_D < 1$ (**collective scattering**)

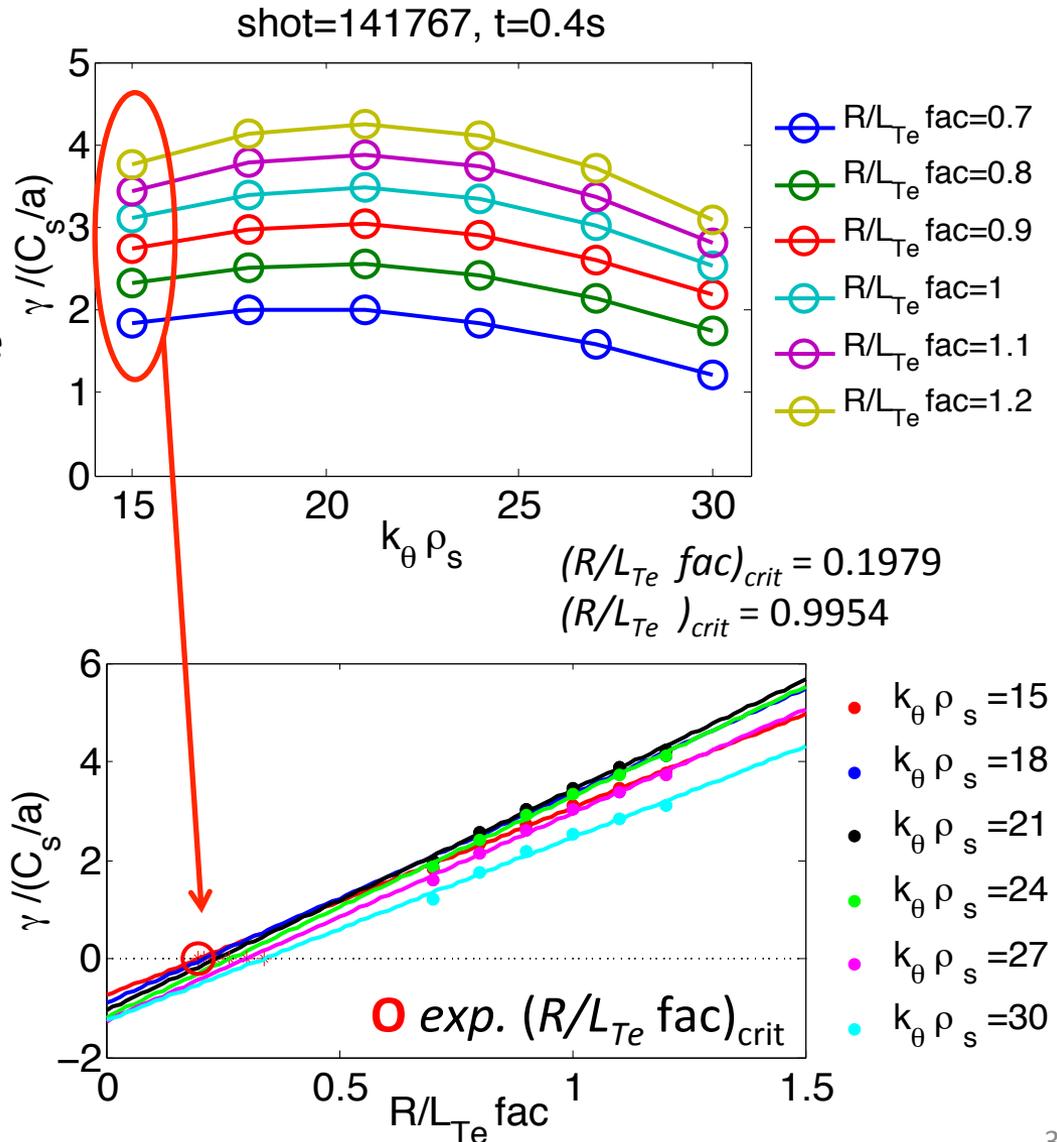
- Scattered power density

$$\frac{d^2 P}{d\Omega d\nu} = P_i r_e^2 L_z |\Pi \cdot \hat{e}|^2 \frac{|\tilde{n}_e(k, \omega)|^2}{VT}$$

r_e classical electron radius
 V, L_z volume and length of scattering volume
 Π polarization tensor
 \hat{e} direction of incident electric field
 T observation time

A Scan on R/L_{Te} is Performed to Compute a Critical Gradient with GS2 Linear Runs

- R/L_{Te} is varied keeping all other quantities constant. The factor is called $(R/L_{Te} \text{ fac})$.
- High-k linear growth rates saturate with decreasing (R/L_e) .
- $(R/L_{Te})_{crit}$ is found to be the minimum R/L_{Te} to satisfy $\gamma = 0$.



Nonlinear GYRO Simulations details

Used numerical parameters based on previous convergence studies [Guttenfelder & Candy, PoP (2011), Ren et al., PoP (2012)]

- 3 kinetic species, D, C, e ($Z_{\text{eff}} \sim 1.85-1.95$)
- Electromagnetic: $A_{||} + B_{||}$, $\beta_e \sim 0.3\%$.
- Collisions ($\nu_{ei} \sim 1 c_s/a$).
- ExB shear ($\gamma_E \sim 0.13-0.16 c_s/a$), used fixed boundary conditions with $\Delta^b = 1 \rho_s$ buffer widths.
- Resolution parameters
 - $L_x \times L_y = 6 \times 4 \rho_s$ ($360 \times 240 \rho_e$).
 - $n_x \times n_y = 192 \times 48$.
 - $k_\theta \rho_s$ [min, max] = [1.5, 73]
 - $k_r \rho_s$ [min, max] = [1.0, 50]
 - $[n_{||}, n_\lambda, n_e] = [14, 12, 12]$