Full-Wave modeling of RF waves propagation and absorption in the presence of beam ions

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Initiative SciDAC-RF

http://www.ornl.gov/fed/scidacrf/

Outline

1) Theoretical introduction

- Why Full-Wave modeling ?
- All-orders spectral description
- How to handle non-Maxwellian distribution functions ?

2) The Mets code

- Main features
- Beam ions modeling: slowing-down distribution
- Equivalent Maxwellian

3) Applications

- Mode converted IBW with Tritium NBI in TFTR
- HHFW with NBI Deuterium beam ions NSTX

4) Conclusions / plans

Plasma-Wave interaction modeling

- Wave Equation: $\nabla\times\nabla\times\mathbf{E}-\frac{\omega^2}{c^2}\left(\mathbf{E}+\frac{i}{\omega\epsilon_0}\mathbf{J}_p\right)=i\omega\mu_0\mathbf{J}_s$

Source Current

- Plasma current: $\mathbf{J}_p(\mathbf{r})=\int d\mathbf{r}'\,\overline{\overline{\boldsymbol{\sigma}}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(r')$

Dielectric tensor

- Integral wave equation (inhomogeneous plasma)

$$\nabla\times\nabla\times\mathbf{E}-\frac{\omega^2}{c^2}\int d^3\mathbf{k}\,\exp(i\mathbf{k}\cdot\mathbf{r})\overline{\overline{\mathbf{K}}}(\mathbf{r},\mathbf{k})\cdot\mathbf{E}(\mathbf{k})=i\omega\mu_0\mathbf{J}_s$$

- Weakly inhomogeneous plasma: □ << L_B
  - Eikonal equation: splitting of fast-varying phase and slow-varying amplitude
  - Dispersion relation D(**r**,**k**)=0 and ray-tracing

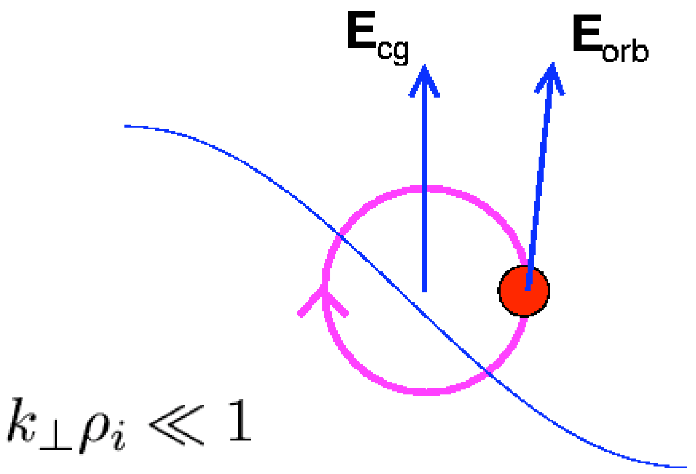
- IC range of frequency:
  - □ # L_B
  - Cut-offs, resonances, inter-modes interactions

→ **Ray-tracing calculation may be inadequate**

→ **Needs for a full-wave analysis**

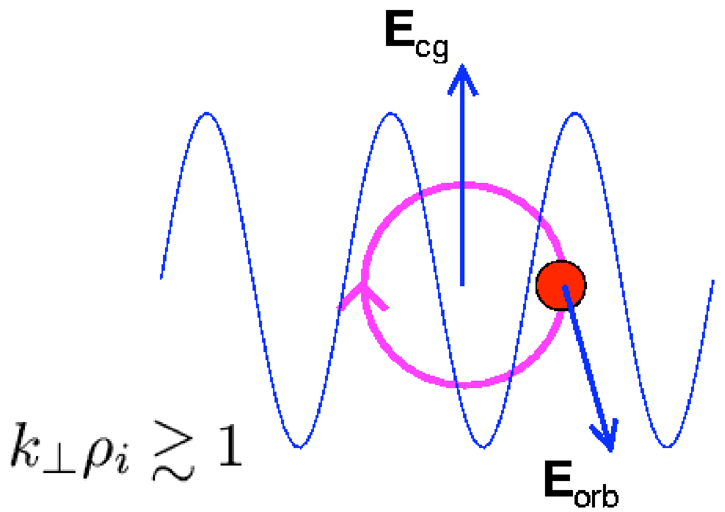
What is an all-orders description ?

2nd order expansion in $k_{\perp}\rho_i$



Perpendicular locality

All-orders description



Perpendicular non-locality

- **All-orders description required:**
    - Reliable modeling of IBW
    - Description of the interaction between wave and fast ions
    - Simulation of heating at harmonics n > 2
- **Drawbacks:**
    - Dielectric tensor and absorption are difficult to compute
    - Interpretation of results often less straightforward
    - Dense matrices: computation time and storage requirements increased

Non-thermal distributions functions

- Dielectric kernel in the local magnetic frame

$$\bar{\bar{\Theta}}_n(x,\mathbf{k}_1,\mathbf{k}_2) \equiv \int_0^\infty d\tau \int_0^\infty dv_{||} e^{\left(i(\omega\tau - n\theta(\tau) - k_{||,1}v_{||}\tau)\right)} \int_{-\infty}^{\infty} dv_\perp \; \bar{\bar{\mathbf{w}}}(x,\mathbf{v},\mathbf{k}_1,\mathbf{k}_2)$$

→ Gyro-averaged energy kernel **W**

→ Dielectric kernel **K: Electromagnetic fields**

→ Energy quantities **P,T**, W_{abs}: **Local absorption**

- General expression for **w** (arbitrary distribution function)

$$\bar{\bar{\mathbf{w}}} \equiv \begin{bmatrix} \frac{v_\perp}{\sqrt{2}} J_{n+1}(\xi_2) \\ \frac{v_\perp}{\sqrt{2}} J_{n-1}(\xi_2) \\ v_{||} J_n(\xi_2) \end{bmatrix} \begin{bmatrix} \frac{\hat{L} f_0}{\sqrt{2}} J_{n+1}(\xi_1) & \frac{\hat{L} f_0}{\sqrt{2}} J_{n-1}(\xi_1) & \hat{L}_n f_0 J_n(\xi_1) \end{bmatrix}$$

$$\xi_{1,2} = k_{\perp 1,2} v_\perp / \Omega_0$$

→ **f_0 arbitrary: numerical computation of velocity integrals**

The Mets code

- Physics features
  - All-orders 1D full wave code
  - No FLR approximation
  - Finite magnetic field gradient effects considered
  - ***Handles arbitrary non-Maxwellian distribution functions***
- One code, two modes

Maxwellian METS →

- Analytical expression for the dielectric tensor
- $1/L_{//} \neq 0$
- Computation time: □# 1mn

Non-Maxwellian METS →

- Velocity integrals performed numerically
- $1/L_{//} = 0$ (No magnetic field gradient effects)
- Computation time: □# 1h

- Two goals
  - Tool for the study of ICRF scenarios
  - Testbed for the implementation of non-Maxwellian distribution capabilities in 2D Full-Wave codes: TORIC, AORSA (SciDAC initiative)

Slowing-down distribution for beam ions

[M. Cox and D.F.H Start, Nucl. Fusion, **24** (1984) 399]

$$f(u,\mu) = \sum_{l=0}^{\infty} a_l(u) P_l(\mu),$$

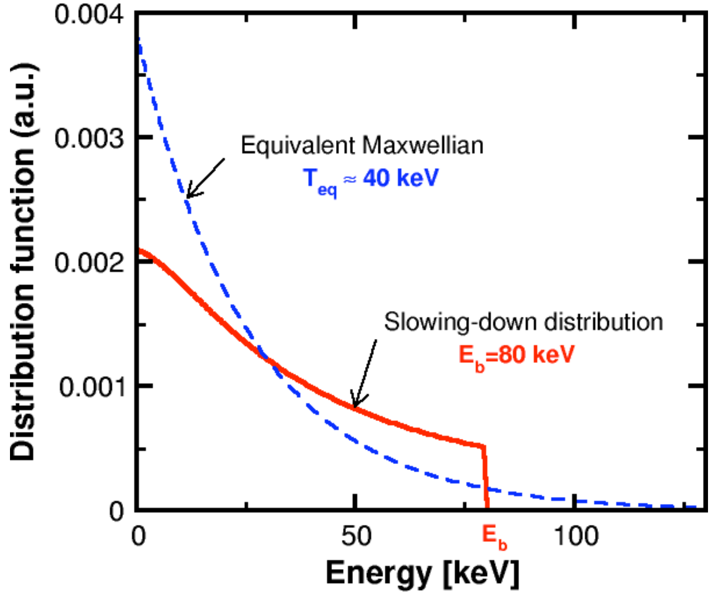
u is normalized to $E_b^{1/2}$

Legendre polynomials

with

Beam angular distribution

$$a_l(u) \equiv (2l+1)\frac{\tau_s}{4\pi(1+u_c^3)} \cdot \frac{K_l}{K_0} \cdot S \cdot A_l(u)\,, \quad K_l \equiv \int_{-1}^{1} d\mu P_l(\mu) K(\mu)$$



Equivalent maxwellian

$$\int_0^{v_b} dv\, \frac{v^2}{2} f_b(v) = \int_0^{\infty} dv\, \frac{v^2}{2} f_{max}(v)$$

$$\longrightarrow \quad T_{eq} = \frac{2m_b}{\ln(1+(u_m^3/u_c^3))} \int_0^{v_m} dv\, \frac{v^4}{v^3+v_c^3}$$

Mode conversion with NBI on TFTR

D-T supershot on TFTR:

B_0=4.7 T, n_{e0}=4.7 10^{19} m^{-3}, T_{e0}=6.8 keV
D,T, H and ^{6}C with $\square_T$=42 %, T_{i0}=31 keV

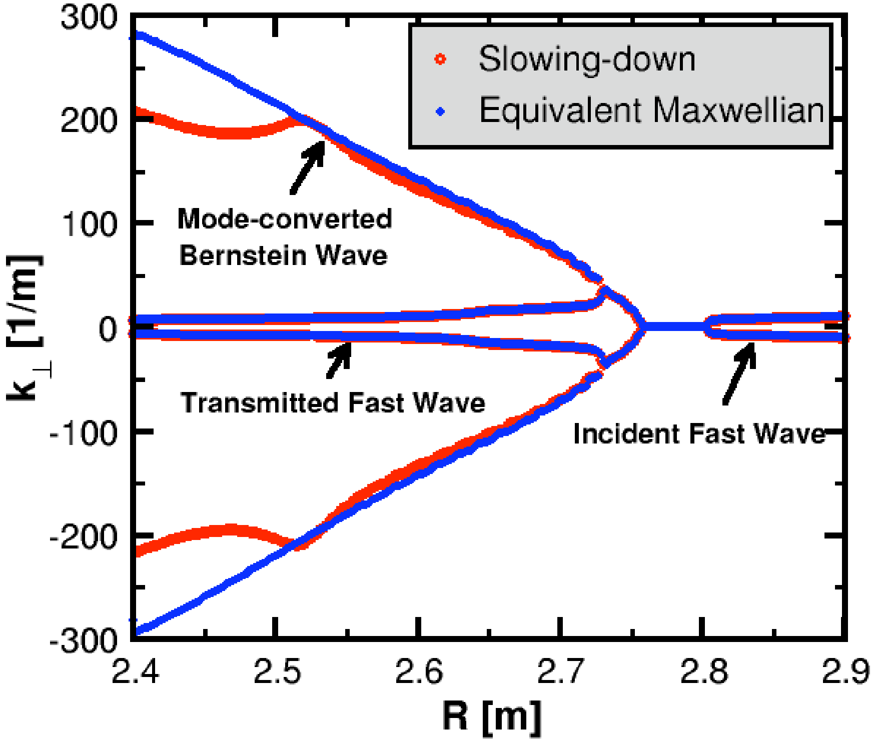
f_{FW}=30 MHz with $k_{//}^{ant}$=7 m^{-1}

Beam Tritium Ions: isotropic slowing-down distribution

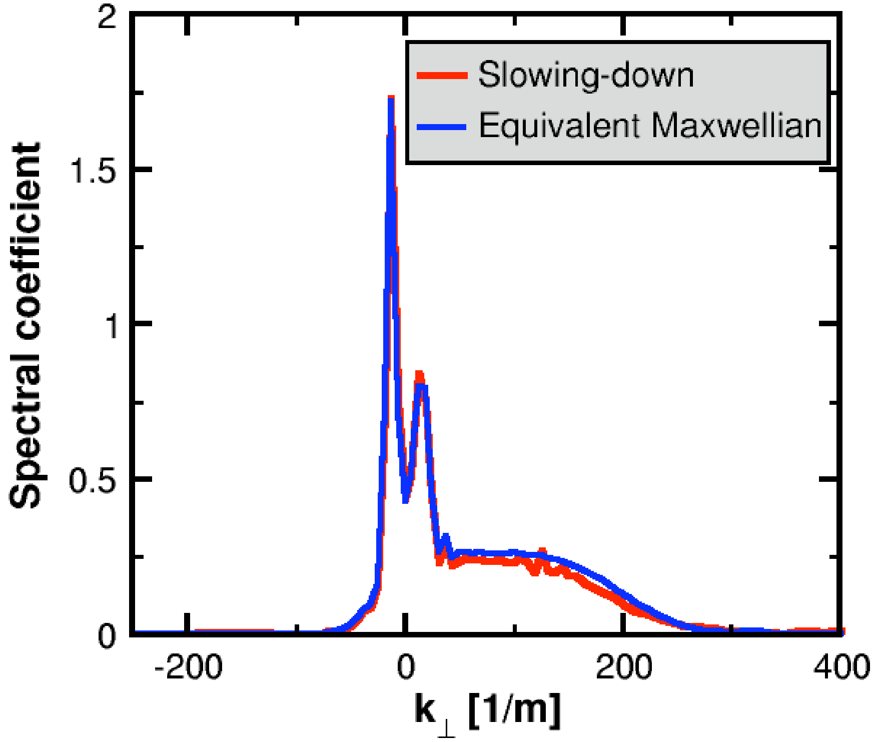
Mode-conversion with NBI in TFTR: propagation

T-NBI ions: isotropic slowing-down distribution

Dispersion relation



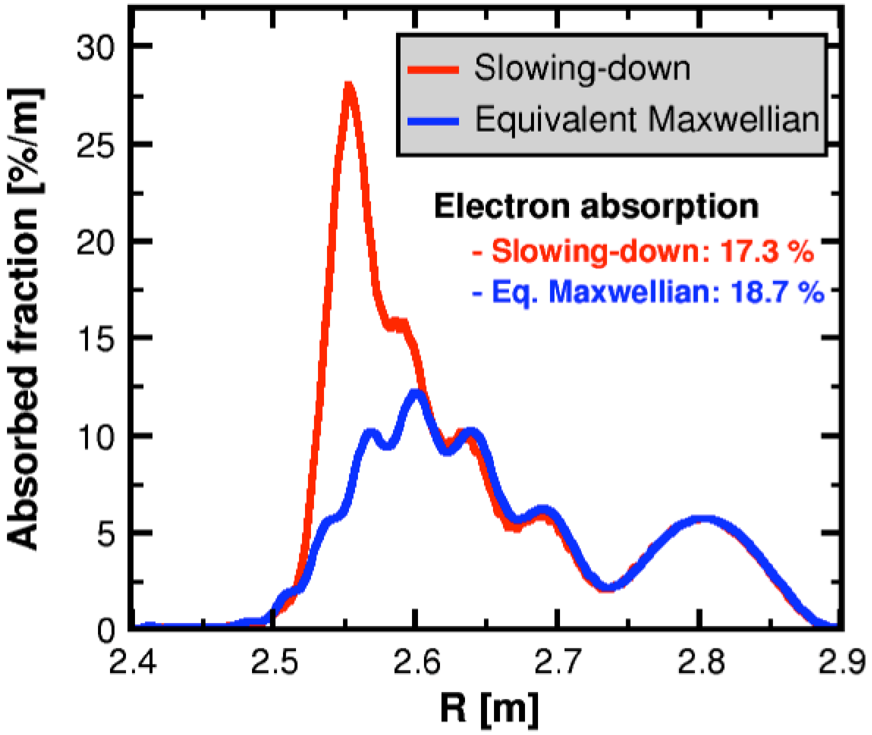
Spectrum



Modification of the Mode-Converted IBW branch for wavenumbers $k_{\perp} > 200 m^{-1}$.

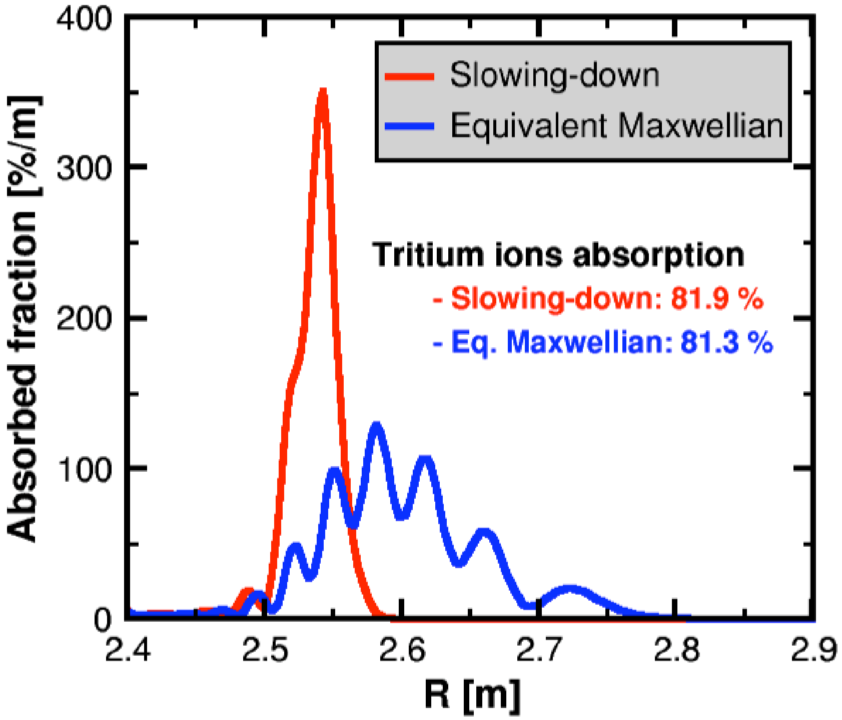
Mode conversion with NBI in TFTR: Absorption

Electron absorption



Electron absorption exhibit roughly similar shapes for both cases except in the absorption region

Tritium beam ions absorption

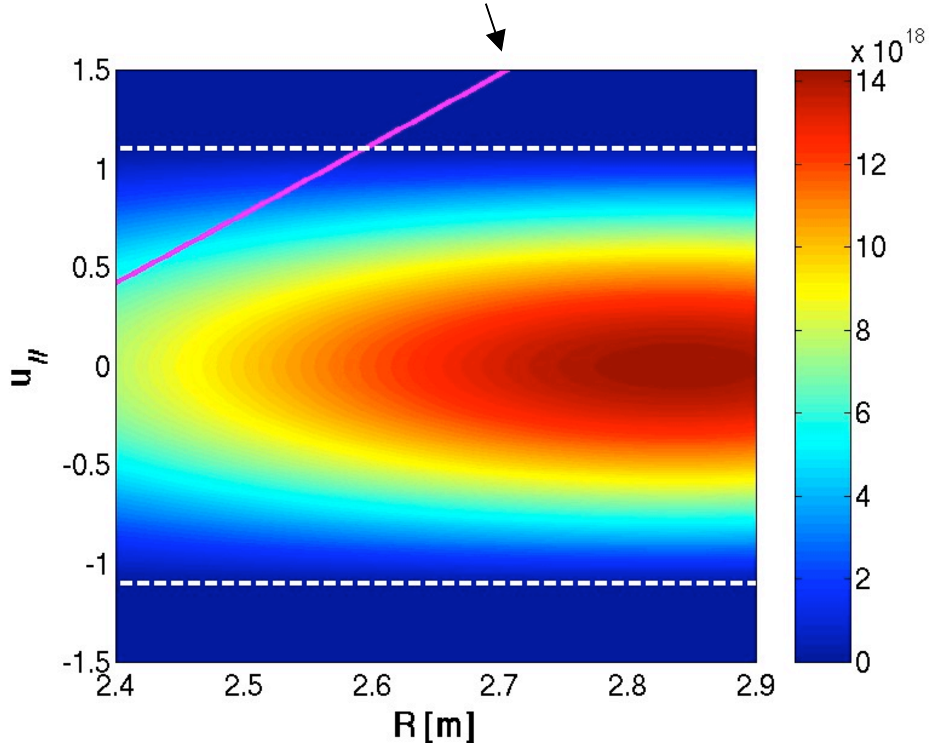


Energetic ions absorption appears to be much more peaked with the slowing-down distribution

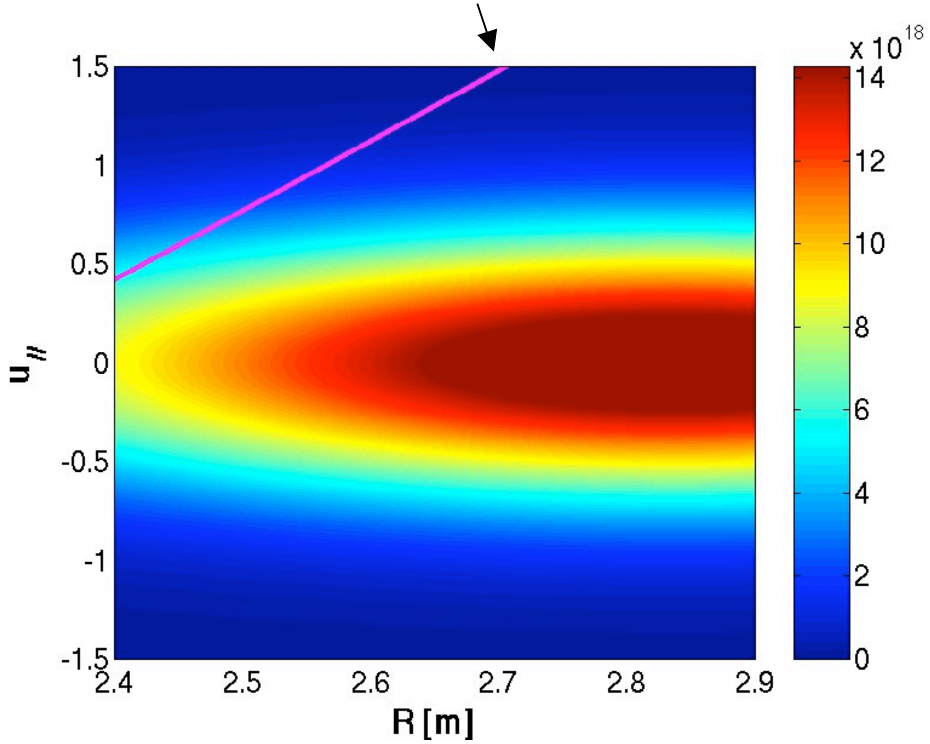
Net absorption similar for both cases, but profiles differ strongly

Distribution function features

Parallel distribution function: $F(u_\parallel) \equiv 2\pi \int_{-\infty}^{\infty} du_\perp u_\perp f(u_\parallel, u_\perp)$

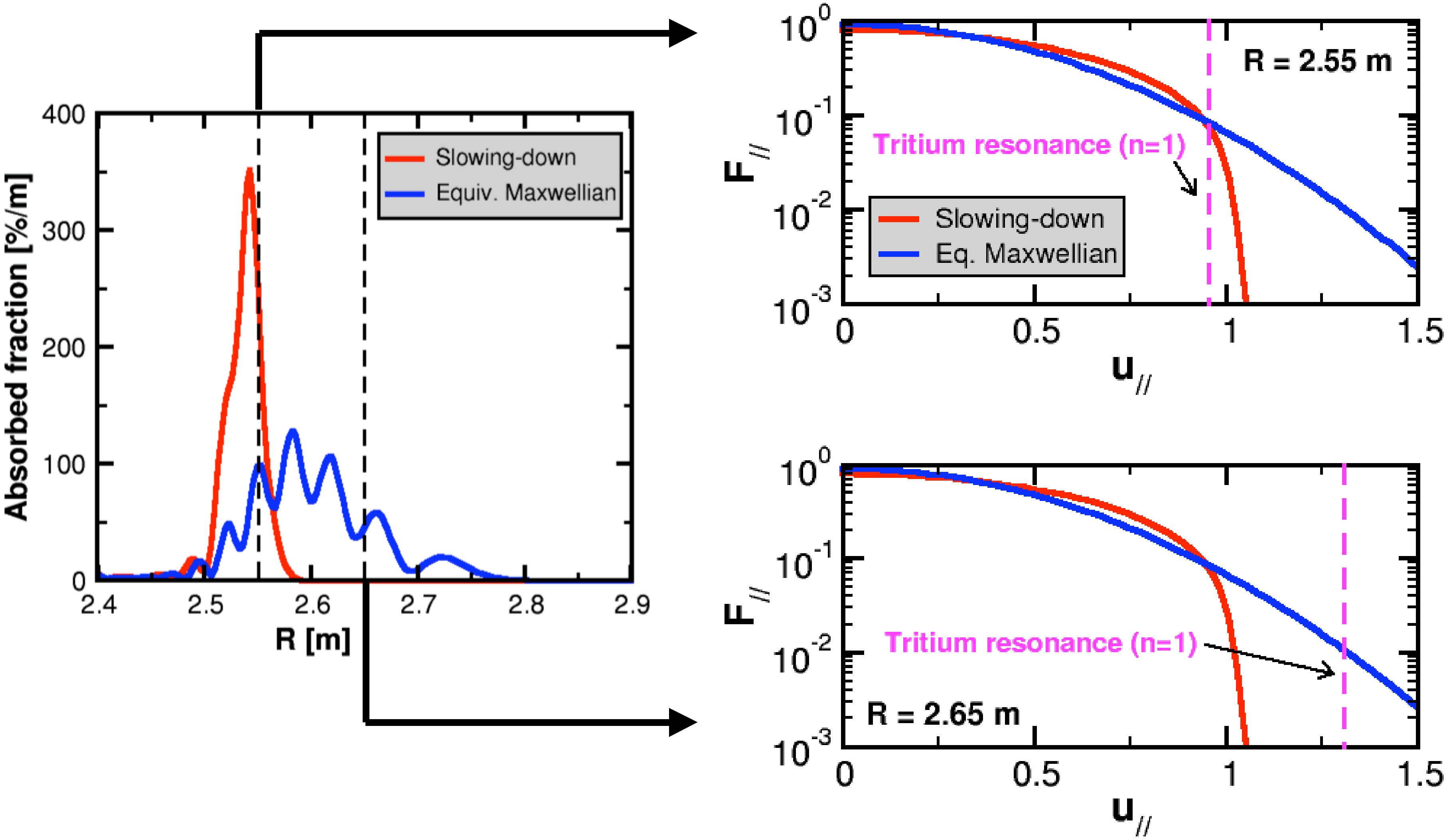

n=1 parallel resonant velocity

Slowing-down distribution


n=1 parallel resonant velocity

Equivalent Maxwellian

Distribution function features (2)



Why does it mostly affect the absorption ?

- Local absorption in an all-orders description

$$W_{abs} = \ldots \mathrm{Im}\left(e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}}\,\mathbf{E}^*(\mathbf{k}_2)\cdot\overset{=}{\mathbf{W}}(\mathbf{r},\mathbf{k}_1,\mathbf{k}_2)\cdot\mathbf{E}(\mathbf{k}_1)\right)$$

- Assuming perpendicular locality: **k**$_1$=**k**$_2$=**k**: $W_{abs} = \ldots \mathbf{E}^*(\mathbf{k})\cdot\overset{=}{\mathbf{W}}^{(a)}\cdot\mathbf{E}(\mathbf{k})$

Anti-hermitian part

- The energy kernel appears as

$$\overset{=}{\mathbf{W}} \propto \int_{-\infty}^{\infty} du_\parallel\,\frac{1}{u_\parallel - u_{\parallel,res}}\int_0^{\infty} du_\perp\,\mathcal{F}\left(\frac{\partial f_0}{\partial u_\parallel},\frac{\partial f_0}{\partial u_\perp},\ldots\right)$$

- And can be treated according to Plemelj formula

$$\overset{=}{\mathbf{W}} \propto \mathcal{P}\left(\int d\mathbf{u}\,\mathcal{F}\right) - i\pi\int du_\perp\,\mathcal{F}\left(\frac{\partial f_0}{\partial u_\parallel},\frac{\partial f_0}{\partial u_\perp}\right)\Bigg|_{u_\parallel=u_{\parallel,res}}$$

The absorption is sensitive to the local values of
the distribution function velocity gradients at $u_{//}=u_{//,res}$

High Harmonic Fast Wave Electron Heating combined with NBI heating in NSTX

NSTX HHFW shot: B_0=0.45 T, n_{e0}=2.75x10^{19} m^{-3}, T_{e0}=1 keV

D, H and ^{6}C with T_{i0}=1 keV

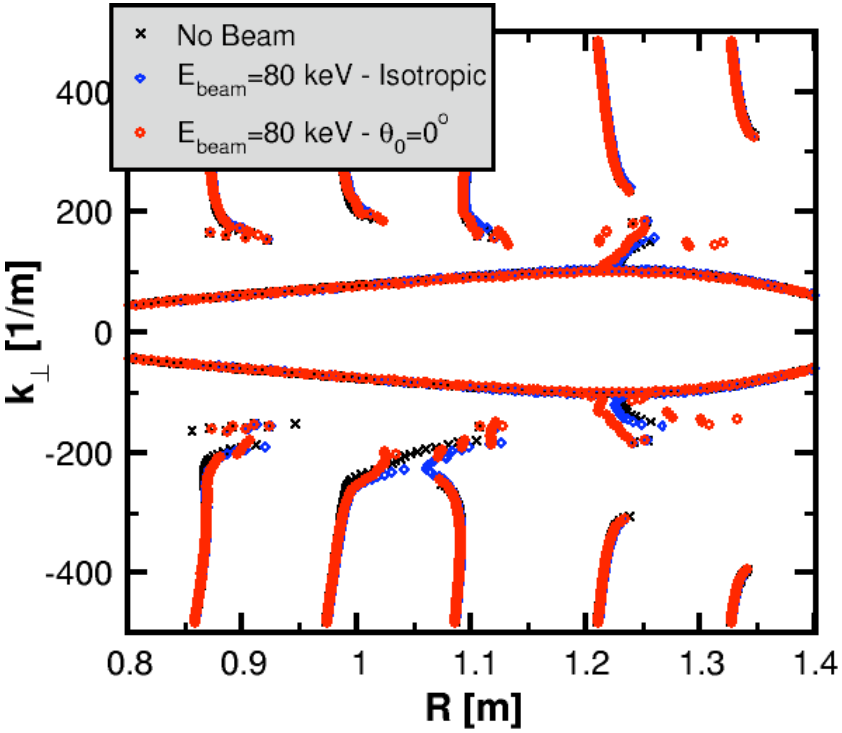
f_{FW}=30 MHz, $k_{//}^{ant}$=14 m^{-1}

D-NBI ions: isotropic / anisotropic slowing-down

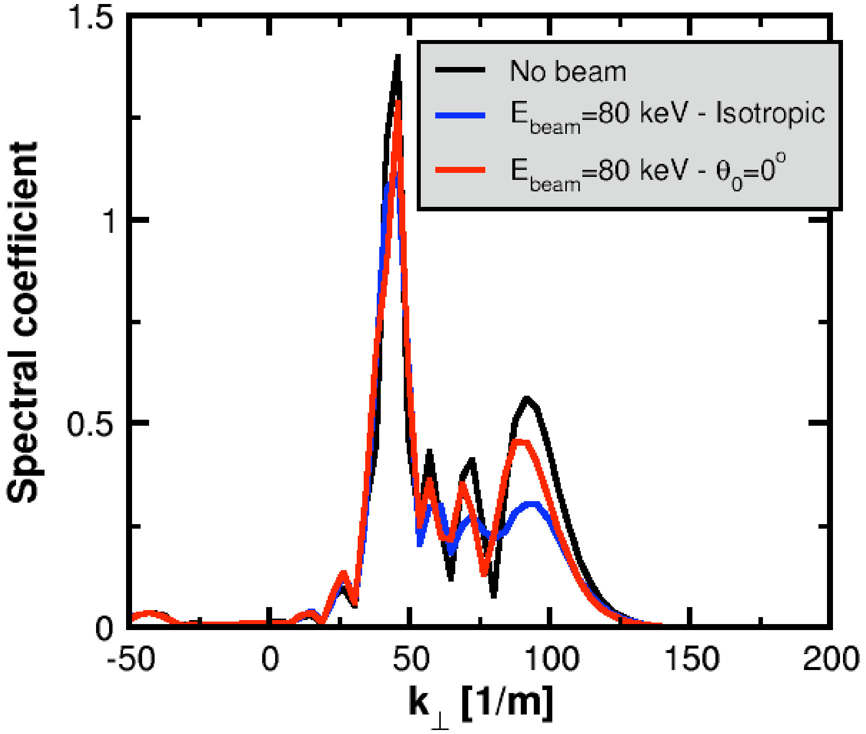
HHFW + NBI on NSTX: Propagation

D-NBI ions: isotropic/anisotropic slowing-down distribution

Dispersion relation



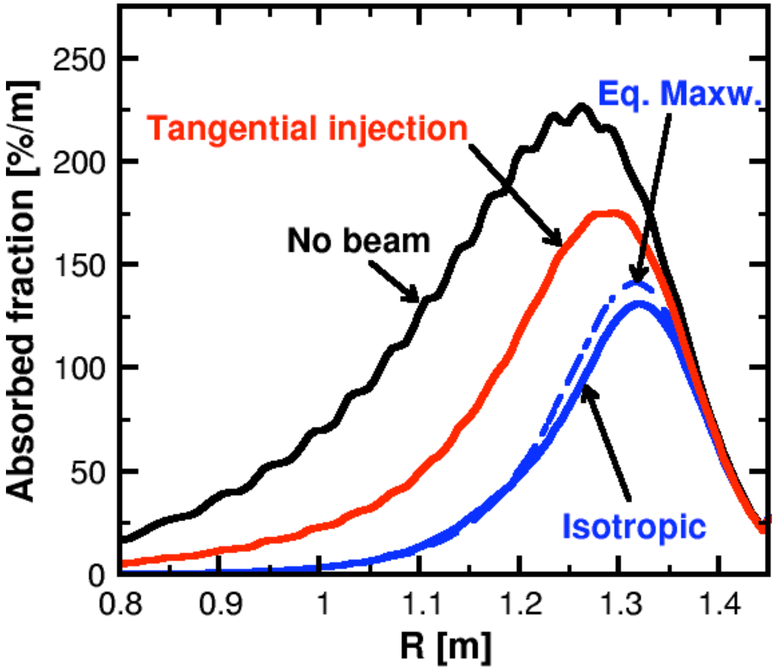
Spectrum



No major changes observed in dispersion relation / spectrum

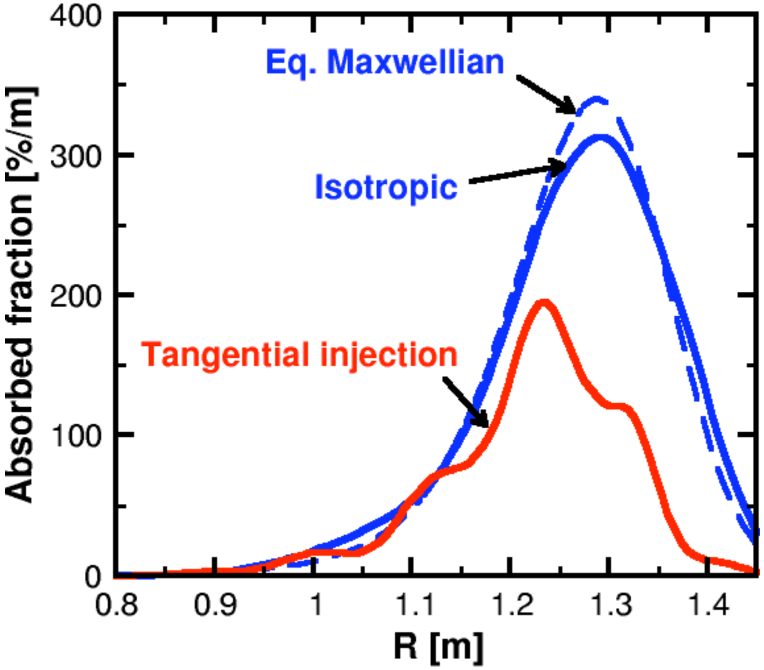
Wave power absorption by fast ions

Electron absorption



- Without beam: **70 %** (per pass)
- Isotropic beam: **24 %**
- Anisotropic beam: **45%**

Deuterium beam ions absorption



- Without beam: **0 %** (per pass)
- Isotropic beam: **70 %**
- Anisotropic beam: **42%**

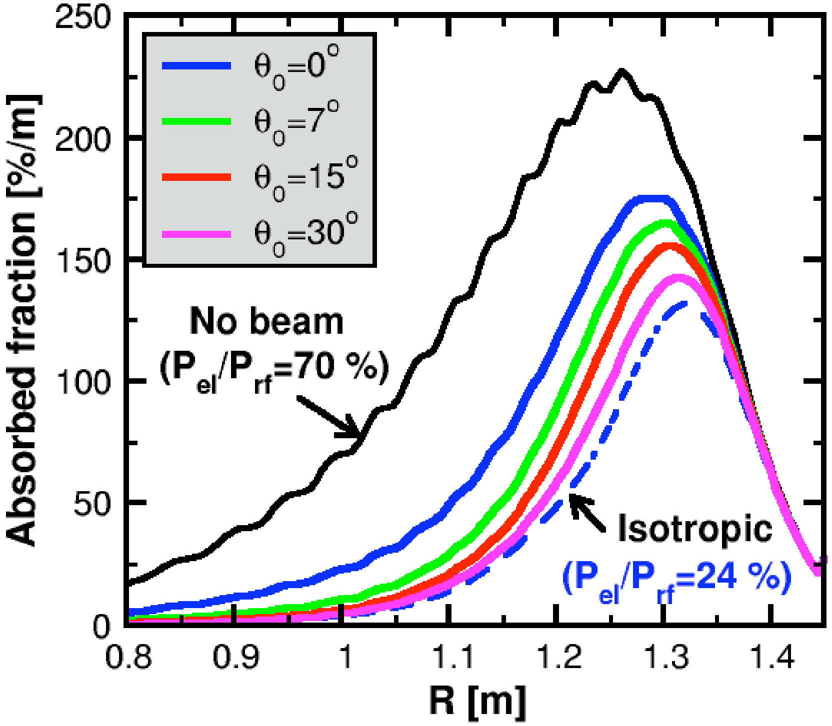
- Isotropic slowing down and equivalent Maxwellian in agreement

- Less fast ion absorption in the case of tangential injection

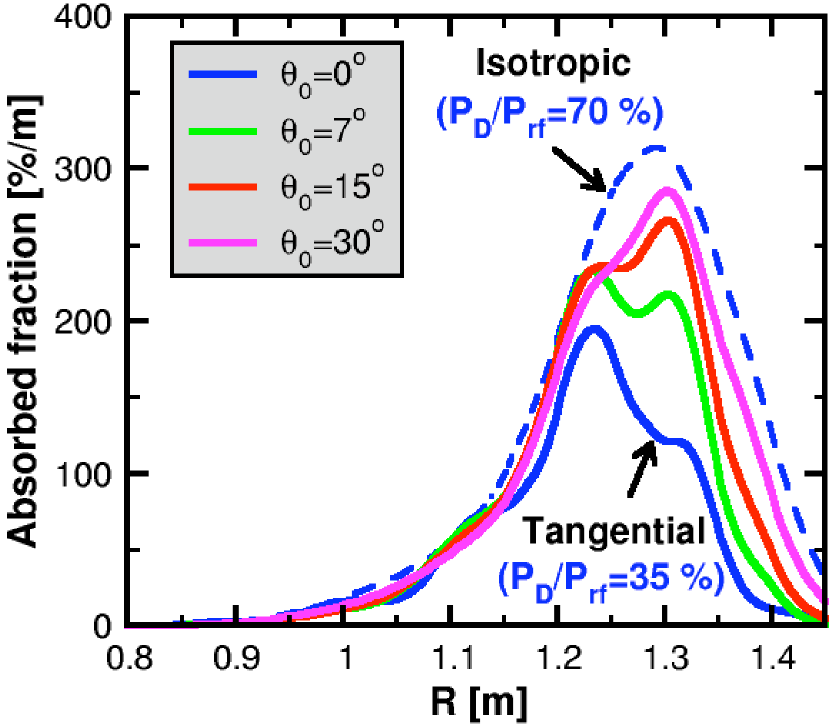
Strong effect of the beam injection angle

Beam angle (θ_0) varied between 0 and 30°

Electron absorption



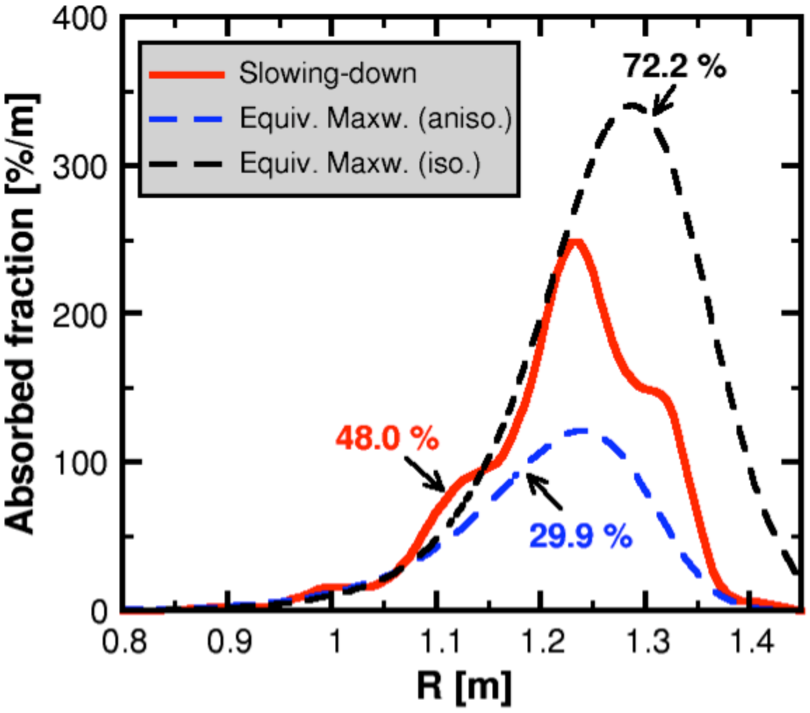
Deuterium beam ions absorption



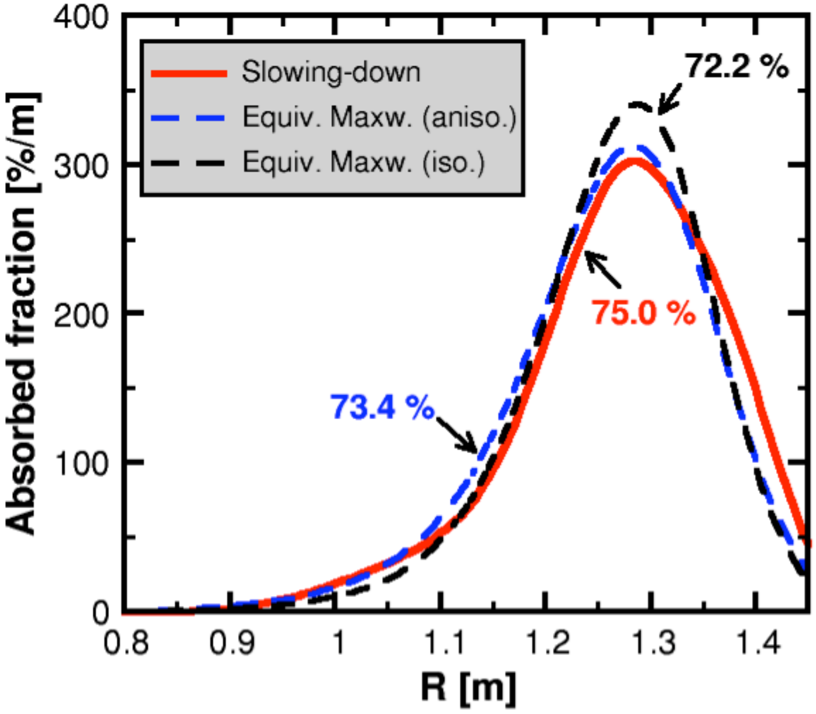
Effect sensitive to the beam pitch-angle distribution, especially for injection angles below 30°.

Anisotropic equivalent Maxwellian

Tangential injection



Perpendicular injection

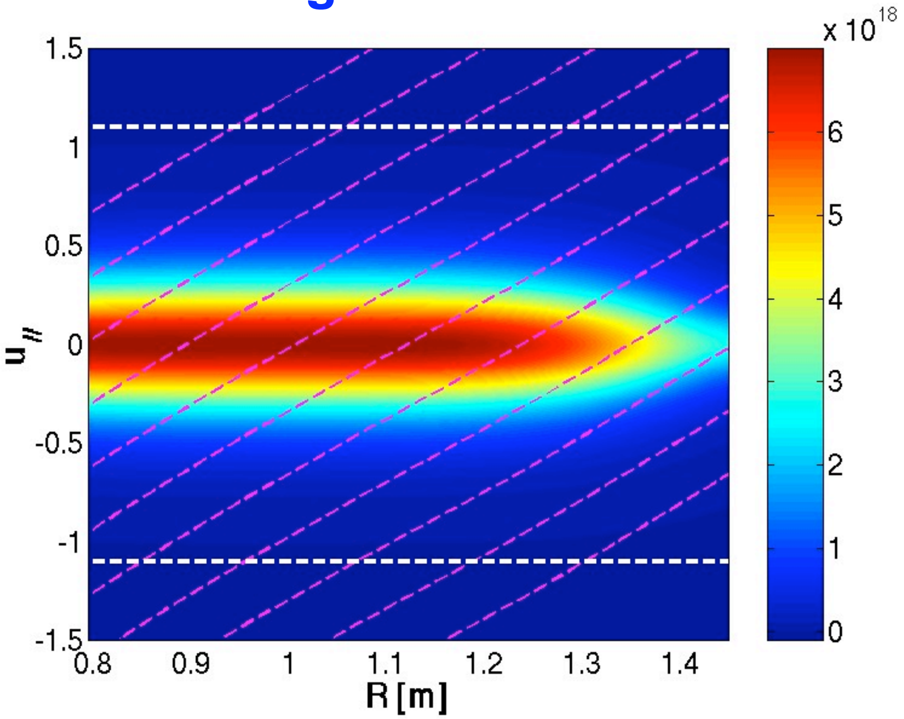


The detailed shape of the distribution function in the parallel velocity direction is fundamental for the absorption

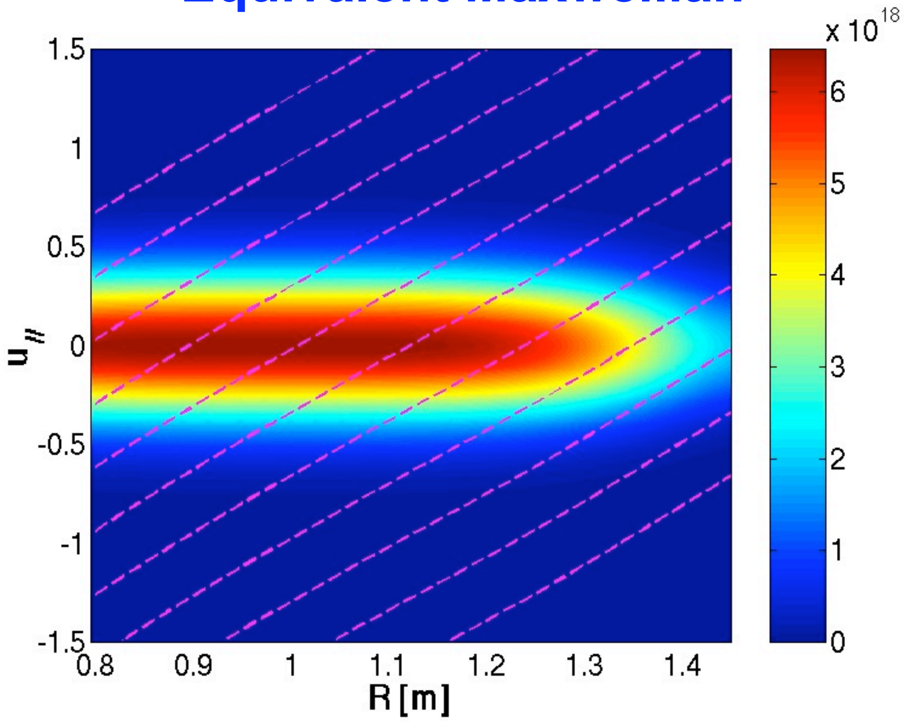
Why does the anisotropic equivalent Maxwellian work for large angles ?

Case of a perpendicular injection ($\square_0=90^o$)

Slowing-down distribution



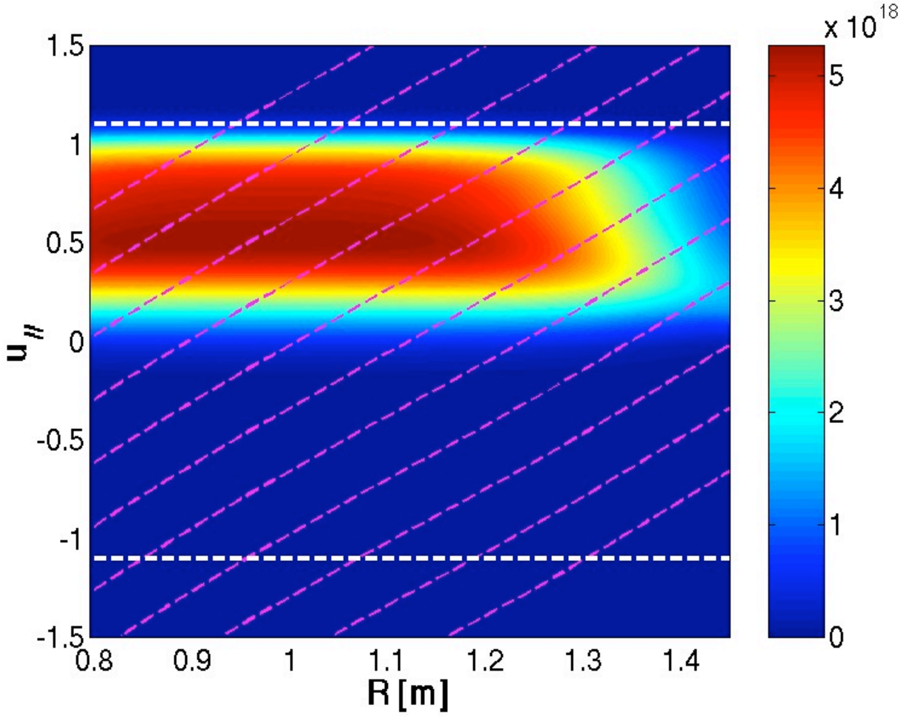
Equivalent Maxwellian



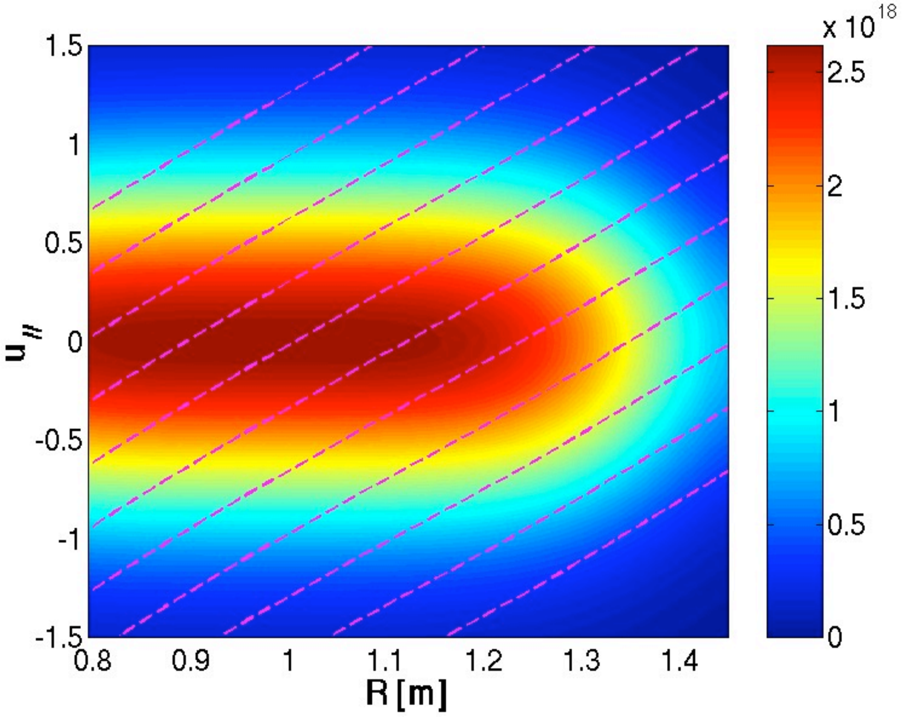
With two temperatures, the equivalent Maxwellian gives a realistic picture of the real distribution function

Why does it fail to describe the absorption for small injection angle ?

Case of a tangential injection ($\square_0$=0°)



Slowing-down distribution



Equivalent Maxwellian

It is not possible to get a good picture of the distribution function for small angles, even with anistropic velocities

Conclusions / Plans

- **Non-Maxwellian distributions implemented in Mets-1D**
  - Numerical treatment of the velocity integrals
  - Thoroughly benchmarked against analytical results

- **Study of ICRF scenarios**
  - Mode conversion with Tritium beam on TFTR
  - HHFW with Deuterium beams on NSTX

- **Inadequacy of the equivalent Maxwellian**
  - Presence of ions faster than the beam injection velocity
  - Anisotropic distribution effects not correctly described

- **Short-term developments**
  - Improvement of the numerical treatment of velocity integrals
  - Alternate function basis for speeding up the computation

- **The next big thing: non-Maxwellian + 2D effects**
  - AORSA-2D: Non-Maxwellian Mets routines just implemented
  - TORIC: Towards a Full-Wave modeling of LHCD