Nonambipolar Transport by Trapped Particles in Tokamaks

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Small nonaxisymmetric perturbations of the magnetic field can greatly change the performance of tokamaks through nonambipolar transport. A number of theories have been developed, but the predictions were not consistent with experimental observations in tokamaks. This Letter provides a resolution, with a generalized analytic treatment of the nonambipolar transport. It is shown that the discrepancy between theory and experiment can be greatly reduced by two effects: (1) the small fraction of trapped particles for which the bounce and precession rates resonate; (2) the nonaxisymmetric variation in the field strength along the perturbed magnetic field lines rather than along the unperturbed magnetic field lines. The expected sensitivity of the International Thermonuclear Experimental Reactor to nonaxisymmetries is also discussed.

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Tokamaks, such as the International Thermonuclear Experimental Reactor (ITER) [1], are sensitive to small nonaxisymmetric magnetic perturbations [2–6]. In order to improve the predictability and the controllability of plasmas in perturbed tokamaks, it is important to understand the fundamental transport associated with nonaxisymmetric perturbations.

In an axisymmetric tokamak, the turning points of a collisionless trapped particle remain on a magnetic surface as the turning points precess toroidally. The magnetic field strength has a periodicity along each magnetic field line, $B(l) = B(l + L)$ with l the distance along a field line and L a constant, so the action $J = \oint M v_{\parallel} dl$ for a particle is a constant on a magnetic surface. When the axisymmetry is broken so $B(l) \neq B(l + L)$, the action for a particle becomes dependent on the toroidal location of its turning point. The conservation of action then implies that the turning point must drift across the magnetic surfaces. The resulting transport depends on the species. Generally ions diffuse faster and produce a net radial current until an ambipolar electric field is established [7]. The radial currents of the nonambipolar diffusion [8] cause a toroidal torque and viscosity, which is often called neoclassical toroidal viscosity (NTV).

Nonambipolar transport has been studied for many years [9–13], and its importance for tokamaks has been recently appreciated [14–16]. Two main regimes were thought important in tokamaks, the $1/\nu$ regime [16] when the $\vec{E} \times \vec{B}$ precession frequency ω_E is low relatively to the collision frequency ν, and the $\nu_{-}\sqrt{\nu}$ regime [17] when ω_E is relatively high. There is a large discrepancy in transport between the two regimes, by several orders of magnitude depending on parameters, but the smaller transport must be chosen as can be readily verified by an approximate connection [18]. The expected transport is then too small for present tokamaks [19], as will be illustrated. The transport may be enhanced by other effects, such as the resonances among ω_E, the magnetic precession ω_B, and/or the bounce frequency ω_b [8,13]. These effects are combined by a generalized analytic treatment in this Letter, which provides a resolution.

The transport by trapped particles can be studied with the bounce-averaged drift-kinetic equation for a perturbed distribution function $f_1(\vec{v}, \vec{x})$. The gyromotions are averaged in the drift-kinetic equation [20], and so particle drift velocity $\vec{v}$ is a function of (E, μ) with the energy E and the magnetic moment $\mu = M v_{\perp}^2/2B$. Consider the drift-kinetic equation

$$v_{\parallel}\hat{b} \cdot \vec{\nabla} f_1 + v_D^{\alpha}\frac{\partial f_1}{\partial \alpha} + v_D^{\psi}\frac{\partial f_0}{\partial \psi} = C[f_1], \tag{1}$$

in the coordinates $\vec{x}(\psi, \vartheta, \alpha \equiv q\vartheta - \varphi)$. Here $(\psi, \vartheta, \varphi)$ are magnetic coordinates with the Jacobian $\mathcal{J}$ for $\vec{B} = \chi' \vec{\nabla}\psi \times \vec{\nabla}\alpha$, where $\chi' = \partial\chi/\partial\psi$ with the poloidal flux χ. The drift $\vec{v}_D$ is decomposed as $v_D^{\alpha} \equiv \vec{v}_D \cdot \vec{\nabla}\alpha$ and $v_D^{\psi} \equiv \vec{v}_D \cdot \vec{\nabla}\psi$. The bounce average is $\langle A \rangle_b \equiv \oint (A dl/v_{\parallel}) / \oint (dl/v_{\parallel}) = (\omega_b/2\pi) \oint d\vartheta A \mathcal{J} B/v_{\parallel}\chi'$ between the turning points with the bounce frequency $\omega_b \equiv 2\pi/\oint d\vartheta(\mathcal{J}B/v_{\parallel}\chi')$.

The bounce average is, however, well defined only when the bouncing orbit is approximately closed. This is the case when the precession is ignorable, $l = 0$ as is assumed in the conventional $1/\nu$ and $\nu_{-}\sqrt{\nu}$ regimes, but also when the particle precesses fast enough to span $l > 0$ times of a full toroidal angle during one bounce. Since the orbit trajectories for each l are different, one can separate the perturbed distribution function for the lth class of particles as

$$f_1 = f_{1l}(\vec{v}, \psi, \alpha) e^{-i2\pi l h(\vec{v}, \vartheta)}, \tag{2}$$

where $h(\vec{v}, \vartheta) = (\int_0^{\vartheta} d\theta v_D^{\alpha} \mathcal{J} B/v_{\parallel}) / (\oint d\theta v_D^{\alpha} \mathcal{J} B/v_{\parallel})$. With the definition of the phase factor $\mathcal{P}^l \equiv e^{i2\pi l h(\vec{v}, \vartheta)}$, the modified collisional operator $C_l[f] \equiv C[f\mathcal{P}^{-l}]\mathcal{P}^l$, and $f_{1l} \propto e^{in\alpha}$ in the presence of nonaxisymmetric field, one can obtain

$$i(l\omega_b - n\langle v_D^\alpha\rangle_b)f_{1l} + \langle C_l[f_{1l}]\rangle_b = \langle v_D^\psi \mathcal{P}^l\rangle_b \frac{\partial f_0}{\partial \psi}. \quad (3)$$

This is a generalized bounce-averaged drift-kinetic equation to be solved for the lth class of particles.

The generalized equation, Eq. (3), implies that a particle in a resonance $l\omega_b - n\langle v_D^\alpha\rangle_b = 0$ effectively does not precess and would drift out radially except for collisions. The radial diffusion through this effective $1/\nu$ behavior is very strong in high-temperature plasmas, and a small fraction of particles that makes the resonance always exists in Maxwellian plasmas. The dominance of these resonating particles on the transport enables one to ignore nonresonant particles $l\omega_b - n\langle v_D^\alpha\rangle_b \neq 0$. Since the lth class of particles has different orbits, their effective radial drifts $\langle v_D^\psi \mathcal{P}^l\rangle_b$ and collisions $\langle C_l[f_{1l}]\rangle_b$ are also different.

One needs to know the drift motions and collisions to solve Eq. (3) for f_{1l}. In the first order of gyroexpansion, the drift velocity is $\vec{v}_D = v_\parallel/B\vec{\nabla}\times(v_\parallel\vec{B}/\omega_g)$ [21], where the gyrofrequency is $\omega_g \equiv eB/M$ and $v_\parallel \equiv \pm(2(U-\mu B - e\phi_e)/M)^{1/2}$ with the total energy U, the electric potential ϕ_e, the charge e and mass M of a species. The nonaxisymmetric part of the precession (α) and the radial drift (ψ) is [8]

$$v_D^{(\alpha,\psi)} = \vec{v}\cdot\vec{\nabla}(\alpha,\psi) = \frac{v_\parallel}{\mathcal{J}B}\frac{\partial}{\partial(\psi,\alpha)}\left(\frac{v_\parallel \mathcal{J}B^2}{\chi'\omega_g}\right), \quad (4)$$

respectively. The bounce-averaged precession becomes

$$\langle v_D^\alpha\rangle_b = -\frac{d\phi_e}{d\chi} + \left\langle \mu\frac{dB}{ed\chi} - (2E - 2\mu B)\frac{d\ln(\mathcal{J}B)}{ed\chi}\right\rangle_b, \quad (5)$$

which includes the electric precession ω_E, and magnetic precession ω_B. The bounce-averaged radial drift is proportional to the spatial variation in the action as $\langle v_D^\psi \mathcal{P}^l\rangle_b = (1/e\chi')(\omega_b/2\pi)(\partial J_l/\partial\alpha)$. The action for the lth class of particles is $J_l = \oint d\vartheta \mathcal{J}BMv_\parallel \mathcal{P}^l/\chi'$ and its spatial variation becomes

$$\frac{\partial J_l}{\partial\alpha} = \frac{2\pi}{\omega_b}\left\langle\left(\frac{2E - 3\mu B}{B}\right)\frac{\partial}{\partial\alpha}(B\mathcal{P}^l)\right\rangle_b. \quad (6)$$

That is, the radial drift occurs due to the symmetry breaking in the action, or equivalently in the effective field strength $B\mathcal{P}^l$ seen by the lth class of particles.

The perturbed distribution function f_{1l} is not analytically tractable due to the complicated collisional operator. Here the simple Krook operator, $C[f_1] = -\nu_K f_1$ with the effective collision frequency ν_K [18] is used to combine the regimes. One can see the validity of this approach in the final solution. Using the drifts and collisions, the solution of Eq. (3) becomes

$$f_{1l} = \frac{(1/e)(\omega_b/2\pi)}{il\omega_b - in(\omega_E + \omega_B) - \nu_K}\left(\frac{\partial J_l}{\partial\alpha}\right)\frac{\partial f_0}{\partial\chi}. \quad (7)$$

The average flux across a magnetic surface is determined by the radial flow as $\Gamma_l = \langle N\vec{u}_l\cdot\vec{\nabla}\psi\rangle$ [16], where the flux average is $\langle A\rangle = \oint d\vartheta d\varphi \mathcal{J}A/\oint d\vartheta d\varphi\mathcal{J}$. Using Eqs. (1) and (2), and by changing variables from $\vec{v}$ to (E, μ) for $f_{1l}(E, \mu, \psi, \alpha)$, one can obtain

$$\Gamma_l = \frac{1}{\mathcal{J}_{00}M^2}\int dE\int d\mu\oint d\varphi\frac{\langle C_l[f_{1l}]\mathcal{P}^{-2l}\rangle_b f_{1l}}{\omega_b\partial f_0/\partial\chi}, \quad (8)$$

where $\mathcal{J}_{00} = 1/(2\pi)^2\oint d\vartheta d\varphi\mathcal{J}$. This is a general expression that one can use to obtain the flux when f_{1l} is known. Using Eq. (7),

$$\Gamma_l = \frac{1}{4\pi^2 e^2 M^2 \mathcal{J}_{00}}\int dE\int d\mu \times\oint d\varphi\frac{\langle|\mathcal{P}^{-l}|^2\rangle_b\nu_K\omega_b}{(l\omega_b - n(\omega_E+\omega_B))^2 + (\nu_K)^2} \times\left|\frac{\partial J_l}{\partial\alpha}\right|^2\frac{\partial f_0}{\partial\chi}. \quad (9)$$

As can be seen, the variation of the field strength through the action and the gradient of the zeroth-order distribution function drives the nonambipolar transport.

The general expressions for f_{1l} and Γ_l in Eqs. (7) and (9) are more tractable if appropriate models and approximations are used. A model of the field can be given by $B = B_0(1-\epsilon\cos\vartheta) + B_0\sum_{nm}\delta_{nm}e^{i(m-nq)\vartheta + in\alpha}$, ignoring the higher-order shaping terms. Only the first order in terms of the inverse-aspect ratio ϵ will be evaluated, so the differences between magnetic coordinate systems can be ignored. The zeroth-order distribution function can be taken by Maxwellian distribution $f_0 = f_M = N/(\sqrt{\pi}v_t)^3 e^{-v^2/v_t^2}$ with $v_t = (2T/M)^{1/2}$.

For convenience, normalized variables $x \equiv E/T$ and $\kappa^2 \equiv (E - \mu B_0(1-\epsilon))/2\mu B_0\epsilon$ will be used instead of (E, μ). The electric precession is independent of (x, κ^2), but the bounce, the magnetic precession frequency and the action integration over φ become

$$\omega_b = \frac{\pi\sqrt{\epsilon}}{2\sqrt{2}}\omega_t\frac{\sqrt{x}}{K(\kappa)} \approx \frac{\pi\sqrt{\epsilon}}{4\sqrt{2}}\omega_t\sqrt{x}, \quad (10)$$

$$\omega_B = \sigma\frac{q^3\omega_t^2}{2\epsilon\omega_g}x\frac{F_{010c}^{-1/2}(\kappa)}{4K(\kappa)} \approx \sigma\frac{q^3\omega_t^2}{4\epsilon\omega_g}x, \quad (11)$$

$$\int d\varphi\left|\frac{\partial J_l}{\partial\alpha}\right|^2 = \frac{\pi(Mv_t qR_0)^2}{2\epsilon}x\sum_{nmm'}n^2\delta_{nmm'}^2 F_{nml}^{-1/2}F_{nm'l}^{-1/2} \quad (12)$$

in the first order in ϵ, using Eqs. (5) and (6). Here the transit frequency $\omega_t = v_t/qR_0$, the sign function σ that $\sigma = +1$ for corotation with plasma current, the complete elliptic integral of the first kind K, and $\delta^2_{nmm'} \equiv \mathrm{Re}(\delta_{nm})\mathrm{Re}(\delta_{nm'}) + \mathrm{Im}(\delta_{nm})\mathrm{Im}(\delta_{nm'})$. The function F^y_{nml} is defined as

$$F^y_{nml}(\kappa) = \int_{-\vartheta_t}^{\vartheta_t} d\vartheta(\kappa^2 - \sin^2(\vartheta/2))^y \cos[\Theta_{nml}(\vartheta)], \tag{13}$$

where $\Theta_{nml}(\vartheta) = (m - nq)\vartheta + 2\pi l h(\vartheta)$ and $\vartheta_t = 2\arcsin(\kappa)$. The approximations $K(\kappa) \approx 2$ and $F^{-1/2}_{010} \approx (1/2)F^{-1/2}_{000} = 2K(\kappa)$ are used above. Also, one can approximate the phase factor $h(\vartheta) = -\sigma K(\kappa, (\pi/2) \times (\vartheta/\vartheta_t))/4K(\kappa) \approx -\sigma\vartheta/2\pi$ by simply assuming the linear behavior of the incomplete elliptic integral of the first kind, $K(\kappa, \vartheta)$, so $\Theta_{nml}(\vartheta) \approx (m - nq - \sigma l)\vartheta$ and $\langle|\mathcal{P}^{-l}|^2\rangle_b \approx 1$.

The effective collisional frequency ν_K is valid if it can represent a more accurate collisional operator such as a pitch-angle operator, that is, $\nu_K f_{1l} \approx \langle C_p[f_{1l}\mathcal{P}^{-l}]\mathcal{P}^l\rangle_b$. If a single harmonic perturbation is applied, it has been shown that [13] $\nu_K \approx (\nu_D/2\epsilon)[1 + (m - nq)^2 + (l/2)^2]$, where the deflection collision frequency ν_D is $\nu_{Da} \approx x^{-3/2}\nu_a \equiv x^{-3/2}\sum_b \nu_{ab}$ for a species a [20]. Here a further approximation is taken $m - nq \approx 0$, so $\nu_K \approx (\nu_D/2\epsilon) \times [1 + (l/2)^2]$. The transport is predominantly driven by resonating particles in the $1/\nu$ regime where the perturbations with $m - nq \approx 0$ give the dominant contribution. This approximation is sufficiently accurate for the $1/\nu$ regime as illustrated in Fig. 1. For plasmas in a pure ν regime, it becomes inaccurate in the presence of multi-

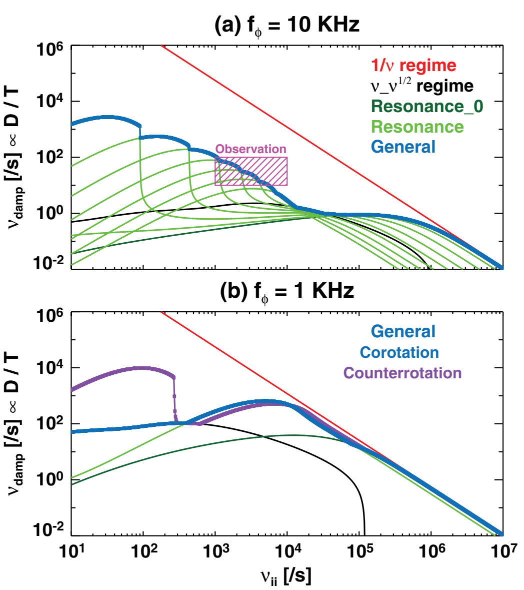

FIG. 1 (color). The typical rotational damping rate (/s) for present tokamaks, which is proportional to diffusivity divided temperature, as a function of ion-ion collision frequency for two different rotations. Each evaluation uses $\Gamma_{1/\nu}$ ($1/\nu$ regime), $\Gamma_{\nu\_\sqrt{\nu}}$ ($\nu\_\nu^{1/2}$ regime), Γ_0 (resonance_0), $l > 0$ Γ_l (resonance), and $\Gamma = \max\{l|\Gamma_l, \Gamma_{\nu\_\sqrt{\nu}}\}$ (general).

harmonic perturbations, and the estimation for $\nu\_\sqrt{\nu}$ regime [17] should be used. Complications of multiharmonic perturbations may lead to stochastic transport [8,11], which are ignored in our study.

One can use the described approximations and obtain the nonambipolar flux, and also the surface-averaged toroidal force density by $\Gamma = (1/eq\chi'^2)\langle\vec{B}_T \cdot \vec{\nabla} \cdot \vec{\Pi}\rangle$ [14]. The toroidal force produces a rotational damping, so it is convenient to use the flow instead of the radial derivatives for the pressure and the electric potential. In the first order gyroexpansion, $1/p(dp/d\chi) + e/T(d\phi_e/d\chi) = -e/T(u^\varphi - qu^\vartheta)$, and the poloidal rotation can be approximated as $u^\vartheta \approx (1/eq)dT_a/d\chi$ [20]. The general formula is then

$$\langle\hat{\phi} \cdot \vec{\nabla} \cdot \vec{\Pi}_l\rangle = \frac{\epsilon^{1/2} p u^\varphi_l}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta^2_{wl} \int_0^\infty dx \mathcal{R}_{1l}, \tag{14}$$

where

$$\delta^2_{wl} = \sum_{nmm'} \delta^2_{nmm'} \frac{F^{-1/2}_{nml} F^{-1/2}_{nm'l}}{4K(\kappa)},$$

$$\mathcal{R}_{1l} = \frac{1}{2} \frac{n^2(1 + (\frac{l}{2})^2)\frac{\nu_a}{2\epsilon} x e^{-x}}{(l\omega_b - n(\omega_E + \omega_B))^2 + ((1 + (\frac{l}{2})^2)\frac{\nu_a}{2\epsilon})^2 x^{-3}},$$

for a species. Note ω_b in Eq. (10) and ω_B in Eq. (11) are also functions of x. The torque is proportional to the toroidal flow u^φ with the neoclassical offset by

$$u^\varphi_l = u^\varphi + c_l \sigma \left| \frac{1}{e} \frac{dT_a}{d\chi} \right|, \tag{15}$$

where the factor $c_l = 1 + \int_0^\infty dx \mathcal{R}_{2l} / \int_0^\infty dx \mathcal{R}_{1l}$ with $\mathcal{R}_{2l} = (x - 5/2)\mathcal{R}_{1l}$. Since the variation is moderate, one can approximate $c_l \approx 2$ between $c_l = 3.5$ when $\nu \to \infty$ and $c_l = 0.5$ when $\omega_E \to \infty$. If the transport in $\nu\_\sqrt{\nu}$ regime [17] is larger than that given by Eq. (14), so the plasma is purely in $\nu\_\sqrt{\nu}$ regime, one can take the maximum of these evaluations as $\Gamma = \max\{l|\Gamma_l, \Gamma_{\nu\_\sqrt{\nu}}\}$. The evaluations for each l and for $\nu\_\sqrt{\nu}$ regime are done independently and include all of the particles.

To understand a typical parametric dependency of Eq. (14) on the collisionality, a set of parameters is chosen: $R_0 = 2$ m, $r = 0.6$ m, $B_0 = 2$ T, $q = 2.2$, and the density $N = 5 \times 10^{19}$ m^{-3}. These parameters are relevant for present tokamaks such as the National Spherical Torus Experiment and DIII-D except the edge region, but also for ITER by a scaling $\langle\hat{\phi} \cdot \vec{\nabla} \cdot \vec{\Pi}_l\rangle \propto 1/R_0$. The temperature is scanned over $T = 0.01$ keV to 100 keV, and two rotations are examined, $\omega_E/2\pi = 1$ kHz relevant for Ohmic plasmas, $\omega_E/2\pi = 10$ kHz relevant for neutral-beam-injection–heated plasmas. The diamagnetic and neoclassical flow are ignored, so $\omega_E/2\pi = f_\phi$. For perturbations, multipoloidal harmonics $\delta_{nm} = 10^{-3}e^{-(m-5)^2/50}$ are applied with $-10 \leq m \leq 20$ and $n = 3$. This spectrum models the actual field from the coils on the outboard side.

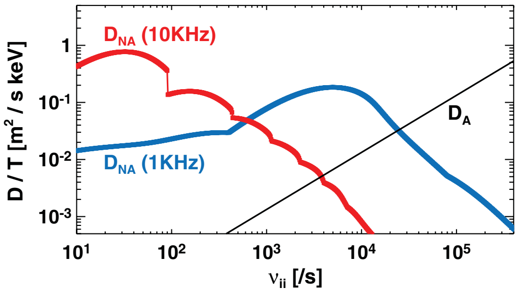

FIG. 2 (color). Comparison between the nonambipolar (D_{NA}) and ambipolar (D_A) diffusions as a function of ion-ion collision frequency for two different rotations.

Figure 1 shows rotational damping rates $\nu_{\mathrm{damp}} = \langle \hat{\phi} \cdot \vec{\nabla} \cdot \vec{\Pi}_l \rangle / 2\pi f_\phi R_0 MN$ as a function of ion-ion collision frequency $\nu = \nu_{ii}$. The $1/\nu$ [16] and $\nu_-\sqrt{\nu}$ [17] calculations are also shown for comparison. Note that $l = 0$ follows almost exactly the $1/\nu$ result for high ν, indicating the accuracy of ν_K. Also, note that there is large discrepancy up to 6 orders of magnitude at low ν between the $1/\nu$ and $\nu_-\sqrt{\nu}$ evaluations. One can see from (a) that the smaller of the two gives the damping rate $\nu_{\mathrm{damp}} \lesssim 1/\mathrm{s}$. However, experiments have shown that plasmas with the given parameters (a) have $\nu_{\mathrm{damp}} = 10$–$100/\mathrm{s}$ in a range of $\nu_{ii} = 10^3$–$10^4/\mathrm{s}$, as roughly marked by a box in Fig. 1(a). Also, a $1/\nu$ behavior has been often observed in this range of the collisionality [4,6], although a simple criteria $\omega_E \gg \nu/\epsilon$ implies that plasmas must be in $\nu_-\sqrt{\nu}$ regime.

The inconsistency can be resolved by the generalized evaluation of Eq. (14). Figure 1(a) shows that the successive l bounce-harmonic resonances strongly enhance the transport for $\nu_{ii} = 10^3$–$10^4/\mathrm{s}$ and give $\nu_{\mathrm{damp}} = 10$–$100/\mathrm{s}$ with a broad $1/\nu$ behavior as all consistent with observations. Note that variations in the field strength must be as large as $\delta \sim 10^{-3}$. This is a relevant value when evaluated along the perturbed (displaced by $\vec{\xi}$) magnetic field lines as $\delta = \delta_E + (\vec{\xi} \cdot \vec{\nabla} B)/B_0$ [22]. If it is evaluated along the unperturbed field lines as in vacuum superposition [19], typically $\delta_E \sim 10^{-4}$ in practice. The δ/δ_E gives another enhancement by a factor of $\sim 10^2$, which is essential in addition to bounce-harmonic resonances to reach the experimental values. The effects of other parameters are weak compared with these two effects. One can also see the case with a low rotation from Fig. 1(b), where only the $l = 1$ resonance occurs for $\nu_{ii} = 10^3$–$10^4/\mathrm{s}$. When ν becomes lower, the plasma enters the $\nu_-\sqrt{\nu}$ regime for the corotating case, but another resonance between the electric and magnetic precession can occur for the counterrotating case. This may degrade the benefit of the counterrotation by the neoclassical flow [6] in ITER.

It is worthwhile to compare the nonambipolar diffusion D_{NA} with the neoclassical ambipolar diffusion $D_A \approx \epsilon^{-3/2} q^2 \rho_e^2 \nu_{ei}$, where ρ_e is the electron gyroradius and ν_{ei} is the electron-ion collisional frequency. The comparison with $\delta \sim 10^{-3}$ in Fig. 2 shows that D_{NA} can be much larger than D_A, and can be comparable to the Bohm-like diffusion in the low collisionality. Also note that the rotation dependency of the nonambipolar transport differs by the collisionality. One can find roughly $D_{NA} \propto 1/\omega_E$ for $\nu_{ii} > 10^3$, but $D_{NA} \propto \omega_E$ for $\nu_{ii} < 10^3$, which implies that the rotational stability in the presence of nonaxisymmetry can greatly change along with the collisionality.

In summary, nonambipolar transport in perturbed tokamaks is discussed with a generalized analytic treatment. The strong enhancement of transport is predicted by the l bounce-harmonic resonances and by the actual variations in the field strength, and significantly improves the consistency between theory and experiment. Nonambipolar transport can be dominant in ITER-relevant regimes, indicating that a strong control of particle and momentum can be utilized, but must be carefully designed not to degrade the energy confinement.

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