The role of kinetic effects, including plasma rotation and energetic particles, in resistive wall mode stability

J.W. Berkery[1], S.A. Sabbagh[1], R. Betti[2], B. Hu[2],
R.E. Bell[3], S.P. Gerhardt[3], J. Manickam[3], and M. Podesta[3]
[1]*Department of Applied Physics and Applied Mathematics, Columbia University, New York, New York 10027, USA*
[2]*Laboratory for Laser Energetics, University of Rochester, Rochester, New York 14623, USA*
[3]*Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543, USA*

Introduction - Kinetic Theory of RWM Stabilization

Continuous, disruption-free operation of high-beta tokamaks requires stabilization of the resistive wall mode (RWM). Theoretically, the RWM is thought to be stabilized by energy dissipation mechanisms that depend on plasma rotation and kinetic effects [1]. Experiments in NSTX show that the RWM can be destabilized in high rotation plasmas while low rotation plasmas can be stable, which calls into question the concept of a simple critical plasma rotation threshold for stability. Kinetic modification of ideal stability is calculated with the `MISK` code, using experimental equilibrium reconstructions. Trapped ions provide the dominant kinetic resonances, allowing a more complex relationship between plasma rotation and RWM stability than simpler critical rotation theories. Energetic particles contribute an important stabilizing effect as well.

When kinetic effects are included, the RWM dispersion relation takes the form $-i\omega\tau_w = -(\delta W_\infty + \delta W_K)/(\delta W_b + \delta W_K)$, where the change in potential energy of the plasma due to kinetic effects, δW_K, results from the assumption of a perturbed kinetic pressure tensor. An expression for δW_K can be derived using a moment of the perturbed distribution function, which results from the solution of the linearized, bounce-averaged drift kinetic equation. The resulting equation contains a frequency-resonance denominator, such that $\delta W_K \sim 1/(\omega_D + l\omega_b - i\nu_{\mathrm{eff}} + \omega_\phi - \omega_* - \omega)$, where the frequencies are precession drift, bounce, collision, plasma rotation, diamagnetic, and the mode frequency, and l is the bounce harmonic. This kinetic term determines the relationship of stability on plasma rotation, by its magnitude compared to the other frequencies for different particle types.

The Role of Plasma Rotation

When the RWM is in a mode-particle resonance with particle's motion, the effect is that energy of the mode can be dissipated, which is stabilizing. Specifically, for $l = 0$, when $\omega_\phi - \omega \approx -\omega_D + \omega_{*i}$, the Doppler-shifted mode frequency is said to be in resonance with the precession drift frequency, δW_K is large and the mode can be stabilized. Similarly for $l \neq 0$ bounce harmonics, $\omega_\phi \approx -l\omega_b + \omega_{*i}$ represents a stabilizing bounce frequency resonance. When the plasma rotation is such that the mode is *off* of these resonances, kinetic stabilization is low and the mode may be unstable. This can occur not only at low rotation, as in classic theories, but also at intermediate rotation levels, in-between the stabilizing resonances [2].

Typically such resonances are most important for trapped thermal ions. For circulating ions there is no $l = 0$ bounce harmonic to allow a strong resonance with the precession drift. For electrons, the collision frequency is typically very large, which leads to a small kinetic stabilizing term. For energetic particles both the bounce and precession drift frequencies are typically much greater than $\omega_\phi - \omega_{*i}$. Therefore energetic particles are not in an energy dissipation mode-particle resonance, and the stabilizing contribution to δW_K from energetic particles is from a real restorative force, and it is approximately independent of ω_ϕ. Stabilization from energetic particles is nevertheless important to consider, and can be broadly separated into two categories: alpha particles which are isotropic in pitch angle, and beam ions which are not.

Energetic Particles with an Isotropic Distribution - Alpha Particles in ITER

As an example of the stabilizing effect of isotropic energetic particles on the RWM, through kinetic effects, we will consider the case of alpha particles in an ITER scenario 4 equilibrium with $\beta_N = 3$ (20% above the $n = 1$ no-wall limit), by scaling $\beta_\alpha/\beta_{\text{total}}$ in `MISK` [3]. Figure 1 indicates that a sufficient population of alpha particles is required to stabilize the RWM at plasma rotation speeds from 0 - 1.8 times that predicted by Polevoi *et al.* [4]. As the alpha particle beta is increased, the growth rate decreases, eventually passing into a stable region. Above 23% alpha particle β, the calculation predicts that the plasma is *stable* to the RWM regardless of the rotation level. At the expected level of $\beta_\alpha/\beta_{\text{total}} = 0.187$, this ITER equilibrium is predicted to just attain marginal stability with $\omega_\phi = \omega_\phi^{\text{Polevoi}}$ (Fig. 1). Note that adding alpha particles does not affect the thermal particle precession drift resonant stabilization that occurs at $\omega_\phi/\omega_\phi^{\text{Polevoi}} = 0.8$.

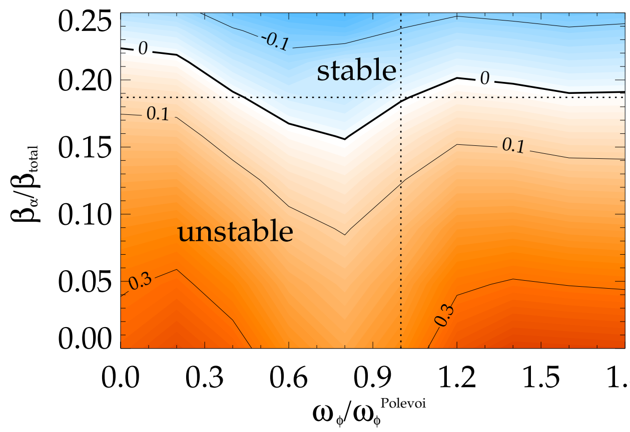

Figure 1: Contours of RWM growth rate for ITER vs. scaled alpha particle β and plasma rotation.

Energetic Particles with an Anisotropic Distribution - Beam Ions in NSTX

Energetic particles add an additional stabilizing term to $\gamma\tau_w$, but they do not affect the thermal particle resonances with plasma rotation [3]. Figure 2a shows the contribution to δW_K as a function of scaled experimental plasma rotation for thermal trapped ions compared to energetic trapped ions for NSTX shot 121083 @ 0.475 s, which has a $\beta_a/\beta_{\text{total}} = 0.176$ according to `TRANSP`. This shot experimentally goes unstable at this time, so ω_ϕ^{exp} is the marginally stable rotation profile. The contribution from energetic particles is significant and mostly real, but is nearly independent of ω_ϕ, as opposed to the obvious resonances displayed in the thermal ion traces. In the present model, the effect of adding energetic particles to the calculation is to decrease the growth rate, as seen in Fig. 2b. At the experimental rotation, the predicted growth rate goes from near marginal to $\gamma\tau_w \approx -0.25$ when isotropic energetic particles are included.

Presently, only trapped beam ions are considered, and with an isotropic distribution function, beam ions are assumed to be spread evenly across pitch angle. If we instead considered an anisotropic distribution function with an analytical Gaussian form [5] $f(\varepsilon,\Psi,\chi) = C(\Psi)/(\varepsilon^{\frac{3}{2}} + \varepsilon_c^{\frac{3}{2}})e^{-(\chi-\chi_0)^2/\delta\chi^2}$, where C is a normalization factor, then depending on the center, χ_0, and width, $\delta\chi$, of the Gaussian, a higher or lower percentage of beam ions might be trapped than the isotropic case would estimate.

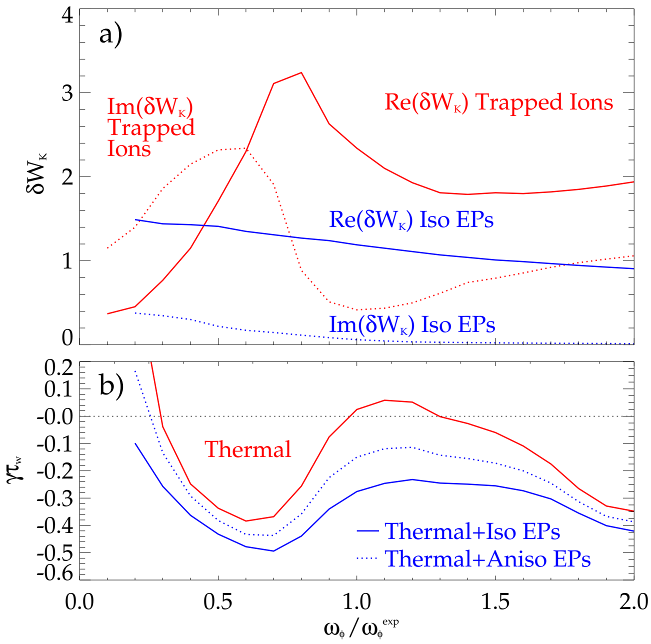

Figure 2: a) δW_K and b) RWM growth rate for an NSTX equilibrium vs. scaled plasma rotation.

This correction to the beam ion calculation could be significant, but it depends on the particular details of the neutral beams for the machine being considered. For NSTX, there are three separate beam sources, with possibly different energies, each has distinct full, half, and one-third energy components, and each can deposit particles at two different χ_0 angles (on the outboard and inboard sides) of a particular surface. Efforts are underway to precisely model the anisotropic energetic particle distribution function as computed by `TRANSP` for NSTX as a linear combination of such Gaussian forms.

For now, a test of the role of anisotropy is performed by considering the a simpler anisotropy model with constants $\chi_0(\Psi) = 0.75$, $\delta\chi(\varepsilon,\Psi) = 0.25$, which puts a lower percentage of particles in the trapped range of χ. This calculation has been included in Fig. 2b. As expected, the anisotropic case with lower trapped fraction leads to greater stability than the isotropic case.

Discussion

From Fig. 2 it can be seen that although the kinetic theory calculation with `MISK` is insightful in illuminating the effects of plasma rotation on stability through mode-particle resonance energy dissipation, and of energetic particles on stability through a real restorative force independent of rotation, a precise quantitative agreement between the calculations and NSTX experimental results has not yet been achieved. Several improvements to the theoretical treatment are being considered in this light. First, the aforementioned improvements to the treatment of anisotropic beam ions may help, as evidenced by the simple model effect shown in Fig. 2b. Second, inclusion of previously neglected theoretical terms, such as centrifugal destabilization, an electrostatic term, and another anisotropy effect which adds a δW term proportional to $-\mu(\tilde{B}_{\|}/B)(\partial f/\partial\mu)$, are being considered. Finally, an improved kinetic theory may help unify the experimental understanding of RWM stability in DIII-D, where a large energetic particle stabilization may be preventing the RWM from going unstable in most cases except for when triggered by a sudden loss of energetic particles through fishbones, and NSTX, where a significantly smaller energetic particle stabilization may be allowing the mode to go unstable more often, and thermal ion frequency resonances with plasma rotation are more clearly seen.

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