Effect of Collisionality on Kinetic Stability of the Resistive Wall Mode

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The impact of collisionless, energy-independent, and energy-dependent collisionality models on the kinetic stability of the resistive wall mode is examined for high pressure plasmas in the National Spherical Torus Experiment. Future devices will have decreased collisionality, which previous stability models predict to be universally destabilizing. In contrast, in kinetic theory reduced ion-ion collisions are shown to lead to a significant stability increase when the plasma rotation frequency is in a stabilizing resonance with the ion precession drift frequency. When the plasma is in a reduced stability state with rotation in between resonances, collisionality will have little effect on stability.

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Introduction.—The resistive wall mode (RWM), a kink mode instability of a tokamak plasma, must be stabilized for future disruption-free magnetically confined fusion operation. Theoretical models have shown that the RWM can be passively stabilized by a combination of plasma toroidal rotation and an energy dissipation mechanism [1], with rotational stabilization garnering the most attention [2,3]. Determining the dependence of stability on collisionality is important, particularly for confident extrapolation of results to future machines such as ITER or a spherical tokamak component test facility [4], which will operate at lower collisionality and at different plasma rotation levels.

Classic models of RWM stability [5,6] did not explicitly include collisionality, but rather a more general dissipation term, which was seen to be stabilizing. The inclusion of neoclassical dissipation [7] in such a model introduced a dependence of the critical plasma rotation for stability on collisionality. It was posited that larger ion-ion collisionality ν_{ii} would allow a smaller toroidal plasma rotation, ω_ϕ while maintaining stability [3]. This dependence was explored in the National Spherical Torus Experiment (NSTX) [8] and found to be qualitatively consistent with experimental results [3]. In the past few years, theoretical models of the RWM including kinetic effects have been introduced [9–12]. Application of this stability model to NSTX equilibria has indicated the importance of a stabilizing resonance between the plasma rotation and the precession drift frequency, and that the marginal stability point should shift from higher to lower rotation as collisionality is increased, consistent with previous conclusions [13–15]. However, a more detailed examination of the interplay of collisionality with kinetic resonances presented here illustrates a key change to the long-standing thought that higher collisionality is generally stabilizing.

Inclusion of collisionality in δW_K.—The RWM energy principle with kinetic effects is [9,10]

$$(\gamma - i\omega_r)\tau_w = -(\delta W_\infty + \delta W_K)/(\delta W_b + \delta W_K), \quad (1)$$

where γ and ω_r are the growth rate and frequency of the mode, τ_w is the wall time (the eddy current decay time of the surrounding conducting structure), δW_∞ is the fluid no-wall potential energy, δW_b is the fluid with-wall potential energy, and δW_K is the kinetic contribution. The kinetic part is derived from the volume integral of a small displacement away from equilibrium $\boldsymbol{\xi}_\perp$ times $\nabla \cdot \tilde{\tilde{\mathbb{P}}}_K$, the perturbed kinetic pressure tensor. The perturbed kinetic pressures are found by taking moments of the perturbed distribution function $\tilde{f}$, which comes from the solution to the linearized drift kinetic equation,

$$d\tilde{f}/dt + (\tilde{\mathbf{F}}/m)\nabla_{\boldsymbol{v}} f = C(\tilde{f}), \quad (2)$$

where $\tilde{\mathbf{F}}$ is the perturbed force. The simplest form of the collision operator is the Krook operator, $C(\tilde{f}) = -\nu_{\rm eff}\tilde{f}$, where $\nu_{\rm eff}$ is the effective collision frequency, which will be defined explicitly in the next section. Using the solution for $\tilde{f}$ from Eq. (2) eventually results in the following form for δW_K (Refs. [9,10,15]):

$$\delta W_K \sim \iint |\chi| \int \lambda \hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} d\hat{\varepsilon} d\chi d\Psi, \quad (3)$$

where λ is the frequency resonance fraction, which for trapped thermal ions is

$$\lambda = \frac{[\Omega_1\Omega_2 + \gamma\nu_{\rm eff} + \gamma^2] + i[\gamma(\Omega_1 - \Omega_2) + \nu_{\rm eff}\Omega_1]}{\Omega_2^2 + (\nu_{\rm eff} + \gamma)^2}. \quad (4)$$

Collisionality enters the kinetic stabilization as a component of a frequency resonance term within an integration over normalized energy ($\hat{\varepsilon} = \varepsilon/T$), pitch angle ($\chi$), and magnetic flux ($\Psi$). In splitting λ into real and imaginary parts we have defined $\Omega_1 = \omega_{*N} + (\hat{\varepsilon} - \frac{3}{2})\omega_{*T} + \omega_E - \omega_r$ and $\Omega_2 = \langle\omega_D\rangle + l\omega_b + \omega_E - \omega_r$. Note that there are contributions to $\mathrm{Im}(\delta W_K)$ from the mode-particle resonance in the integral of Eq. (3) when $\Omega_2 \to 0$, and from nonresonant dissipation effects when γ, $\nu_{\mathrm{eff}} \neq 0$.

The collisionality term's relative magnitude with respect to the other frequencies—the density and temperature gradient parts of the diamagnetic frequency (ω_{*N} and ω_{*T}), the $E \times B$ frequency ($\omega_E = \omega_\phi - \omega_{*N} - \omega_{*T}$), the bounce-averaged precession drift frequency ($\langle \omega_D \rangle$), and the harmonic times the bounce frequency ($l\omega_b$)—determine its importance in stabilization. Therefore, it is not possible *a priori* to say whether decreased collisionality will decrease stability for a given rotation level, as was theorized in simpler models [3,7]. Instead, theoretical calculations of kinetic stability of the resistive wall mode have shown that changing collisionality through temperature and density can either increase *or* decrease the stability, depending on the plasma rotation [13,14].

Let us now construct a simple analytic model (and make the assumption that γ is much smaller than the other relevant frequencies) and separate it into two cases, $\Omega_2 \gtrsim \nu_{\mathrm{eff}}$ and $\Omega_2 \ll \nu_{\mathrm{eff}}$. In the former case, $\mathrm{Im}(\delta W_K)$ is dominated by a collisional dissipation term, while the latter case $\mathrm{Im}(\delta W_K)$ is dominated by a rotational resonance term. Therefore we will separate plasmas with particular rotation profiles into "off-resonance" ($\Omega_2 \gtrsim \nu_{\mathrm{eff}}$) and "on-resonance" ($\Omega_2 \ll \nu_{\mathrm{eff}}$) cases. This is a simplification, as any plasma rotation profile could cause resonance to occur with particles of particular pitch angle, energy, and radial location, but it is a useful exercise to examine the parametric dependence of δW_K on ν_{eff}. We find

$$\delta W_K^{\mathrm{off}} \sim \frac{\Omega_1 \Omega_2}{\Omega_2^2 + \nu_{\mathrm{eff}}^2} + i \frac{\nu_{\mathrm{eff}} \Omega_1}{\Omega_2^2 + \nu_{\mathrm{eff}}^2}, \qquad (5)$$

$$\delta W_K^{\mathrm{on}} \sim \frac{\Omega_1 \Omega_2}{\nu_{\mathrm{eff}}^2} + i \left(\text{resonant term} + \frac{\Omega_1}{\nu_{\mathrm{eff}}} \right). \qquad (6)$$

Therefore, for off-resonance plasmas, collisionality more strongly affects the imaginary term, and decreased collisionality tends to decrease $\mathrm{Im}(\delta W_K)$ while increasing $\mathrm{Re}(\delta W_K)$ slightly. For on-resonance cases, collisionality more strongly affects the real term, and decreased collisionality increases $\mathrm{Re}(\delta W_K)$. The "resonant term" above results from the residue of the integration of $\sim 1/\Omega_2$ as $\Omega_2 \to 0$.

Collisionality models.—There are several possibilities that can be considered for ν_{eff} in Eq. (2) for momentum-transferring collisions between test particles j and bulk ions i. Three simple expressions are collisionless, $\nu_0 = 0$, no energy dependence (SI units) [16],

$$\nu_1(\Psi) = \frac{\sqrt{2} n_i m_{ji}^{1/2} Z_i^2 Z_j^2 e^4 \ln\Lambda}{12 \pi^{3/2} \epsilon_0^2 m_j T_j^{3/2}} \epsilon_r^{-1}, \qquad (7)$$

and simple energy dependence, $\nu_2(\Psi, \varepsilon) = \nu_1 \hat{\varepsilon}^{-(3/2)}$, where $m_{ji} = m_j m_i/(m_j + m_i)$, Z is the charge state, and Λ is the "plasma parameter." Density (n) and temperature (T) are assumed to be one-dimensional spatial functions of Ψ. Including ϵ_r, the inverse aspect ratio, makes ν_{eff} the frequency of collisions causing a trapped ion to take a scattering step on the order of the banana width [17].

The MISK code [10,14] uses the simple energy dependence, ν_2. By including an energy dependence, the effect of reduced collisionality for particles in the high energy tail of the distribution function is included [10]. In contrast, other codes, such as MARS-K [11], consider both ions and electrons to be collisionless ($\nu_{\mathrm{eff}} = \nu_0$). A potentially more accurate way of including collisionality is through a Lorentz operator with complex energy dependence, and a pitch-angle dependence [18].

Note that a typical value of ν_1 is about 1 kHz for NSTX over most of the profile from the axis towards the edge. The collisionality is much larger for electrons ($\nu_{ei}/\nu_{ii} \approx \sqrt{2m_i/m_e} \gg 1$); therefore, it will have a bigger impact on the electron calculation [19]. Conversely, collisions of energetic particles with the bulk ions are extremely rare [$\nu_{ai}/\nu_{ii} = (T_i/\varepsilon_a)^{3/2} \ll 1$], so energetic particle collisionality is expected to be irrelevant [15].

Effect of collisionality model on experimental stability calculations.—The MISK code is used to calculate the various components of δW_K, including trapped ions, trapped electrons, circulating ions, Alfvén layers, and energetic particles (beam ions) for the NSTX discharge 140 132 at 0.704 s. This equilibrium is from just before the RWM goes unstable in the experiment and has experimental rotation and collisionality profiles, defined as $\omega_\phi^{\mathrm{exp}}$ and ν^{exp}, which are in the typical operational range. MISK uses a perturbative approach for this calculation; the RWM eigenfunction was taken directly from the PEST code [20] and assumed to be unchanged by the kinetic effects. The calculation was performed with each of the three collisionality models and for off- and on-resonance cases. For the off-resonance case $1.5\omega_\phi^{\mathrm{exp}}$, which falls in between ω_D and ω_b resonances, was used. For the on-resonance case, a value of $0.2\omega_\phi^{\mathrm{exp}}$, which is in resonance with ω_D, was used.

We can now consider the individual components of δW_K for both off- and on-resonance cases (Fig. 1). The energetic particle component, as expected, is not affected by the model used. Alfvén layer contributions [10,14] are not discussed here; they are also minimally affected.

The largest difference between the collisionless and the two collisional models is that when collisions are added the trapped electron component becomes nearly zero. Rather than acting to dissipate the energy of the mode, electron-ion collisions are so frequent that they dissipate the normally stabilizing kinetic electron pressure perturbations, thus reducing the importance of electrons to RWM stabilization. This is expected for NSTX, because the electron ν_{eff} in the denominator is large compared to other frequencies; however, in future devices with lower overall collisionality this may not be the case. Indeed, in a previous

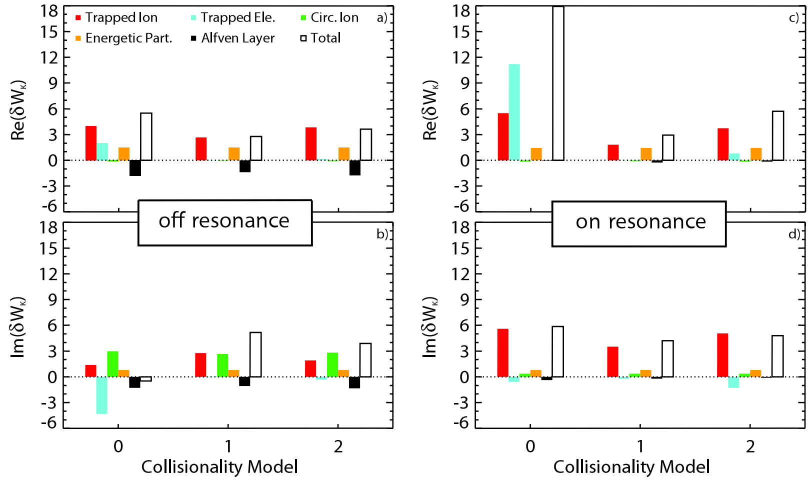

FIG. 1 (color online). Calculated $\mathrm{Re}(\delta W_K)$ and $\mathrm{Im}(\delta W_K)$ for each ν_{eff} model, for NSTX shot 140 132 at 0.704 s off resonance (a),(b) and on resonance (c),(d).

MISK calculation for ITER [15] the ion and electron contributions to δW_K were roughly of the same order. One difference of using ν_2 instead of ν_1 is that for high energy suprathermal electrons, ν_{eff} is reduced, making the electron $\mathrm{Re}(\delta W_K)$ increase slightly.

The on-resonance case selected here refers to a resonance between the trapped ion precession drift frequency and the Doppler-shifted mode frequency for the zero bounce harmonic, $l = 0$. Since there is no equivalent to $l = 0$ for circulating ions, the transit frequency is always present in Ω_2, so δW_K is often small for circulating ions, and it is relatively unaffected by collisionality.

Therefore, besides collisionless electrons, the only component significantly affected by collisionality is the trapped thermal ion. This component should follow the parametric dependence of Eqs. (5) and (6), and indeed we find that, as expected, in the off-resonance case both $\mathrm{Re}(\delta W_K)$ and $\mathrm{Im}(\delta W_K)$ are affected, while in the on-resonance case $\mathrm{Re}(\delta W_K)$ is more strongly affected. In both cases the values calculated with ν_1 are farther from the collisionless case than ν_2 is, because the energy dependence reduces ν_2 for suprathermal particles.

Effect of collisionality magnitude on experimental stability calculations.—It is interesting to see what effect the magnitude of the collision frequency has on stability as well, especially in light of future machines which will have lower collisionality. Figure 2 shows a stability diagram [14] for the off- and on-resonance cases, plotting the values $\mathrm{Re}(\delta W_K)$ vs $\mathrm{Im}(\delta W_K)$ for each of the collisionality models. The shaded region is predicted to be unstable ($\gamma > 0$) and the lines are contours of constant normalized growth rate $\gamma\tau_w$ calculated from Eq. (1) using MISK, with δW_b and δW_∞ from PEST. Using ν_2 as the best model, it is scaled from 1/10 to 10 times ν^{exp}. Here we assume that ν_{eff} changes but n_i and T_i remain constant in the rest of the calculation. Although this is an artificial situation, it allows us to examine the role of collisionality independent of other variables, in contrast to the analysis that was previously performed [13,14]. Changes of even a factor of 2 can have a significant effect. Note that the ν_0 point is a continuation of the trends to zero [for example, the negative $\mathrm{Im}(\delta W_K)$ point for ν_0 in Figs. 2(a) and 1(b)].

As expected from Eq. (5), in the off-resonance case, decreased ν_{eff} increases $\mathrm{Im}(\delta W_K)$ while also increasing $\mathrm{Re}(\delta W_K)$ [Fig. 2(a)]. The net effect is that the calculation tends to move along contours of constant γ, and therefore the stability stays relatively constant. For the on-resonance case [Fig. 2(b)], decreased ν_{eff} increases $\mathrm{Re}(\delta W_K)$ while

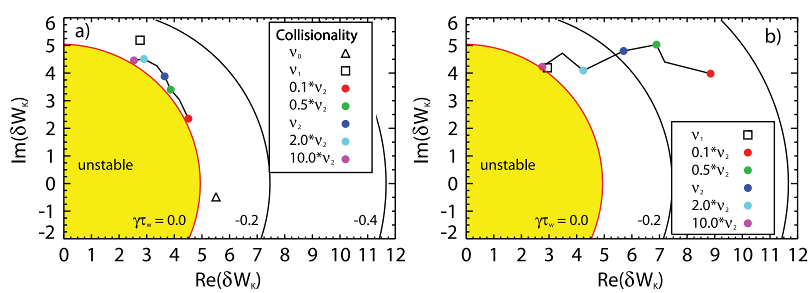

FIG. 2 (color online). Stability diagram for NSTX shot 140 132 at 0.704 s off (a) and on (b) resonance, showing δW_K from MISK for the three different collisionality models and a range of scaled ν_2. In (b) the ν_0 point is off scale to the right.

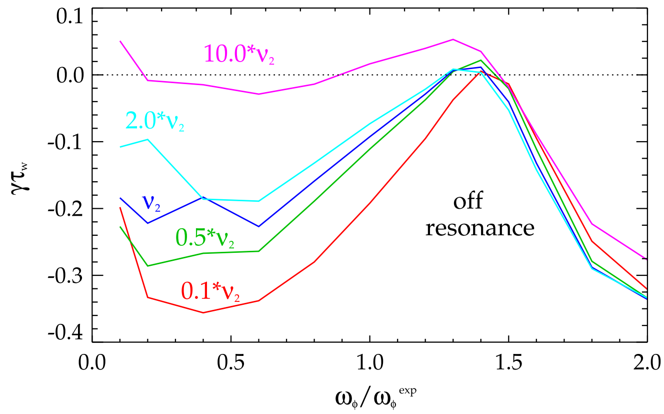

FIG. 3 (color online). $\gamma\tau_w$ vs $\omega_\phi/\omega_\phi^{\rm exp}$ for NSTX shot 140 132 at 0.704 s at various levels of scaled ν_2.

$\mathrm{Im}(\delta W_K)$ is less affected, as expected from Eq. (6). The result is an increase in stability.

Figure 3 shows normalized growth rate vs scaled experimental rotation for the given NSTX discharge for the same variations of experimental ν_2 used in Fig. 2. One can clearly see that a stabilizing resonance with the ion precession drift frequency occurs at low rotation ($0.2 < \omega_\phi/\omega_\phi^{\rm exp} < 0.7$) and with bounce and transit frequencies at higher rotation ($\omega_\phi/\omega_\phi^{\rm exp} > 1.6$) while in between these there exists a narrow range of off-resonance rotation profiles with marginal stability ($1.3 < \omega_\phi/\omega_\phi^{\rm exp} < 1.5$).

In general, from Eqs. (5) and (6), and Figs. 2 and 3, we can say that with other parameters equal, the reduced collisionality expected in future devices can enhance the RWM stability of on-resonance plasmas (an effect that can also be seen in Fig. 6 of Ref. [14]), while leaving the reduced stability of off-resonance plasmas roughly unchanged. In conjunction with kinetic resonances, collisions play a different role that can appear contradictory when compared to simpler models. This is because reduced collisions reduce the collisional dissipation that is important when plasma rotational resonances are not present, but also reduce the damping of resonant kinetic stabilizing effects, allowing them to be more powerful—two competing effects. This behavior is in contrast to the predictions of previous models [3,7], where decreased $\nu_{\rm eff}$ was thought to be universally destabilizing.

Conclusion.—Collisionality appears in the frequency resonance term of δW_K in kinetic RWM stability theory. Electrons collide with ions so frequently that their contribution to δW_K becomes small. Ion-ion collision frequency, however, is in the range of other important plasma frequencies, making the choice of collisionality model important. Changing from a collisionless case to an energy-dependent collision model affects the trapped ion δW_K when the plasma is off resonance and mainly the trapped ion $\mathrm{Re}(\delta W_K)$ when it is on resonance. Finally, varying the ion-ion collisionality by an order of magnitude can have a significant effect on the calculated growth rate of the RWM. In future devices with lower collisionality the amount of variation of the plasma stability will increase for a given change in plasma rotation, making it especially important to avoid the more sudden approach to unstable off-resonance ω_ϕ through plasma rotation profile control, or through active mode control when such unstable ω_ϕ profiles occur.

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