Predictions and observations of global beta-induced Alfvén-acoustic modes in JET and NSTX

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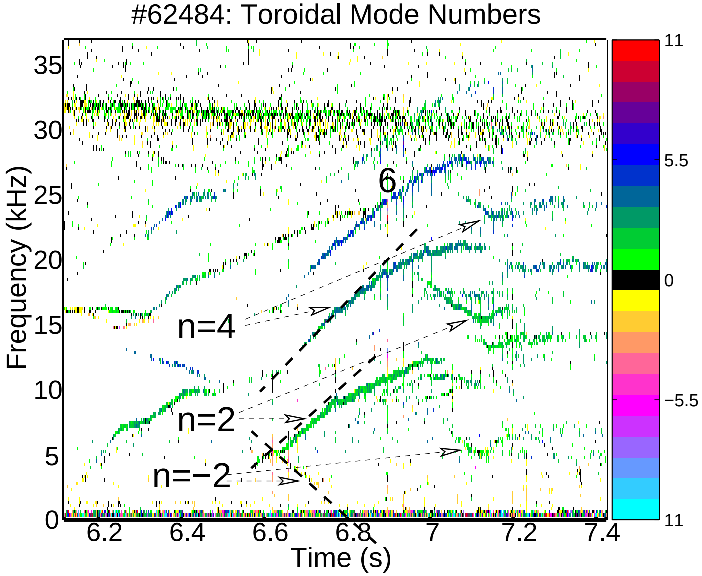
34^{th} EPS Conference on Plasma Physics, July 2-6, Warsaw, 2007

EFDA EUROPEAN FUSION DEVELOPMENT AGREEMENT JET

PPPL PRINCETON PLASMA PHYSICS LABORATORY

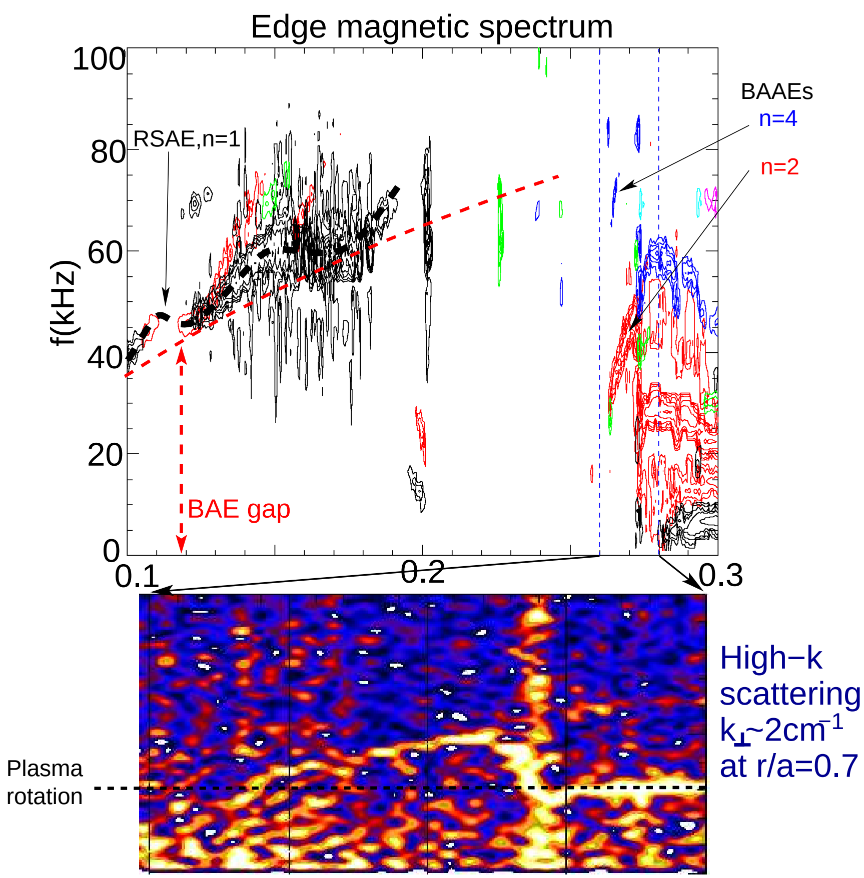
New experimental observations on JET and NSTX motivate low frequency mode study



- JET:
  2MW ICRH in low plama density $\Rightarrow$ high H-minority beta:
  $\beta_H \sim \tau_{se} \sim n_e^{-1}$;
  $\beta_H \sim \beta_{plasma}(\sim 2\%)$.
- New chirping frequency activity $f_{pl} = 0 - 20kHz$ (for RSAE, Alfvénic cascades $f_{pl} \neq 0$)
- Structure at upper limit (another mode, same n).
- Only even n's were observed.

Frequency is much lower than RSAE/TAE frequency, v_A/qR:
$\omega_* = 0.5kHz$ is small

Variety of MHD instabilities are routinely observed in NSTX



- Edge Magnetic spectrum and internal high-k scattering see the same low-f activity as in JET, $t = 0.27s$.
- only even numbers, sweeping frequency, $n = 2, 4$
- reversed shear q-profile from MSE
- $f_{TAE} = v_A/2qR \simeq 90kHz$
- $t = 0.262sec$, $n = 2$ mode frequency $f_{lab} \simeq 30 - 50\,kHz$, $f_{pl} \simeq f_{lab} - n \times 15 = 0 - 20\,kHz$, $\Rightarrow$ too low for TAE for observed $n = 2$ activity.

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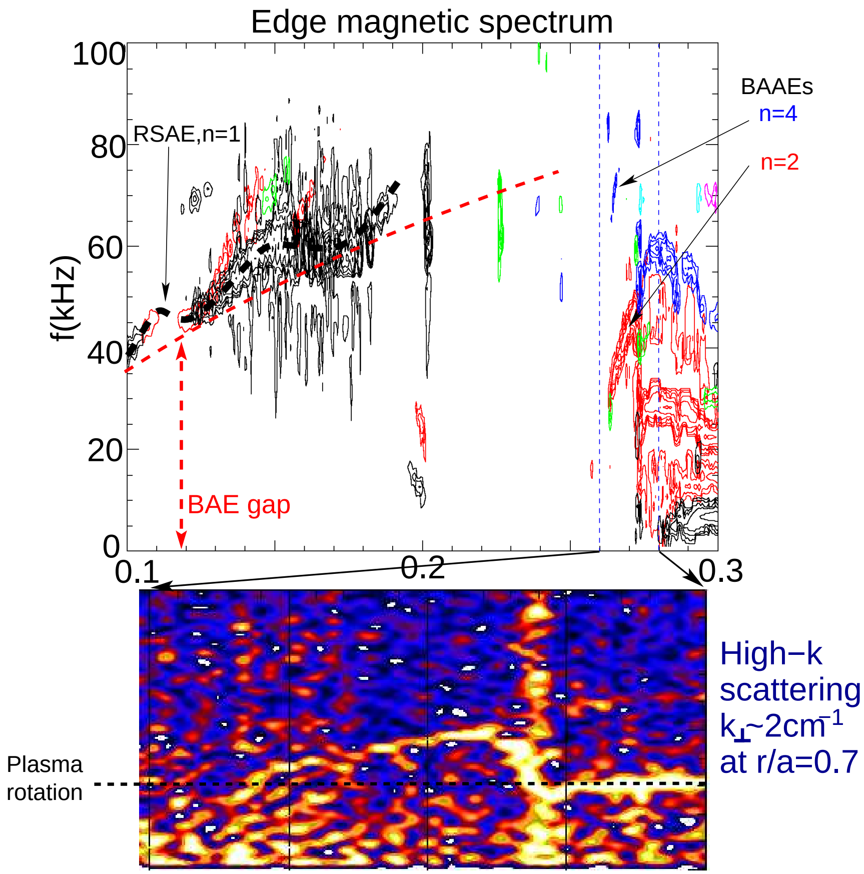

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What are these modes: EPMs, fishbones, KBM, TAEs?

What is the importance of low-f instabilities?

- New class of instabilities called here Beta-induced Alfvén Acoustic Eigenmode (BAAE) helps to study two fundamental MHD waves: Alfvén and acoustic.

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- Due to coupling to acoustic branch strong interaction with thermal ions is expected:
  - $\Rightarrow$ strong drive due to fast ions and strong damping due to thermal ions,
  - $\Rightarrow$ potential for energy channeling from beam ions directly to thermal ions (α-channeling).

TALK OUTLINE

1. Theory of Alfvén - acoustic continuum in ideal MHD
2. JET analysis and data comparison
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Theory of Alfvén/acoustic continuum

Simplified shear Alfvén and acoustic coupled equations in low-β, large aspect ratio plasma, low ω_*, (Cheng, Chance '86):

$$\Omega^2 y + \partial_{\|}^2 y \quad + \gamma\beta \sin\theta z \quad = 0 \; (Alfvenic) \tag{1}$$

$$\Omega^2 \left(1 + \frac{\gamma\beta}{2}\right) z + \frac{\gamma\beta}{2} \partial_{\|}^2 z \quad + 2\Omega^2 \sin\theta y \quad = 0 \; (acoustic), \tag{2}$$

where $\Omega \equiv \omega R_0 / v_A$, $y \equiv \xi_s \varepsilon / q$, $\xi_s \equiv \vec{\xi} \cdot \frac{[\mathbf{B} \times \nabla\psi]}{|\nabla\psi|^2}$ and $z \equiv \nabla \cdot \vec{\xi}$, $\hat{k}_{\|} \equiv i\partial_{\|}$.

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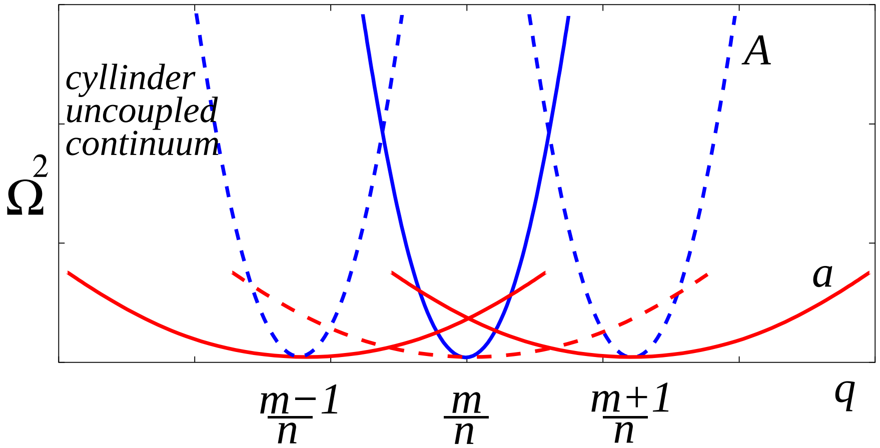
Various solutions follows (Winsor'68, Goedbloed'75, Mikhailovski'75,'98, Chu'92, Turnbull '92, Zonca'96, van der Holst'00, Breizman'05, Berk'06):

- Pure acoustic modes (AMs) $\Omega^2 = \frac{1}{2}\gamma\beta k_\parallel^2$.
- Pure Alfvénic branch $\Omega^2 = k_\parallel^2 + \gamma\beta\left(1 + 1/2q^2\right)$.
- GAMs: $\Omega^2 = \gamma\beta\left(1 + 1/2q^2\right)$ in the assumption of $\Omega^2 \geq \gamma\beta$.
- Modified shear Alfvén branch $\Omega^2 = k_0^2/\left(1 + 2q^2\right)$ exists for $\Omega^2 \ll \gamma\beta$.

Alfvén/acoustic coupling in toroidal equilibrium (schematic)

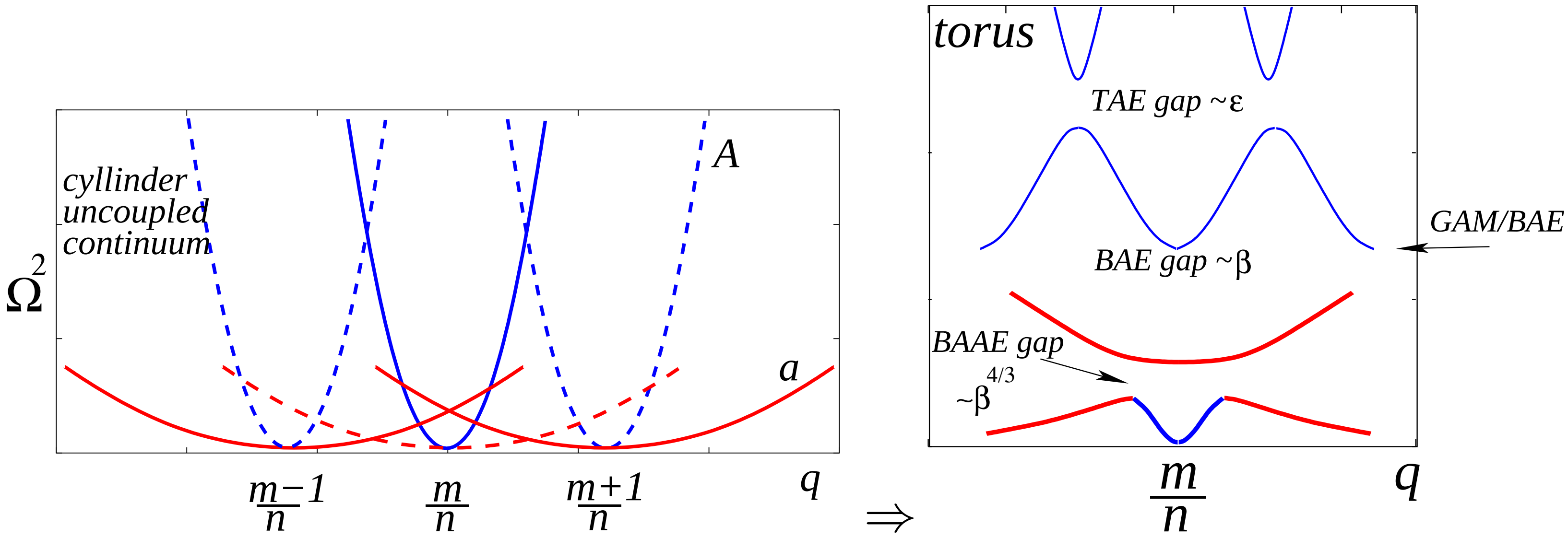
- Alfvén (A) continuum at low frequency: $\Omega^2 = k^2_{0,\pm 1}$
- Acoustic (a) branch $\Omega^2 = \gamma\beta k^2_{0,\pm 1}/2\,(1+\delta)$



Alfvén/acoustic coupling in toroidal equilibrium (schematic)

- Alfvén (A) continuum at low frequency: $\Omega^2 = k_{0,\pm1}^2 \,/\, \left(1+2q^2\right)$ (modified)

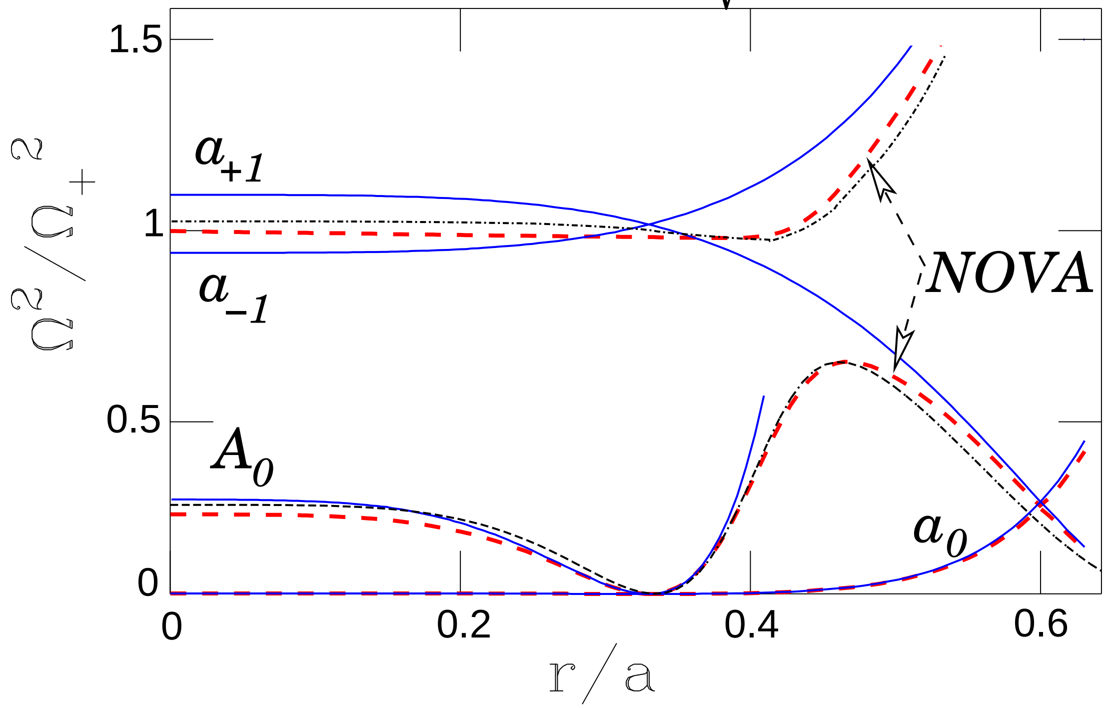
- Acoustic (a) branch $\Omega^2 = \gamma\beta k_{0,\pm1}^2 / 2\left(1+\delta\right)$ is coupled via $m \pm 1$ sidebands with modified Alfvén continuum (m harmonic) due to geodesic curvature and pressure.



Analytic dispersion for Alfvén/acoustic continuum gap is derived

JET *PPPL*

- Consider JET, monotonic q-profile, ten times higher aspect ratio.
- $\Omega_+ = \sqrt{\gamma\beta/2}/q_r$ (compare with GAM $\Omega = \sqrt{\gamma\beta\left(1+1/2q^2\right)}$).



Analytic dispersion - black, dashed lines.

NOVA continuum is in good agreement with theory.

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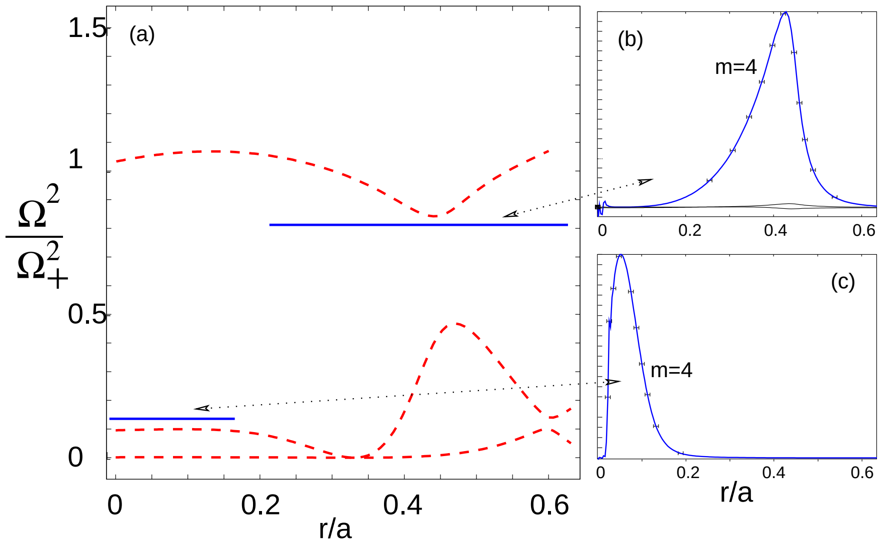
JET plasma analysis: two global BAAE modes are found numerically



- Monotonic $q-$profile (EFIT), $q_0 \simeq 1$, $q_a = 4$.
- Core localized and gap BAAEs are found with one dominant poloidal harmonic.
- Core localized BAAE has $\omega = v_A k_\parallel / \sqrt{1 + 2q_{min}^2}|_{r=0}$.
- Gap BAAE has $\Omega_+ \simeq v_A \sqrt{\gamma\beta/2}/q_{min}R$.
- $n = 4$, $r\xi_n$ is shown.
- $\nabla\xi$, $m \pm 1$ sidebands are present ($\sim \xi_\theta/a$).

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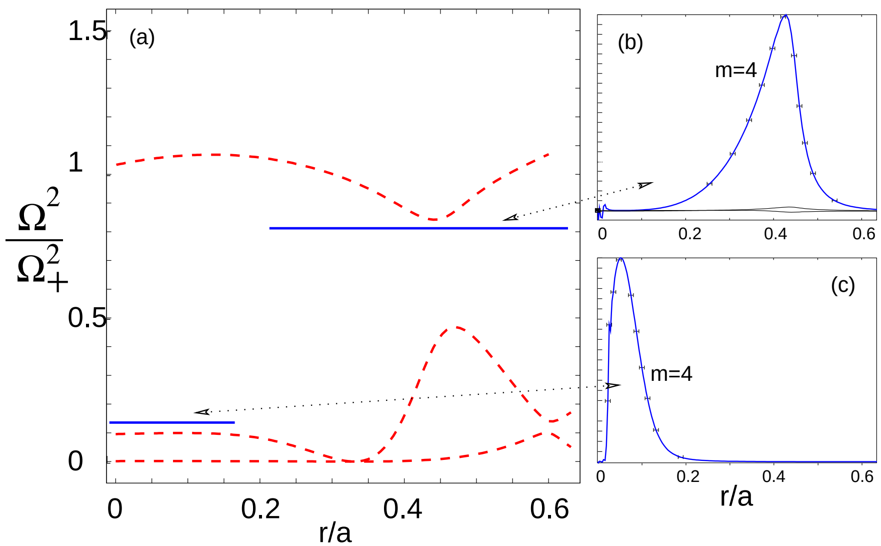

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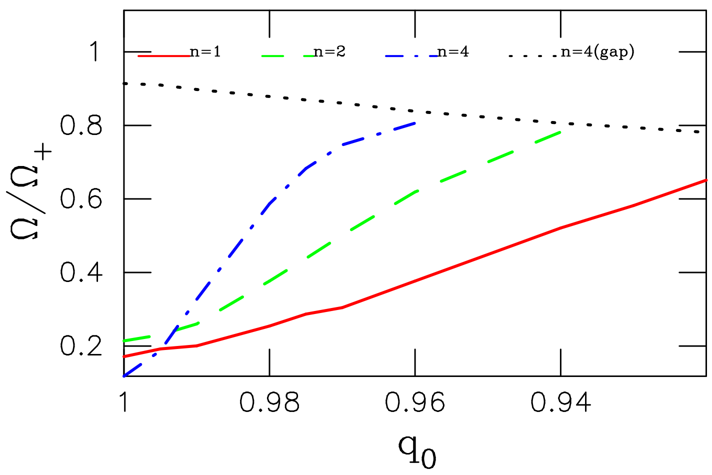
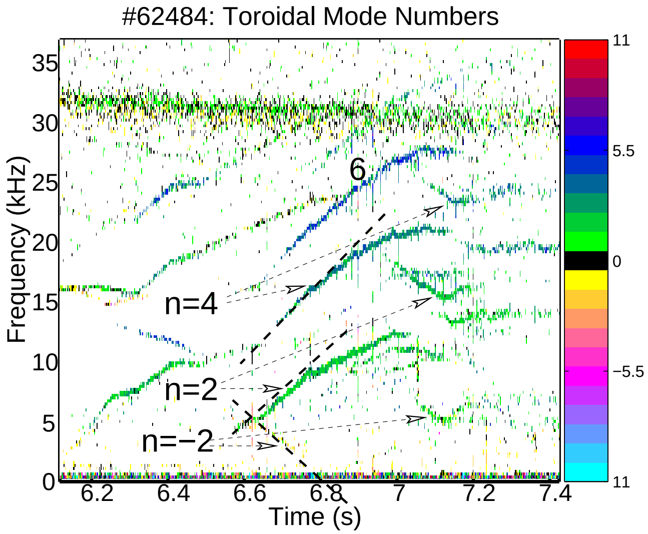
BAAE frequency is related to q_{min} value $\Rightarrow$ useful for diagnostic

Relaxing q-profile results in BAAE frequency up-sweep

- Core BAAE activity is predicted to have sweeping frequency
- Up-chirp is limited by the gap, $\Omega_+ \simeq v_A\sqrt{\gamma\beta/2}/qR$.
  $\Omega_+ = 31kHz$ for pure electron plasma.
- Rotation is significant $f_{rot} = 2.5kHz$.





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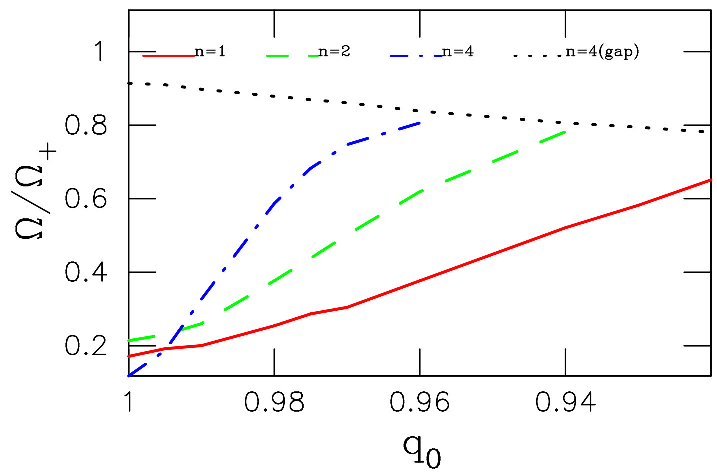

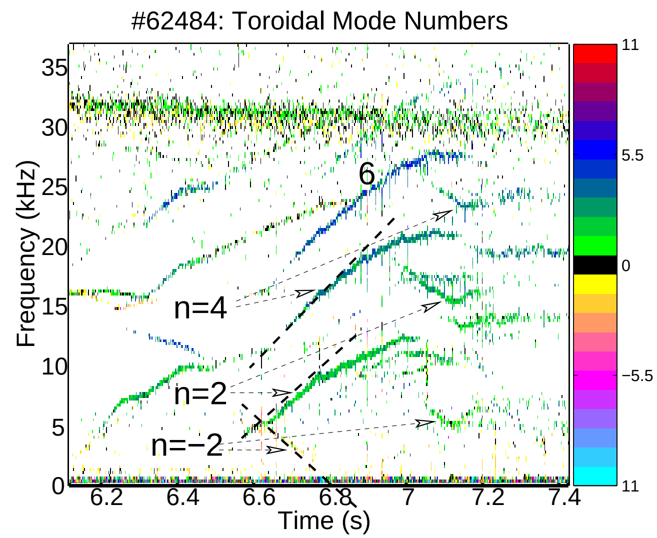

NOVA predicts $f_{BAAE} = 24.8kHz$ against observed $14kHz$, all n's exist ($q_0 = 1$).
Possible way to resolve this is to assume local negative shear with $q_{min} = 3/2$:
1) frequency $\sim q^{-1}$, goes down to $\sim 16kHz$,
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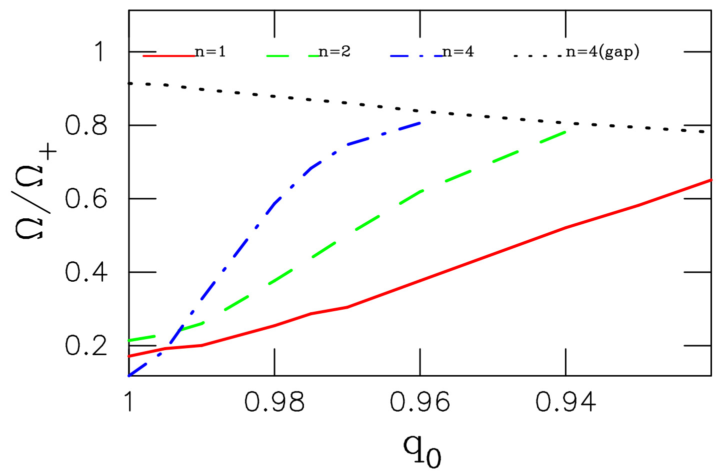

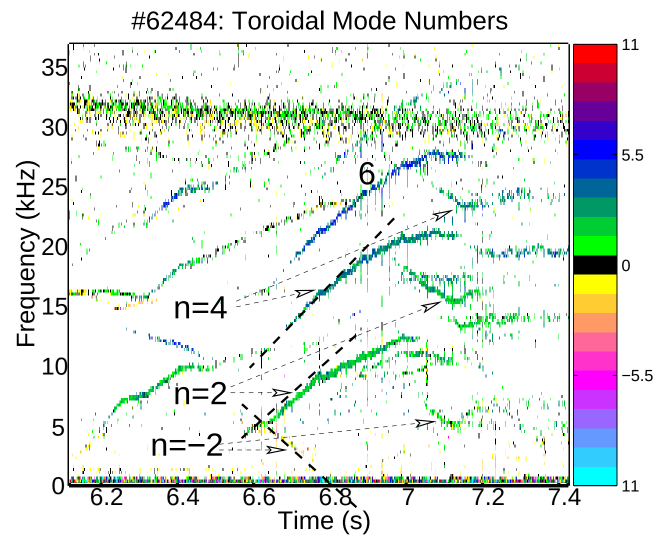

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MSE was available only later in the shot.

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NSTX experiments with MSE address frequency mismatch

- low density $n_e \simeq 3\times 10^{19} m^{-3}$, $P_{NBI} = 2MW$, $E_{NBI} = 90keV$.
- $16?$ channel MSE measures q profile (reversed shear).
- Goal is to validate theory.

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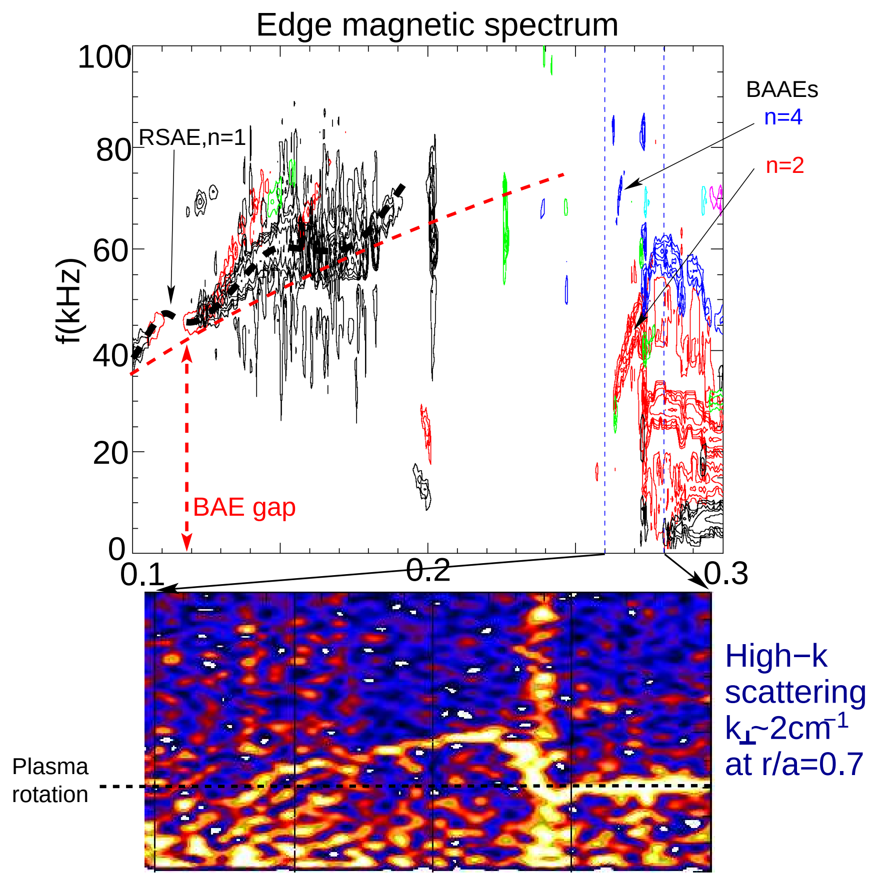
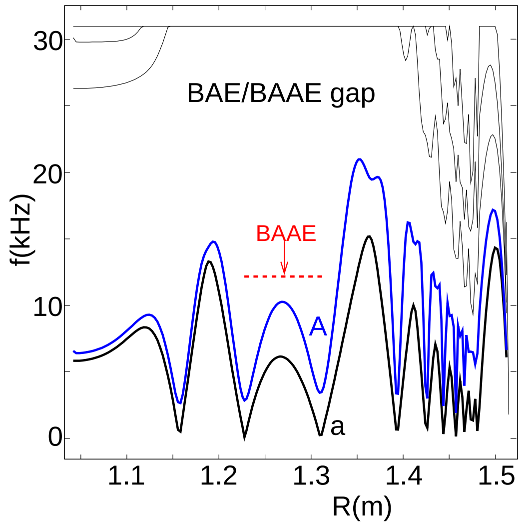
- 16? channel MSE measures q profile (reversed shear).
- Goal is to validate theory.
- Low frequency oscillations (BAAEs) are seen unstable:
  - Characteristic upshift frequency evolution from zero (plasma frame).
  - BAAEs reside in wider BAE gap $f \sim \sqrt{\beta_{pl}}$.
  - High-k component of BAAE at $r/a = 0.7 \Rightarrow$ conversion to KAW (see H.Park poster).

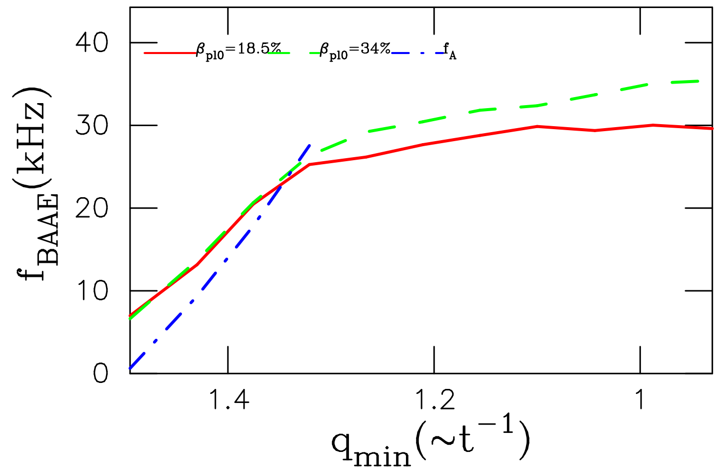


TAE/RSAEs are suppressed (see E. Fredrickson poster) and BAAEs are excited by beams in high-β NSTX plasmas (typically $\beta_{pl} >\sim 15\%$).

Global BAAE modes are found at q_{min} surface in NSTX

JET — *PPPL*





MSE measured inversed q-profile is used in NOVA modeling.

- At high-$\beta_0 = 34\%$, BAE gap opens up to TAE gap.

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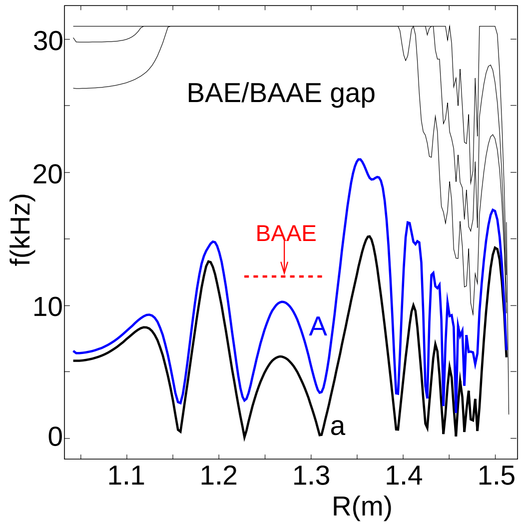
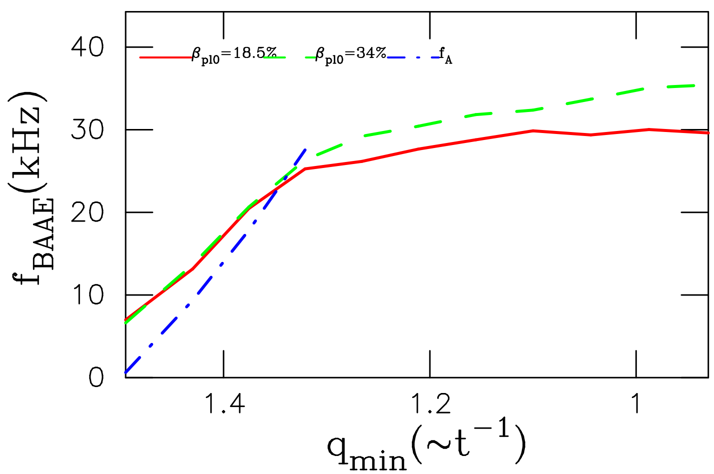
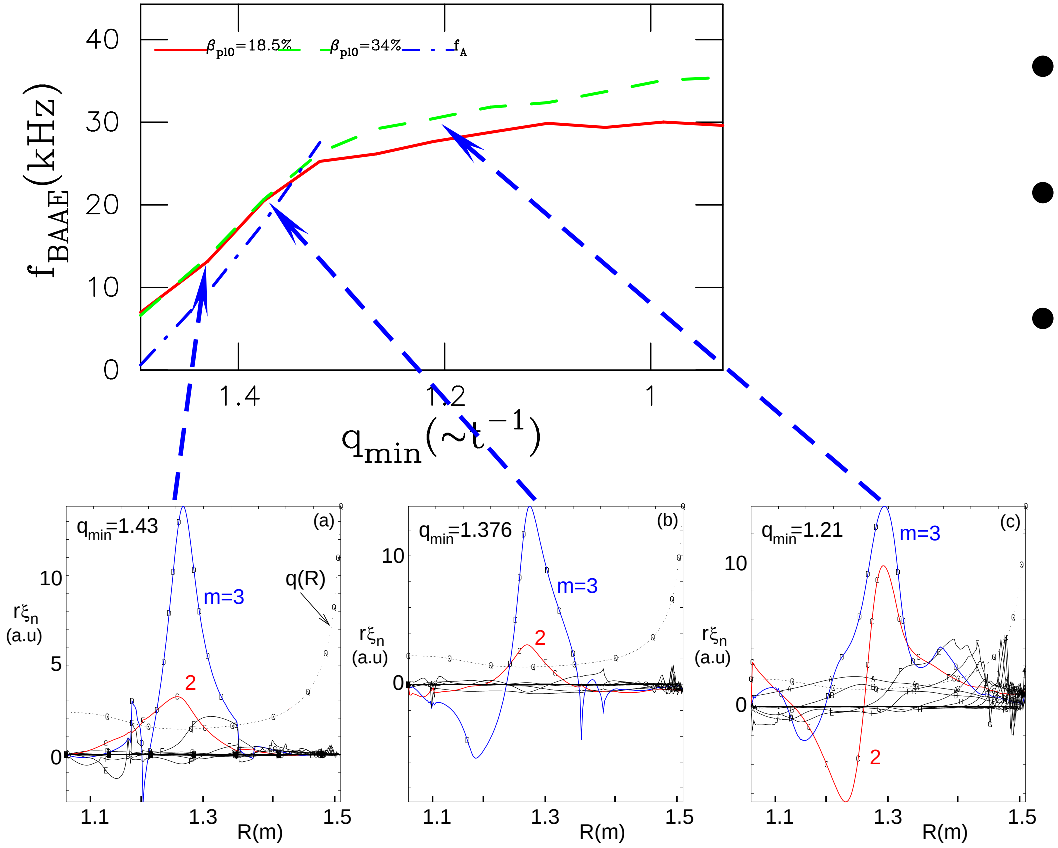




MSE measured inversed q-profile is used in NOVA modeling.

- At high-$\beta_0 = 34\%$, BAE gap opens up to TAE gap.
- Two Alfvén/acoustic (A/a) continuum branches are found with $\Omega^2 < \gamma\beta$, $n = 2$.
- Low shear BAAE frequency does not depend on β for q close to rational
- f_{BAAE} is close to theory dispersion $f_A = v_A k_{\parallel} / \sqrt{1 + 2q_{min}^2}|_{r=0}$.

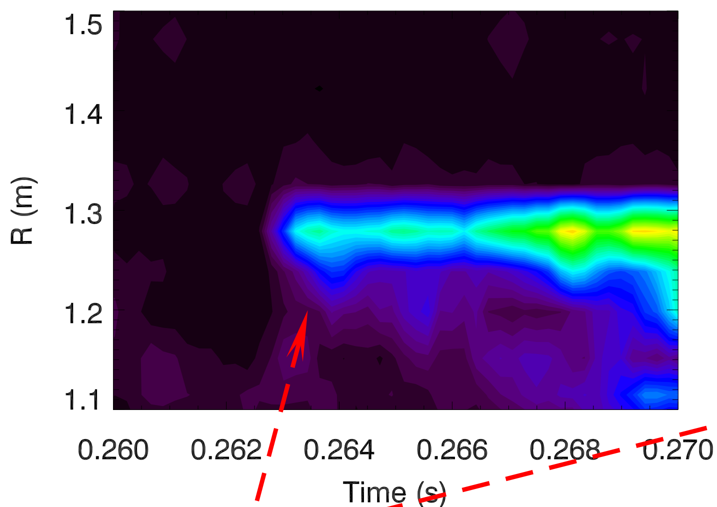
NOVA: BAAE broadens radially as q_{min} decreases

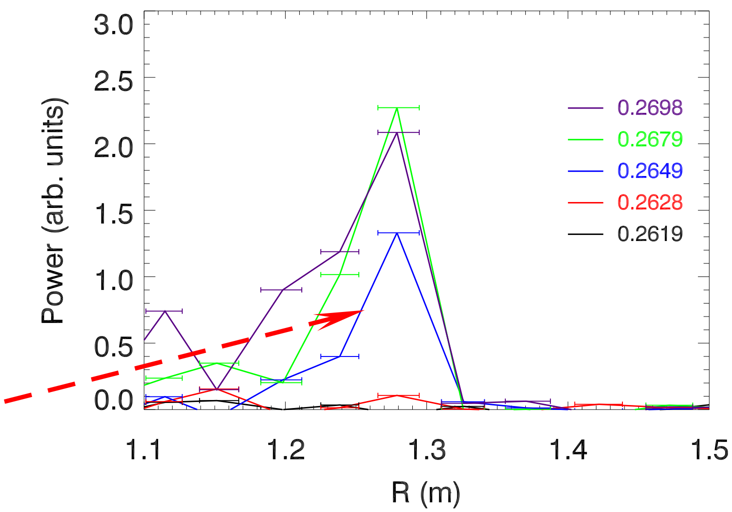


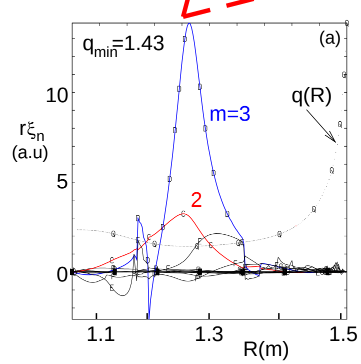
- BAAE frequency sweeps as q-profile relaxes.
- One dominant harmonic $m = nq_{min} = 3$.
- BAAEs interact with the continuum.

Ultra SXR measures the same radial structure broadening

Raw USXR signal (∼BAAE structure) Radial profile evolution







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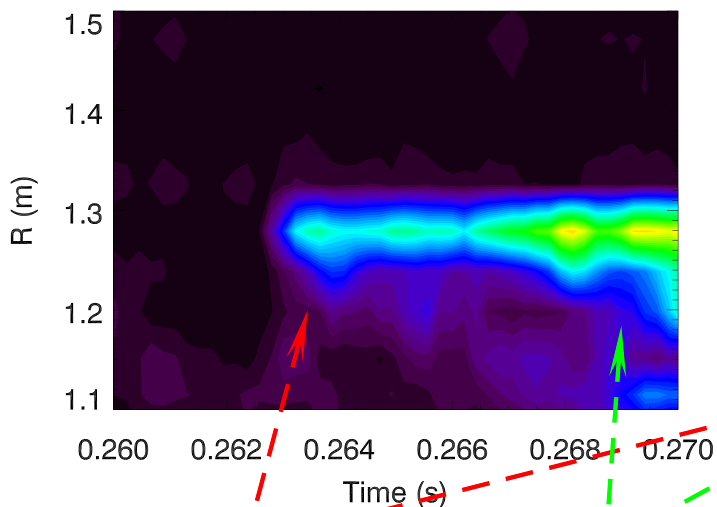
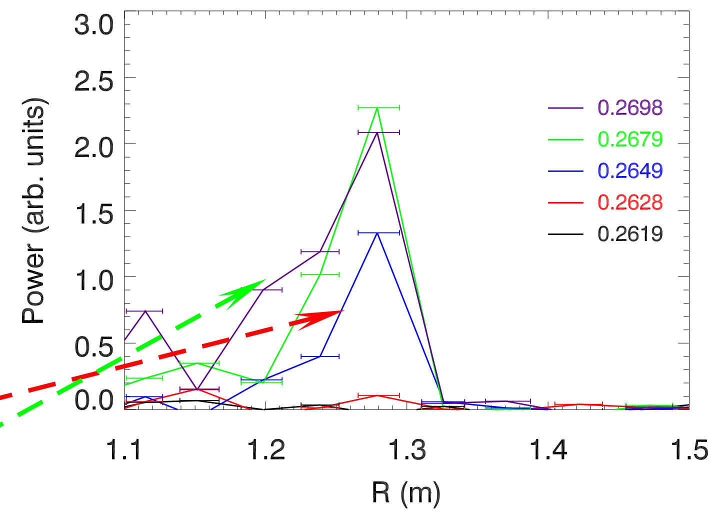
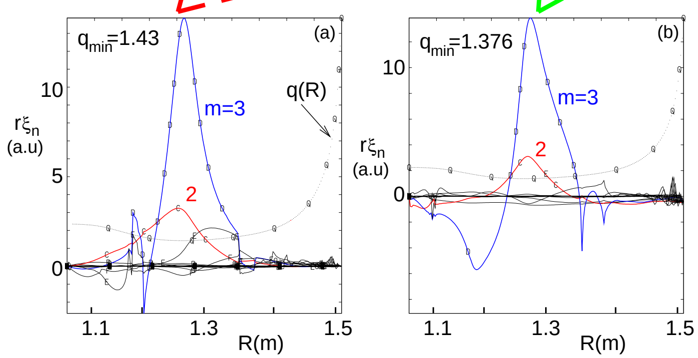


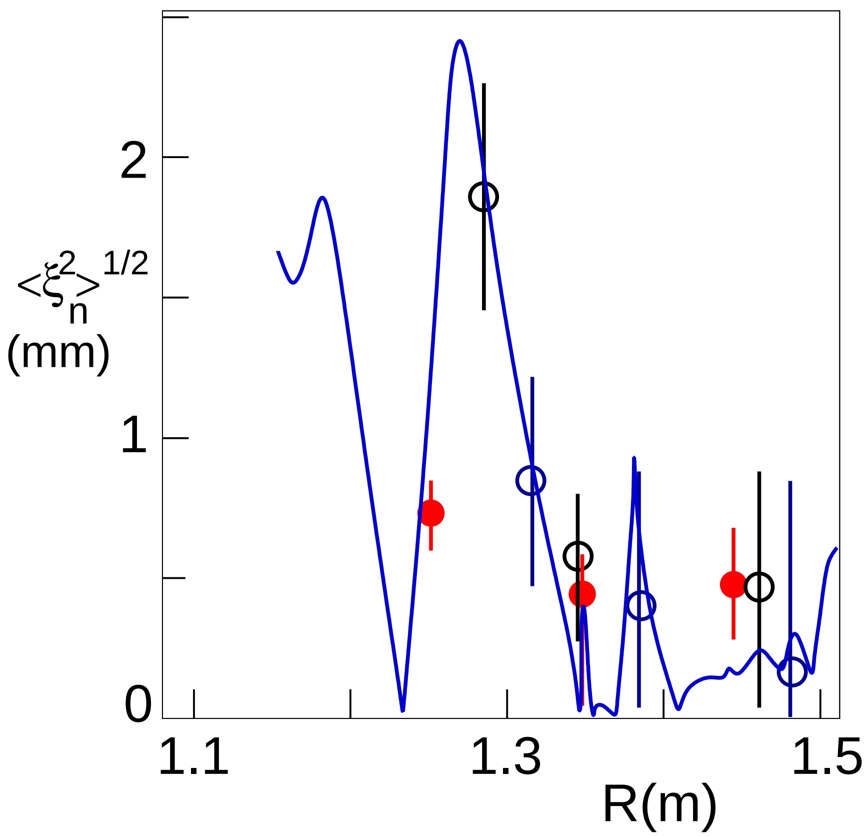
Radial profile evolution





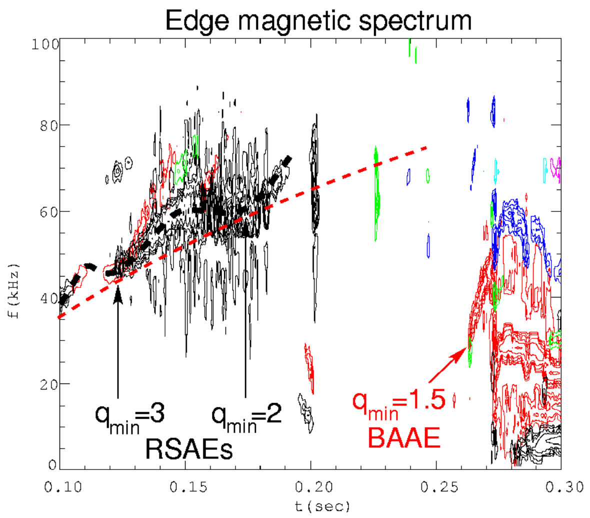
BAAE broadens as q_{min} decreases

Reflectometer confirms localized BAAE structure



- Three plasmas with different densities $n_e = 3 - 3.8 \times 10^{19} m^{-3}$,
  - each plasma has three point measurement
- Vertical axis refers to red solid circles #123816.
- Two other plasma measurements were normalized.
- Measurements are taken at signal maximum.
- Internal fluctuations level $\delta n / n \sim 2 \times 10^{-3}$.

BAAEs complement RSAEs for MHD spectroscopy



- In low- to mediaum-β plasma BAAEs complemets RSAE/TAEs.
- In high-β plasma BAAEs may be the only tool for MHD spectroscopy.
- Zero BAAE frequency point (plasma frame) indicates rational q_{min}.
- BAAE activity is terminated by some IRE at $t = 0.275s$.
  Potential interplay of beam driven instabilities with internal $m = 3/n = 2$ kink-like instability - similar to TAE/sawtooth nonlinear interplay (Bernabei'01, Sharapov'06).
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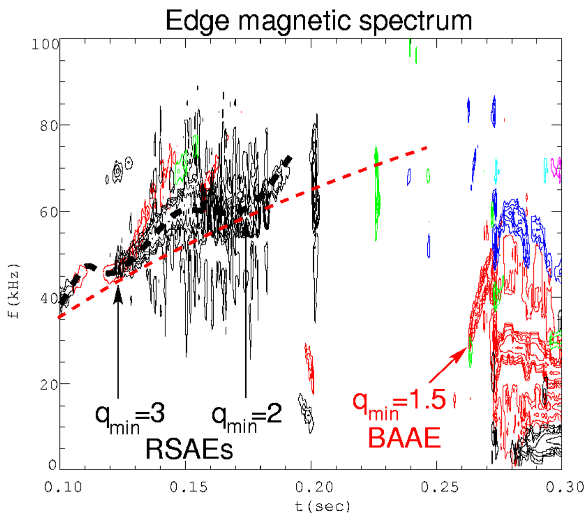

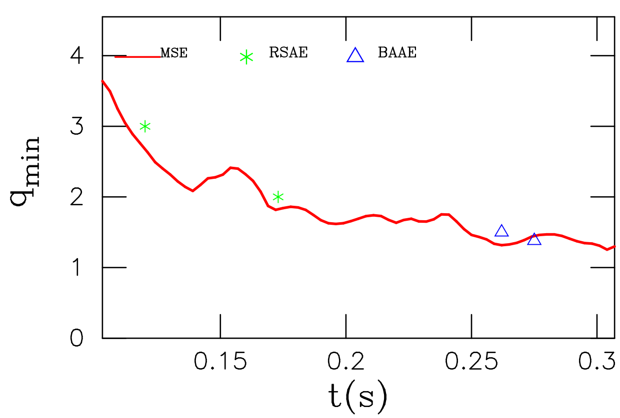

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  - the existence of geodesic curvature induced gaps in the Alfvén/acoustic continuum below GAM frequency (van der Holst'00),
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  - low-n global beta-induced Alfvén/acoustic eigenmodes - BAAE are found,
  - BAAEs exist in finite beta plasma within wider BAE gap.
- BAAEs are different from BAEs (Heidbrink-Turnbull-Chu-Huysmans) interpretation as BAAEs require compressibility effect, i.e. sound wave coupling:
  - frequency can sweep up from almost zero in reversed shear.
  - frequency is lower $0 < \Omega < \sqrt{\gamma\beta/2}/q_{min}$ vs. $\Omega = \sqrt{\gamma\beta\left(1+1/2q_{min}^2\right)}$ for BAE/GAM.
  - both low shear gap BAAEs can coexist (similar to RSAE/TAEs)

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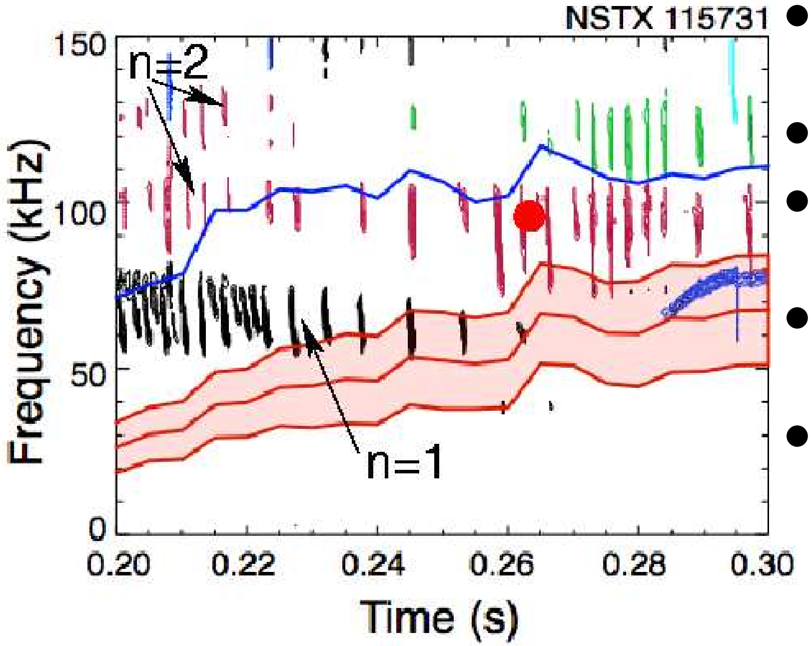
  - low-n global beta-induced Alfvén/acoustic eigenmodes - BAAE are found,
  - BAAEs exist in finite beta plasma within wider BAE gap.
- BAAEs are different from BAEs (Heidbrink-Turnbull-Chu-Huysmans) interpretation as BAAEs require compressibility effect, i.e. sound wave coupling:
  - frequency can sweep up from almost zero in reversed shear.
  - frequency is lower $0 < \Omega < \sqrt{\gamma\beta/2}/q_{min}$ vs. $\Omega = \sqrt{\gamma\beta\left(1+1/2q_{min}^2\right)}$ for BAE/GAM.
  - both low shear gap BAAEs can coexist (similar to RSAE/TAEs)
- Kinetic modification of MHD theory is required for new global modes (Zonca'96, Mikhailovski'98):
  - damping is expected to be strong due to phase velocity of acoustic component close to thermal ion velocity.
  - dominant electron plasma is expected to be favorable for BAAE existence.

Summary (continued)

- NOVA analysis shows existence of global BAAEs in ICRH JET and NBI NSTX plasmas.
- Qualitatively NOVA predicts BAAE frequency evolution in agreement with observations on both tokamaks.
- In NSTX $n = 2$ BAAE internal structure, frequency and their evolution are in agreemnt with NOVA.
  - MSE measurements on NSTX seem to validate theory and MDH (q_{min}) spectroscopy via BAAEs.
  - Maybe usefull for burning plasmas, ITER.
- For pure electron plasma (lowest f) BAAE frequency is above the measured value in JET by factor ~ 1.77.
- Need to reconcile theory and experiment:
  - may imply local reversed shear witeh $q_{min} = 1.5$ but strong indications exist for $q_0 = 1$,
  - possible redistribution of the current drive due to:
    - MHD activity H-minority transport,
    - ICRH current drive,
    - runaway electrons in low density JET plasma.

Thank you !

TAEs can transform to BAAE at strong drive



- Pink shaded area is theoretical BAAE gap scaling with rotation and $n = 2$.
- Upper blue curve is core n=2 TAE frequency.
- Red dot is NOVA BAAE frequency for $n = 2$: consistent with observations.
- At the start of the discharge TAE are unstable, $f \simeq 100kHz$.
- Later TAEs transform to EPM - BAAE:
  - Chen'94, Cheng '95, Gorelenkov '03.
  - Toroidal rotation is strongly sheared and may affect the mode localization.