Theory and observations of low frequency eigenmodes due to Alfvén acoustic coupling in toroidal fusion plasmas

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PPPL PRINCETON PLASMA PHYSICS LABORATORY

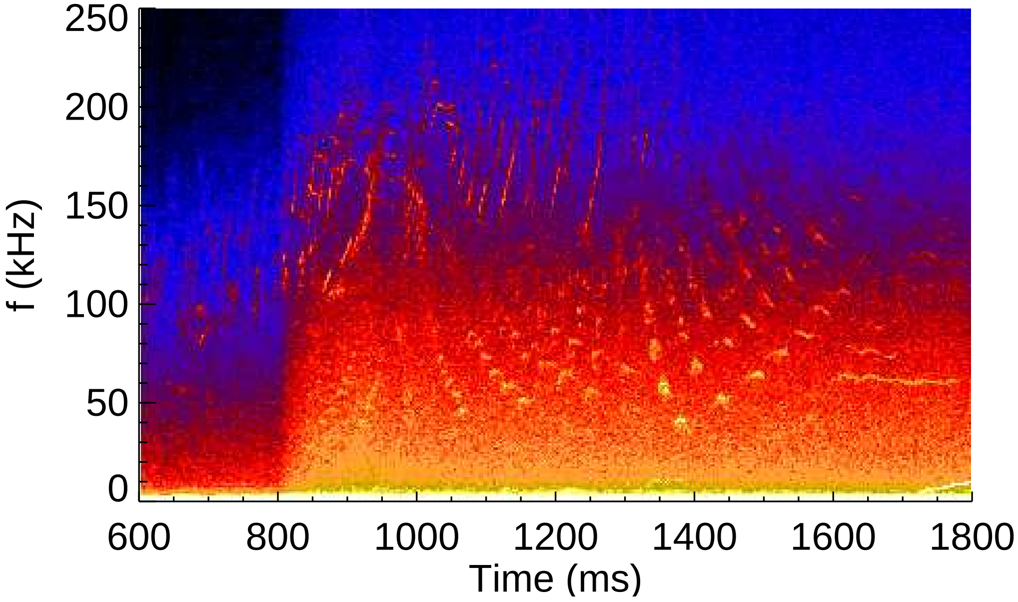
EFDA EUROPEAN FUSION DEVELOPMENT AGREEMENT JET

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

Interpretation of new experimental observations on JET, NSTX and DIII-D requires low frequency instability studies

Strongly heated plasmas with energetic particles reveal complicated MHD spectra with multiple instabilities

DIII-D ECE spectrum



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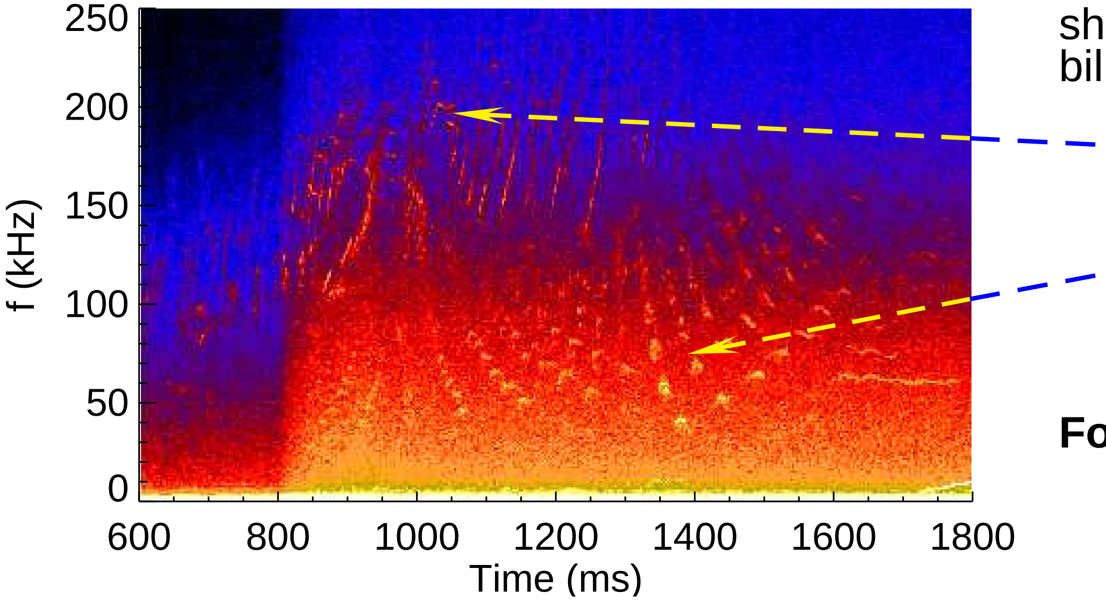

DIII-D ECE spectrum shows two sets of MHD instabilities:

- TAE/RSAEs and
  (toroidicity-induced/reversed shear AEs - Alfvén Cascades)
- new BAAE modes
  (Beta-induced Alfvén-acoustic Eigenmodes)

For both

- characteristic frequency is below TAE ($\sim 200kHz$)
- frequency sweep start correlates with rational $q(t)$
- indicates RS plasmas

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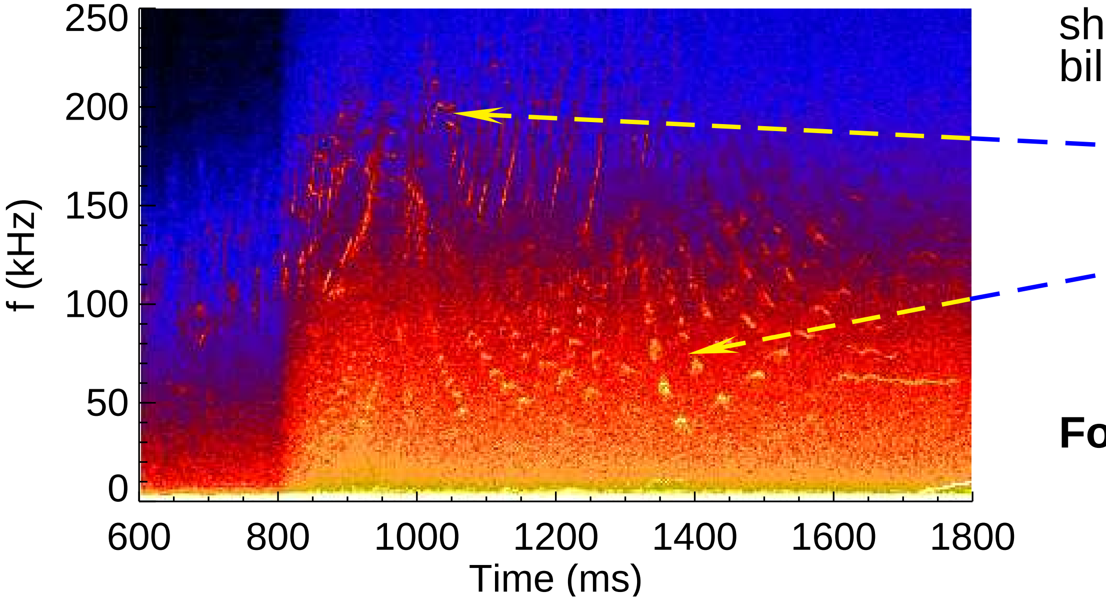

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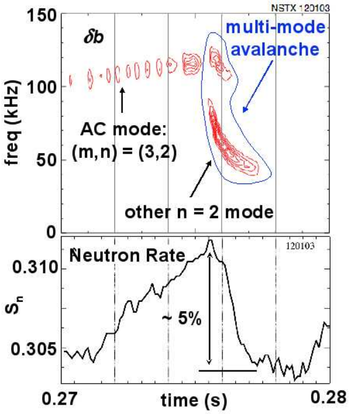
β and geodesic curvature are responsible for Alfvén-acoustic mode coupling at low frequencies $\Rightarrow$ has to be understood

Motivation to study low-f instabilities

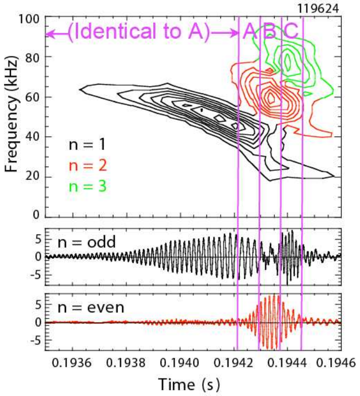
- Various *AE and a new class of instabilities called here Beta-induced Alfvén Acoustic Eigenmode (BAAE) help to study two fundamental MHD waves: Alfvén and acoustic *(Gorelenkov, APS'06, EPS'07)*.

Motivation to study low-f instabilities

- Various *AE and a new class of instabilities called here Beta-induced Alfvén Acoustic Eigenmode (BAAE) help to study two fundamental MHD waves: Alfvén and acoustic *(Gorelenkov, APS'06, EPS'07)*.
- Energetic Particle (EP) driven low-f instabilities lead to radial EP transport:



  - RSAEs form avalanches
  - induce neutron (EP) losses(5%)

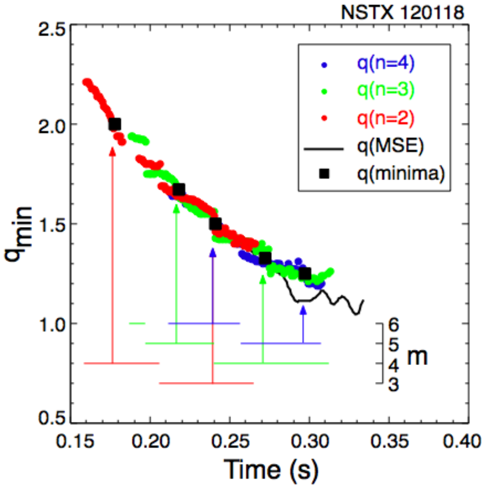


  - BAAEs form avalanches
  - induce losses (13% this case)

see Fredrickson, EX/6-3, M. Van Zeeland, EX/6-2

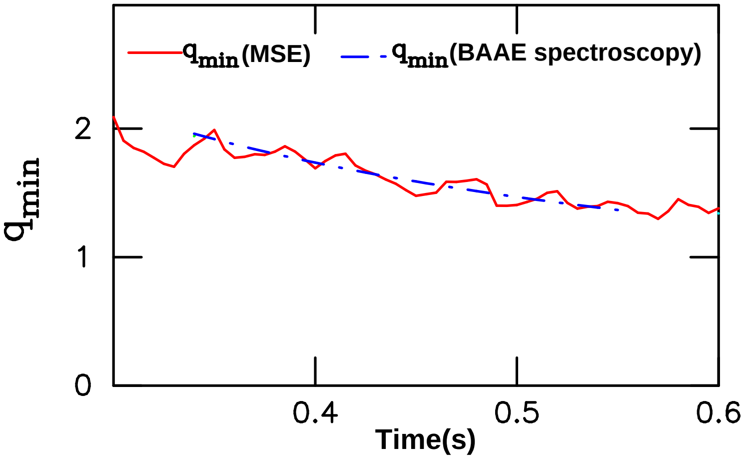
Motivation to study low-f instabilities (continued)

q_{min} from RSAEs



(E.Fredrickson, PoP'07)

q_{min} from BAAEs



- MHD spectroscopy application to infer q-profile is confirmed by MSE in NSTX
  (see also Rimini, EX/1-2 for MHD spectroscopy in JET)
- *AEs are expected in burning plasmas

Talk outline

1. Theory of Alfvén - acoustic continuum in ideal MHD
   - frequency relations of various *AE
2. Suppression of RSAE sweep in NSTX
3. Kinetic theory of Alfvén - acoustic (BAAE) continuum
4. New class of plasma instabilities called Beta - induced Alfvén - Acoustic global Eigenmodes (BAAEs) are studied in tokamaks
5. Discussion and Summary

Alfvén/acoustic continuum bounds global modes $D(r)=0, \ \ [(\partial_r D(r)\,\partial_r - S)\,\phi = 0]$

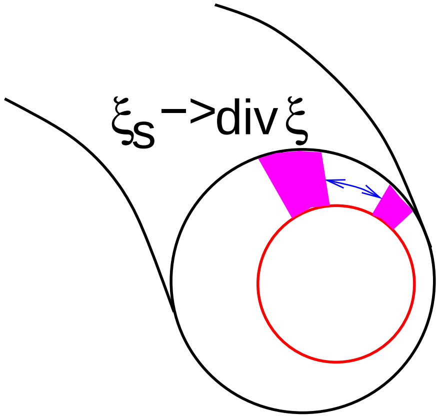
Shear Alfvén and acoustic continuum MHD equations capture main effects in low-β, large aspect ratio plasma, low ω_*, *(Cheng, Chance, PFl ’86):*

$$\Omega^2 y + \partial_\parallel^2 y \quad + \gamma\beta \sin\theta\, z \quad = 0\ (Alfvenic) \tag{1}$$

$$\Omega^2\left(1+\frac{\gamma\beta}{2}\right) z + \frac{\gamma\beta}{2}\partial_\parallel^2 z \quad + 2\Omega^2 \sin\theta\, y \quad = 0\ (acoustic), \tag{2}$$

where $\Omega \equiv \omega R/v_A$, $y \equiv \xi_s \varepsilon/q$, $\xi_s \equiv \vec{\xi}\cdot\frac{[\mathbf{B}\times\nabla\psi]}{|\nabla\psi|^2}$ and $z \equiv \nabla\cdot\vec{\xi}$, $\hat{k}_\parallel \equiv i\partial_\parallel/R$.

Geodesic curvature coupling: m Alfvénic and $m \pm 1$ acoustic harmonics.



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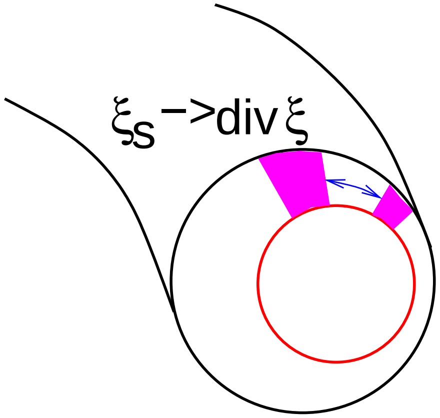
Geodesic curvature coupling: m Alfvénic and $m\pm 1$ acoustic harmonics.

Various solutions exist*

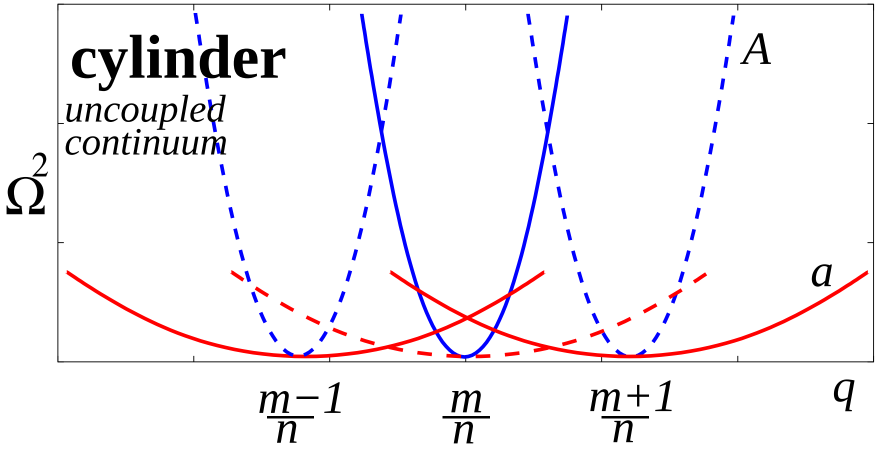


- uncoupled ***acoustic* (a)** $\Omega^2 = \frac{1}{2}\gamma\beta k_\parallel^2 R^2$
  and ***Alfvénic* (A)** branches $\Omega^2 = k_\parallel^2 R^2 + \Omega_{GAM}^2$.
- ***GAM*s**: $\Omega_{GAM}^2 = \gamma\beta\left(1+1/2q^2\right)$
- ***modified shear Alfvén*** branch $\Omega^2 = k_\parallel^2 R^2/\left(1+2q^2\right)$

**Winsor'68, Mikhailovski'75,'98, Chu'92, Zonca'96, van der Holst'00, Smolyakov'08*

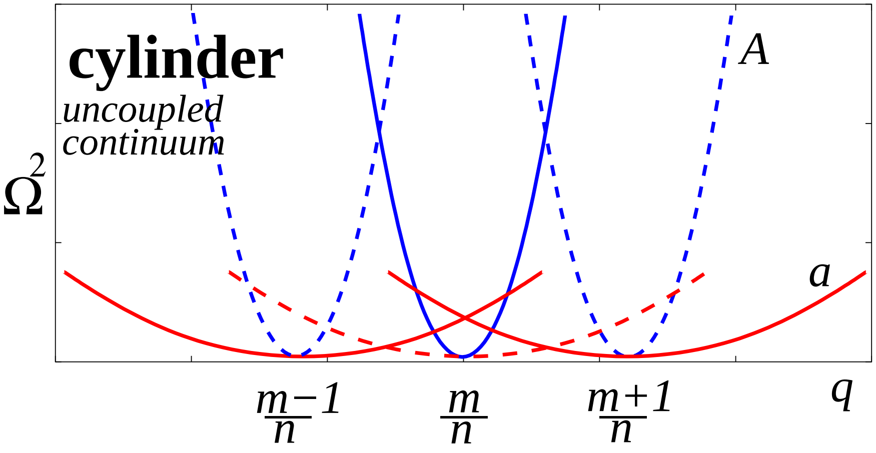
Alfvén/acoustic coupling in toroidal equilibrium (schematic)

- *Alfvén* (A) continuum at low frequency: $\Omega^2 = k^2_{\parallel\pm1}R^2$
- *Acoustic* (a) branch $\Omega^2 = \gamma\beta k^2_{\parallel\pm1}R^2/2\,(1+\delta)$

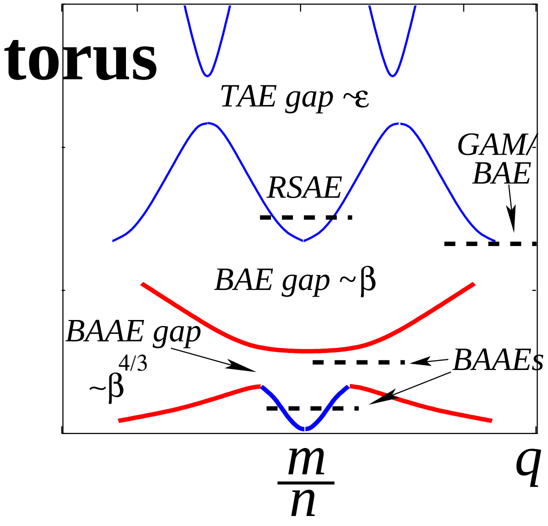


Alfvén/acoustic coupling in toroidal equilibrium (schematic)

- *Alfvén* (A) continuum at low frequency: $\Omega^2 = k_{\|\pm1}^2 R^2 \,/\, \left(1+2q^2\right)$ (modified)
- *Acoustic* (a) branch $\Omega^2 = \gamma\beta k_{\|\pm1}^2 R^2/2\,(1+\delta)$ is coupled via $m\pm1$ sidebands with *modified Alfvén* continuum (m harmonic)



$\Rightarrow$



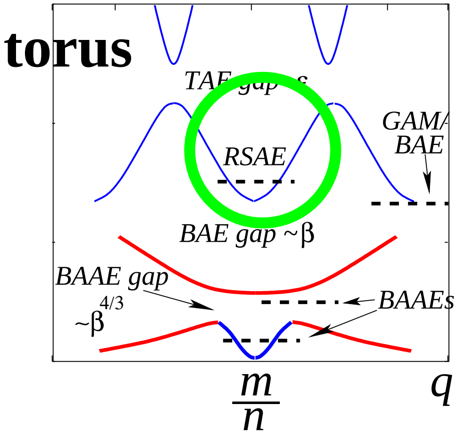
Global modes exist in A-a continuum gaps

(van der Holst'00)

Lower (below TAE) gaps are due to β and geodesic curvature effects

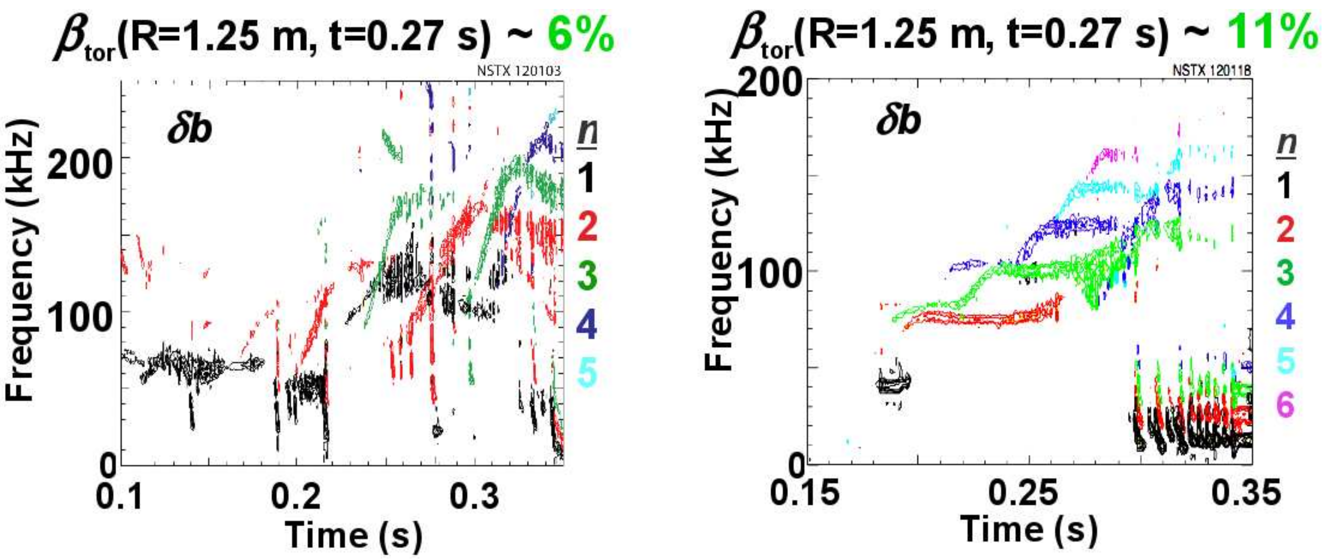
TALK OUTLINE

1. Theory of Alfvén - acoustic continuum in ideal MHD
    - MHD Alfvén - acoustic continuum is key to understand *AE "zoology"
2. **Suppression of RSAEs in NSTX**



3. Kinetic theory of Alfvén - acoustic continuum
4. Beta - induced Alfvén - Acoustic global Eigenmodes (BAAEs)
    - JET
    - NSTX
    - DIII-D
5. Discussion and Summary

Increasing β significantly changes RSAE spectrum evolution in NSTX



- RSAEs historically not observed in ST typical conditions:
  - NSTX regularly operates reverse-shear with $\beta \sim 20\%$, $R/a = 1/0.8$, $P_{NBI} = 2-6MW$.
- Increasing β (and $\nabla\beta$) reduces frequency sweep
- sweep suppression helps to reduce EP transport
- losses are observed when RSAEs form avalanches

β suppresses RSAE sweep; $\nabla\beta$ upshift RSAE frequency in NSTX

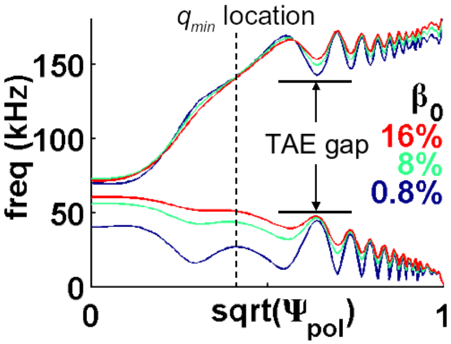
NOVA, MHD code is applied *(Crocker, APS'07)*

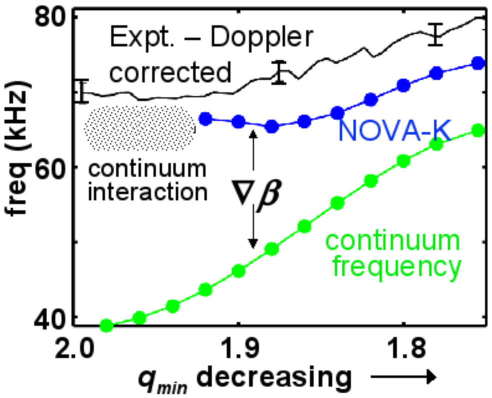
1. Increasing β elevates continuum near $q_{min} \Rightarrow$ raises RSAE frequency towards TAE gap.
   - Frequency sweep suppression is expected when

$$\omega_{GAM}^2 = \gamma\beta\omega_A^2 > \frac{1}{4q^2}\omega_A^2 = \omega_{TAE}^2$$

2. Both theory and data show that $\nabla\beta$ contribution to the RSAE (sweep) frequency is strong *(Fu, PoP'06, Gorelenkov, PPCF'06, Gorelenkov, Sherwood'08)*

$$\Delta f_{\nabla\beta} = f_{GAM}\sqrt{-\frac{r\partial\beta}{\gamma\beta\partial r}\left(1-q^{-2}\right)} \sim f_{GAM}$$





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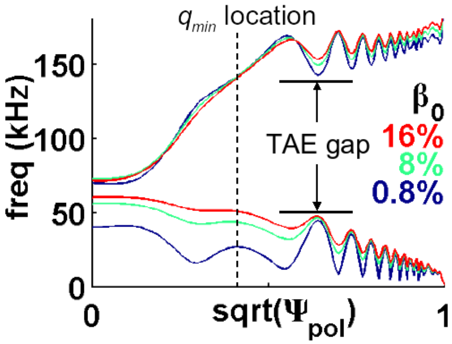

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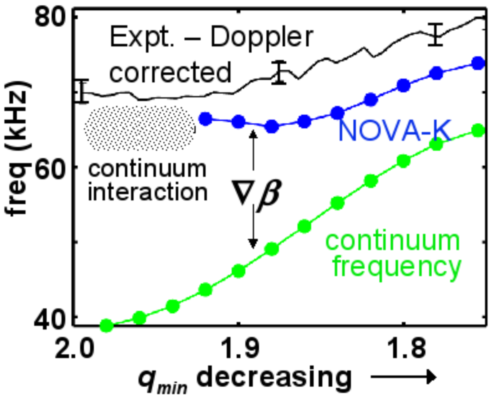

Ideal MHD codes can be used for RSAE modeling in ST conditions: high β, high ε.

Kinetic theory is required for 1) proper frequency normalization and 2) to account for mode - continuum interaction

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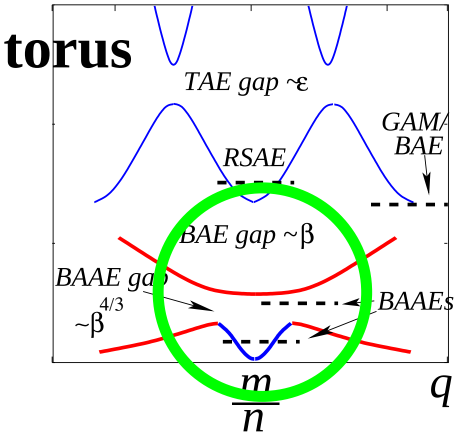
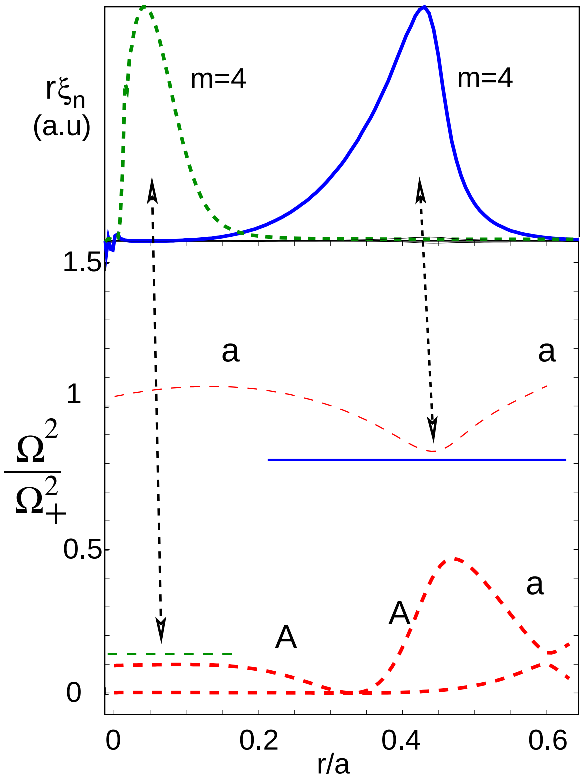
   - MHD Alfvén - acoustic continuum is key to understand *AE “zoology”
2. Suppression of RSAE sweep in NSTX
3. **Kinetic theory of Alfvén - acoustic continuum**



4. Beta - induced Alfvén - Acoustic global Eigenmodes (BAAEs)
   - JET
   - NSTX
   - DIII-D
5. Discussion and Summary

Extremum points of Alfvén - acoustic continuum determine global mode localization

Ideal MHD (NOVA) results



- Core localized and gap BAAEs are found with one dominant poloidal harmonic *(Gorelenkov,PLA'07):*
  - monotonic $q-$profile (EFIT, JET), $q_0 \geq 1$, $q_a = 4$.

  1. low shear sweeping BAAE (A):
     $\omega \simeq v_A k_\parallel / \sqrt{1+2q_{min}^2}|_{r=0}$
  2. gap BAAE:
     $\Omega_+ \simeq \sqrt{\gamma\beta/2}/q_{min}$,
     $\gamma = (T_e + 7T_i/4)/(T_e+T_i)$

- $\nabla\xi$, $m \pm 1$ sidebands are present ($\sim \xi_\theta/a$).

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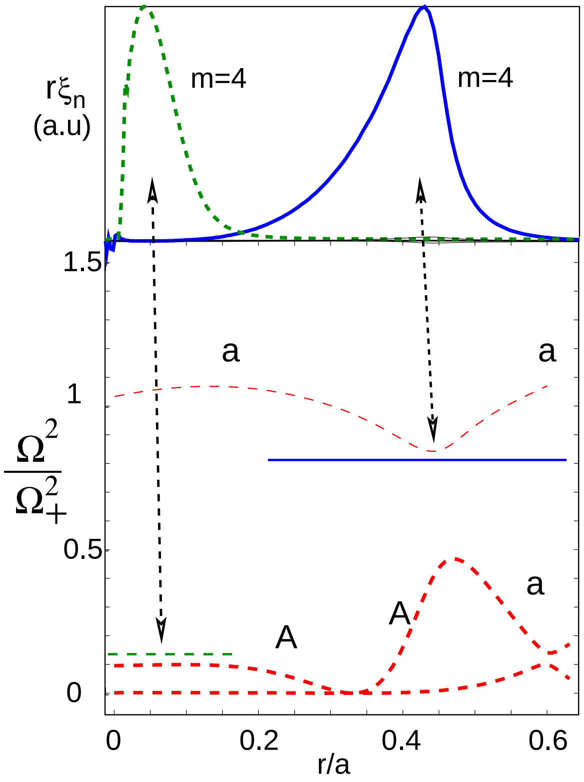

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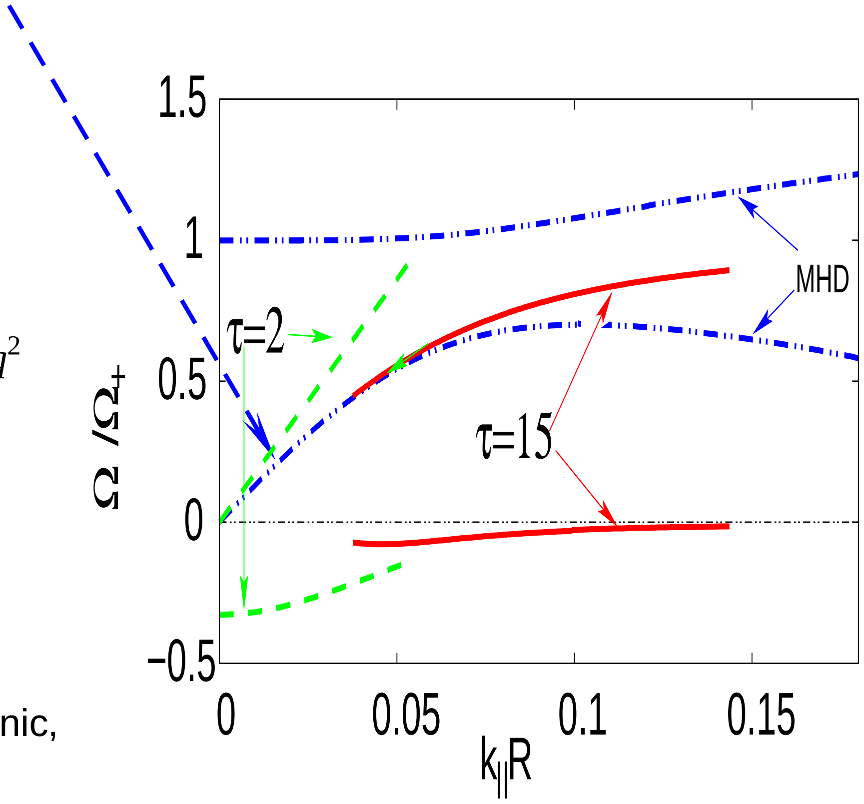
How MHD results hold in the presence of kinetic effects, acoustic coupling?

Kinetic theory modifies MHD dispersion of BAAEs

General low frequency kinetic dispersion relation is obtained: *Zonca et.al. PPCF'96, Mikhailovskii et.al. Pl.Phys.Rep'99*

Two cases are of interest for the **modified Alfvén branch** (sweeping f BAAEs) ($\tau\beta_i/2 = 0.25\%$, $\tau \simeq T_e/T_i$)



1. $\tau > 2\xi_i^2 \gg 1$, $\xi_i = \omega/k_{\parallel\pm1}v_{Ti}$
   results similar to MHD, $\tau = 15 \rightarrow$

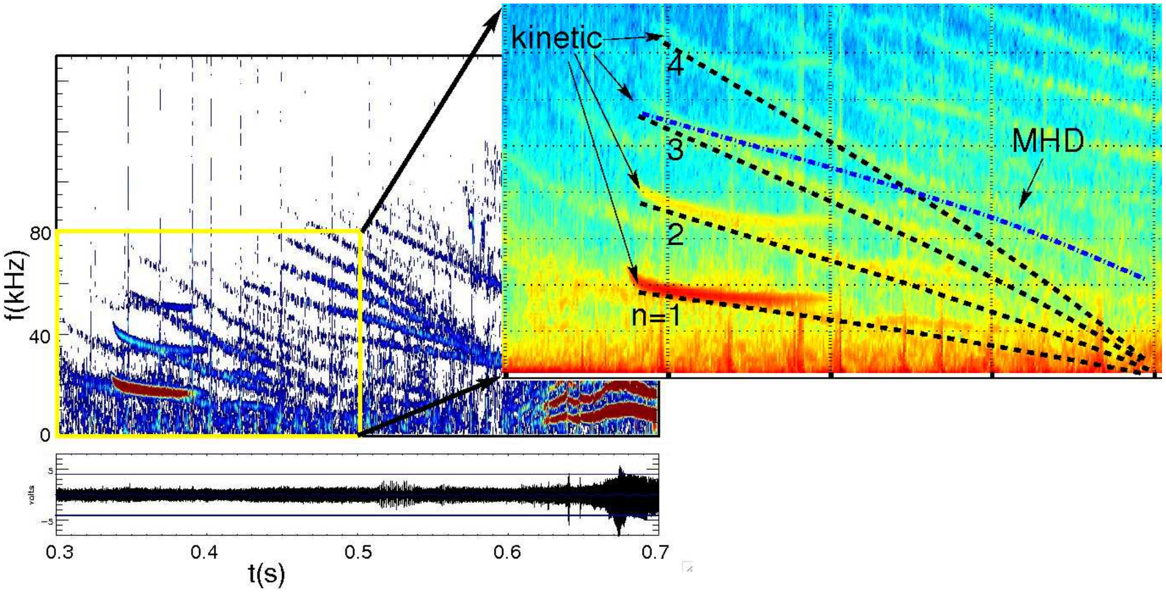
$$\frac{k_{\parallel}^2R^2}{\Omega^2} \simeq 1+2q^2\left(1+e^{-\xi_i^2}\frac{i\xi_i^3\sqrt{\pi}}{2}\right) \simeq 1+2q^2$$

2. $T_i \sim T_e$, $\xi_{\pm i} \ll 1$,

$$\frac{k_{\parallel}^2R^2}{\Omega^2} \simeq 1+q^2\left(\frac{1}{2}+\frac{\pi}{8}\right)+\frac{iq^2\sqrt{\pi}}{\xi_s\sqrt{2}}$$

- phase velocity is different from Alfvénic, depends on T_e/T_i.

NSTX multiple BAAE frequency measurements confirm kinetic dispersion



q_{min} is from MSE: $f = f_{BAAE} + n f_{rot}$, $n < 0$, $n = -1 \div -4$.

Applied modified Alfvénic dispersion with rotation $f_{rot}(q_{min}) = 19 - 23kHz$, $\omega_{*n=1} \simeq 2kHz \ll f_{BAAE}$

Modified Alfvénic wave dispersion agrees better with the kinetic dispersion at $T_i = T_e$:
$f_{BAAE} = v_A k_\| / 2\pi \sqrt{1 + q_{min}^2 (1/2 + \pi/8)}$ vs MHD $v_A k_\| / 2\pi \sqrt{1 + 2q_{min}^2}$.

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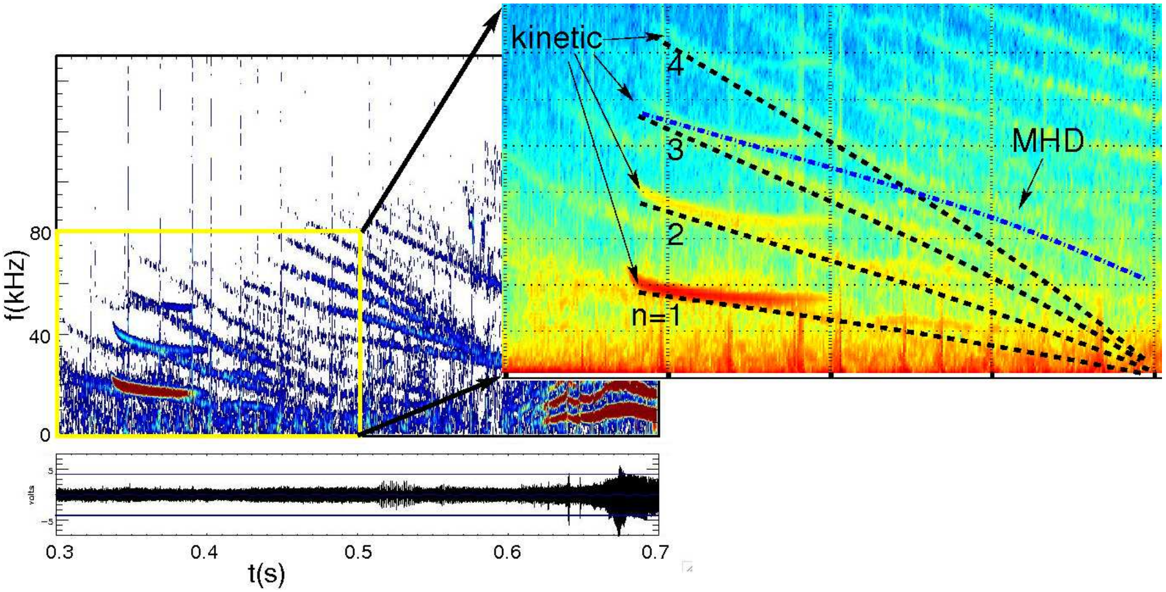

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Kinetics improves and complements MHD framework for BAAE studies: i) proper acoustic wave dispersion, ii) ion Landau damping

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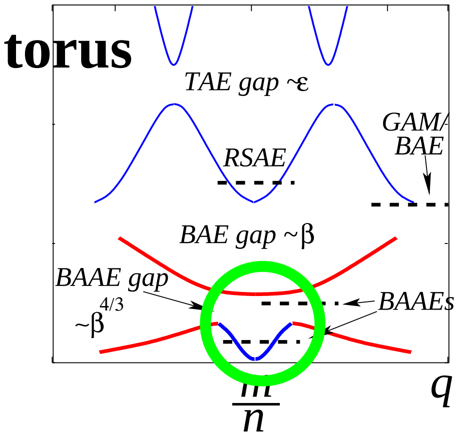
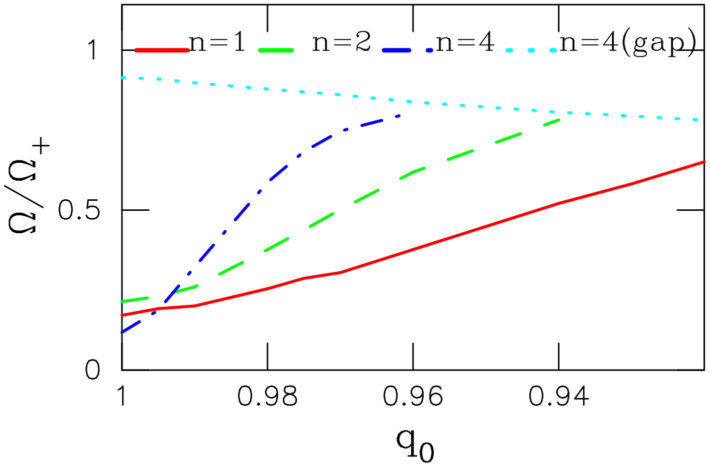
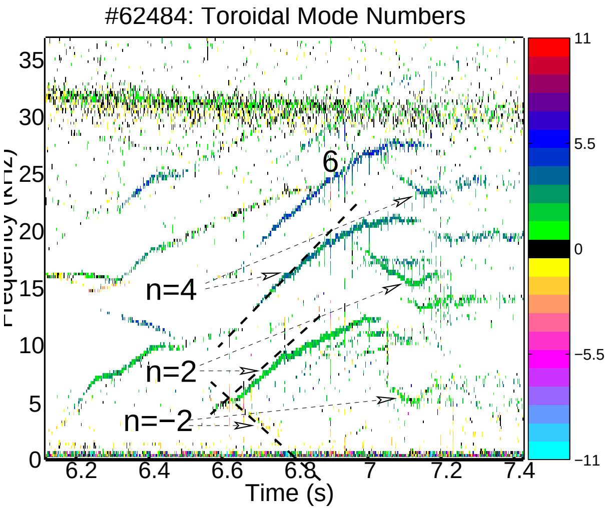


   - in JET, NSTX and DIII-D
5. Discussion and Summary

In JET decreasing q_{min} results in BAAE frequency up-sweep

- Core BAAE activity is predicted to have sweeping frequency ($T_e \gg T_i$)
  - Up-chirp is limited by the gap, $\Omega_+ \simeq \sqrt{\gamma\beta/2}/q$.
  - Core BAAE evolution frequency is close to modified Alfvén branch.
- Rotation is inferred $f_{rot} = 2.5kHz$.





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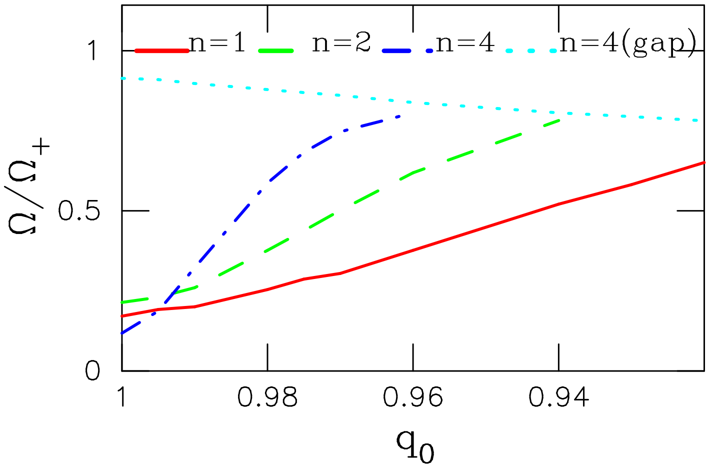

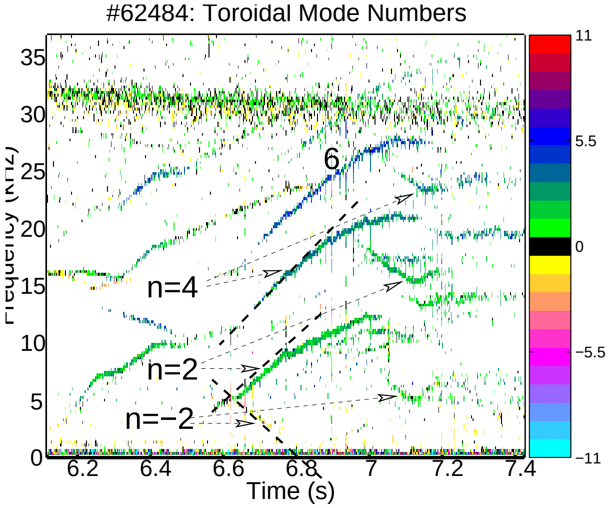

NOVA predicts gap $f_{BAAE} = 16.5kHz$ against observed $14kHz$ if $q_{min} = 1.5$.
$\Rightarrow$ only even m's are expected: $m = nq_{min}$ is integer.
Caveats: q was not measured.

NSTX experiments with MSE study BAAE frequency dependencies

- Low density $n_e \simeq 3 \times 10^{19} m^{-3}$, $P_{NBI} = 2MW$, $E_{NBI} = 90keV$.
- 12 channel MSE measures q profile (reversed shear).
  - helps to validate theory.

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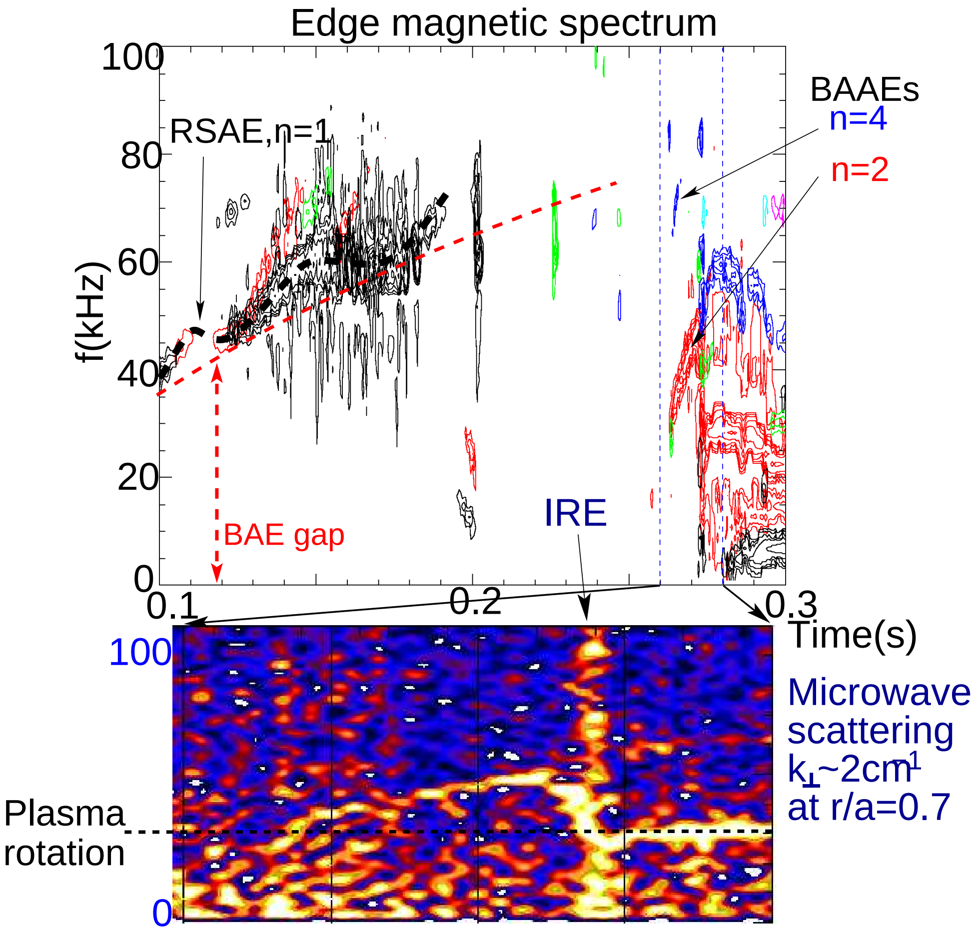
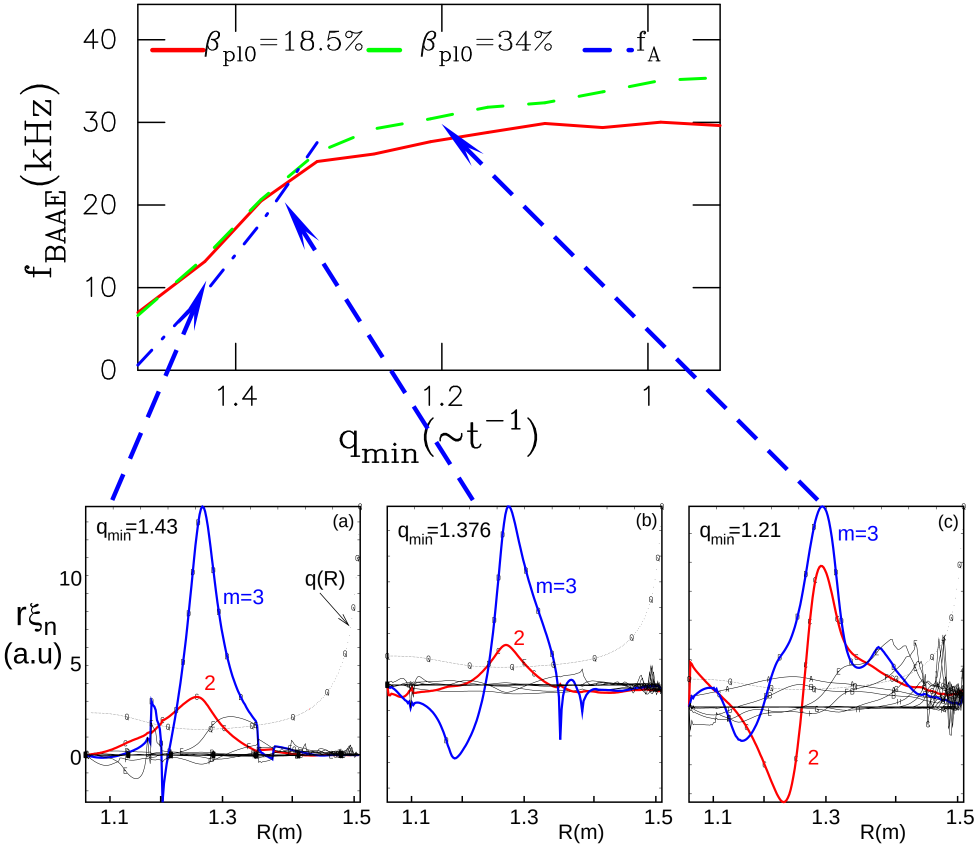
- 12 channel MSE measures q profile (reversed shear).
  - helps to validate theory.
- Low frequency oscillations (BAAEs) are seen unstable:
  - Characteristic upshift frequency evolution from zero (plasma frame).
  - Modes are localized to q_{min} surface.
- High-k diagnostic sees BAAEs at $r/a = 0.7$ *(H.Park, APS'07)*.



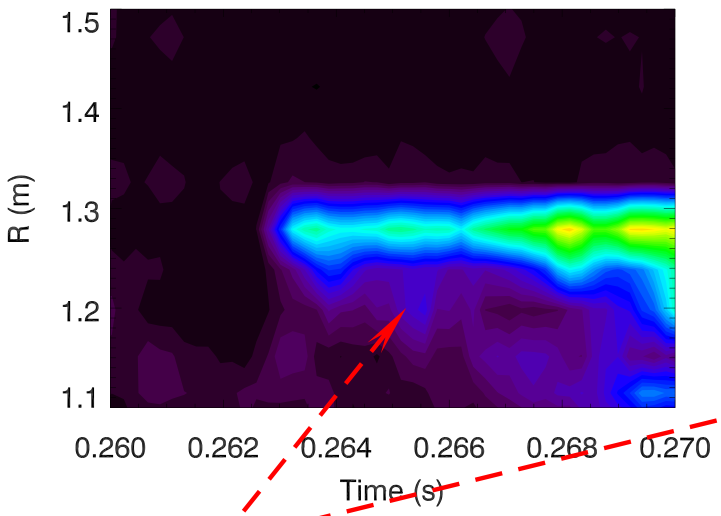
NOVA: BAAE broadens radially as q_{min} decreases

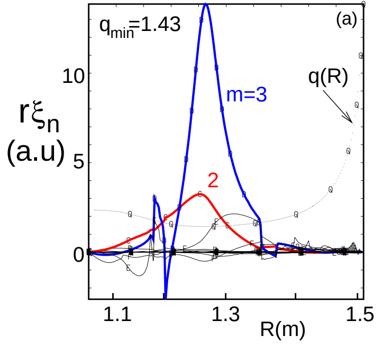


- BAAE frequency sweeps as q-profile relaxes.
  - f does not depend on beta (as expected) near rational q_{min} (=1.5).
  - ξ_r has one dominant harmonic $m = nq_{min} = 3$.
- f_{BAAE} is close to modified Alfvén branch $f_A = v_A k_\parallel / 2\pi\sqrt{1+2q_{min}^2}$
- Continuously transforms to gap mod
- BAAEs interact with the continuum.

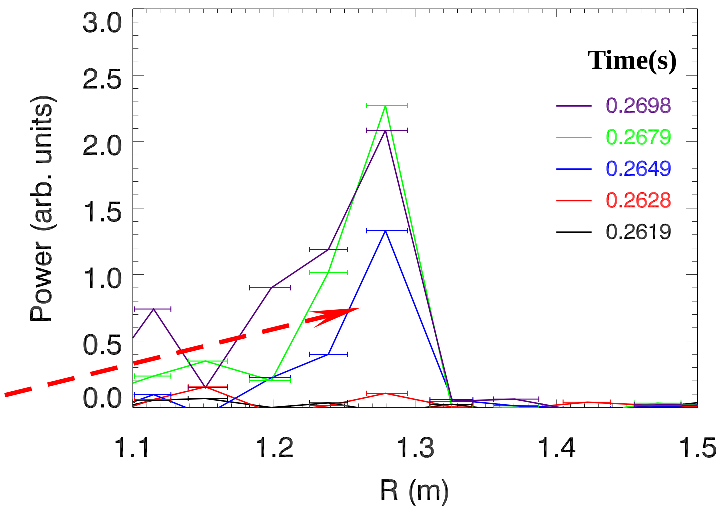
Ultra SXR measures the same radial structure broadening

Raw USXR signal (∼BAAE structure, *Tritz, JHU*)





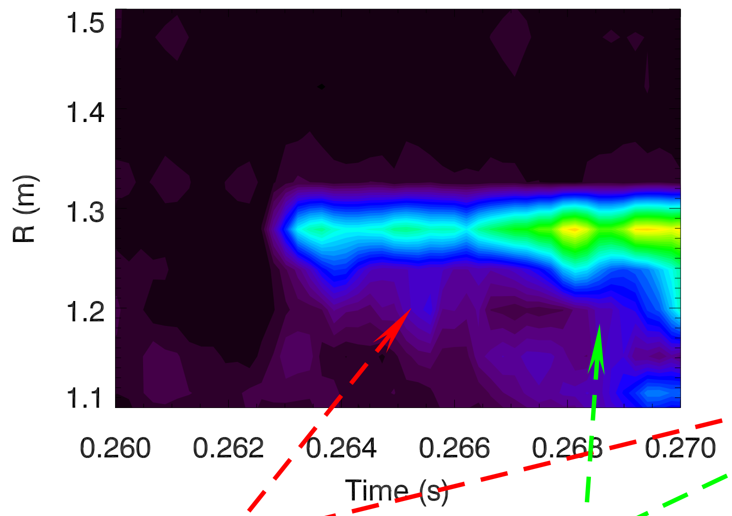
Radial profile evolution

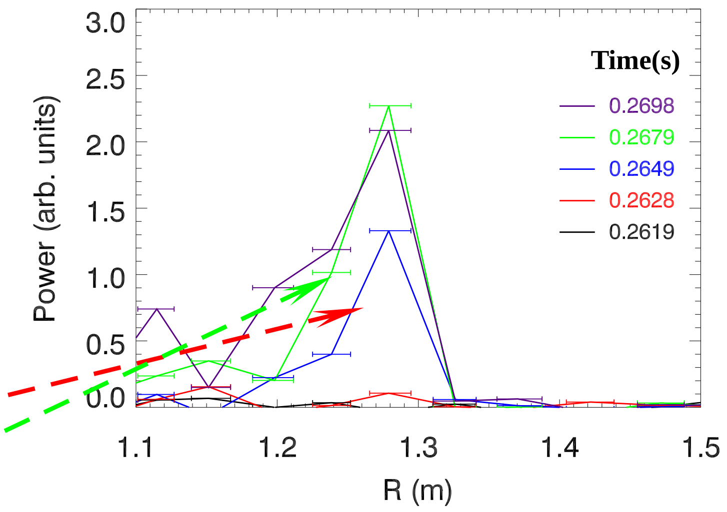


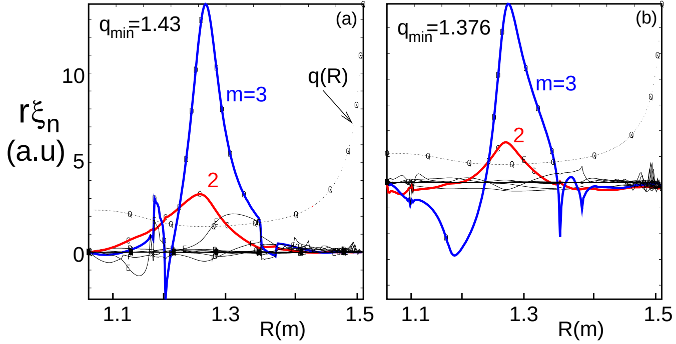
Ultra SXR measures the same radial structure broadening

Raw USXR signal (∼BAAE structure, *Tritz, JHU*)

Radial profile evolution

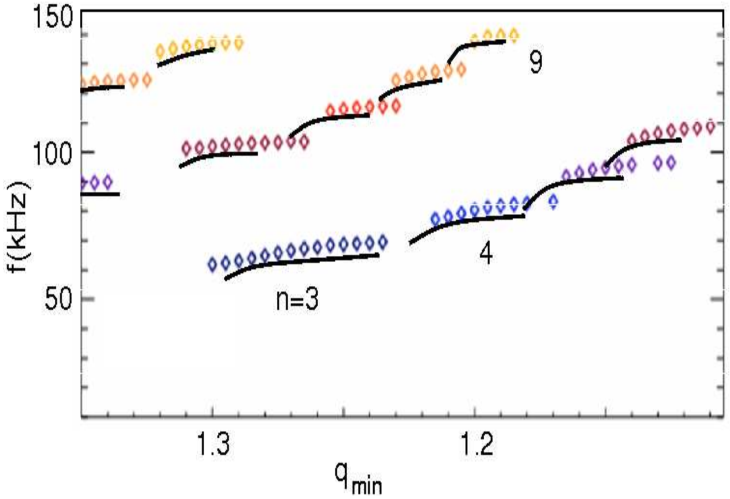


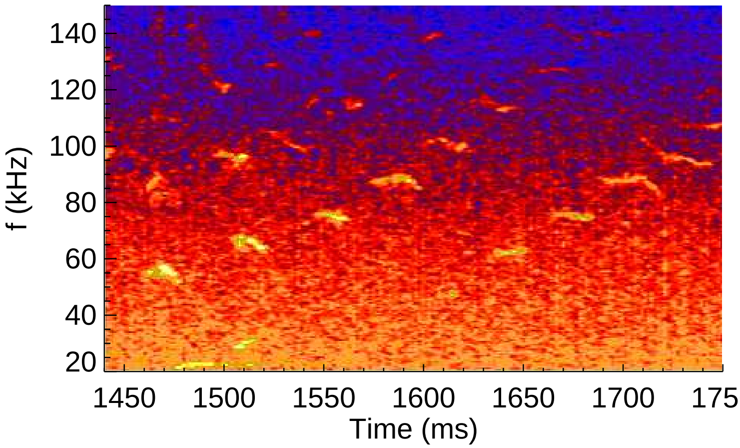




BAAE broadens as q_{min} decreases

For DIIID NOVA predicts BAAE frequencies with the same patterns as measured





- Numerically (using NOVA MHD code) BAAEs are found inside Alfvén-acoustic continuum gaps (points) - not as sweeping modes
  - this is due to strong β profile variation and shear effects
  - modes interacting with the continuum are not resolved
  - kinetic theory renormalization gives similar frequencies (shown as lines)
- Kinetic theory predicts Landau damping for gap BAAEs, $\gamma/\omega \simeq 25\%$ at $\tau = n_i z_i^2 T_e/T_i n_e = 2$ (only those frequencies are plotted).
- Uncertainties (n numbers) do not allow more accurate frequency comparison

Conclusions

1. RSAEs frequency sweep is suppressed in NSTX at high pressure when $f_{GAM} \geq f_{TAE}$.
   - ideal MHD describes sweep suppression
2. New low frequency BAAE modes are observed and studied within MHD and kinetic theory.
   - global modes exist in geodesic curvature induced Alfvén/acoustic continuum gaps
   - low-n global beta-induced Alfvén/acoustic eigenmodes - BAAE are found numerically,
   - BAAE frequency is $0 < \Omega < \sqrt{\gamma\beta/2}/q_{min}$ vs. $\Omega = \sqrt{\gamma\beta\left(1+1/2q_{min}^2\right)}$ for BAE/GAM.
3. Kinetic modification of MHD theory is important for BAAEs
   - ion Landau damping of the gap BAAEs $\gamma/\omega < 25\%$ is expected if $T_e/T_i > 2$.
   - NSTX results show good agreement between measured frequency and kinetic theory.
4. Due to coupling to acoustic branch thermal ions are expected to interact strongly:
   - strong fast ion drive and strong damping on thermal ions,
   - potential for energy channeling from beam ions directly to thermal ions *(α-channeling, Fisch, PRL'93, hot-ion mode, LiWall).*
5. Both RSAEs and BAAEs can be used to infer q_{min} values.