MHD Simulation of Relaxation to a Flipped ST Configuration in Helicity-driven Systems

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Abstract

To understand comprehensively the relaxation in helicity-driven low aspect ratio toroidal systems, the dynamics of spherical torus (ST) plasmas, when the external toroidal magnetic field (TF) is decreased and its direction is reversed, has been investigated using three-dimensional magnetohydrodynamic (MHD) numerical simulations. In result, it has been demonstrated that the ST plasma relaxes to a flipped ST state, accompanied by the self-reversal of magnetic fields. This result is in agreement with recent observations in the HIST device. Our simulation reveals the detailed dynamics during the relaxation. The growth of the $n = 1$ mode in the central open flux and the following magnetic reconnection event between open and closed flux are observed and it is found that the transition from the ST to the flipped ST configuration corresponds to the relaxation of high λ plasma in central open flux region to low λ state.

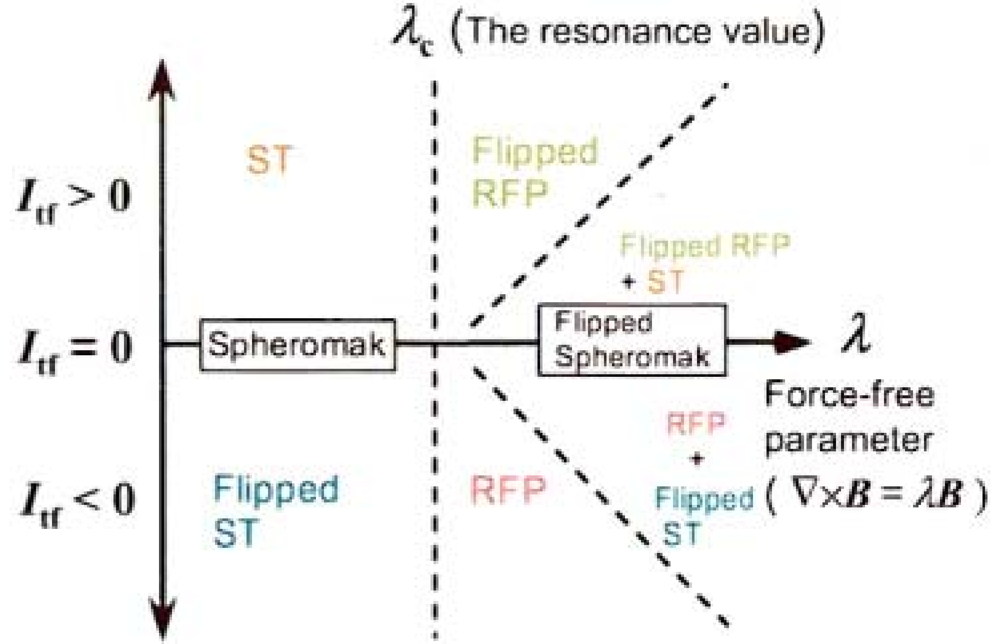
Introduction

The primary objective of this study is to understand comprehensively the MHD relaxation dynamics in helicity-driven low aspect ratio toroidal systems (ST, Spheromak, Spherical RFP, *etc.*).

- Helicity-driven relaxation theory refers to the existence of flipped ST states in the regime of TF coil current $I_{tf} < 0$.

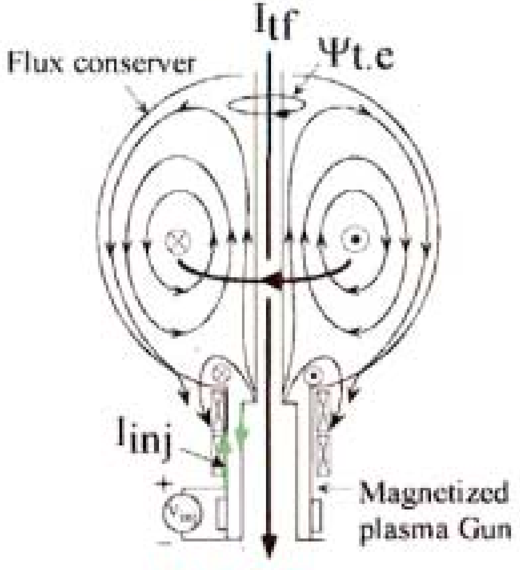
Helicity-driven relaxation theory:
TAYLOR, J. B., *et al.*,
Nucl. Fusion **29** (1989), 219.

Prediction of the flipped ST :
BROWNING, P.K., *et al.*,
Plasma Phys. Control. Fusion **35** (1993) 1563.

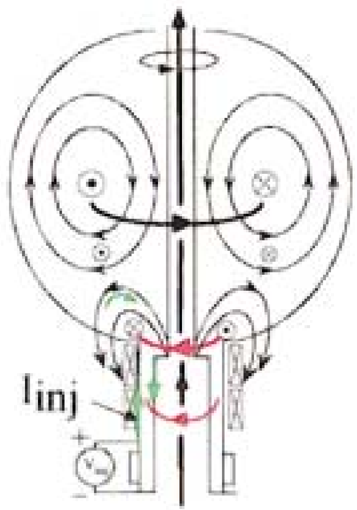


$\lambda -$TF diagram obtained from the equilibrium analysis of helicity-driven relaxed configurations

CHI in a ST and a Flipped ST

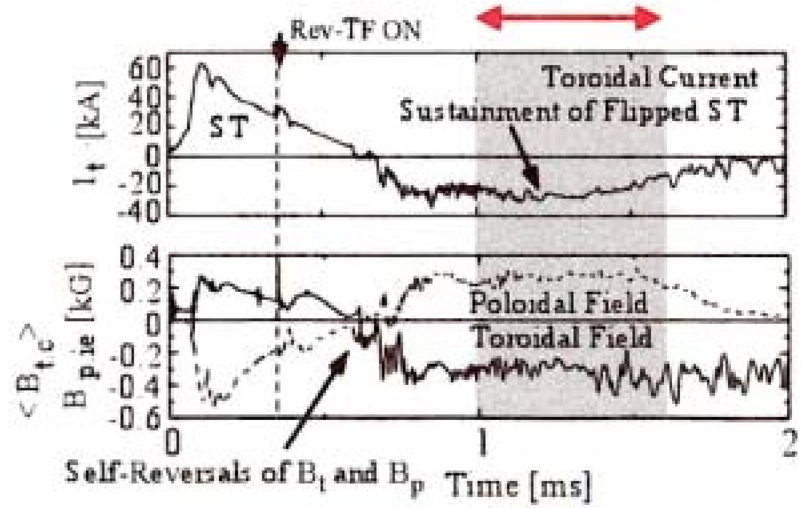


(a) ST



(b) Flipped ST

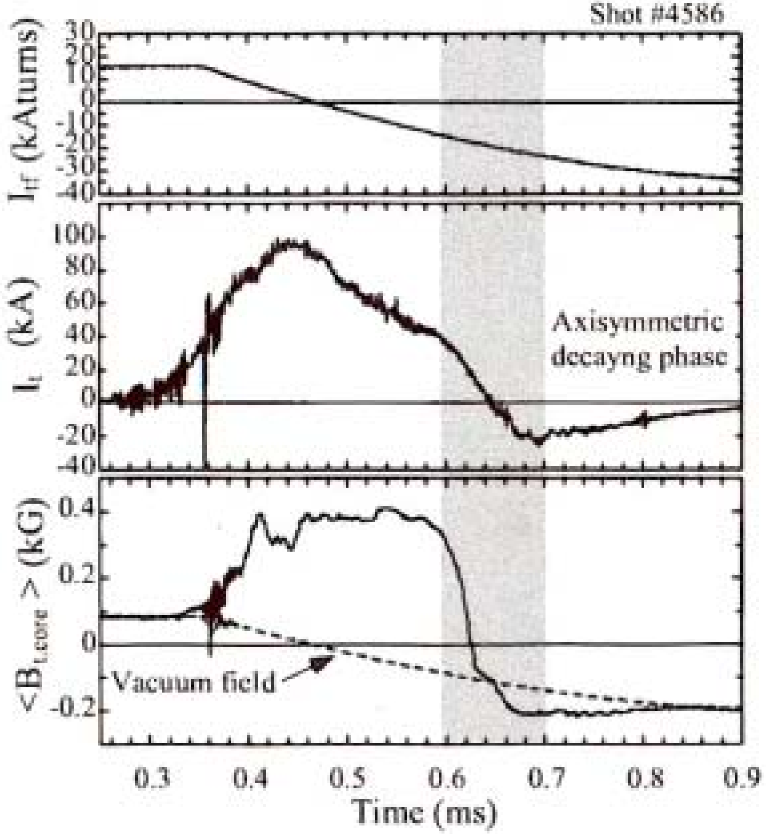
- In recent HIST experiments, it has been demonstrated that the flipped ST plasma can be successfully sustained by Coaxial Helicity Injection (CHI).

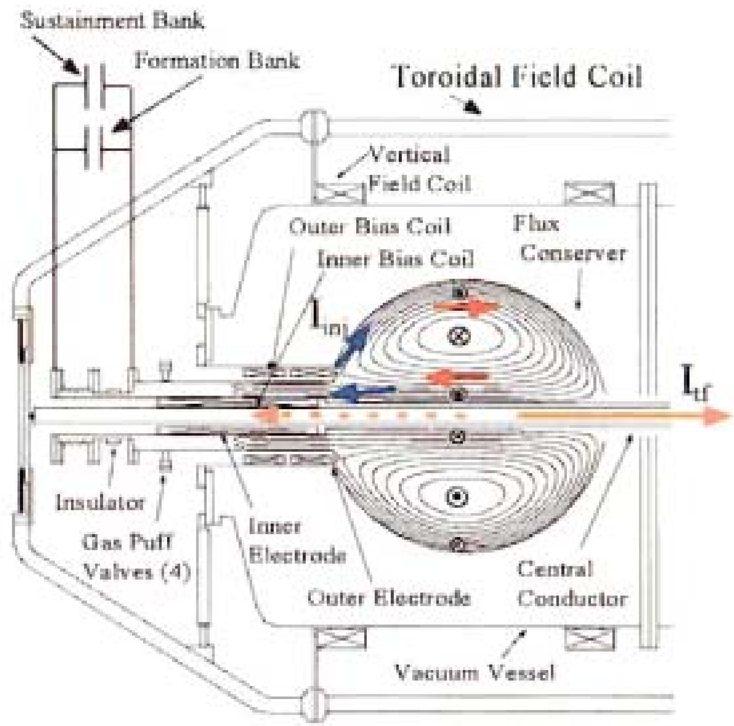


<u>Notable significance</u>

- During the sustainment, the field in the core region is not quite disturbed because there is no the central open flux encircling the closed flux.
- Therefore, we do not have to think of whether closed flux surfaces can be produced by CHI in the flipped ST.

Flipped ST formation





HIST device (Himeji)

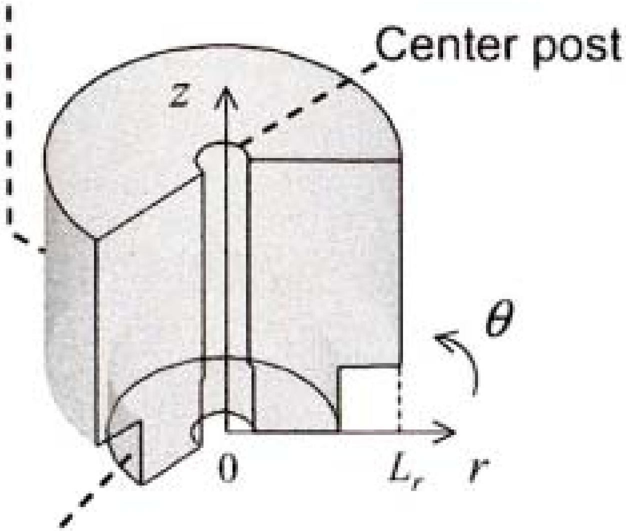
- In the HIST experiments, it has been found that the flipped ST can be formed from a ST plasma as the direction of external toroidal field (TF) is reversed.
- In this work, the self-reversal dynamics of magnetic polarity is clarified using three-dimensional MHD numerical simulations.

Geometry

Confinement region

$$0.15L_r \leq r \leq L_r\ ,\ 0.5L_r \leq z \leq 2L_r$$

$$(\ N_r \times N_\theta \times N_z\ ) = (\ 69 \times 64 \times 121\ )$$



Gun region

$$0.175L_r \leq r \leq 0.65L_r\ ,\ 0 \leq z < 0.5L_r$$

$$(\ N_r \times N_\theta \times N_z\ ) = (\ 39 \times 64 \times 40\ )$$

Velocity at the boundary

$$V = 0$$

Magnetic field at the boundary

Open field lines are fixed on inner and outer boundaries of a gun region.

$$\begin{cases} B_\perp = 0 & \text{(Top, bottom, and inner and outer boundaries of a confinement region)} \\ B_\perp \neq 0 & \text{(Inner and outer boundaries of a gun region)} \end{cases}$$

$$\frac{\partial B_\perp}{\partial t} = 0$$

External toroidal field is decreased and its direction is reversed with a time constant of τ_r.

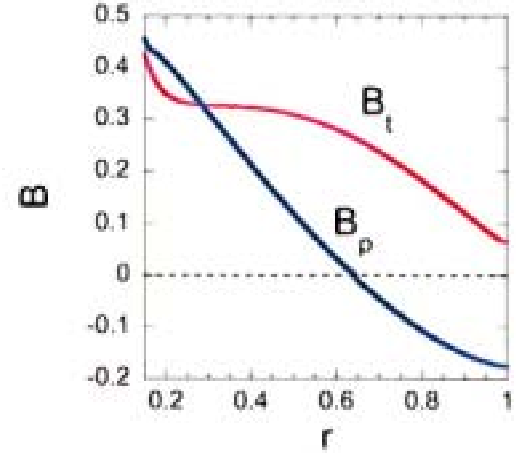
$$\frac{\partial}{\partial t} B_\theta = - \frac{B_{\theta,\mathrm{init}}}{\tau_r} \qquad \text{(Center post)}$$

Initial condition

Grad-Shafranov numerical equilibrium

$$\mu_0 j_\theta = -\frac{1}{r}\left\{ r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \frac{\partial^2 \psi}{\partial z^2}\right\} = \mu_0 r\frac{\mathrm{d}P}{\mathrm{d}\psi} + \frac{I}{r}\frac{\mathrm{d}I}{\mathrm{d}\psi}$$

$$B_r = -\frac{1}{r}\frac{\partial \psi}{\partial z} \quad , \quad B_\theta = \frac{I}{r} \quad , \quad B_z = \frac{1}{r}\frac{\partial \psi}{\partial r}$$

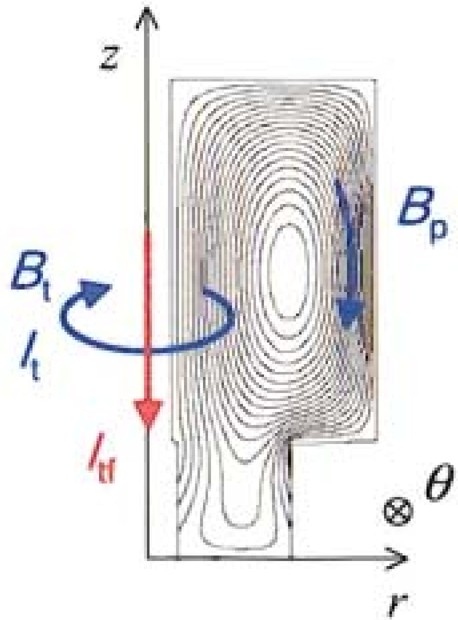


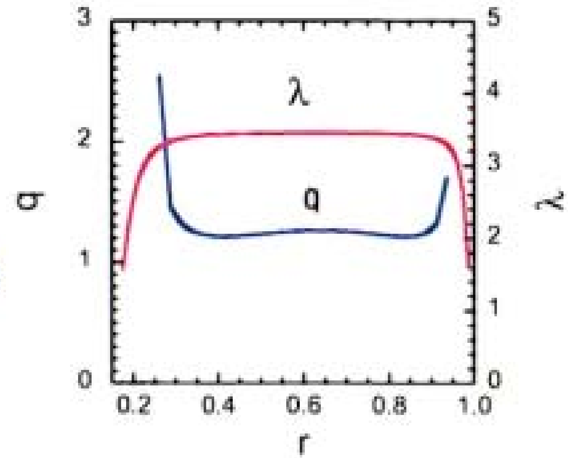
Assumption about $P(\psi)$ and $I(\psi)$

$$\frac{\mathrm{d}P}{\mathrm{d}\psi} = 0$$

$$I = \Lambda\left(\sqrt{\psi^2 + \delta^2} - \delta\right) + \frac{I_{\mathrm{tf}}}{2\pi}$$

- Open magnetic flux penetrates the inner and outer boundaries of a gun region.
- $\lambda\ (\equiv \boldsymbol{j}\cdot\boldsymbol{B}\,/\,B^2)$ profile is close to the Taylor state.
- The safety factor on axis q_0>1.





$\Lambda = 3.47$, $\delta = 0.1\,\psi_0$,
$I_{\mathrm{tf}} / I_{\mathrm{t}} = 0.4$

* All values are normalized

MHD equations

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} = -\nabla \cdot \rho \boldsymbol{v}\boldsymbol{v} + \boldsymbol{j} \times \boldsymbol{B} - \nabla p - \nabla \cdot \Pi$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p\boldsymbol{v} - \kappa \nabla T) - (\gamma - 1)\left(p\nabla \cdot \boldsymbol{v} + \Pi : \nabla \boldsymbol{v} - \eta j^2\right)$$

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \eta \boldsymbol{j}$$

$$\boldsymbol{j} = \nabla \times \boldsymbol{B}$$

$$T = p / \rho$$

$$\Pi = \nu \left(\frac{2}{3} (\nabla \cdot \boldsymbol{v}) \mathbf{I} - \nabla \boldsymbol{v} - {}^{t}(\nabla \boldsymbol{v}) \right)$$

ρ : Constant for simplicity

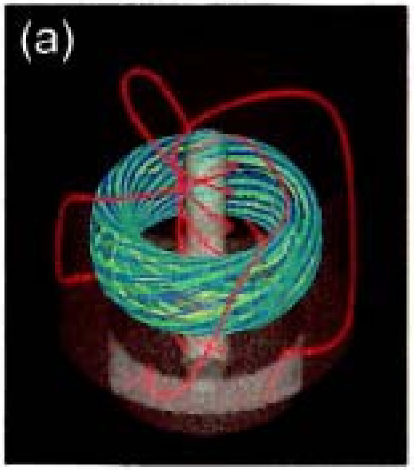
Spatial derivatives : The second-order finite differences method

Time integration : The fourth-order Runge-Kutta-Gill method

Parameters

	Normalized	[Real unit]
Cylinder radius (Confinement region) L_r	1.0	[0.5 m]
Mass density ρ	1.0	[5×10^{19} /m^3]
Characteristic magnetic field B_0	1.0	[0.2 T]
Initial toroidal current I_t	1.0	[80 kA]
Initial pressure p (temperature T)	1.0×10^{-2} (1.0×10^{-2})	[40 eV]
Initial TF current I_{tf}	0.4	[32 kA]
Initial force-free parameter on axis λ_0	3.49	[6.98 /m]
Resistivity η	1.0×10^{-4}	[3.9×10^{-5} Ωm]
Viscosity ν	1.0×10^{-3}	[2.6×10^{-5} kg/m·sec]
Conductivity κ	1.0×10^{-3}	[0.32 W/m·K]
Alfvén time τ_A	1.0	[0.81 μsec]
Reversing time constant of TF τ_r	50	[40 μsec]
Time limit for the reversal t_l	100	[81 μsec]

Spatial structure of magnetic field lines



$t = 0\ \tau_A$

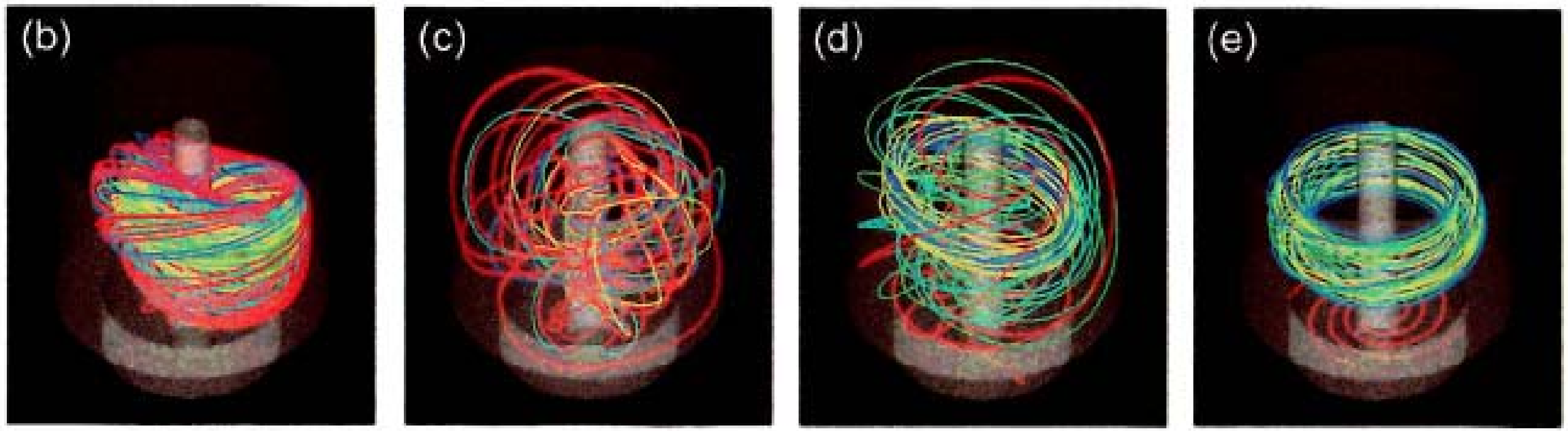
(a) Helicity-driven ST configuration
---- Open field lines (red) surround closed field lines

(b) The $n = 1$ distortion and
magnetic reconnection between open and closed fields

(c) Collapse of initial flux surfaces

(d) Partial generation of a torus-shaped core field

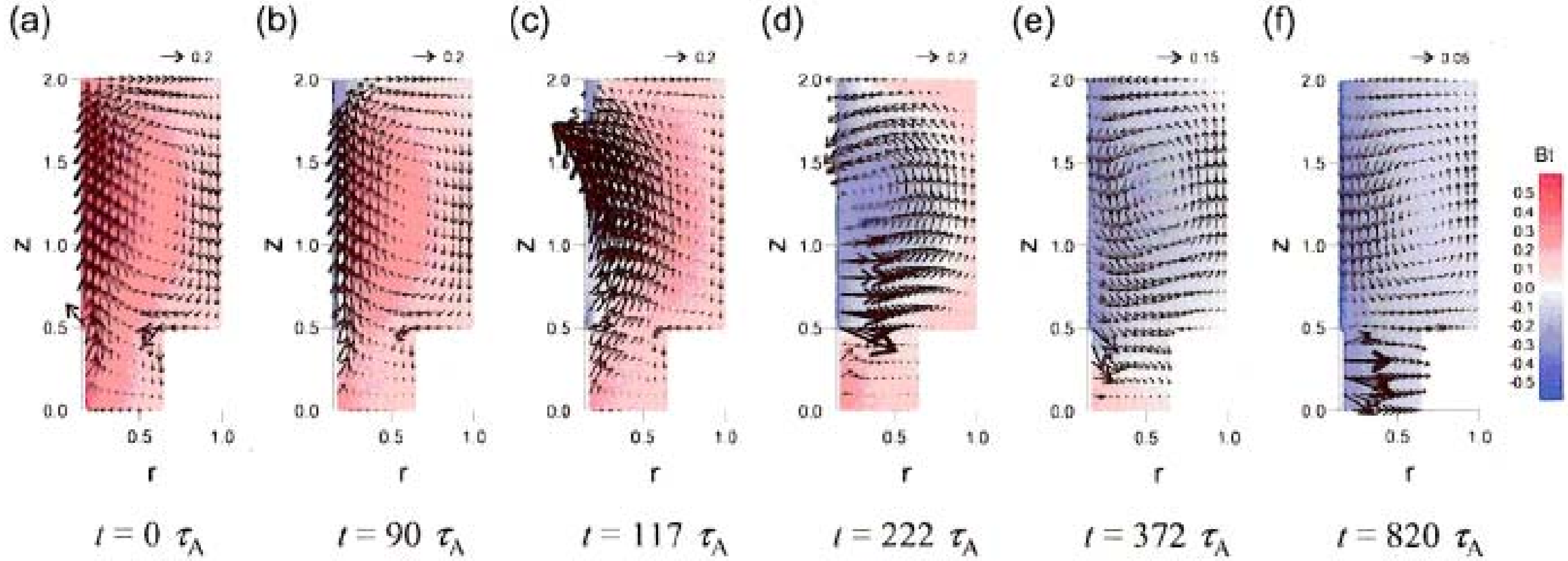
(e) Self-organization of an axisymmetric steady configuration
---- Open field lines directly connects without encircling the closed flux



$t = 117\ \tau_A$ $t = 222\ \tau_A$ $t = 485\ \tau_A$ $t = 820\ \tau_A$

[TF just reverses at $t = 100\ \tau_A$] (Field lines are classified by color so that it may be legible.)

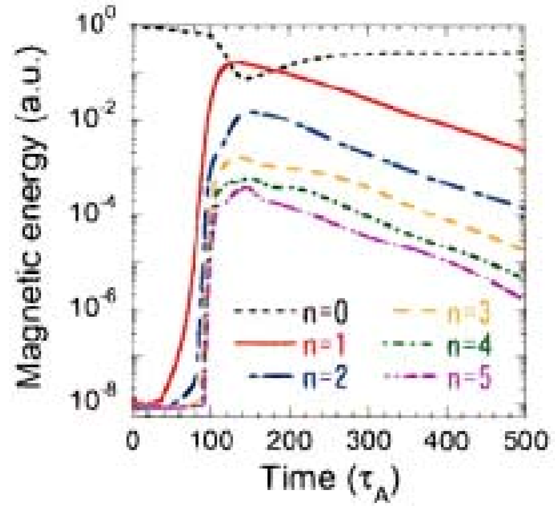
Vector and contour plots of magnetic fields



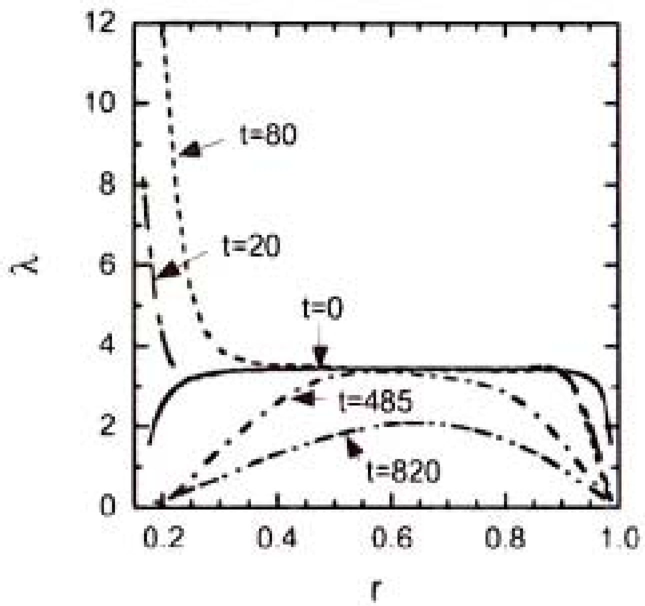
- We confirm that the direction of poloidal magnetic field as well as toroidal magnetic field reverses.

We have demonstrated the formation of the flipped ST configuration.

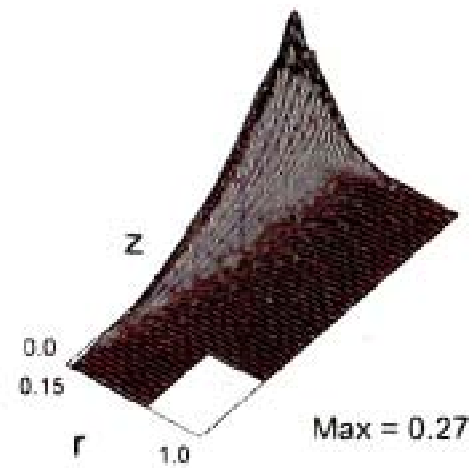
- We see a large distortion of fields around the center post, mainly $n = 1$, as the toroidal field at the inner edge is reversed.



Evolution of λ profile



$n = 1$ magnetic energy at $t = 80\ \tau_A$



- Enhancement of λ in the open flux region around the center post makes the $n = 1$ mode unstable there.

  —— That is ensured by stability analysis for the configurations of different λ in the open and closed flux region. [D. P. Brennan *et al.* Phys. Plasmas **9**, 3526 (2002)]

- During the transition to the flipped ST configuration, an unstable plasma of high λ in the open flux region relaxes to a stable one of low λ.

Summary

- The dynamics of a ST plasma, when the external toroidal magnetic field is reversed, is investigated using 3-D MHD numerical simulations.

In result,

- We have demonstrated that the ST plasma relaxes to a flipped ST state, accompanied by self-reversal of both paramagnetic toroidal field and poloidal field.
- The self-reversal of magnetic polarity is caused by the growth of $n = 1$ mode in central open flux and the following reconnection event between open and closed flux.
- The enhancement of λ in the central open flux is responsible for the growth of $n = 1$ mode there.
- The formation of the flipped ST configuration corresponds to the relaxation of high λ plasma in central open flux region to low λ state.

Progress and future work

- Quantitative analyses of magnetic helicity and magnetic energy will be conducted to assure that the self-reversal of fields indicates *global helicity conservation*.
- Sustainment mechanism for the flipped ST configuration by CHI will be investigated.