

Response of Tokamaks to Non-axisymmetric Magnetic Perturbations

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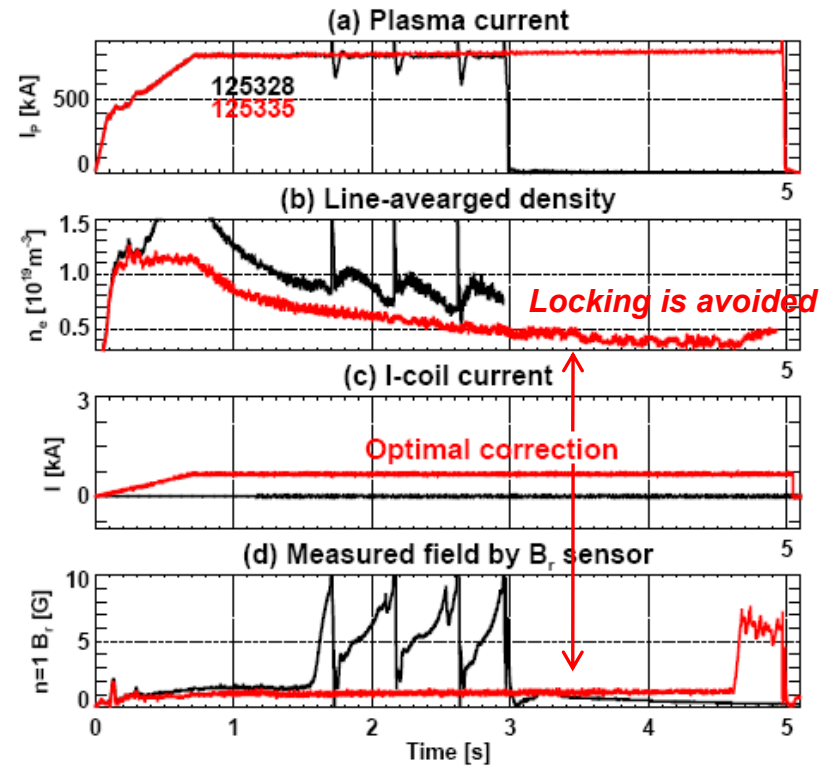
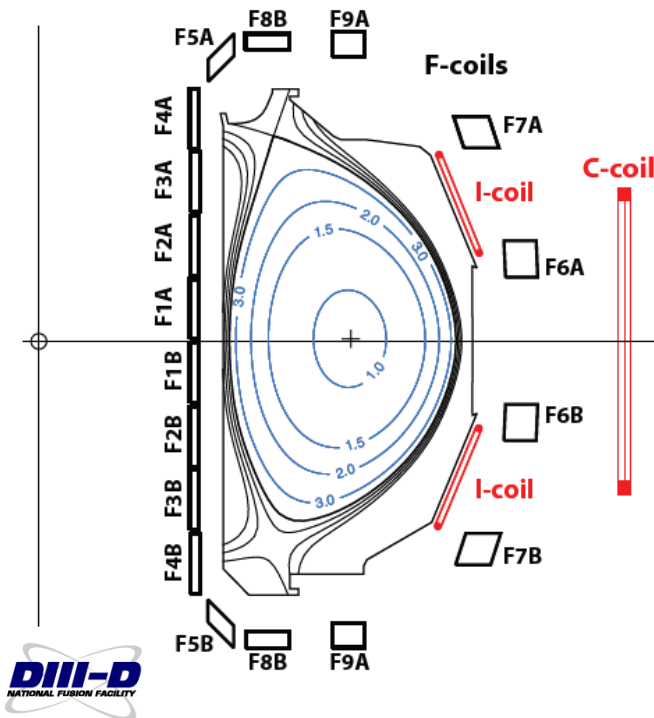
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Motivation

- Tokamaks are not in perfect axisymmetry
 - Small non-axisymmetric (3D) fields are unavoidable in tokamaks
 - Non-axisymmetric fields are typically bad (Locking, tearing, braking), but also can be good if controlled (RMP ELM suppression)
- Theoretical understanding for 3D equilibria is required
 - Non-ideal 3D codes (M3D, NIMROD,...) : Not free boundary yet...
 - Ideal 3D codes (VMEC,...) : Slow and not optimal to tokamaks
 - Vacuum 3D codes (Vacuum superposition) : Not in force-balance
- So, we use ideal perturbed equilibria :
 - Pros: Free-boundary, precise at rational surfaces, in force-balance
 - Cons: Ideal evolution, No inner-layer dynamics, etc

DIII-D example of plasma responses to non-axisymmetric magnetic perturbations

- DIII-D has $n=1$ intrinsic error fields
 - Error fields can cause locking (opening of islands)
 - C-coil and I-coil can mitigate locking effects



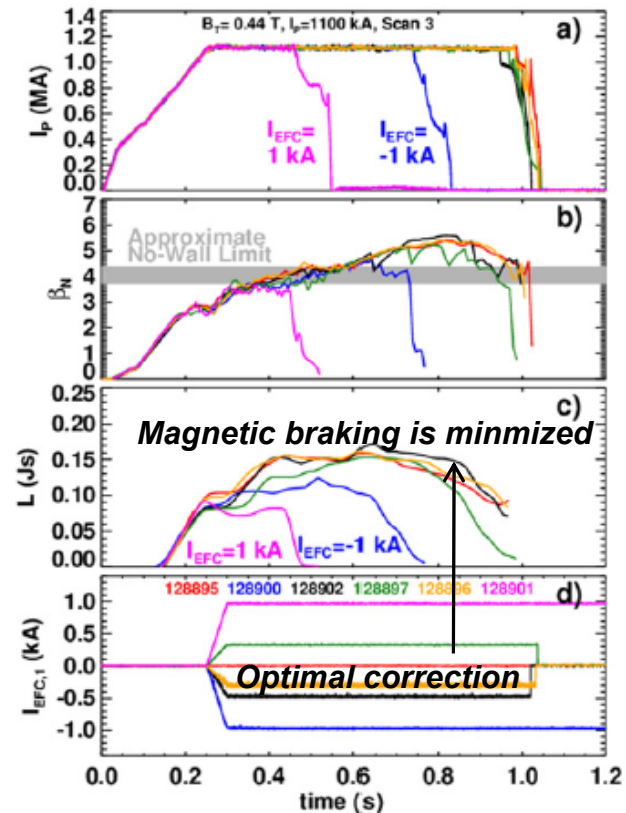
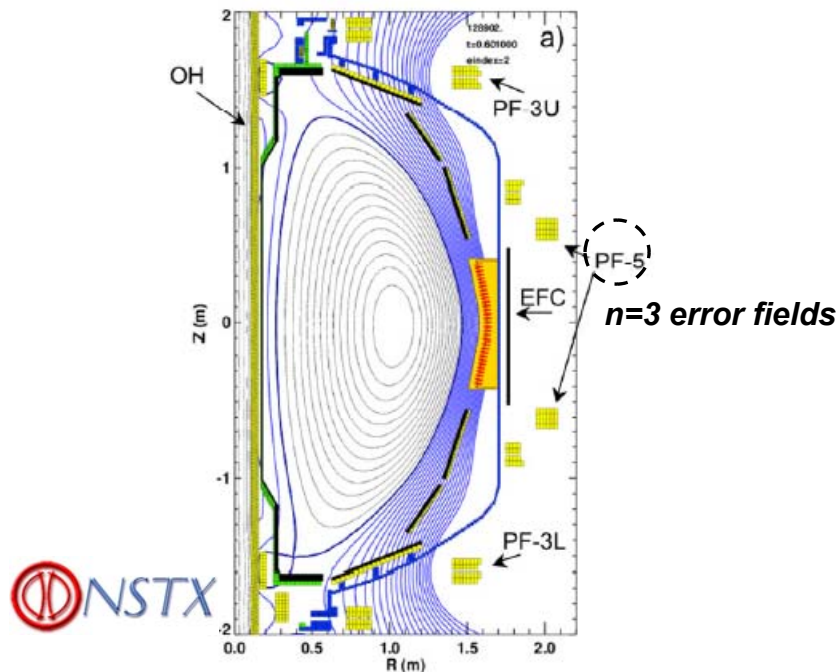
[Park, PRL 99, 195003 (2007)] [Park, submitted to POP (2010)]

NSTX example of plasma responses to non-axisymmetric magnetic perturbations

- NSTX has $n=1$ and $n=3$ intrinsic error fields
 - $n=3$ error fields can cause magnetic braking (rotation damping)
 - EFC correction can mitigate both effects

[Menard, NF 50, 045008 (2010)]

[Gerhardt, PPCF 52, 104003 (2010)]



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- Development of Ideal Perturbed Equilibrium Code (IPEC)
- IPEC applications to tokamaks
 - Plasma responses to applied perturbations
 - Error field corrections and plasma locking
 - Non-ambipolar transport and magnetic braking
- Control of non-axisymmetric perturbations
 - Dominant external field distribution and overlap field
 - **ITER error field study**
- Summary and Future Work

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IPEC solves ideal free-boundary perturbed equilibria

- Given an axisymmetric equilibrium, $\vec{\nabla} p_0 = \vec{j}_0 \times \vec{B}_0$, and given an non-axisymmetric field, $\delta \vec{B}^x(\vec{x})$, IPEC solves

[Park, POP 14, 052110 (2007)]

$$\vec{F}[\vec{\xi}] = \vec{0} = \delta \vec{j} \times \vec{B}_0 + \vec{j}_0 \times \delta \vec{B} - \vec{\nabla} \delta p$$

$$\delta \vec{B} = \vec{\nabla} \times (\vec{\xi} \times \vec{B}_0) \quad \text{and} \quad \delta \vec{j} = (\vec{\nabla} \times \delta \vec{B}) / \mu_0$$

$$\delta p = -\vec{\xi} \cdot \vec{\nabla} p_0 - \gamma p_0 (\vec{\nabla} \cdot \vec{\xi})$$

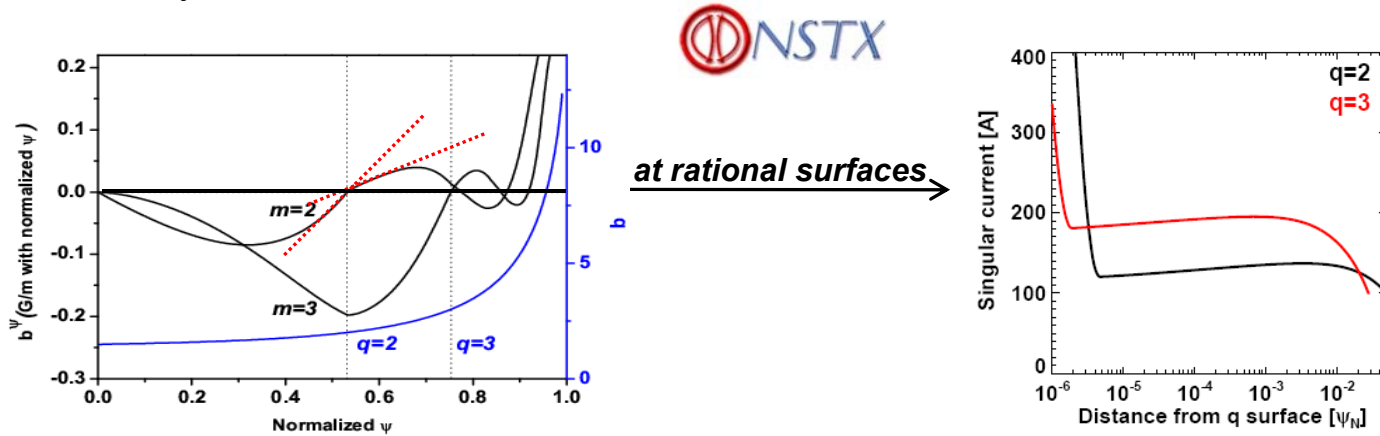
$p_0(\psi)$ and $q_0(\psi)$ profiles are preserved (ideal constraints)

- External field is given, and the total field is determined by virtual surface currents (external boundary condition)
 - External (vacuum) field : $\delta \vec{B}^x$
 - Total (External + Plasma) field : $\delta \vec{B} = \delta \vec{B}^x + \delta \vec{B}^p$
- Then, DCON stability code solves ideal fixed-boundary perturbed equilibria without islands (internal boundary condition)

Ideal constraints give total resonant fields driving islands

- Ideal constraints (no islands) give shielding current by the jump in the tangential field at the rational surface :

$$\delta \vec{j}_{\parallel} = \frac{i \Delta_{mn} e^{i(m\vartheta - n\varphi)}}{\mu_0 n \int B_0^2 / |\nabla \psi|^3 da} \delta(\psi - \psi_{mn}) \vec{B}_0 \quad \text{with the jump } \Delta_{mn} = \left[\frac{\partial}{\partial \psi} \left(\frac{\delta \vec{B} \cdot \vec{\nabla} \psi}{\vec{B}_0 \cdot \vec{\nabla} \vartheta} \right) \right]_{mn}$$

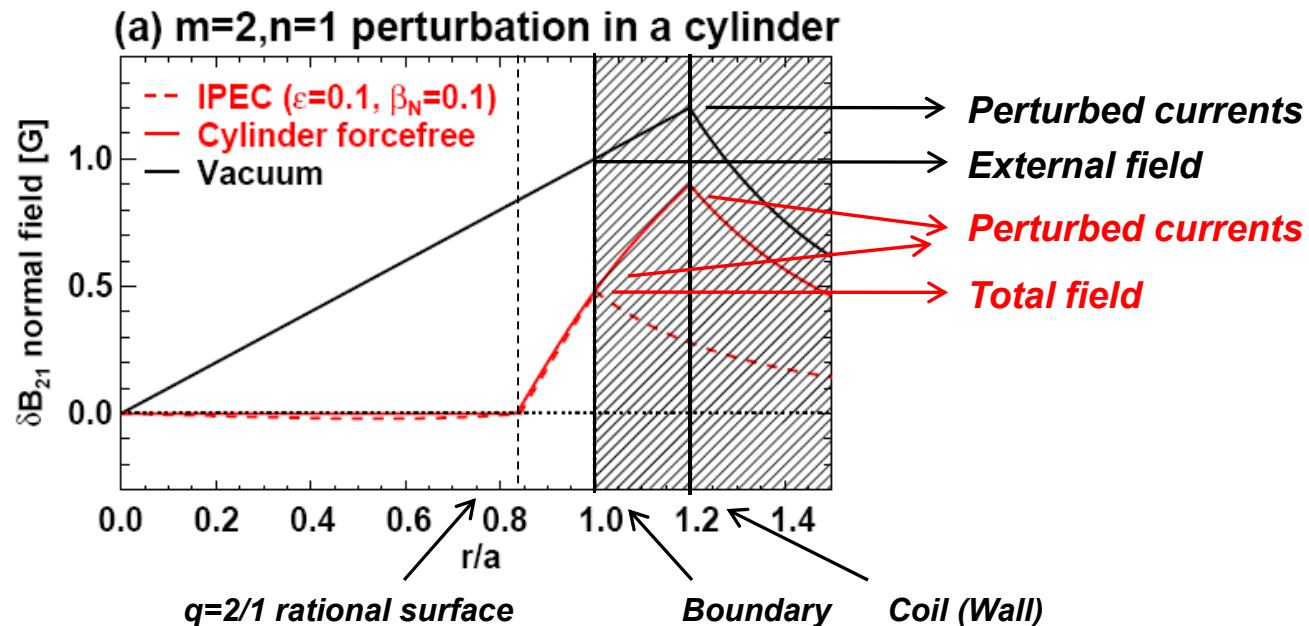


- The field suppressed by the shielding current : $\vec{\nabla} \times \delta \vec{B} = \mu_0 \delta \vec{j}_{\parallel}$
- The total resonant field would drive islands without shielding currents :

$$\delta B_{mn} \equiv \frac{\int \delta \vec{B} \cdot d\vec{a} e^{-i(m\vartheta - n\varphi)}}{\int da}$$

IPEC free-boundary solutions have been benchmarked in simple limits

- IPEC solutions are verified in a cylinder



[Park, POP 16, 056115 (2009)]

- IPEC free boundary equilibria are benchmarked with cylindrical force-free solutions (and also with CAS3D in simple limits)
- IPEC and vacuum solutions are very different even in simple limits

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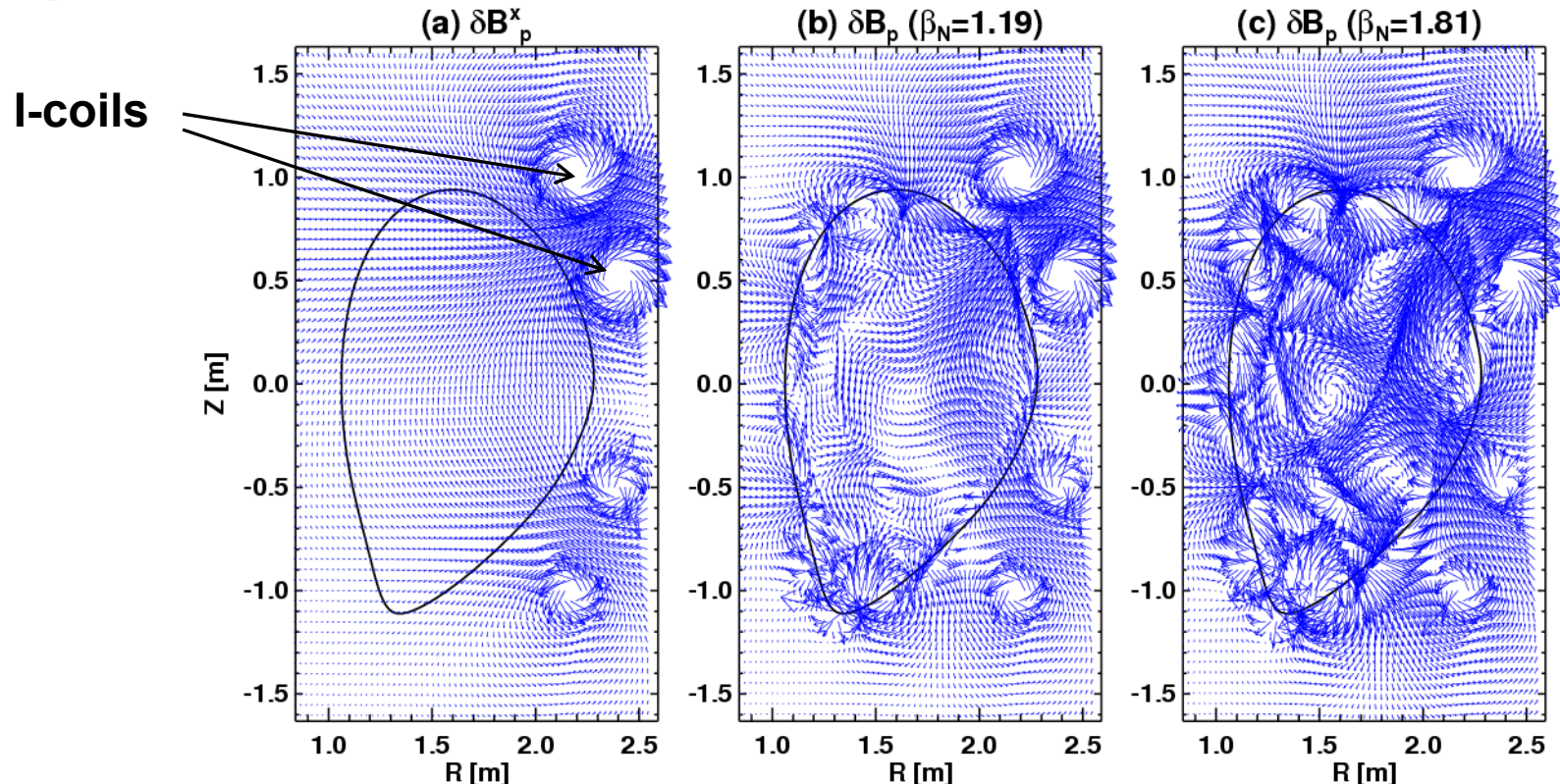
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Tokamak responses can be very strong by poloidal coupling, amplification or shielding

- Ideally perturbed plasma currents can strongly modify vacuum field in tokamaks and give strong plasma responses

DIII-D NATIONAL FUSION FACILITY **n=1 RFA**

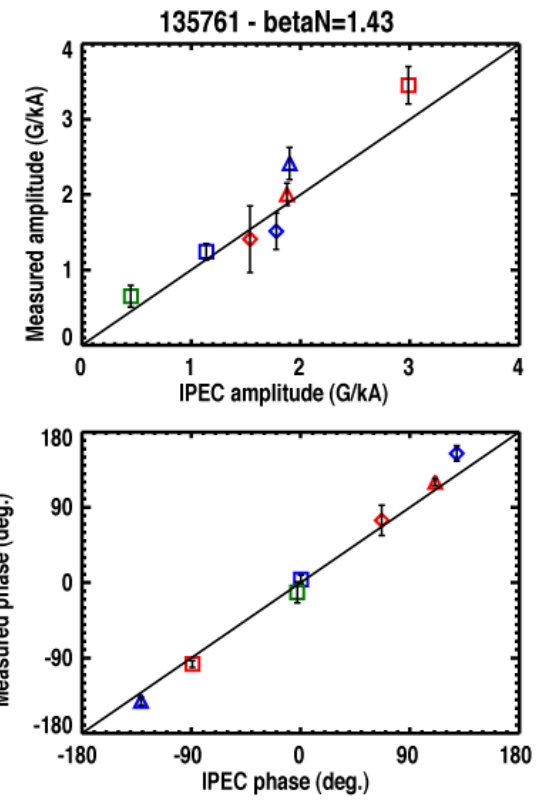
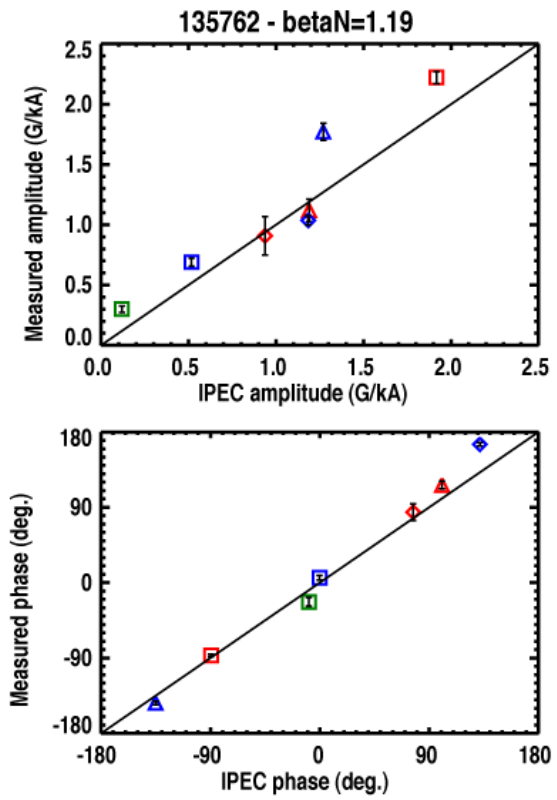
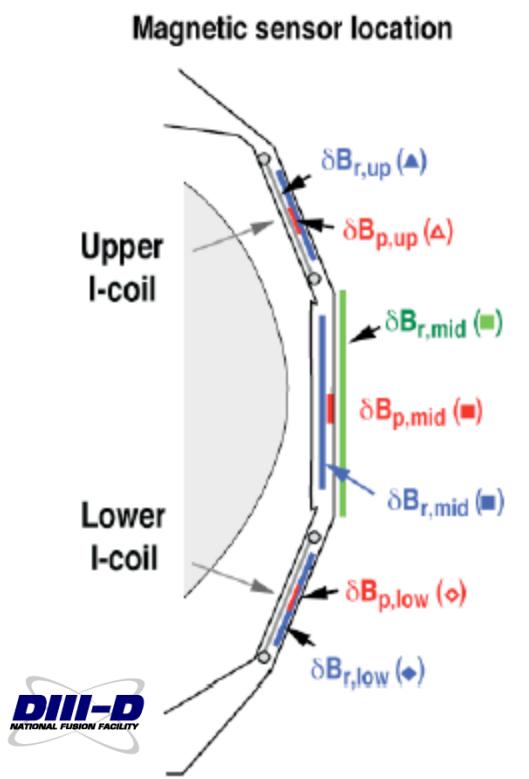
[Lanctot, Reimerdes, Park]



Ideal predictions are validated by plasma response measurements when plasma is stable

- IPEC, MARS-F codes and DIII-D plasma response measurements show good agreements when plasma is stable

n=1 RFA



[Lanctot, Reimerdes, Park]

[For MARS-F applications, see BI3.00002 by Lanctot]

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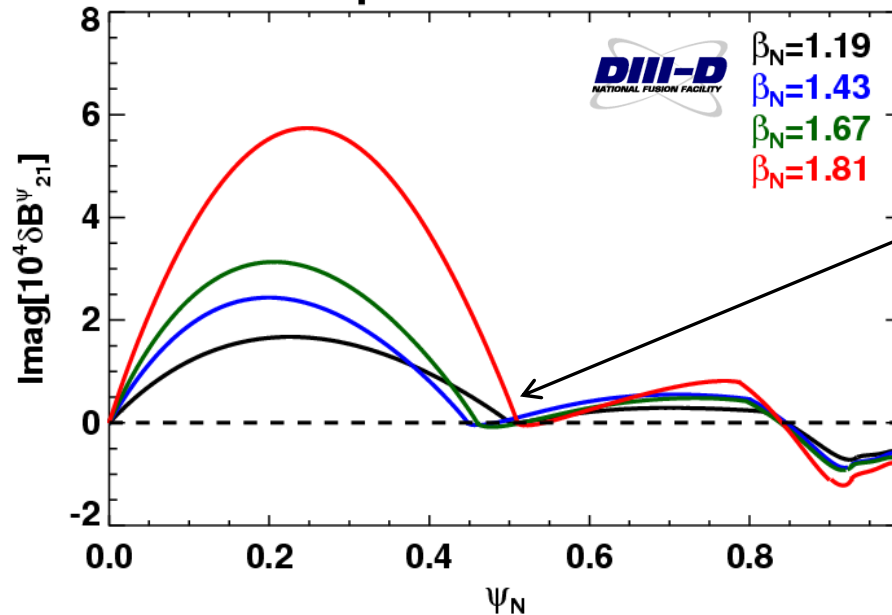
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IPEC total resonant fields give the drive of magnetic islands and locking

- IPEC total resonant fields are self-consistent in ideal limit and give the drive of magnetic islands and locking
- This is valid before the onset of locking

[Lanctot, Reimerdes, Park]

n=1 components of normal field

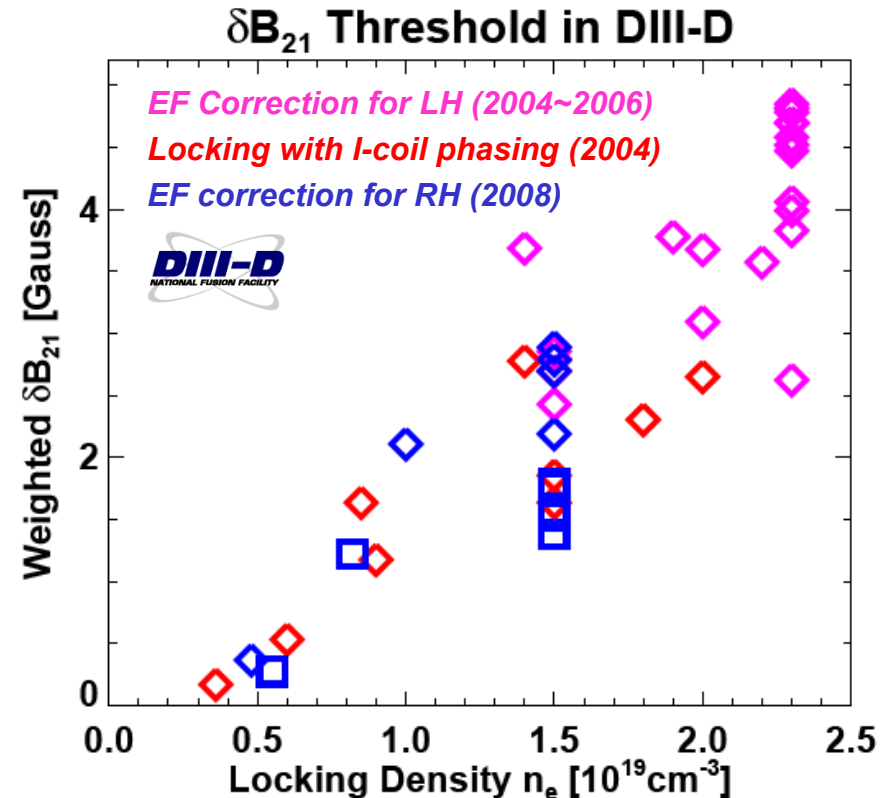
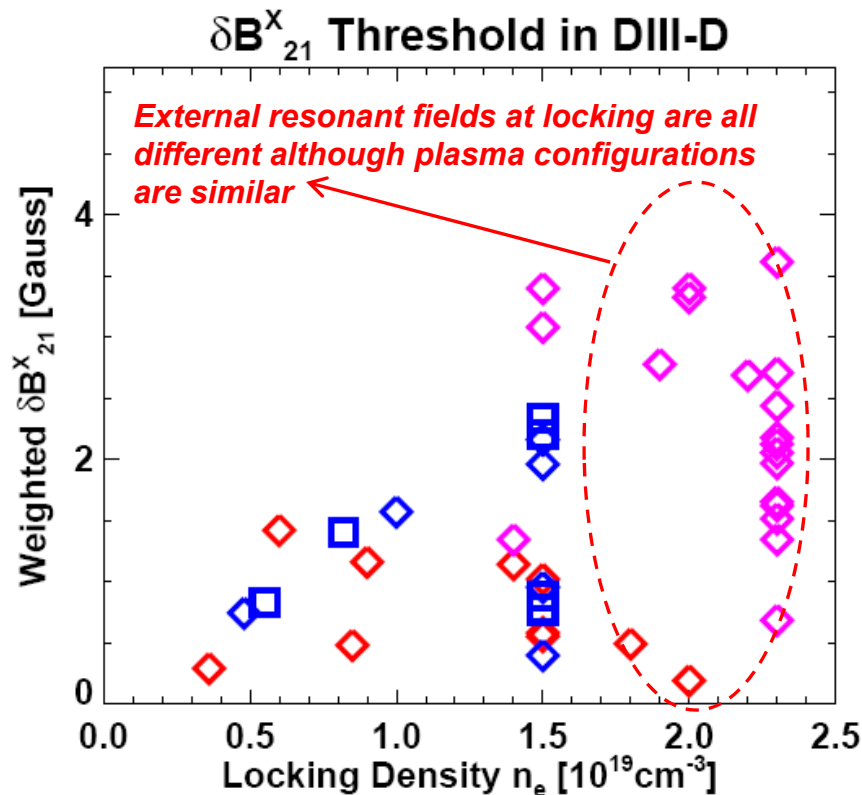


Total resonant fields are suppressed, but become larger when plasma has higher pressure. Required shielding currents also become larger, and islands can open if plasma can not maintain shielding currents

Total resonant fields explain much better error field correction results than vacuum methods

- External resonant field (based on standard vacuum superposition, $\delta B^P=0$) showed often paradoxical results
- IPEC resonant field restored good parametric correlation as expected

[Schaffer, LaHaye, Scoville, Park]



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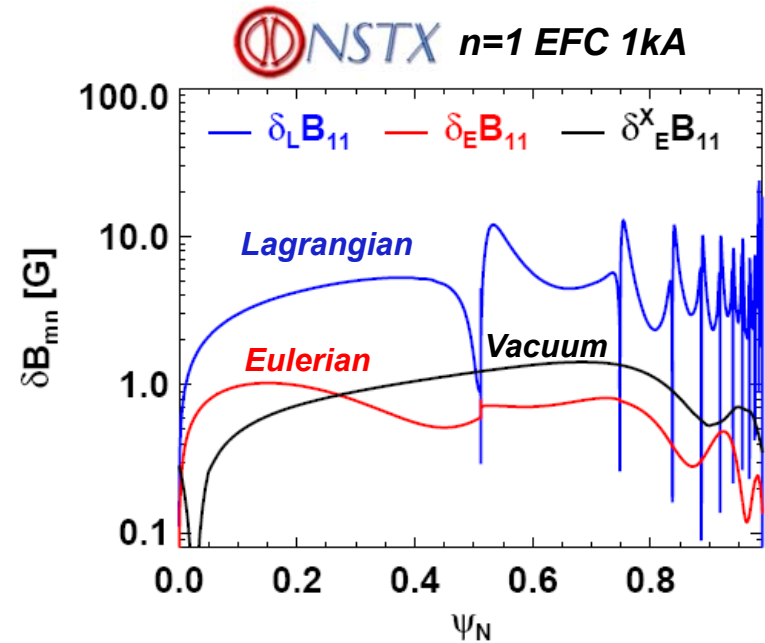
Non-axisymmetric variation in $|B|$ produces non-ambipolar transport

- Action is dependent on toroidal location in the presence of the non-axisymmetric variation in the field strength
- Action must be conserved, so a particle must have an additional radial non-ambipolar transport (Called NTV transport in tokamaks)
- Important variation occurs by the variation in the field strength along the perturbed field lines, not along unperturbed field lines : Need perturbed equilibrium calculations

Lagrangian $\delta_L B \equiv \delta_E B + \vec{\xi} \cdot \vec{\nabla} B_0$

Eulerian $\delta_E B \equiv \delta \vec{B} \cdot \hat{b}_0$

Vacuum $\delta_E B^x \equiv \delta \vec{B}^x \cdot \hat{b}_0$



[Boozer, POP 13, 044501 (2009)]

[Park, POP 16, 056115 (2009)]

NTV formula across different regimes has been derived with additional approximations

- New NTV formula has been derived with effective Krook collisional operators to combine different regimes [Park, PRL 102, 065002 (2009)]

– NTV has the form of

$$\tau_{\varphi} \cong C [\delta B]^2 \frac{v_{eff}}{(\ell \omega_{\ell bounce} - n \omega_{E \times B} - n \omega_{\nabla B})^2 + v_{eff}^2} (\omega_{rot} - \omega_{neo})$$

– NTV formula gives the toroidal torque density as

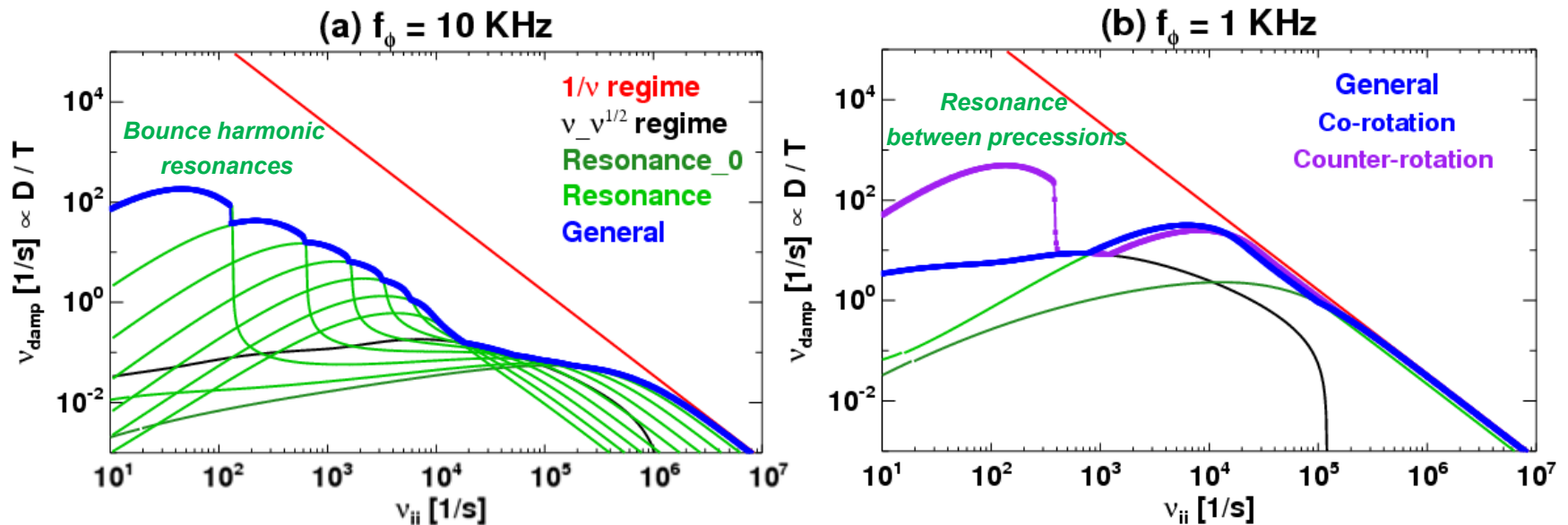
$$\langle \hat{\phi} \cdot \vec{\nabla} \cdot \vec{\Pi}_a \rangle_{\ell} = \underbrace{\frac{\epsilon^{-1/2} p_a}{\sqrt{2} \pi^{3/2} R_0}}_C \underbrace{\int_0^1 d\kappa^2 \delta_{w,\ell}^2}_{\delta B^2} \underbrace{\int_0^{\infty} dx \mathcal{R}_{a1\ell}}_{Resonance} \underbrace{\left[u^{\varphi} + 2.0 \sigma \left| \frac{1}{e} \frac{dT_a}{d\chi} \right| \right]}_{\omega_{rot} - \omega_{neo}}$$

$$\delta_{w,\ell}^2 = \sum_{nmm'} \delta_{nmm'}^2 \frac{F_{nm\ell}^{-1/2} F_{nm'\ell}^{-1/2}}{4K(\kappa)}, \quad F_{nm\ell}^y \equiv \int_{-\vartheta_t}^{\vartheta_t} d\vartheta (\kappa^2 - \sin^2(\vartheta/2))^y \cos(m - nq - \sigma\ell)\vartheta$$

$$\mathcal{R}_{a1\ell} = \frac{1}{2} \frac{n^2 (1 + (\frac{\ell}{2})^2)^{\frac{\nu_a}{2\epsilon}} x (x - \frac{5}{2})^y e^{-x}}{\left[\ell \frac{\pi \sqrt{\epsilon}}{4\sqrt{2}} \omega_{ta} \sqrt{x} - n \omega_E - n \sigma \frac{q^3}{4\epsilon} (\omega_{ta}^2 / \omega_{ga}) x \right]^2 + \left[(1 + (\frac{\ell}{2})^2)^{\frac{\nu_a}{2\epsilon}} \right]^2 x^{-3}}$$

Combined NTV gives right order of magnitudes as observations

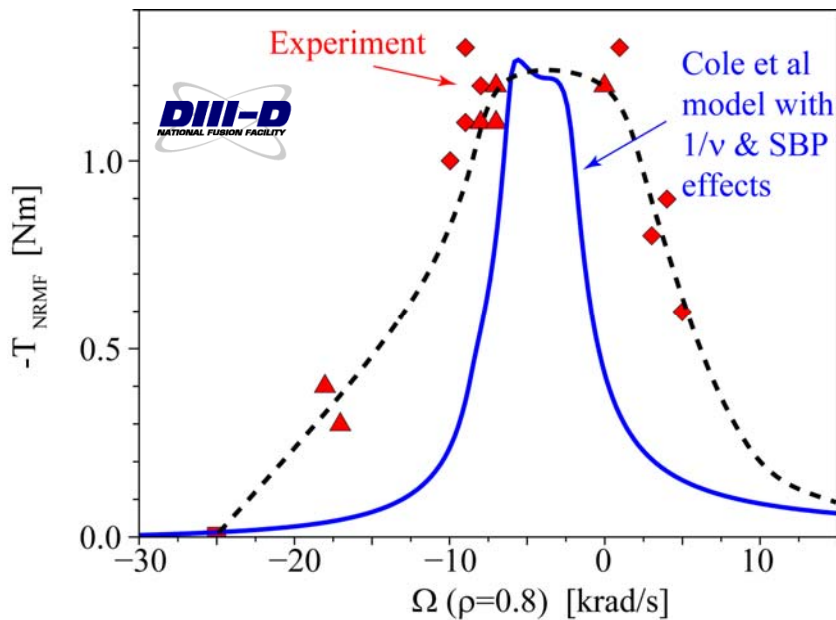
- Combined formula gives $10\sim 100\text{ s}^{-1}$ damping rates as observed in relevant parametric spaces of tokamak magnetic braking experiments
- Various predictions can be made :
 - NTV can be stronger in lower ν , due to bounce-harmonic resonances
 - NTV can be stronger by counter-rotation in lower ν and lower rotations, due to resonances between electric and magnetic precessions



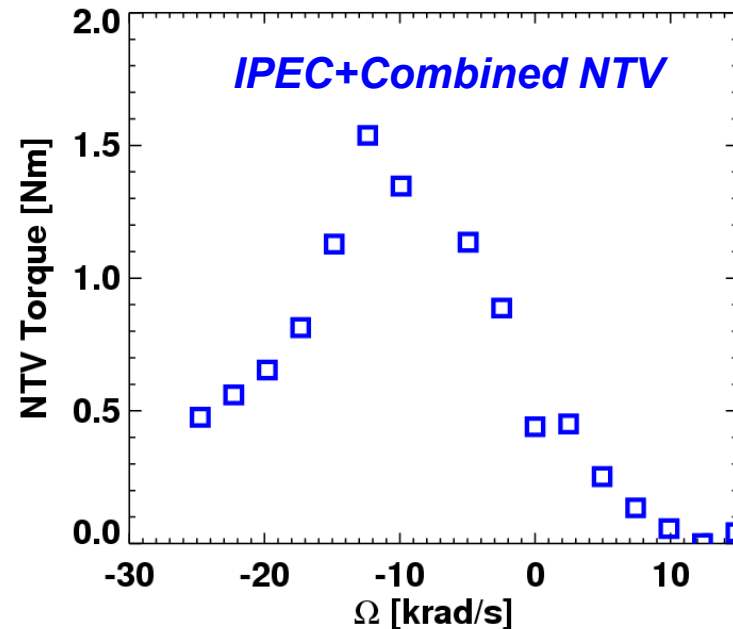
Combined NTV theory with IPEC field is being compared with magnetic braking experiments

- DIII-D magnetic braking experiments have shown resonant behaviors in NTV when rotations are low
- SuperBanana Plateau (SBP) theory predicts the resonances
- IPEC field + combined NTV theory also gives good predictions using actual fields and all profiles in experiments

[See PI2.00005 by Cole]



[J.-K. Park, A. M. Garofalo, W. Solomon]



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Non-axisymmetric external perturbations can be decomposed by their importance for physics

- A physically important quantity, such as total resonant field, can be represented by a matrix coupled to external representation

Total resonant field at rational surfaces

$$\delta B_{rmn} \equiv \frac{\int \delta \vec{B} \cdot d\vec{a}_r e^{-i(m\vartheta - n\varphi)}}{\int da_r}$$

← IPEC

External field on the boundary

$$\delta B_{bmn}^x = \frac{\int \delta \vec{B}^x \cdot d\vec{a}_b^{1/2} e^{-i(m\vartheta - n\varphi)}}{\sqrt{\int da_b}}$$

$$\vec{B}_r [R] = \vec{C} [R \times M] \cdot \vec{\Phi}_b^x [M]$$

- SVD analysis gives new decomposition of external field based on its importance on the physical quantity

$$\vec{C} = \left(\vec{U}_{b1}^x s_1 \vec{V}_{b1}^{x\dagger}, \dots, \vec{U}_{bR}^x s_R \vec{V}_{bR}^{x\dagger} \right) \quad \left| \vec{V}_{bi}^x \right| = 1$$

- The first mode is defined the “dominant external field”, since the first mode is more important often by an order of magnitude than others

Dominant external field is highly robust across different tokamak configurations

- Dominant external field is highly robust across various tokamak configurations and plasma parameters [Park, NF 48 045006 (2008)]

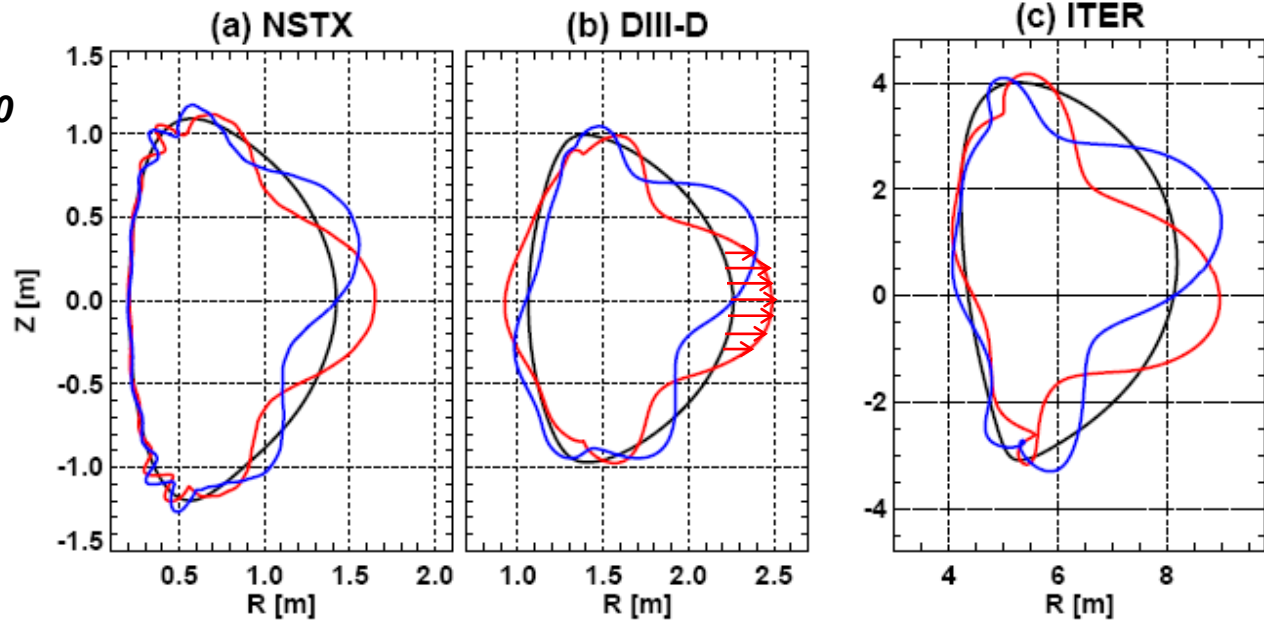
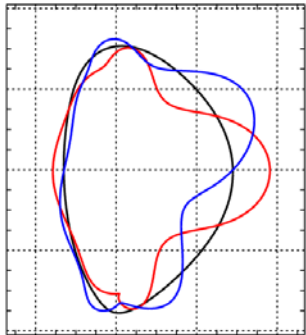
Shape of the dominant external field

<Cosine part (red) and Sine part (blue)> on the plasma boundary

$$\vec{V}_{b1}^x \rightarrow \delta \vec{B}^x \cdot \hat{n}_b = A(\theta) \cos(n\phi) + B(\theta) \sin(n\phi)$$

CMOD

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Error field effects can be understood based on overlap with the dominant external field

- Given error field can be decomposed

$$\vec{\Phi}_b^{xerr} = \left(\vec{\Phi}_b^{xerr} \cdot \vec{V}_{b1}^x \right) \vec{V}_{b1}^x + \dots$$

- If the dominant external field is almost only important distribution,

$$s_1 \gg s_i, \quad \vec{C} \cdot \vec{V}_{bi}^x \quad (i \geq 2) \cong 0$$

- Then the total resonant field is

$$\left| \vec{B}_r \right| = \left| \left(\vec{C} \cdot \vec{V}_{b1}^x \right) \left(\vec{\Phi}_b^{xerr} \cdot \vec{V}_{b1}^x \right) \right| = s_1 \left| \left(\vec{\Phi}_b^{xerr} \cdot \vec{V}_{b1}^x \right) \right|$$

- Overlap field is defined as

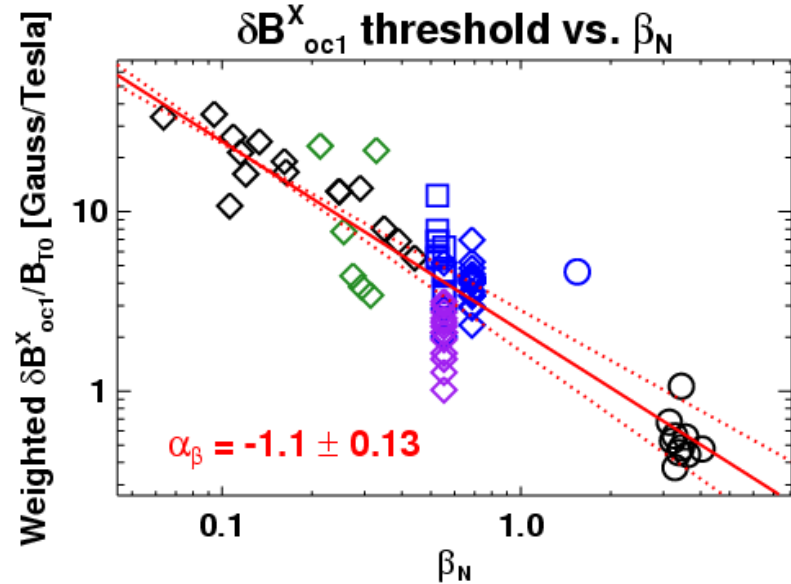
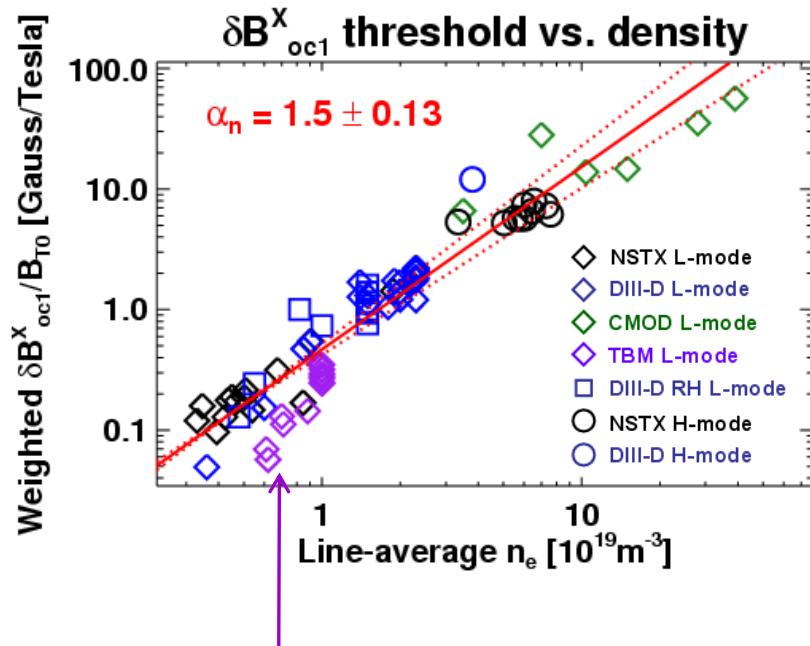
$$\delta B_o^x \equiv \left(\vec{\Phi}_b^{xerr} \cdot \vec{V}_{b1}^x \right)$$

- Overlap field is the external representation of the total resonant field

Scaling for locking drive becomes more reliable with overlap field

- Tokamak locking scaling based on overlap field :

$$\frac{\delta B_{oc1}^x}{B_{T0}} \leq 0.4 \times 10^{-4} \left(n [10^{19} m^{-3}] \right)^{1.5} \left(B_{T0} [T] \right)^{-1.9} \left(R_0 [m] \right)^{1.2} \beta_N^{-1.1}$$



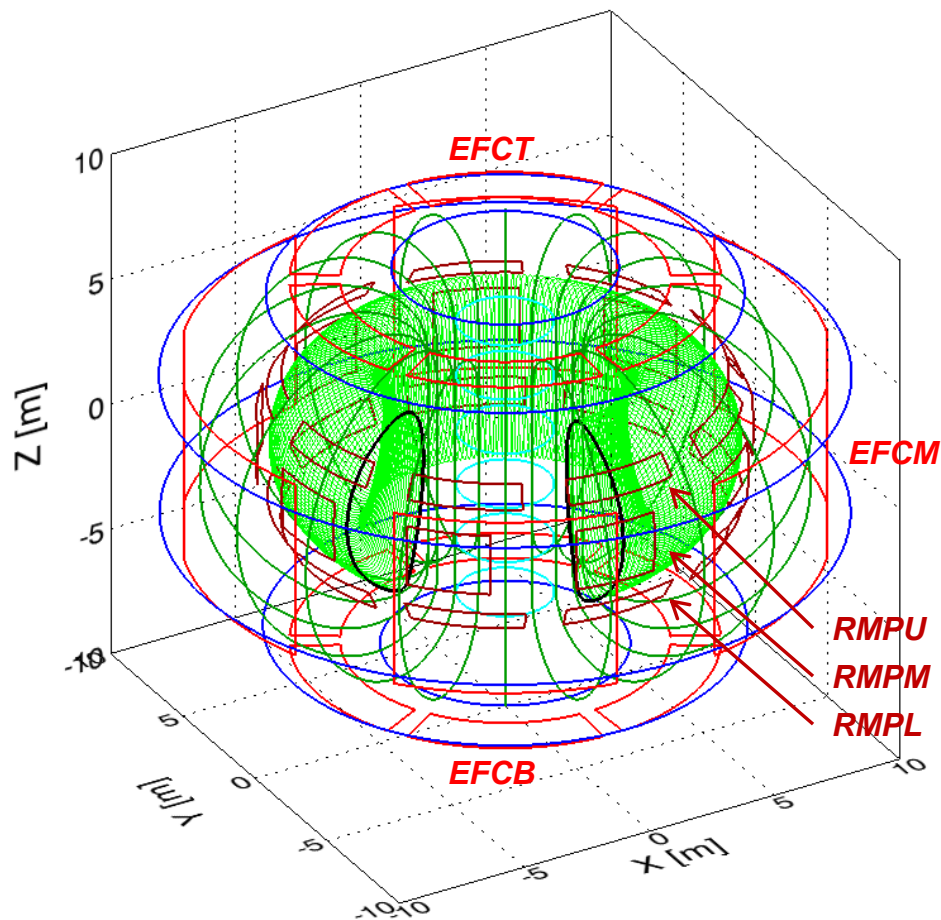
[For TBM cases, see BI3.00001 by Schaffer, and also see XO4.00003 on Friday]

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ITER error field correction coil capability has been revisited with new methods

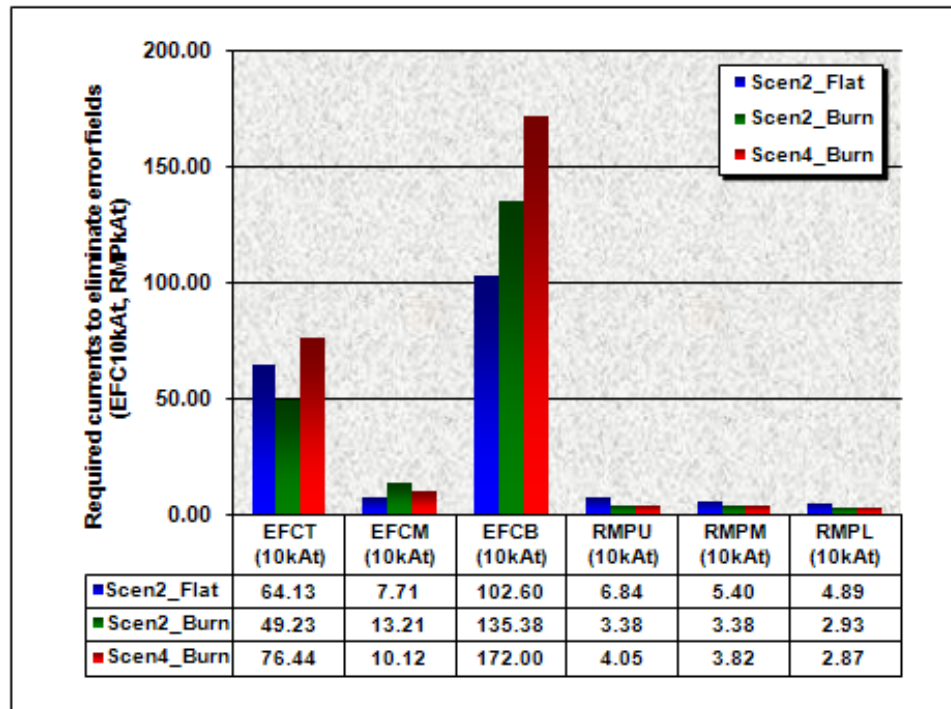
- Potentially ITER may have 3x6 Error Field Correction Coils (EFCC) and 3x9 Resonant Magnetic Perturbation Coils (RMPC)
- EFCC (or RMPC) must have capability to compensate error fields due to primary magnet (CS/PF/TF) coil distortions
- Coil capability has been revisited in order to include ideal plasma response effects



Worst error fields and required corrections were estimated based on overlap field

- Overlap field of each component shows
 - Worst error fields come from PF3~4 tilt
 - Correction capability of each correction : RMPU,M,L > EFCM >> EFCT,B

Required currents to eliminate the worst overlap fields



One conclusion : EFCT and EFCB coils are very inefficient for resonant field correction

- EFCT and EFCB coils are less efficient than EFCM by an order of magnitude for the resonant fields
- Are EFCT and EFCB coils still useful in terms of other physics?
- Even if so, can RMP coils do much better?
- Now we like to compare (EFCM+EFCT+EFCB) and (EFCM+RMPI+RMPL) with (EFCM only)

Other physics? : NTV is the best known physics element for non-resonant fields

- Neoclassical Toroidal Viscosity (NTV) causes plasma rotation damping and destabilizes various MHD
- NTV estimation presently requires perturbed equilibrium calculations for the Lagrangian variation in the field strength with various bounce harmonic integrations

$$\tau_{\phi} \cong C [\delta B]^2 \frac{v_{eff}}{(\ell \omega_{\ell bounce} - n \omega_{E \times B} - n \omega_{\nabla B})^2 + v_{eff}^2} (\omega_{rot} - \omega_{neo})$$

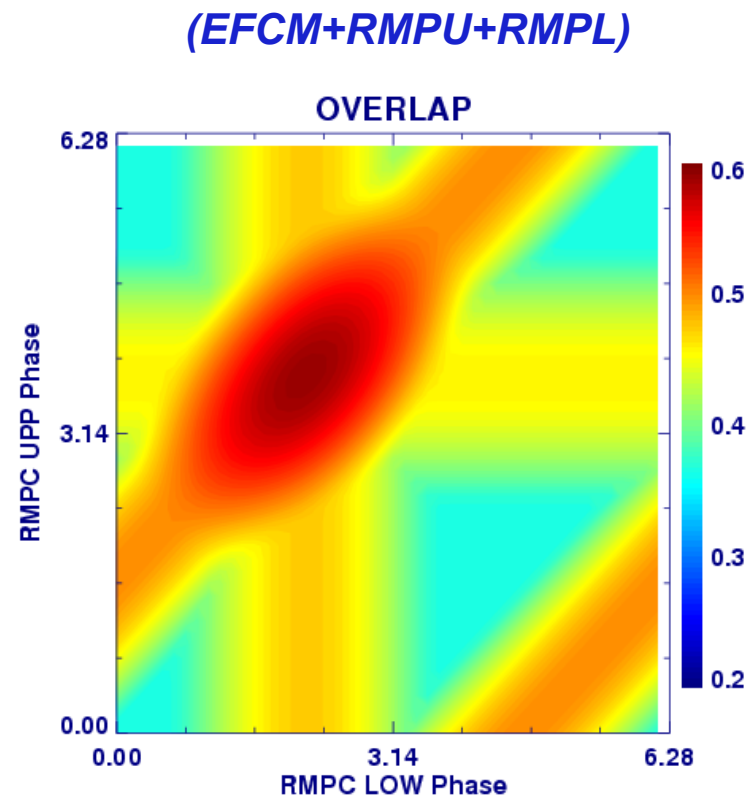
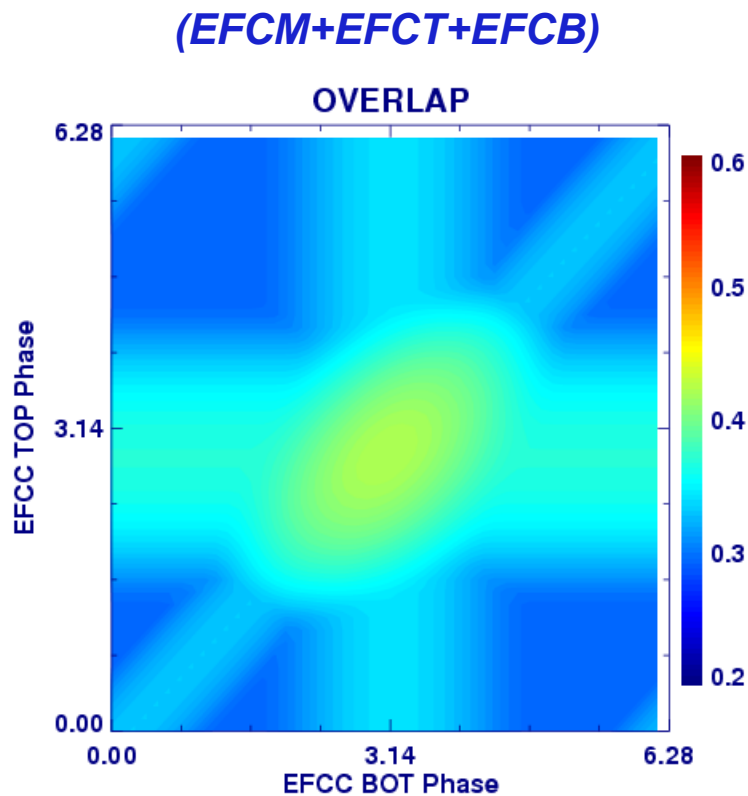
- That is, NTV is not easy to optimize
- So, we use overlap approximation : Correction should simultaneously
 - Remove the overlap field (remove the resonant components)
 - Maximize the overlap (minimize the non-resonant components)

$$C_o \equiv \frac{(\vec{\Phi}_b^{xerr} \cdot \vec{\Phi}_{b1}^x)}{|\vec{\Phi}_b^{xerr}| |\vec{\Phi}_{b1}^x|}$$

$$\text{Overlap (0-1)} = \frac{\text{Overlap field}}{\text{RMS surface average of the field}} \approx \frac{\text{Resonant component}}{\text{Sum of non-resonant component}}$$

Corrections were optimized based on overlap

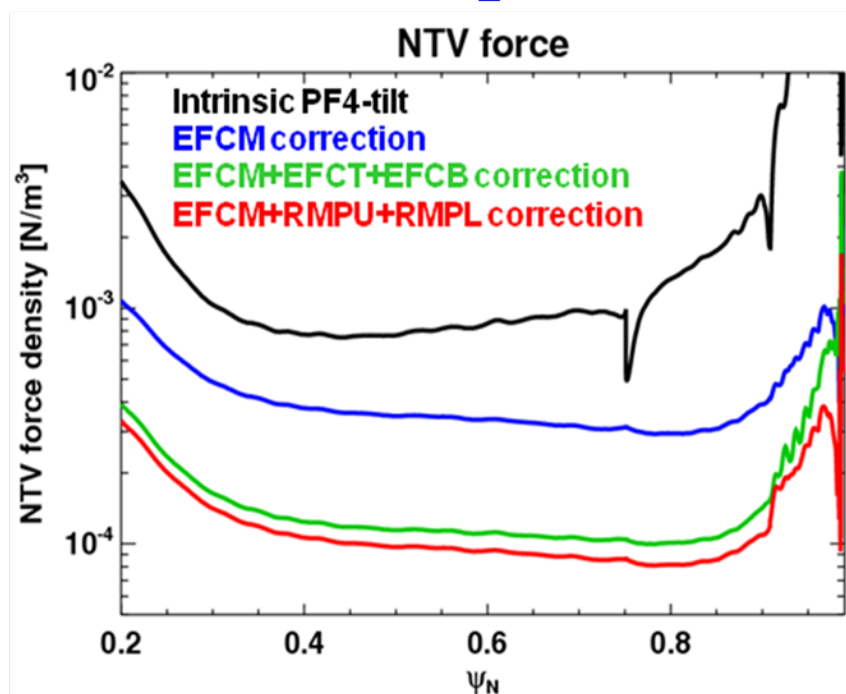
- EFCT (RMPU) and EFCB (RMPL) amplitudes and phases (relatively to a fixed EFCM amplitude and phase) were optimized to maximize overlap



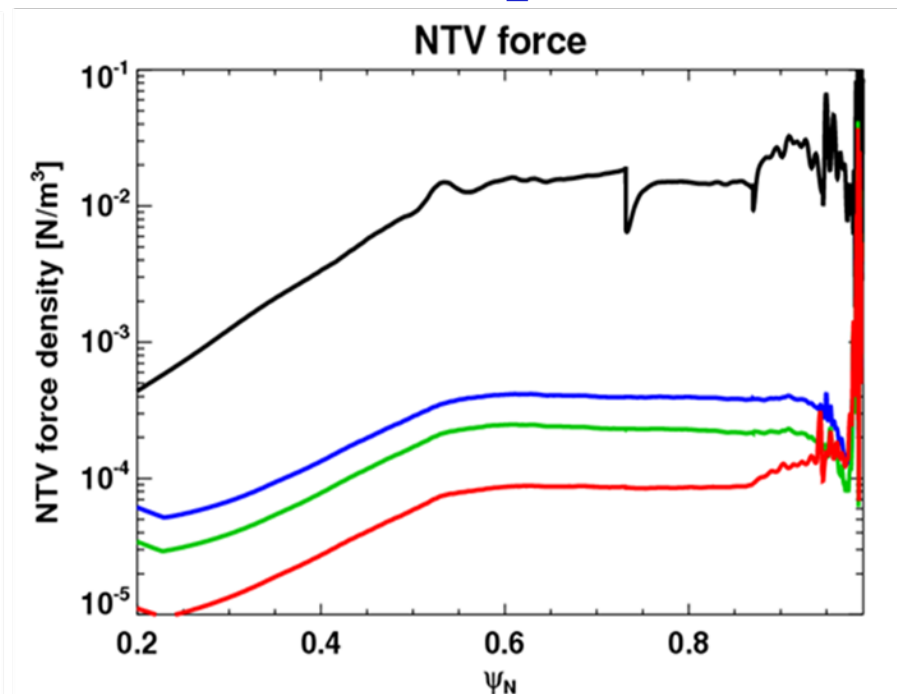
Optimized corrections give the best capability for both resonant and non-resonant components

- EFCM gives NTV reduction by 1~2 orders of magnitude
 - This indicates reduction of resonant components is also very important for NTV
- Further NTV reduction by a factor of 1~3 is possible by optimization
- However, required currents are **EFCM+EFCT+EFCB (95+164+257kAt)**
>> EFCM only (132kAt) > EFCM+RMPU+RMPL (71+23+23kAt)

Scen2_Burn



Scen4_Burn



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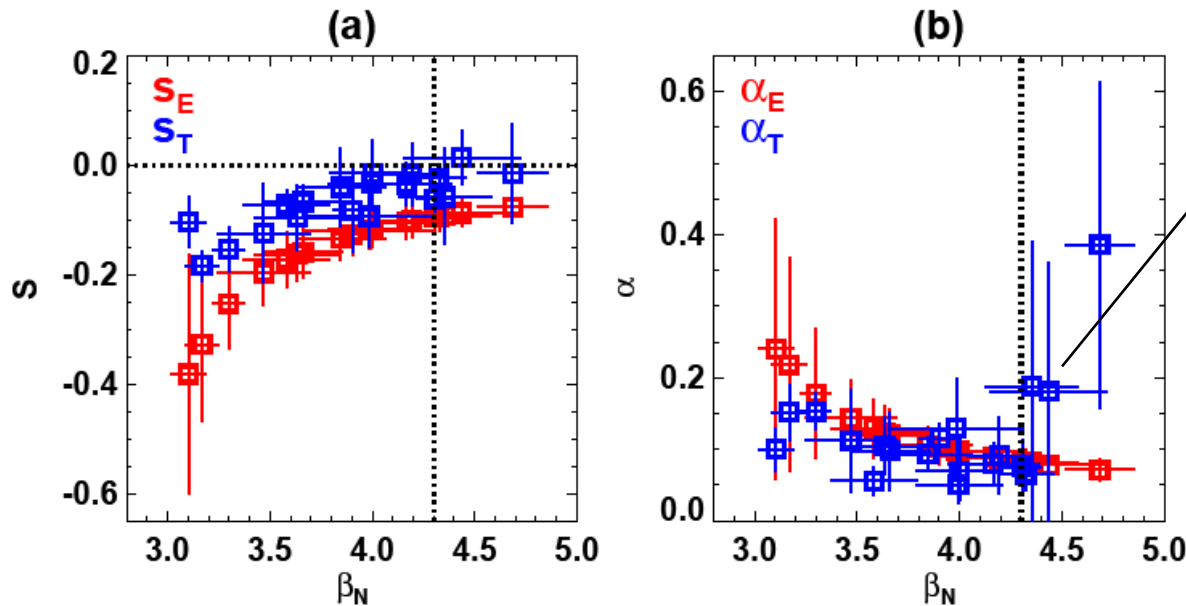
High- β plasmas require perturbed equilibrium calculations consistently with non-ideal forces

- Locking and NTV give a toroidal torque, which is not included in IPEC, and so IPEC is not consistent when the torque is large

$$\vec{\nabla} p_0 = \vec{j}_0 \times \vec{B}_0 - \left(\vec{\nabla} \delta p - \delta \vec{j} \times \vec{B}_0 - \vec{j}_0 \times \delta \vec{B} + \delta \vec{f} \right) \text{ where } \delta \vec{f} = \vec{\nabla} \cdot \vec{\Pi} + \dots$$

- Importance of the torque (non-ideal force) can be estimated as

$$s \equiv -\frac{\delta W}{\delta W_V} \quad \text{and} \quad \alpha \equiv -\frac{T_\phi}{2\delta W_V}$$



*In high beta plasmas,
The torque becomes too large,
indicating the importance of the
currents associated with the
torque in perturbed equilibria :
Importance of tensor pressure
perturbed equilibria*

[Park , POP 16, 082512 (2009)]

Summary and Future Work

- IPEC solves free-boundary ideal perturbed equilibria
- IPEC applications showed that plasma response to non-axisymmetric fields are important in tokamaks
 - Vacuum superposition approximation is generally not valid
- Combined NTV with ideal perturbed equilibria give reasonable explanations and predictions for non-ambipolar transport in tokamaks
- Dominant external field and overlap give useful tools to understand control of non-axisymmetric magnetic perturbations
 - Revision of ITER error field study has shown (EFCM+RMPL) is the best configuration for both resonant and non-resonant components
- Perturbed equilibria should include torques to be more self-consistent