Data-Driven Modeling of the Magnetic Profile and Rotation Profile for Advanced Tokamak Scenarios in DIII-D

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Abstract

First-principle predictive models based on flux averaged transport equations often yield complex expressions not suitable for real-time control. As an alternative to first-principle modeling, data-driven modeling techniques involving system identification have the potential to obtain low-complexity, dynamic models without the need for ad hoc assumptions. This work focuses on the evolution of the toroidal rotation and safety factor profiles in response to magnetic, heating and current-drive systems. Experiments are conducted during the current flattop, in which the actuators are modulated in open-loop to obtain data for the model identification. The plasma profiles are discretized in the spatial coordinate by Galerkin projection. Then a linear model is generated by the prediction error method to relate the rotation and safety factor profiles to the actuators according to a least squares fit.

Objectives

- Simple linear models based on system identification (data-driven modelling) are desired for control implementations
- Models of magnetic profile and toroidal rotation profile in response to certain inputs; the neutral beams, the gyrotron power, and the plasma current.
- A model for coupled evolution of the magnetic profile and rotation profile

Why magnetic profile control?

- Achieving sustained tokamak operation.
- Non-inductive sources of current are required for steady state operation.
- Setting up a suitable toroidal current profile can lead to self-generated, non inductive current (bootstrap current).
- Controlling the current profile will therefore be important to achieving steady-state reactor operation.

Why rotation profile control?

- Plasma performance while operating in high pressure conditions is limited by transport phenomena and Magnetohydrodynamic (MHD) instabilities.
- Optimizing some of the plasma profiles such as the toroidal rotation profile can improve plasma performance.
- For example, increasing bulk fluid rotation around the tokamak produces a velocity gradient. The velocity results in a sheared plasma flow reducing turbulence and improving heat confinement.

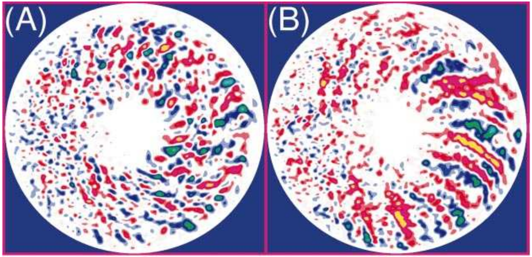


Figure: Simulation of turbulence with (A) and without (B) sheared plasma flow.

[1] M. De Bock, *Understanding and controlling plasma rotation in tokamaks*, Doctoral Thesis, Technische Universiteit Eindhoven, 2007

Plasma profile

- First-principle models based on transport equations yield complex expressions not suitable for control.
- Use linear models based on system identification instead.
- By taking the surface average over the magnetic flux surfaces, axisymmetric plasma transport equations can be represented by one dimensional nonlinear parabolic PDEs whose variables are dependent on both time and normalized radius $\hat{\rho}$.
- Around certain trajectories the PDEs can be linearized as

$$\frac{\partial x(\hat{\rho}, t)}{\partial t} = \mathcal{A}(\hat{\rho})x(\hat{\rho}, t) + \mathcal{B}(\hat{\rho})u(t) + \mathcal{K}(\hat{\rho}, t)e(\hat{\rho}, t), \tag{1}$$

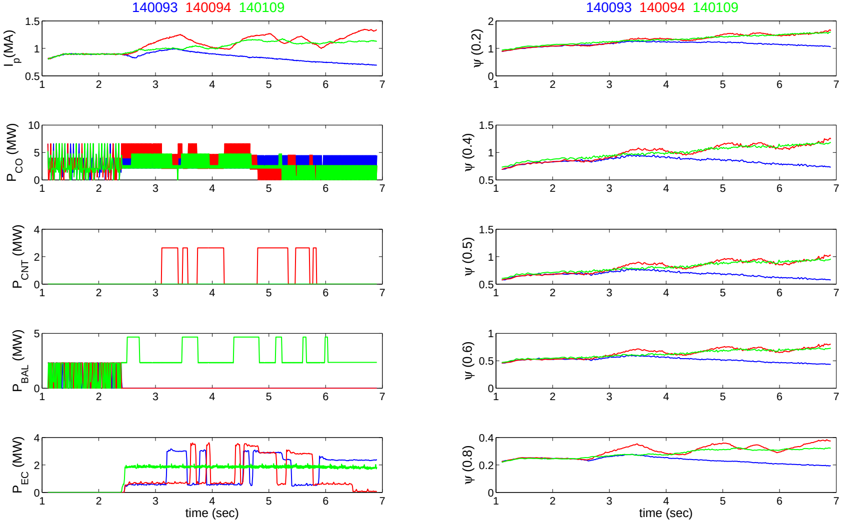
where $x(\hat{\rho}, t)$ represents a collection of physical variables such as the poloidal magnetic flux profile $\psi(\hat{\rho}, t)$ or the rotation profile $V_\phi(\hat{\rho}, t)$.

$\mathcal{A}(\hat{\rho}), \mathcal{B}(\hat{\rho})$, and $\mathcal{K}(\hat{\rho})$ are infinite dimensional operators.

Model identification

- To perform model Identification data is collected during high confinement (H-mode).
- The reference plasma state: Plasma current $I_p = 0.9$ MA, 65% boot strap current, (H-mode): $3.5 < \beta_N < 3.9$ (β_N: measure of pressure).
- Actuators modulated in open loop according to predefined waveforms around the values for the reference discharge.
- During each discharge one actuator is modulated while the other actuators are held constant and equal to the values for the reference discharge.

Actuators modulated to quantify plasma response

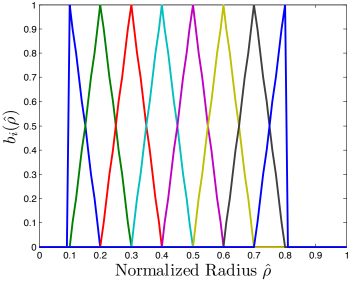


Discretization by Galerkin projection

- The infinite dimensional system (1) can be discretized by projecting the distributed variable $x(\hat{\rho}, t)$ onto a basis function space.
- The Galerkin projection reads

$$x(\hat{\rho}, t) \approx \sum_{i=1}^{N} G_i(t) b_i(\hat{\rho}), \tag{2}$$

- where $b_i(\hat{\rho})$ are the basis functions. Piece-wise linear functions with $i = 1, 2, ...N$.



Discretization by Galerkin projection

- The expansion coefficients, $G_i(t)$, called the Galerkin coefficients represent the model points.
- To determine the Galerkin coefficients, we multiply both sides of the expansion equation (2) with any basis function $b_j(\hat{\rho})$, $j = 1, 2, ...N$ and integrate over the spacial coordinate to obtain,

$$\int_0^1 x(\hat{\rho}, t) b_j(\hat{\rho}) d\hat{\rho} = \int_0^1 \Big[\sum_{i=1}^{N} G_i(t) b_i(\hat{\rho}) \Big] b_j(\hat{\rho}) d\hat{\rho}, \tag{3}$$

- If the basis functions are orthonormal, i.e. $\int_0^1 b_i(\hat{\rho}) b_j(\hat{\rho}) d\hat{\rho} = \delta_{ij}$, then the coefficients G_i can be computed explicitly. Otherwise the coefficients are obtained by solving a matrix equation.

Model identification

- After discretization we have a lumped parameter model, which reads:

$$\frac{dX(t)}{dt} = AX(t) + Bu(t) + Ke(t), \tag{4}$$

  where $X(t)$ is the vector of Galerkin coefficients.

Control Actuators

- 1. Co-current NBI
- 2. Counter-current NBI
- 3. Balanced NBI
- 4. Total ECRH and ECCD power from all the gyrotrons
- 5. Loop voltage

Then the model is fit to experimental data according to a least squares fit.

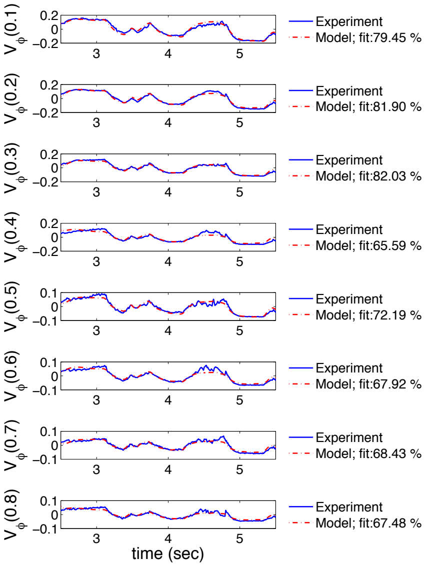
Model identification

- System identification of the toroidal rotation profile carried out for 8 Galerkin coefficients computed at $\hat{\rho} = 0.1, 0.2, ...0.8$.
- Identification shots were organized into various groups; 1 group for little modulation and 1 group for each set of shots with a particular actuator modulated.
- The identification was then carried out in a step-wise procedure.
  - 1. Initial estimation of A was obtained using the group with little modulation
  - 2. Holding the slowest eigenmodes constant, the columns of the B matrix were estimated in subsequent steps, one column at at time.
  - 3. Each column estimated with the actuator corresponding to that column.
- The estimation process is carried out by fitting A and B to the data according to a least squares fit by minimizing the norm $h(Q) = \frac{1}{2} tr(Q)$

$$Q(\theta) = \frac{1}{N} \sum_{k=1}^{N} \epsilon(k, \theta) \epsilon^T(k, \theta) \tag{5}$$

where ϵ is the prediction error $(X(k)|_{\text{measured}} - X(k)|_{\text{model}})$, k is the sample, and θ are the parameters to be determined.

Fitted model describes rotation response accurately



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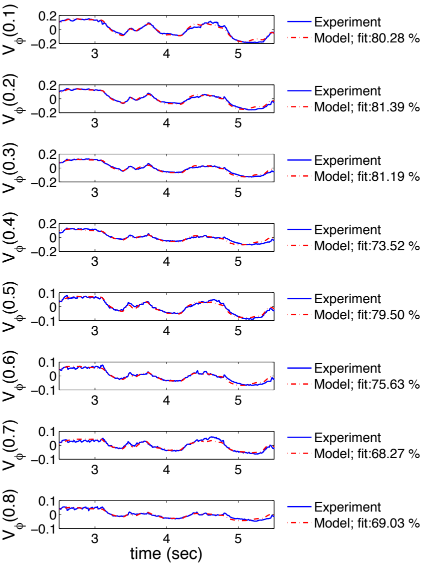

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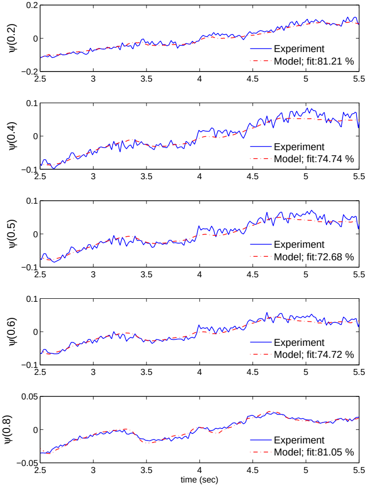
Fitted model describes ψ response accurately

- The same identification process is carried out for the ψ profile (the poloidal flux relative to the boundary value)



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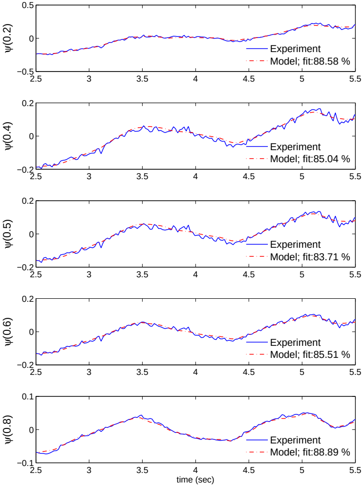

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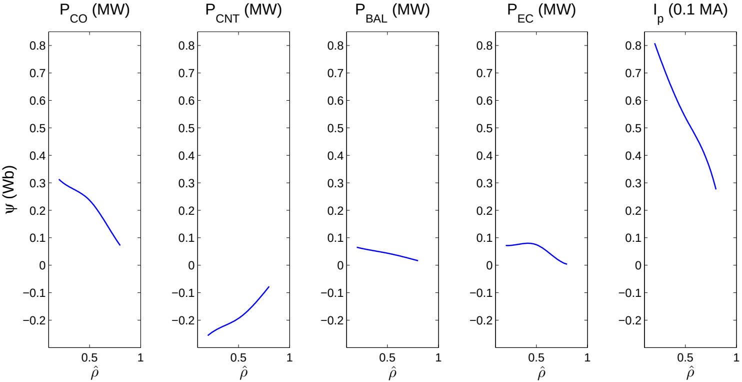
Steady state gains: magnetic profile ψ

- The estimated steady state gain matrix $K_{sg} = -A^{-1}B$



- Steady state response of flux profile to unit change of the various inputs.
- The plasma current and co-current NBI are the most capable in adjusting the profile.
- The opposing affects of co-current and counter-current NBI are expected.

Two time scale coupled model

- The linearized coupled model between the magnetic profile ψ and a kinetic profile V_ϕ can be written as

$$\frac{\partial \psi}{\partial t} = A_{11}\psi(t) + A_{12}V_\phi(t) + B_1 u(t) \quad (6)$$

$$\epsilon\frac{\partial V_\phi}{\partial t} = A_{21}\psi(t) + A_{22}V_\phi(t) + B_2 u(t) \quad (7)$$

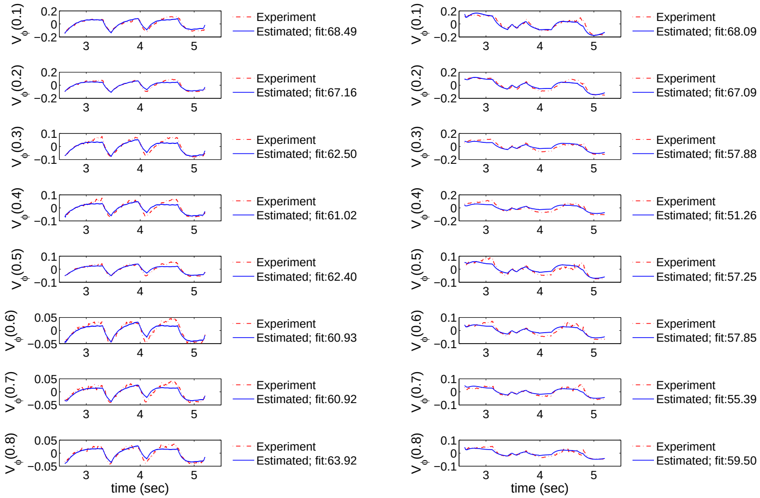
- ϵ is the ratio between energy confinement time and the characteristic resistive diffusion time ($\epsilon \ll 1$)
- In the limit $\epsilon \to 0$ the model can be decomposed into slow and fast models of the form

$$\frac{\partial \psi}{\partial t} = A_{\text{slow}}\,\psi + B_{\text{slow}}\,u_{\text{slow}} \quad \text{and} \quad V_\phi|_{\text{slow}} = C_{\text{slow}}\,V_\phi|_{\text{slow}} + D_{\text{slow}}\,u_{\text{slow}} \quad (8)$$

$$\frac{\partial V_\phi|_{\text{fast}}}{\partial t} = A_{\text{fast}}\,V_\phi|_{\text{fast}} + B_{\text{fast}}\,u_{\text{fast}} \quad (9)$$

Two time scale model: $V_\phi|_{\text{slow}} + V_\phi|_{\text{fast}}$, fitted model

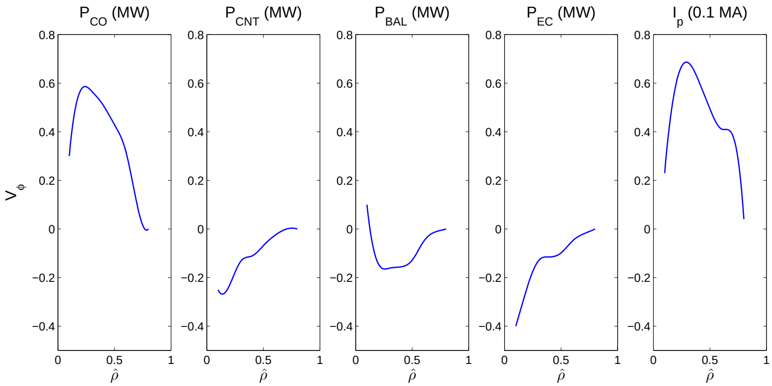
- Using the previous model determined for ψ as A_{slow}, B_{slow}



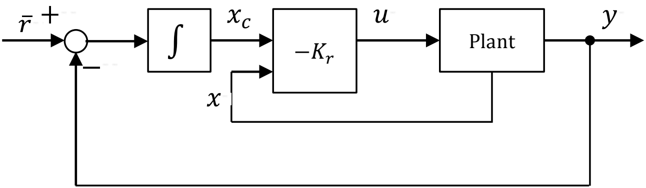
(g) Shot 140076 (h) Shot 140092

Static gains: rotation profile (V_ϕ)

- The estimated steady state gain matrix $K_{sg} = -C_{\text{slow}} A_{\text{slow}}^{-1} B_{\text{slow}} + D_{\text{slow}}$



Optimal state feedback controller with integral action (Proportional + Integral control)



- To design the controller above, the plant is augmented with the integrator states $x_c = \int \bar{r} - y$:

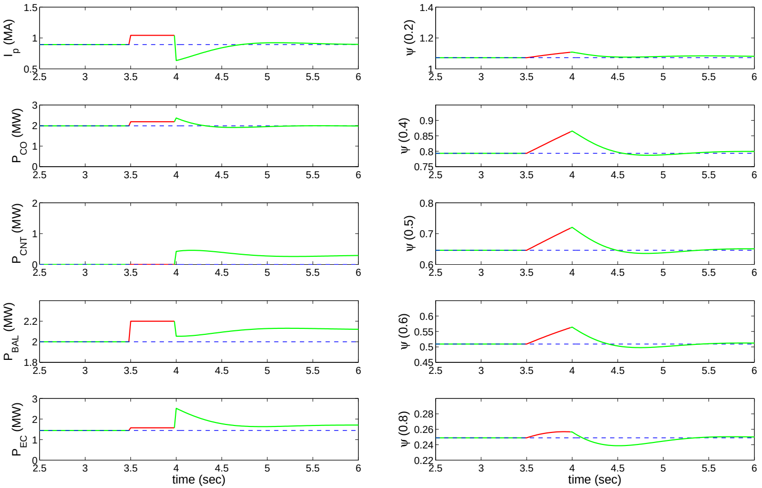
$$\dot{\bar{x}} = \begin{bmatrix} -C & 0 \\ A & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} -D \\ B \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{r} \tag{10}$$

- Then using the augmented plant a simple state feedback control law of the form $u(t) = -K_r \bar{x}(t)$ is determined to minimize the cost functional

$$J = E\left\{ \lim_{T\to\infty} \frac{1}{T} \int_0^T \left[\bar{x}^T \bar{Q} \bar{x} + u^T \bar{R} u \right] dt \right\} \tag{11}$$

Control simulation: successful disturbance rejection

- Input disturbance at $t = 3.5$ s. The green period indicates the feedback is turned on and the red period indicates the feedback is turned off.
- The dash blue line is the target profile.

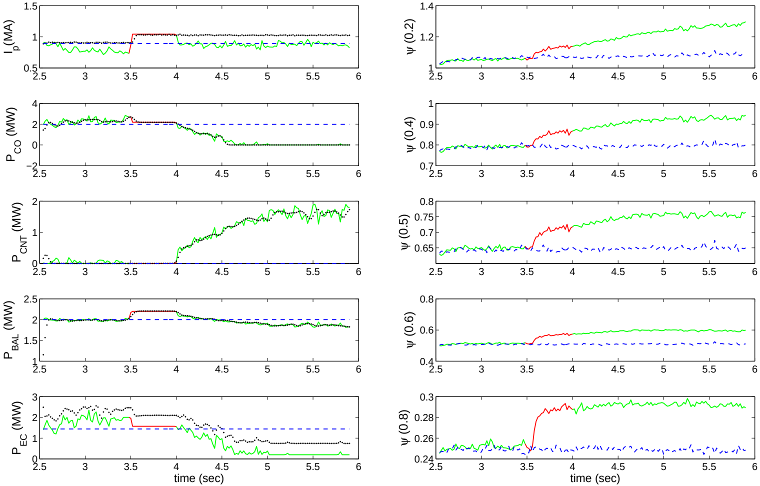


Control simulation result

- Input disturbances are applied to all the actuators except the counter-current beam with a magnitude of about $10 - 15$ % of their respective feedforward values.
- The feedback is turned off for 0.5 s to allow time for the disturbance to perturb the system.
- At $t = 4.5$ s the feedback is turned back on to regulate the states back to their reference trajectories.

Experimental result

- Input disturbance at $t = 3.5$ s. The green period indicates the feedback is turned on and the red period indicates the feedback is turned off.
- (left) The black dots represent the delivered inputs, the green-red line represent the requested inputs. (Blue) target profile.



Experimental result

- The experimental procedure is identical to the simulation.
- From 2.5 $-$ 3.5 s the control performs well, holding the ψ-profile tight with the desired target.
- At 3.5 s an input disturbance is introduced and allowed to perturb the system for 0.5 s without feedback control (red portion).
- At 3.5 s the feedback is turned back on.
- Unfortunately, a mistake with the DIII-D settings disallowed the plasma current from going down.
- The neutral beams and gyrotrons are adjusted in the correct directions as predicted by the model, but the failure in I_p actuation results in poor control.
- A second experimental attempt is scheduled for December 2011.

Future Work

- Incorporate coupled evolution of toroidal rotation profile with the poloidal magnetic flux profile and the temperature profile.
- Use identified models in conjunction with Magnetohydrodynamic (MHD) stability models in development by Yongkyoon In (Fartech) to further study the stabilizing affects of the profile optimization on MHD instabilities.