Multivariable Multi-Model-based Magnetic Control System for the Current Ramp-up Phase in the National Spherical Torus Experiment (NSTX)

Wenyu Shi, Justin Barton, Majed Alsarheed and Eugenio Schuster

Department of Mechanical Engineering and Mechanics, Lehigh University

50th IEEE Conference on Decision and Control and European Control Conference

This work was supported by the NSF CAREER award program (ECCS-0645086), and US DOE DE-FG03-99ER54522.

December 13, 2011

Outline

- Introduction
- Description of NSTX
  - Axisymmetric Plasma Response Model
  - Uncertain Model for Current Ramp-up phase
- Control System Design
  - Control System Structure
  - Plasma Current & Position Controller
  - Plasma Shape and X-point Location Controller
    - Singular Value Decomposition (SVD)
    - Design of μ Synthesis Controller
- Simulation Results
- Conclusion

Introduction

- Magnetic control in tokamaks refers to controlling the magnetic fields to maintain or change the plasma position, shape and current.
- The tokamak is a pulsed machine. In each pulse the plasma is created, its current is ramped up to a constant reference value, and eventually the current is ramped down and the plasma is terminated.
- The magnetohydrodynamic (MHD) equilibrium continually evolves during the ramp-up phase of the discharge. As a consequence, the plasma response model obtained via linearization around the changing MHD equilibrium evolves as well.
- The plasma shape requirements in a practical, highly-efficient tokamak are very stringent, and the extreme shapes that must be achieved. The objectives of plasma shape control:
  - Maintain the plasma within the containment vessel
  - Shape the plasma to satisfy certain operational objectives
  - Stabilize the vertical instability

NSTX Tokamak

- National Spherical Torus Experiment (NSTX) at the Princeton Plasma Physics Laboratory (PPPL) presents a unique control challenge relative to other tokamaks in that there are no shaping control coils on the inboard radius of the plasma.

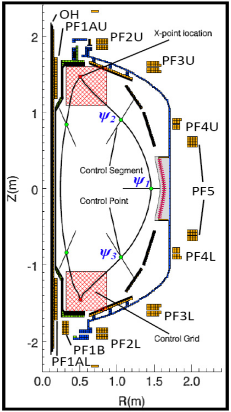


Figure: Poloidal Cross Section of NSTX

Axisymmetric Plasma Response Model

- The NSTX system, which is composed of the plasma, shaping coils, and passive structure, can be described for a particular plasma equilibrium by a linearized axisymmetric plasma response model.

$$
\begin{aligned}
M^*_{cc}\dot{I}_c + R_c I_c + M^*_{cv}\dot{I}_v + M^*_{cp}\dot{I}_p &= V_c \\
M^*_{vv}\dot{I}_v + R_v I_v + M^*_{vc}\dot{I}_c + M^*_{vp}\dot{I}_p &= 0 \\
M^*_{pp}\dot{I}_p + R_p I_p + M^*_{pc}\dot{I}_c + M^*_{pv}\dot{I}_v &= V_{no}
\end{aligned}
\tag{1}
$$

  - I_c, I_v, and I_p represent the currents in the PF coils, vessel, and plasma.
  - V_c represents the vector of voltages applied to the PF coils, and V_{no} represents the effective voltage applied to drive plasma current by noninductive sources (no noninductive current source is considered in this work, i.e., $V_{no} = 0$).
  - R_c, R_v, and R_p represent the PF coils, vessel, and plasma resistances.
  - $M^*_{ab} = M_{ab} + X_{ab}$ represents the plasma-modified mutual inductance matrix where $a, b \in \{c, v, p\}$.
  - M_{ab} is the conductor-to-conductor mutual inductance.
  - X_{ab} describes a plasma motion-mediated inductance, linearized around the plasma equilibrium. X_{ab}, representing changes in flux due to plasma motion, are functions only of the equilibrium current distribution J^{eq}_{ϕ} and vacuum magnetic field B^{eq}.

Axisymmetric Plasma Response Model

- The output equation is expressed as

$$\delta y = C_{I_c}\delta I_c + C_{I_v}\delta I_v + C_{I_p}\delta I_p \qquad (2)$$

  - The matrices C_{I_s} are defined as $C_{I_s} = \frac{\partial y}{\partial I_s} + \frac{\partial y}{\partial R_p}\frac{\partial R_p}{\partial I_s} + \frac{\partial y}{\partial Z_p}\frac{\partial Z_p}{\partial I_s}$, where the first term is the "direct" response ($I_s \in \{I_c, I_v, I_p\}$). The remaining terms are responses due to motion of the plasma.
  - R_p and Z_p denote the radial and vertical positions of the plasma.
- The linearized plasma response model around a particular plasma equilibrium is written in state space form

$$\dot{x}^p = Ax^p + Bu^p, \qquad \delta y^p = C\delta x^p, \qquad (3)$$

  with $x^p = [I_c^T \ I_v^T \ I_p^T]^T$ and $u^p = [V_c^T \ 0 \ V_{no}^T]^T$.
- We define $\delta y^p = y^p - y^p_{eq}$ and $\delta x^p = x^p - x^p_{eq}$ where y^p_{eq} and x^p_{eq} are the values of the equilibrium outputs and states from which the model is derived.
- The output vector $y^p \in \Re^p$ ($p = 8$) represents the fluxes ψ_1, ψ_2, ψ_3 at the control points, the magnetic field B_r and B_z at the desired X point location, the plasma radial and vertical positions R_p and Z_p, and the plasma current I_p.

Singular Value Decomposition (SVD)

- As $t \to \infty$ the system can reach the steady state.

$$\bar{y} = \bar{P}\bar{u} \qquad P(s) = C(sI - A)^{-1}B + D \qquad \bar{P} = P(0) \tag{4}$$

- The weight matrices are introduced to the transfer matrix, and a singular value decomposition (SVD) is carried out in input-output expression:

$$\tilde{P} = Q^{1/2}\bar{P}R^{-1/2} = USV^T \qquad \bar{y} = Q^{-1/2}\tilde{P}R^{1/2}\bar{u} \tag{5}$$

  where $Q \in \Re^{5\times 5}$, $R \in \Re^{9\times 9}$ are positive define weight matrix for tracking error and control effort.

- The SVD is introduced to simplify the calculation

$$\begin{array}{l} \bar{y}^* = S^{-1}U^TQ^{1/2}\bar{y} \\ \bar{u}^* = V^TR^{1/2}\bar{u} \end{array} \Rightarrow \bar{y}^* = S^{-1}U^TQ^{1/2}Q^{-1/2}USV^TR^{1/2}\bar{u} = \bar{u}^* \tag{6}$$

  where $U \in \Re^{p\times m}$, $V \in \Re^{m\times m}$, and $S = \text{diag}[\sigma_1, \sigma_2, \ldots, \sigma_m] \in \Re^{m\times m}$.

- The performance index and tracking error can be expressed as:

$$\bar{e} = \lim_{t\to\infty} e(t) = \bar{r} - \bar{y} = Q^{-1/2}US(\bar{r}^* - \bar{y}^*)$$

$$\bar{J} = \lim_{t\to\infty} e^T(t)Qe(t) = (\bar{r}^* - \bar{y}^*)^T S^2(\bar{r}^* - \bar{y}^*) = \sum_{i=1}^{m} \sigma_i^2(\bar{r}^* - \bar{y}^*)^2 \tag{7}$$

  where r is the reference value and $\bar{r}^* = S^{-1}U^TQ^{1/2}\bar{r}$.

Uncertain Model for Current Ramp-up Phase

- During the plasma current ramp-up phase, 26 scenario points from the experimental shot #124616 from 91 ms to 391 ms are chosen to describe the plasma equilibrium evolution.
- A frequency response study for the family of 26 decoupled plasma models obtained for the current ramp-up phase shows that the models do not have a large difference in magnitude, as shown in the figure below.

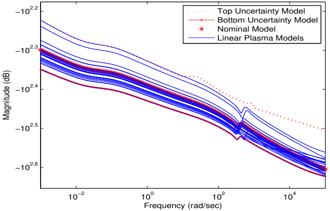


Figure: Frequency Study of Plasma Models

- The model has the highest magnitude, denoted as P_{top}.
- The model has the lowest magnitude, denoted as P_{bottom}.
- The model, which has the mean magnitude, is chosen as the nominal model, denoted as P_0.

Uncertain Model for Current Ramp-up Phase

- The family of plasma models can be considered as a one time-varying state-space model, which is written as an uncertain state-space model and formulated into a robust control framework.
- By defining the matrices

$$M_0 = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix}, \qquad \Delta M_i = \begin{bmatrix} \Delta A_i & \Delta B_i \\ \Delta C_i & \Delta D_i \end{bmatrix} \tag{8}$$

where $\Delta A_i = A_i - A_0$, $\Delta B_i = B_i - B_0$, $\Delta C_i = C_i - C_0$, $\Delta D_i = D_i - D_0$, and $i \in 1,2$ refers to the top and bottom models respectively, the state-space system matrices are now written as uncertain matrices as

$$A = A_0 + \sum_{i=1}^{2} \delta_i \Delta A_i \;\; B = B_0 + \sum_{i=1}^{2} \delta_i \Delta B_i \;\; C = C_0 + \sum_{i=1}^{2} \delta_i \Delta C_i \;\; D = D_0 + \sum_{i=1}^{2} \delta_i \Delta D_i.$$

By conducting a frequency analysis of the uncertain model of the above system, the uncertain model is proven to capture the behavior of the family of plasma models.

Uncertain Model for Current Ramp-up Phase

- By using singular value decomposition and grouping terms, the matrix ΔM_i is expressed as

$$\Delta M_i = U_i \Sigma_i V_i^T = (U_i \sqrt{\Sigma})(\sqrt{\Sigma} V_i^T) = \begin{bmatrix} L_i \\ W_i \end{bmatrix} \begin{bmatrix} R_i \\ Z_i \end{bmatrix}^T . \tag{9}$$

- By employing (9), the uncertainty is written as

$$\delta_i \Delta M_i = \begin{bmatrix} L_i \\ W_i \end{bmatrix} \begin{bmatrix} \delta_i I_{q_i} \end{bmatrix} \begin{bmatrix} R_i \\ Z_i \end{bmatrix}^T \tag{10}$$

where q_i is the rank of the matrix ΔM_i. The system matrix M is finally expressed as

$$M = M_0 + \sum_{i=1}^{2} \delta_i \Delta M_i = H_{11} + H_{12} \Delta H_{21} \tag{11}$$

where

$$H_{11} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} \quad H_{12} = \begin{bmatrix} L_1 & L_2 \\ W_1 & W_2 \end{bmatrix} \quad H_{21} = \begin{bmatrix} R_1^T & Z_1^T \\ R_2^T & Z_2^T \end{bmatrix} \quad \Delta = \begin{bmatrix} \delta_1 I_{q_1} & 0 \\ 0 & \delta_2 I_{q_2} \end{bmatrix} .$$

- the transfer function $P(s)$ between the output y and the input u is next expressed as

$$P(s) = F_u \left(F_l(H, \Delta), \frac{1}{s} I \right) = F_l \left(F_u \left(H, \frac{1}{s} I \right), \Delta \right) \tag{12}$$

Control System of NSTX

- The proposed control architecture shown in in the figure below is composed of three loops:
  - The first loop is devoted to plasma current regulation (PID controller).
  - The second loop is dedicated to plasma radial and vertical position stabilization (Adaptive PID controller).
  - The third loop is used to control the plasma shape and X-point location (Multi-Input-Multi-Output (MIMO) robust controller).

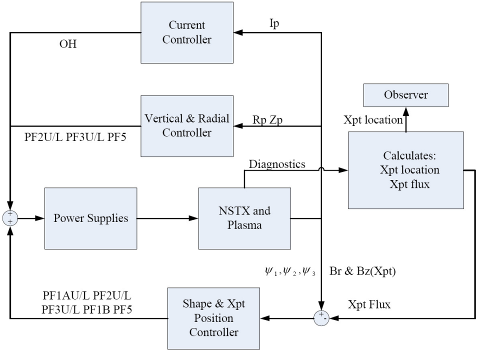


Figure: NSTX control system architecture

Plasma Current & Position Control of NSTX

- The ohmic (OH) coil is dedicated to plasma current regulation. The proposed plasma current controller is written as

$$V_{OH} = G_P^{I_p} \Delta I_p + G_I^{I_p} \int_0^t \Delta I_p dt + G_D^{I_p} \frac{d \Delta I_p}{dt}, \tag{13}$$

where $\Delta I_p = I_p - I_P^{ref}$ with I_P^{ref} denoting the reference plasma current. The parameters $G_P^{I_p}$, $G_I^{I_p}$, and $G_D^{I_p}$ are the plasma current PID gains.

- In general, the goal of position control is to minimize the closed-loop cost function $J_a(k_c) = e_a(k_c)^2/2$, where k_c is an adjusted parameter. The error e_a is defined as $e_a(k_c, t) = r_a(t) - y_a(k_c, t)$, where $r_a(t)$ is the reference and $y_a(k_c, t)$ is the system output, which will be defined below as the actual radial and vertical positions of the plasma.
- In order to make J_a small, it is reasonable to change k_c in the direction of the negative gradient of J_a, which is defined as

$$\dot{k}_c = \frac{dk_c}{dt} = -\lambda \frac{\partial J_a}{\partial k_c} = -\lambda \frac{\partial J_a}{\partial e_a} \frac{\partial e_a}{\partial k_c} = \lambda e_a \frac{\partial y_a}{\partial k_c}, \tag{14}$$

where λ is the step length, and $\partial y_a / \partial k_c$ is the sensitivity derivative.

Plasma Position Control of NSTX

- The output is expressed as $y_a = P_a u_a$, where $u_a = k_c K_{PID} e_a$. The goal is to make $y_a(k_c, t) = r_a(t)$ by choosing the optimal value k_c^*. The optimal reference is $r_a = P_a k_c^* K_{PID} e_a = (k_c^*/k_c) P_a k_c K_{PID} e_a = (k_c^*/k_c) y_a$. The adjusted parameter k_c is therefore expressed as

$$\dot{k}_c = \lambda e_a \frac{\partial y_a}{\partial k_c} = \lambda e_a \frac{\partial \frac{(r_a k_c)}{k_c^*}}{\partial k_c} = \frac{\lambda}{k_c^*} e_a r_a = \nu e_a r_a, \tag{15}$$

where ν is the adaptive gain. Using (15), the proposed radial position controller is then written as

$$\Delta V_{PF2_R}^{U/L} = \Delta V_{PF3_R}^{U/L} = \Delta V_{PF5_R} = V_{R_p} \tag{16}$$

$$V_{R_p} = k_{cr}(G_P^{R_p} \Delta R_P + G_I^{R_p} \int_0^t \Delta R_P dt + G_D^{R_p} \frac{d\Delta R_P}{dt}) \tag{17}$$

where $\dot{k}_{cr} = \nu_r \Delta R_p R_P^{ref}$ denotes the radial adjusted parameter.

- The proposed vertical position controller is written as

$$\Delta V_{PF2_Z}^{j} = \Delta V_{PF3_Z}^{j} = V_{Z_p}(j) \tag{18}$$

$$V_{Z_p}(j) = (-1)^j k_{cz}(G_P^{Z_p} \Delta Z_P + G_I^{Z_p} \int_0^t \Delta Z_P + G_D^{Z_p} \frac{d\Delta Z_P}{dt}) \tag{19}$$

where $\dot{k}_{cz} = \nu_z \Delta Z_p Z_P^{ref}$ denotes the vertical adjusted parameter.

Design of the Shape Controller for Ramp-up Phase

- The control goal is to design a $k \times k$ feedback controller K, where k is the number of significant singular values, that can stabilize the system and keep the tracking error $e_s^* = r_s^* - y_s^*$ small. The corresponding block diagram of the system is shown below.

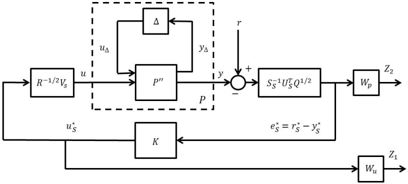


Figure: Shape Control System Design Structure

- The weight functions $W_p(s)$ and $W_u(s)$ are parameterized as

$$W_p(s) = K_p \left(\frac{\frac{s}{M_1} + w_{b1}}{s + w_{b1} A_1} \right)^2, \; W_u(s) = K_u \left(\frac{s + w_{b2} A_2}{\frac{s}{M_2} + w_{b2}} \right)^2$$

  and the coefficients M_i, A_i, w_{bi}, for $i \in 1,2$, as well as K_p and K_u, are design parameters defined based on frequency-domain specifications.

Design of the Shape Controller for Ramp-up Phase

- The closed-loop transfer function from the external input r_s^* to the external outputs $\begin{bmatrix} Z_1^T & Z_2^T \end{bmatrix}^T$ is defined as

$$T_{zr} = F_u(N, \Delta), \tag{20}$$

where $N = F_l(P^*, K)$, and P^* is the transfer function from $[u_\Delta^T \; r_s^{*T} \; u_s^{*T}]^T$ to $[y_\Delta^T \; Z_1^T \; Z_2^T \; e_s^{*T}]^T$. We seek a controller $K(s)$ that robustly stabilizes the system and minimizes the H_∞ norm of the transfer function $T_{zr}(N, \Delta)$, i.e.,

$$\min_{K(s)} \| T_{zr}(N, \Delta) \|_\infty = \min_{K(s)} (\sup_\omega \bar{\sigma}[T_{zr}(N, \Delta)(j\omega)]) \tag{21}$$

where $\bar{\sigma}$ represents the maximum singular value.
- The *DK*-iteration method, which combines H_∞ synthesis and μ analysis, can be used to obtain an iterative solution.
  - K step: Synthesize an H_∞ controller for the scaled problem, $\min_K \left\| DN(K)D^{-1} \right\|_\infty$ with fixed $D(s)$.
  - D step: Find $D(j\omega)$ to minimize $\bar{\sigma}(DND^{-1})$ at each frequency with fixed N.
  - Fit the magnitude of each element of $D(j\omega)$ to a stable and minimum-phase transfer function $D(s)$ and go to step 1.

Design of the Shape Controller for Ramp-up Phase

- The robust stability is determined by the structured singular value
$$\mu(N_{11}(j\omega)) = 1/\min\{k_m | \det(I - k_m N_{11}\Delta) = 0\} \quad (22)$$
where N_{11} is the transfer function from u_Δ to y_Δ. The system is robustly stable for all allowable perturbations if and only if $\mu(N_{11}(j\omega)) < 1, \forall\omega$.

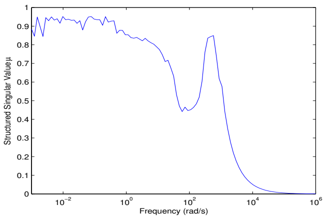
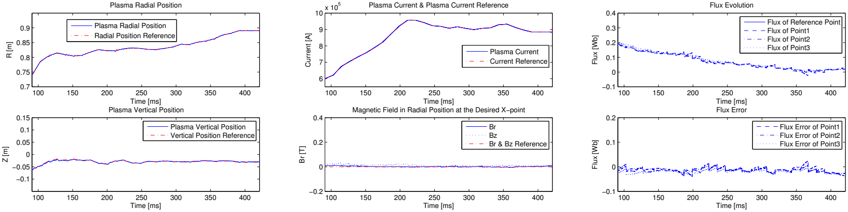


Figure: Structured Singular Value μ versus Frequency

- Finally, the overall plasma shape and Xpt location controller is written as
$$\hat{K}(s) = \frac{U(s)}{E(s)} = R^{-1/2} V_s K(s) S_s^{-1} U_s^T Q^{1/2} \quad (23)$$
The contribution to the coil voltages is written as
$$V_{Shape} = [\Delta V_{PF1A_s}^{U/L} \, \Delta V_{PF1B_s} \, \Delta V_{PF2_s}^{U/L} \, \Delta V_{PF3_s}^{U/L} \, \Delta V_{PF5_s}]^T = \mathscr{L}^{-1}\{\hat{K}(s)E(s)\}$$
where $\mathscr{L}^{-1}$ denotes the inverse Laplace transform.

Simulation Results During Current Ramp-up Phase

- Figure (a) shows the time responses for the plasma radial and vertical positions.
- Figure (b) (top) shows the time evolution of the plasma current, and the tracking error is less than 0.5%. The components of magnetic field at the desired X-point are shown in Figure (b) (bottom).
- Figure (c) (top) shows both the flux at the X-point and the flux at the three control points (ψ_1, ψ_2, and ψ_3), and the flux at the control points tracks the flux at the X-point. Figure (c) (bottom) shows the tracking errors.



(a) (b) (c)

Figure: (a) Plasma radial and vertical position; (b) Plasma current and magnetic field; (c) Magnetic flux & flux error at the control points

Simulation Results During Current Ramp-up Phase

- A series of four plasma boundary shapes at different times are shown in the figure below. The blue circles represent the control points, the blue asterisk represents the actual location of the X-point, and the red asterisk represents the reference location of the X-point.

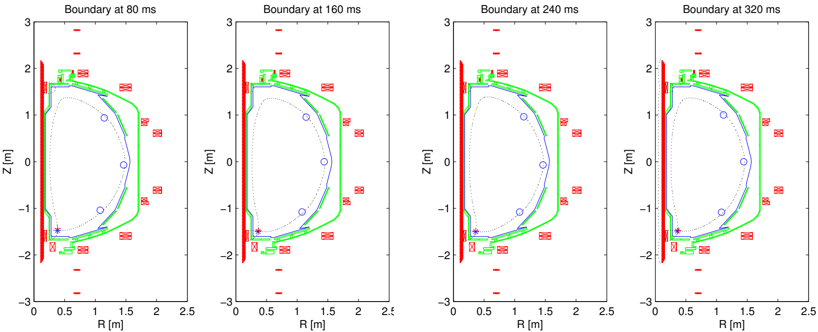


Figure: Plasma Boundary at 80 ms, 160 ms, 240 ms and 320 ms