A synthetic diagnostic for validation of nonlinear ETG simulations against measurement with coherent electromagnetic scattering

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Why are synthetic diagnostics needed?

- Measured and simulated plasma quantities may differ
- Measured and simulated quantities may be in different domains (as in the case of scattering experiments)
- Diagnostics ‘filter’ plasma quantities by a Transfer Function

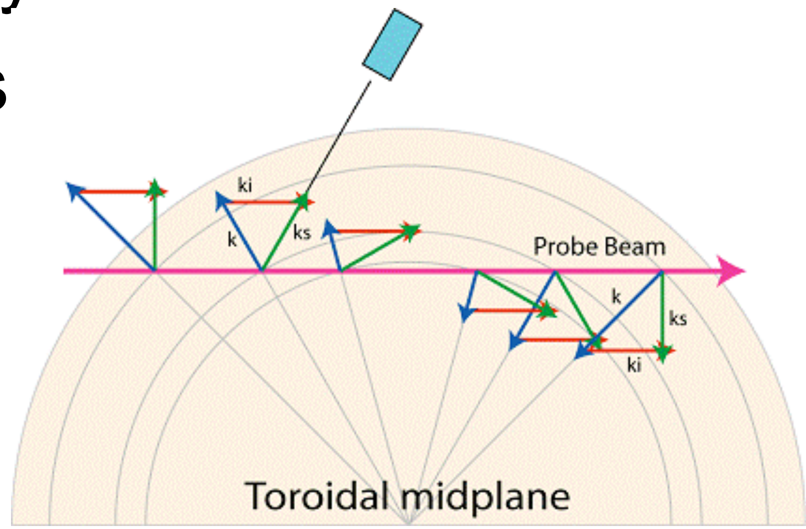
A synthetic diagnostic *simulates* the experimental setup to provide a filter to the numerical output of simulations.

A synthetic diagnostic is itself a model

A synthetic diagnostic for coherent scattering should take into account:

- Measured and simulated density fluctuations in different domains

$$\tilde{n}_{SIM}(r,\theta,\phi,t) \qquad \tilde{n}_{HK}(\vec{k},t)$$



- Interpretation of measurements is based on a model

- Should be suitable for use in *predictive* mode to quantify uncertainties on measured spectra

Outline

- General issues with theory-exp comparison
- Structure of the high-k synthetic diagnostic
- Application to NSTX plasma discharge #124901
- Further implementation and applications

Theory-exp comparison is based on frequency and/or wavenumber spectra

$P_{HK}(k_\perp^j, \omega)$ Discrete in $k_\perp$, good statistics in ω

$P_{SIM}(k_r, k_\theta, \omega)$ Wide range in (k_r, k_θ), poor statistics in ω

Comparison based on P($k_\perp$)

✗ Limited range of experimental values

✗ Detector calibration needed for a quantitative comparison

Comparison based on P(ω)

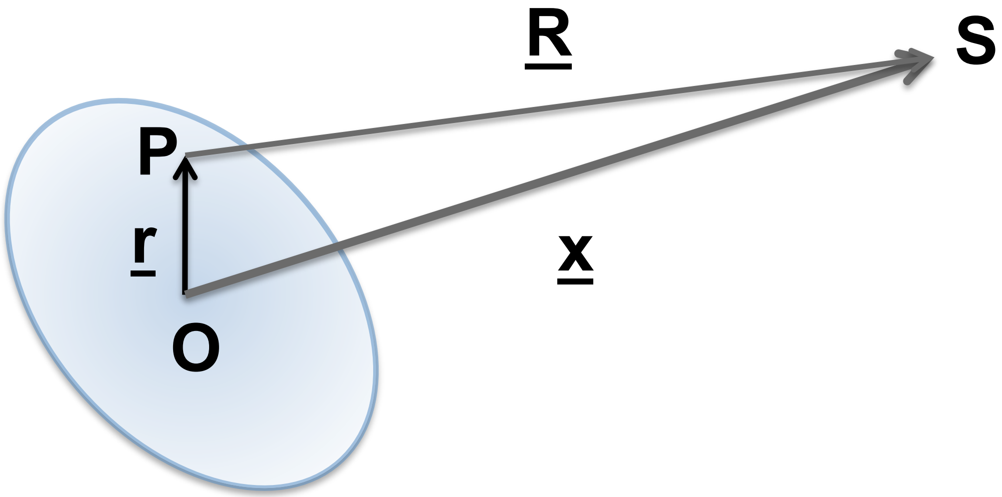
✗ Short time series in simulations ($\Delta\omega$ small enough to resolve low-ω)

✓ Comparison still possible when data are not calibrated

Basic Requirements:

1. Computation of spectra from simulations (statistical accuracy, efficiency)
2. Selection of (k_r, k_θ) from simulated spectra to mimic real diagnostic

The ingredients for the synthetic diagnostic are contained in the expression for the measured electric field

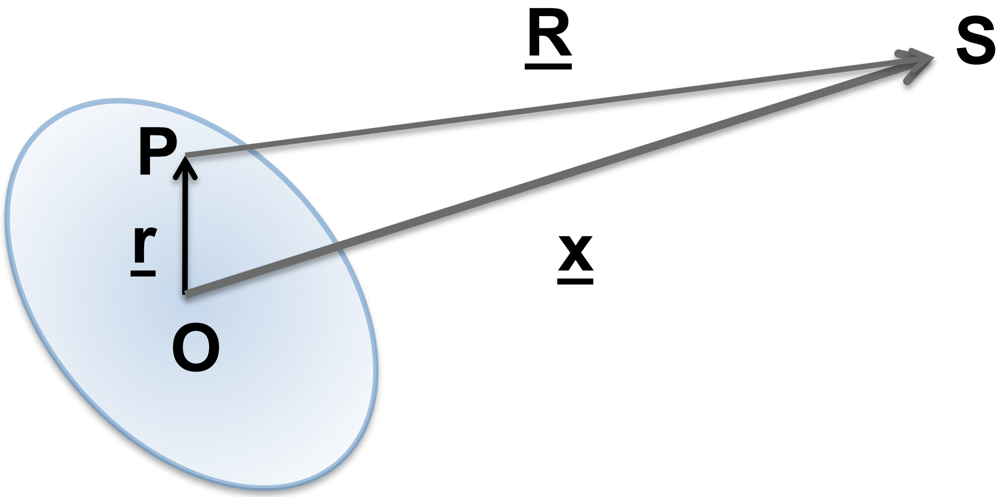


$$\vec{E}_s = \left[\frac{r_e}{R}\hat{s} \times \hat{s} \times \vec{E}_i\right]$$

$$\vec{E}_i(\mathbf{r},t) = \mathbf{E}_i(\mathbf{r}_\perp)\, e^{i(\vec{k}_i \cdot \vec{r} - \omega_i t)}$$

$$\vec{E}_s(\nu_s) = \frac{r_e}{x} e^{i\vec{k}_s \cdot \vec{x}} \, (\hat{s}\hat{s} - \mathbf{1}) \cdot \int_{T'} dt' \int_V d^3 r' \, \mathbf{E}_i(\mathbf{r}_\perp)\, e^{i(\omega t' - \vec{k} \cdot \vec{r})}\, \tilde{n}(\vec{r}\,', t')$$

The ingredients for the synthetic diagnostic are contained in the expression for the measured electric field



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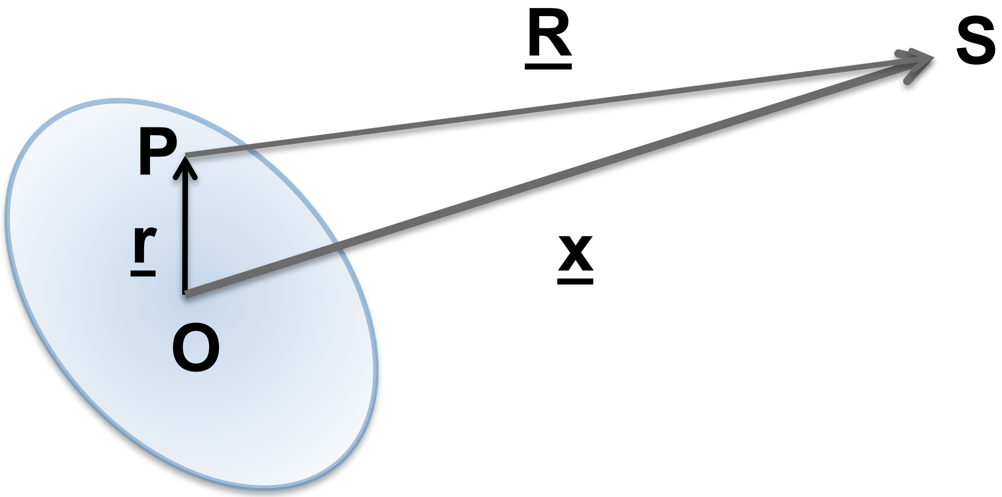
Direction & amplitude of k_s

The ingredients for the synthetic diagnostic are contained in the expression for the measured electric field



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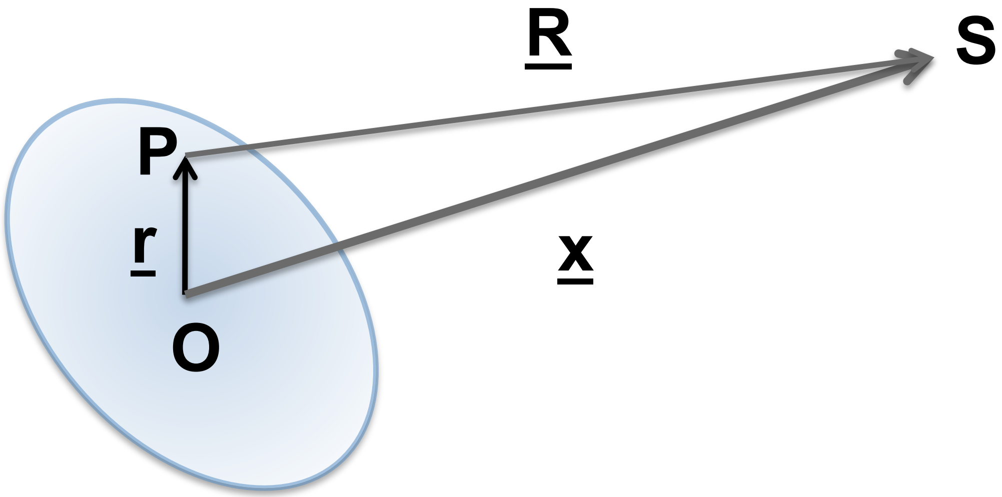
Direction & amplitude of k_s

Amplitude profile of beam

(size of the scattering volume)

The ingredients for the synthetic diagnostic are contained in the expression for the measured electric field



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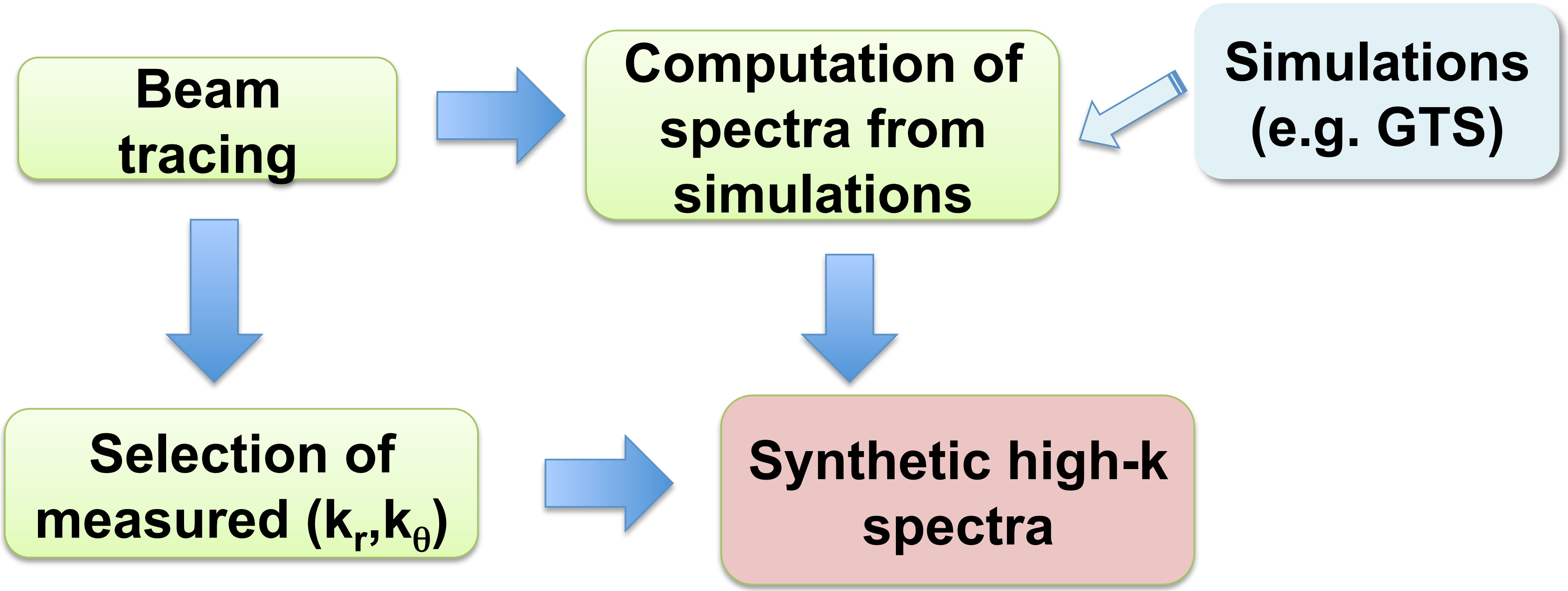
Direction & amplitude of k_s

Fourier Transform of density fluctuations weighted by the beam intensity

There are three blocks in this synthetic diagnostic



- standalone
- applicable to other simulations (ITG, fluid)
- applicable (with limitations) to lower frequency beams
- can be used in predictive mode

Block 1: computation of spectra

INPUT:

From beam tracing

Scattering volume
center @ (R_s,z_s) size a_R , a_z

From simulations

$\tilde{n}(r,\theta,\phi_i,t_j)$ $\phi_1 \ldots \phi_M$ M=64
$t_1 \ldots t_N$ N=340

Construct a Gaussian Function (GF) in (R,z) space

Toroidal separation > size of scatt. volume
=> *Each poloidal plane is independent*

Interface with the numerical code

Interpolate density fluctuations

Interpolate GF onto (R,z) grid

Fourier Trasforms

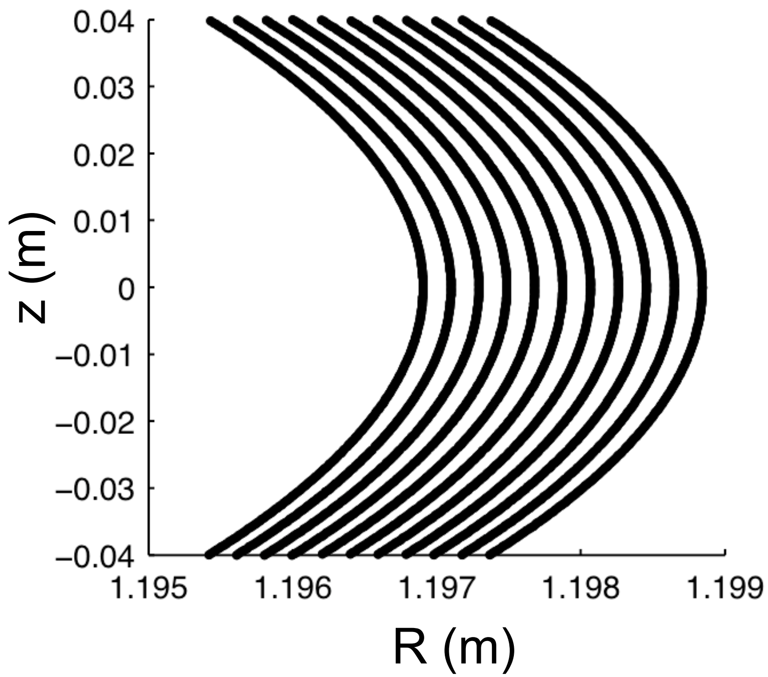
Weight ñ by GF

FT^{θ} $N_r x N x M$

FT^{r} $N_k^{\theta} x N x M$

FT^{t} $N_k^{\theta} x N_k^{r} x M$

k_θ spectra are computed in real space along the diamagnetic direction



- Along each flux surface in real space (R,z) construct a diamagnetic trajectory:

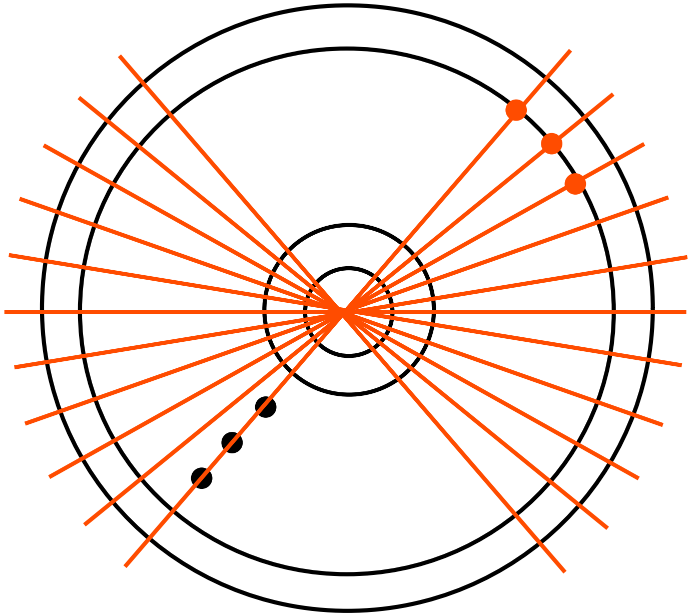
$$ds_j = \sqrt{(R_{j+1} - R_j)^2 + (z_{j+1} - z_j)^2}$$

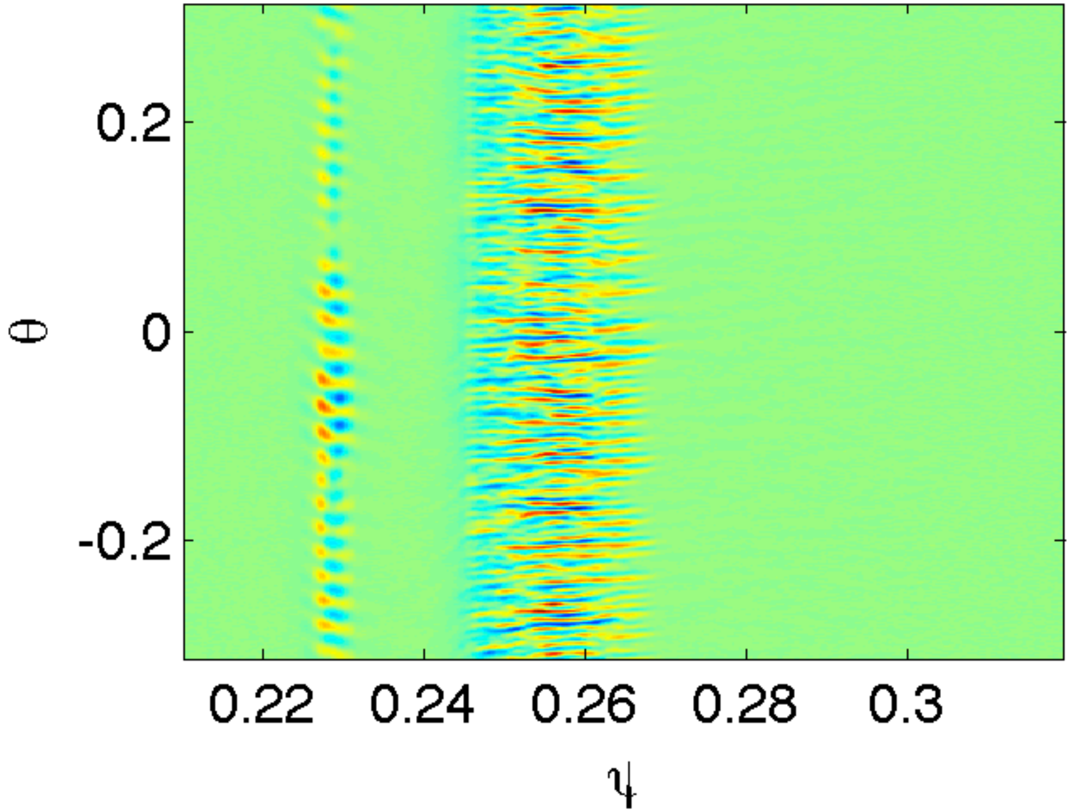
- Interpolate along this trajectory using the same *ds* for all flux surfaces (to have the same k_N and Δk_θ)
- The Fourier components depend only on the value of R at midplane

$$\tilde{n}(R_{mid}, k_\theta, t_i)$$

✘ Computation of k_r spectra requires interpolation of Fourier components along R

Near term: density fluctuations interpolated directly in flux coordinates





Original grid: $\Delta\theta$ uniform along each flux surface (but it depends on surface)

1D interpolation along θ => distribute data along rays •••

1D interpolation along ψ => values chosen to have uniform ΔR at midplane •••

This part will be included in the GTS code, as an operation on the stored output

Block 2: The beam tracing is a key element of the high-k synthetic diagnostic

Find location and extension of scattering volume

$$\mathbf{E}_i(\mathbf{r}_\perp) \to E_0\, e^{-r_\perp^2/a^2}$$

Windowing for Fourier Transform (FT)

$$\int_V d^3r\, \boxed{\mathbf{E}_i(\mathbf{r}_\perp)\,\tilde{n}(\vec{r},t)}\, e^{i(\omega t-\vec{k}\cdot\vec{r})}$$

$$\vec{k}_s, \vec{k}_i$$

Inside the scattering volume

$$\vec{k} = \vec{k}_s - \vec{k}_i$$

$$k = 2k_i \sin(\theta_s/2)$$

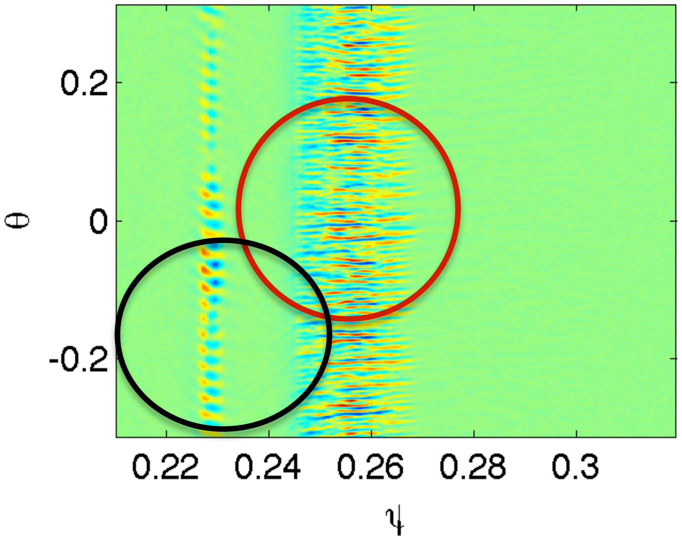
Find *Instrument Selectivity Function*

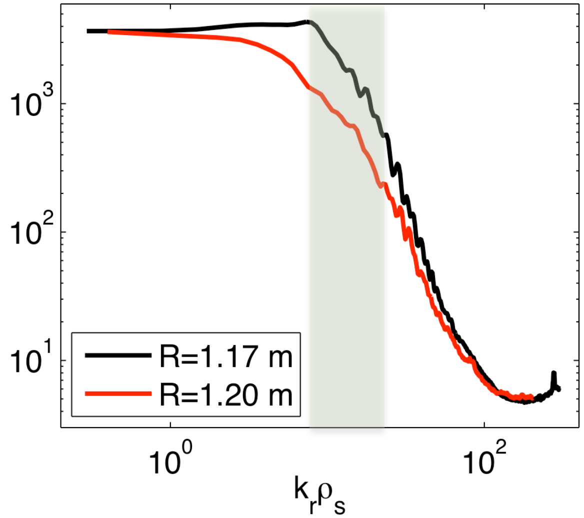
(k_r,k_θ) selection

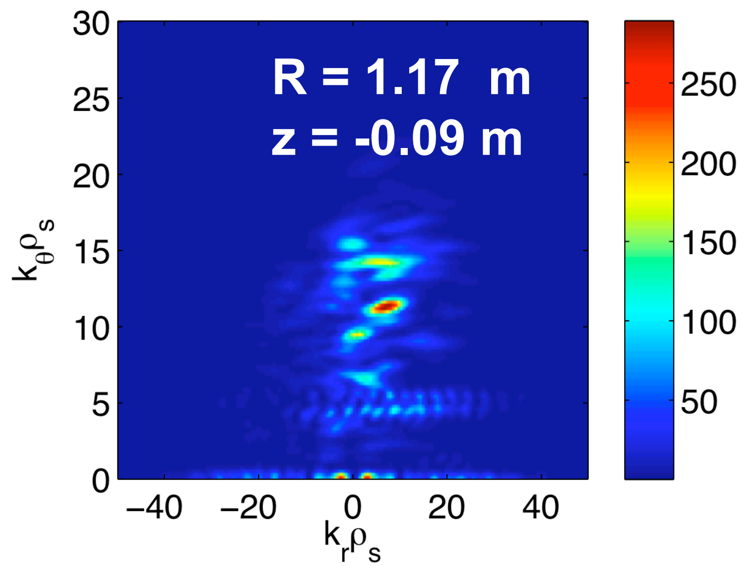
Note: beam tracing (vs. *ray* tracing) is required for an accurate reconstruction of

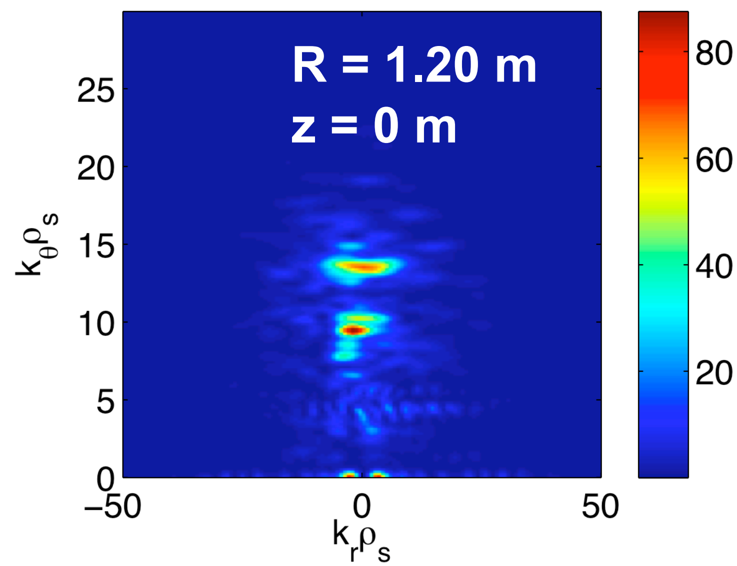
- Scattering volume
- Instrument Selectivity Function

Small changes in the position of scattering may significantly affect the spectra





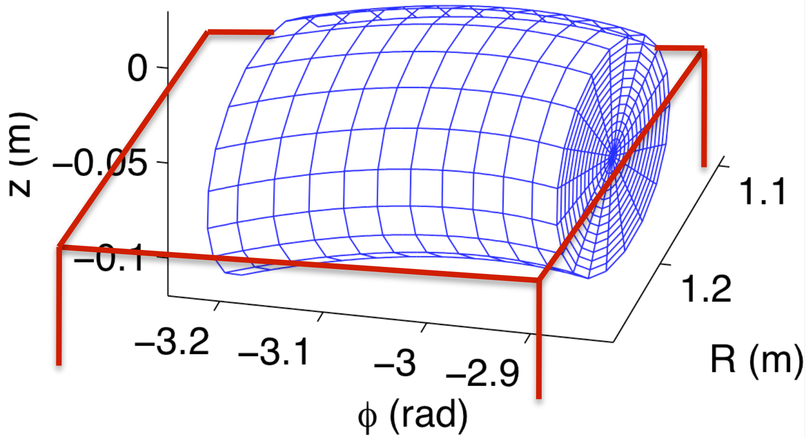




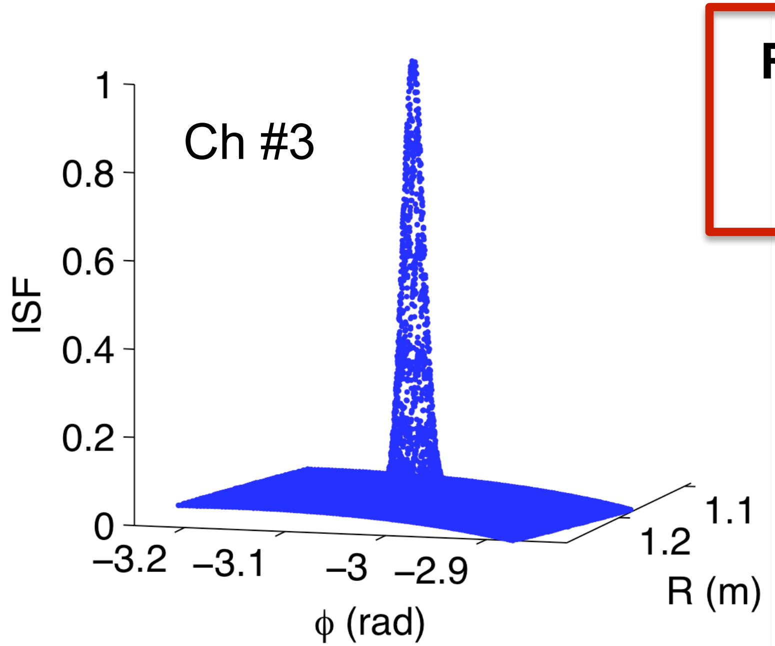
NOTE: No selection of (k_r,k_θ) yet

The measured k's are weighted by an Instrument Selectivity Function (ISF)

[E. Mazzucato, Phys. Plasmas **10** 753 (2003)]

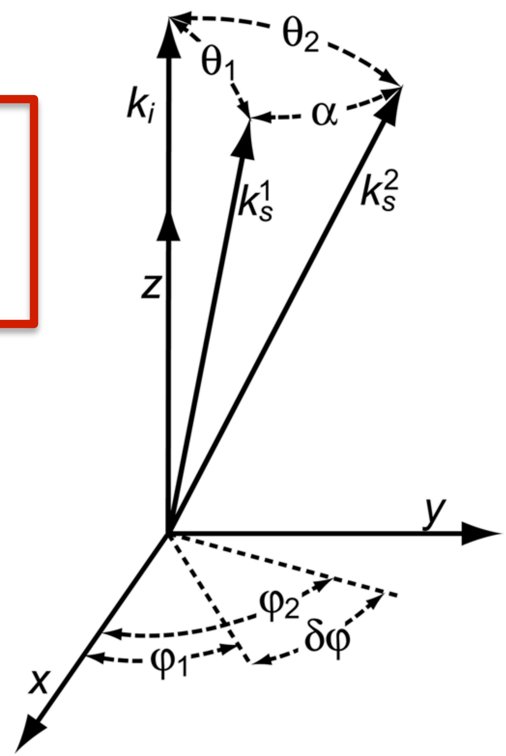


- First, take a toroidal length $L = \dfrac{2a}{\sin(\theta_s)}$
- Then, compute the ISF for all k_i, θ_s, within this volume

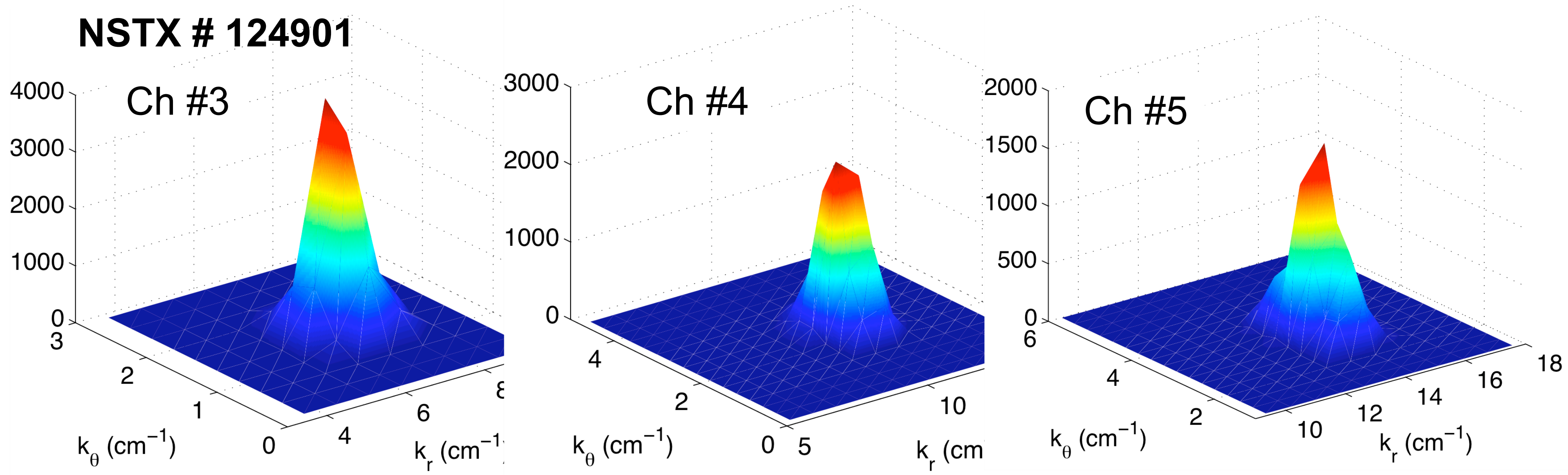


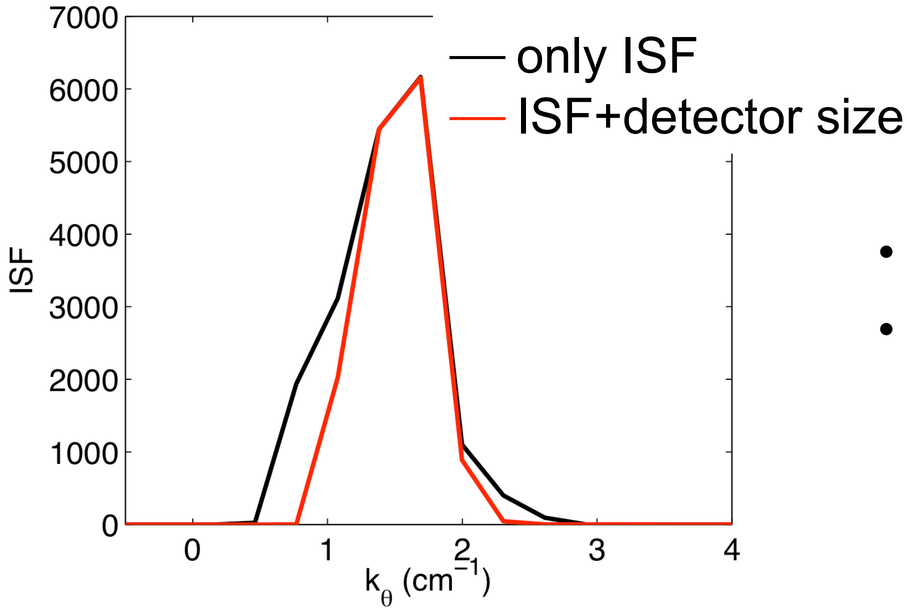
Relative collection efficiency

$$F = \exp(-\alpha^2/\alpha_0^2)$$



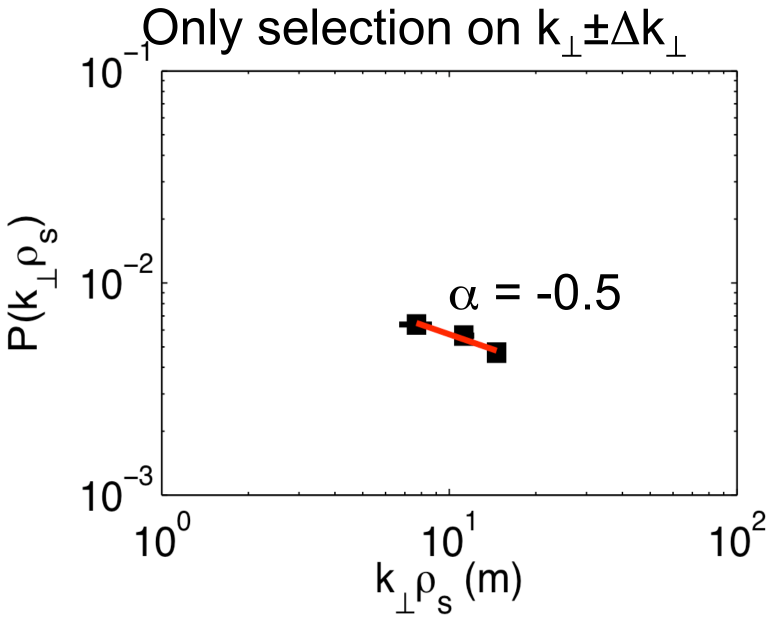
The ISF bounds the measured (k_r,k_θ) range





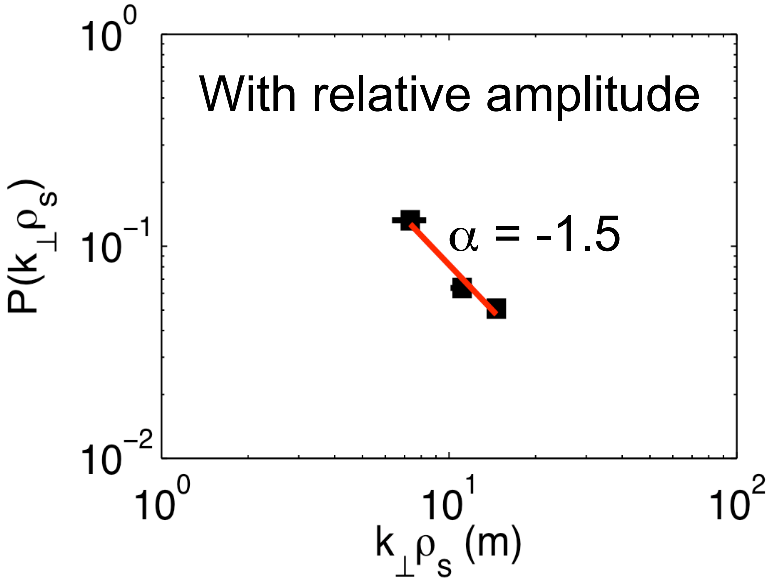
- Finite size of detector should also be included
- **The relative amplitude decreases with increasing scattering angle**

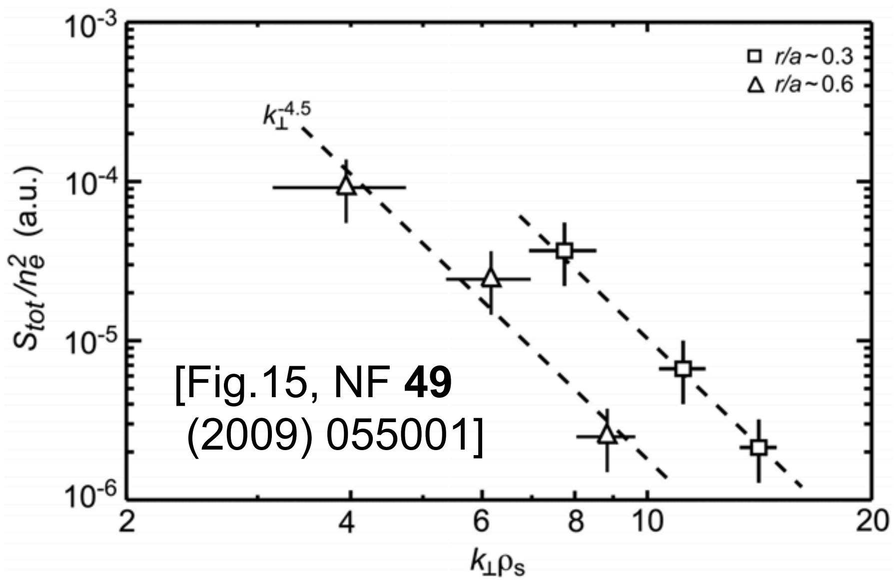
The relative amplitude must be taken into account when computing the spectral index



Spectral index still lower than $\alpha_{EXP} = -4.5$

Estimate difficult because of reduced range of $k_\perp$





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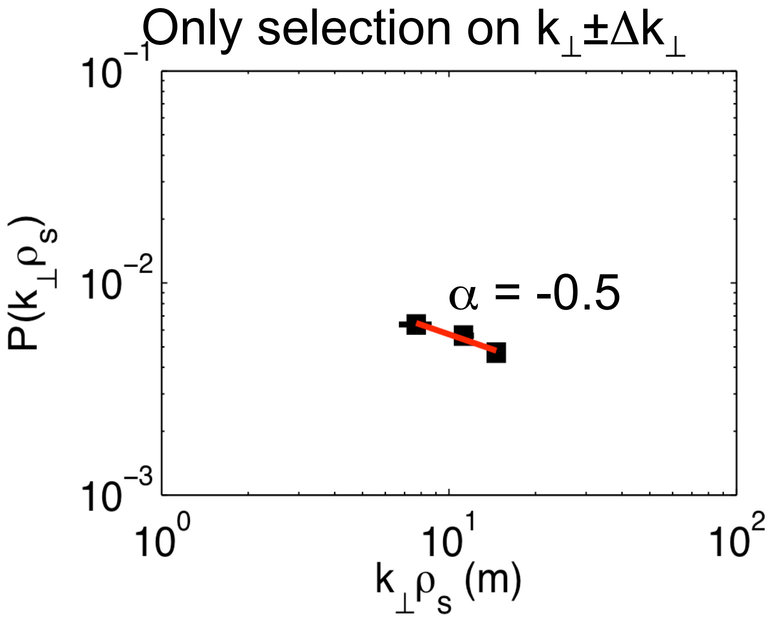
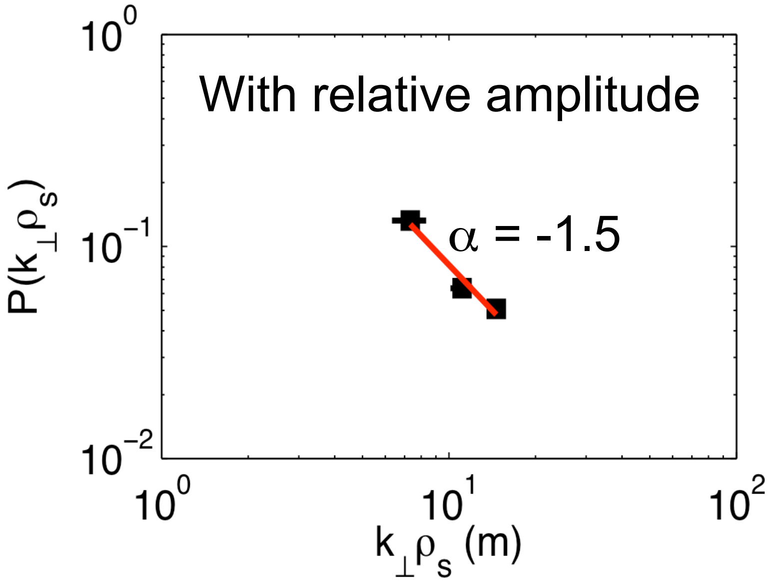
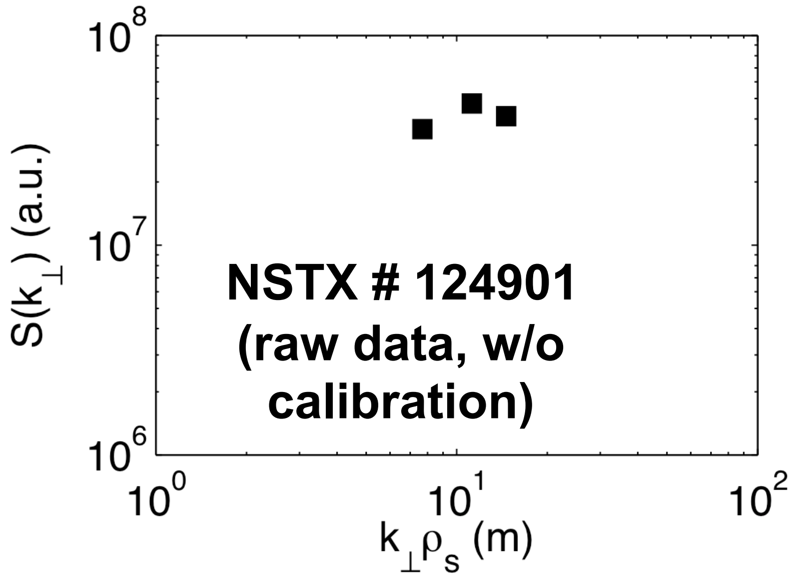


Spectral index still lower than $\alpha_{EXP} = -4.5$

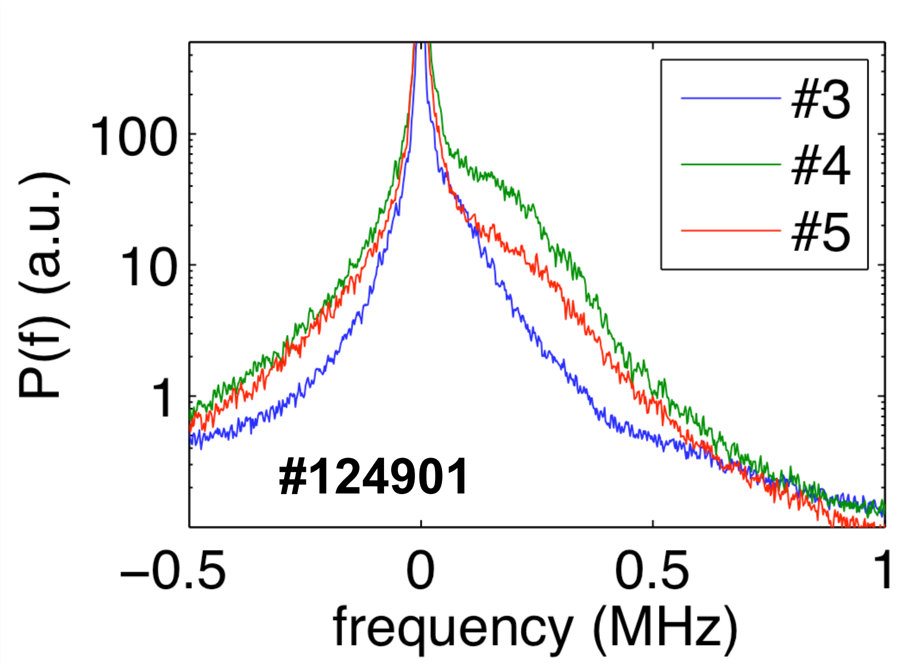
Estimate difficult because of reduced range of $k_\perp$

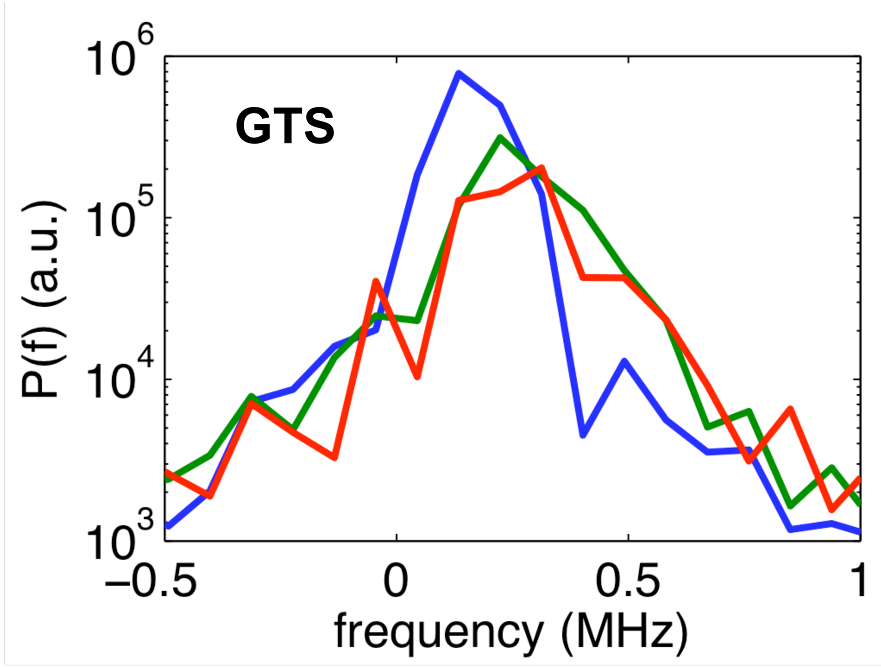
Comparison difficult because data are not calibrated for this shot





Similar features observed in measured and simulated spectra





- maximum spectral amplitude below 0.5 MHz
- broader spectra at larger wavenumbers (ch #4-5 compared to #3)

Analysis on multiple planes required for statistical significance of spectra

Sources of uncertainties

A synthetic diagnostic for coherent scattering relies on model

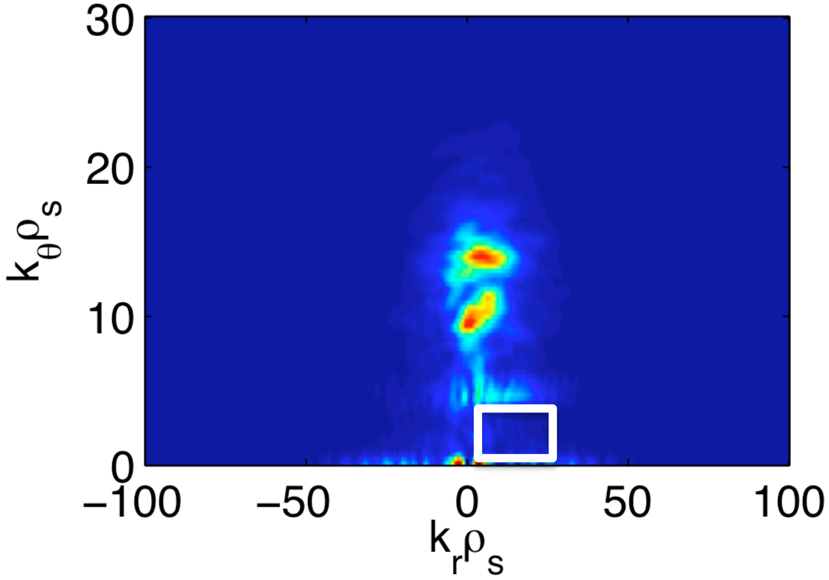
- Ray tracing results depend on
  - Density profile
  - Magnetic equilibrium reconstruction

$\Rightarrow$ Uncertainties from the input profiles

$\Rightarrow$ may affect the prediction of position of scattering

$\Rightarrow$ Sensitivity studies should be performed for the synthetic high-k

The synthetic diagnostic can be used in predictive mode



Starting from the present configuration:

• How does the measured spectrum look like when injection/detection angles are changed?

• How do changes in the simulated spectrum affect the measured spectrum in the range of lower k?

If we want to measure the spectrum in the wavenumber range where simulations do predict streamers:

• What is the most suitable geometrical configuration ?

• How many channels are needed?

• Which distance between channels?

Summary

A synthetic high-k diagnostic is being developed that:

- Consists of standalone blocks (applicable to exp and to various codes)
- Reproduces conditions close to the experiment

  (Beam propagation and spread, selection of **k** using an ISF)
- Computation of spectra is optimized to
  - minimize errors due to interpolation
  - maximize efficiency
- Can be used in interpretive mode or in predictive mode

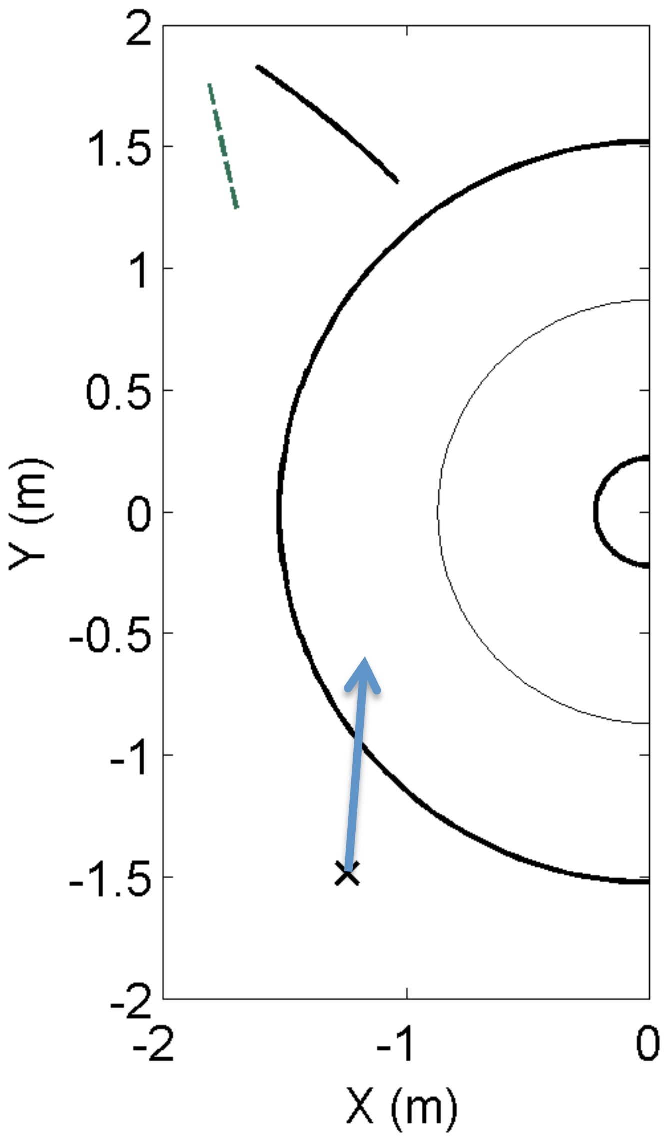
Implementation and future work

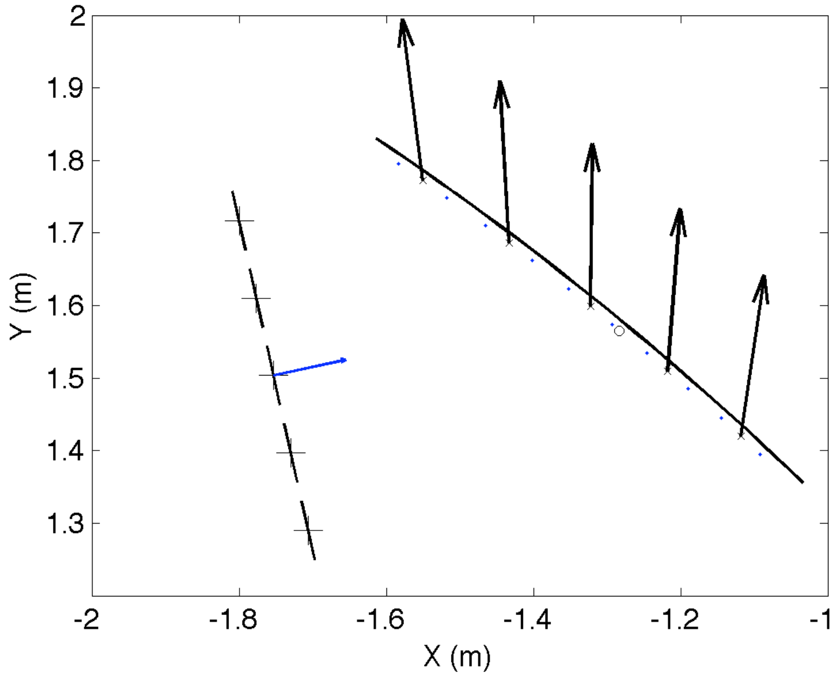
- Implement the Instrument Selectivity Function for general injection and detection geometry
  - better model for the detector transmission function
- Include fluctuations profile in the ray tracing to estimate uncertainties in the ISF (it may be computational heavy)
- Study the sensitivity of the synthetic diagnostic to plasma parameters for different experimental configurations

Backup slides

	k_r (cm^{-1})	k_θ(cm^{-1})	θ_s(rad)	
# 3	7.41	2.07	0.132	Central ray
	7.13	1.73	0.126	Detector axis
	7.0±0.7	1.5±0.4		ISF
# 4	10.84	2.92	0.193	Central ray
	10.82	2.53	0.191	Detector axis
	10.7±0.9	2.4±0.5		ISF
# 5	14.12	3.75	0.251	Central ray
	14.23	3.27	0.251	Detector axis
	14.1±0.8	3.2±0.5		ISF

Experimental layout





Input parameters for ray tracing

- Launching geometry
- Receiving geometry
- Size of receiving windows

A beam tracing code accounts for spreading

[Novak and Orefice, Phys. Plasmas **1** 1242 (1994)]

$$\vec{E}_i(\mathbf{r},t) = \mathbf{E}_i(\mathbf{r}_\perp)\, e^{i[k_0 S(\mathbf{r}) - \omega_i t]}$$

$$S = R + iI$$

$$\Re : \quad (\nabla R)^2 - (\nabla I)^2 = N^2$$

$$\Im : \quad \nabla R \cdot \nabla I = 0$$

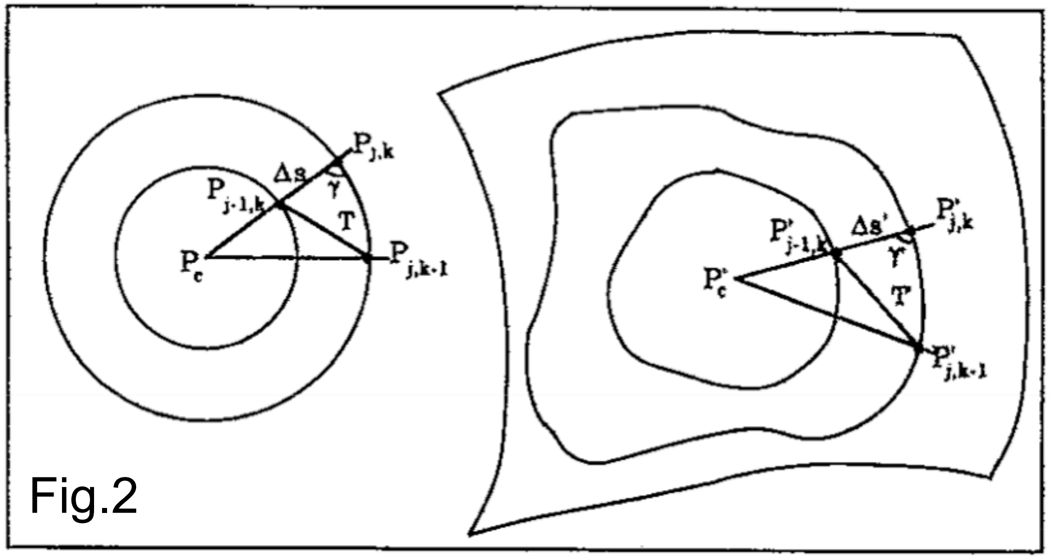


Fig.2

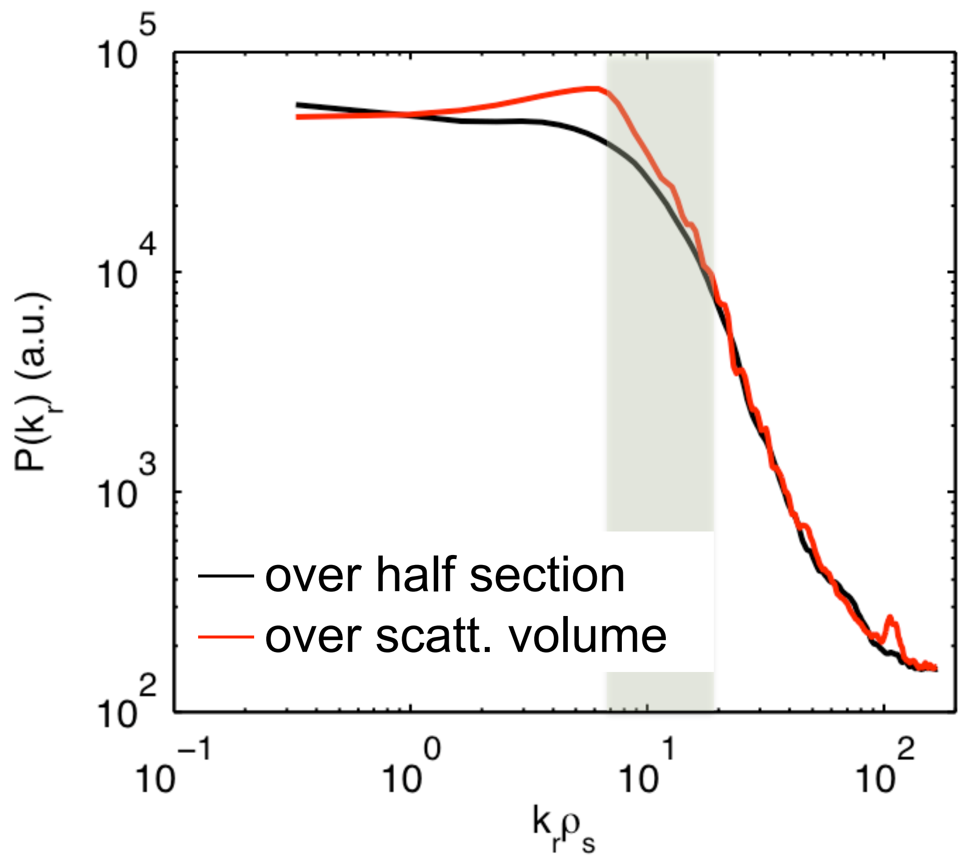
$$\frac{d(\nabla I)^2}{ds_i} \equiv \frac{\left[\nabla I(P_i)\right]^2 - \left[\nabla I(P)\right]^2}{ds_i} = \frac{1}{ds_i}\left(dx_i \frac{\partial}{\partial x} + dy_i \frac{\partial}{\partial y} + dz_i \frac{\partial}{\partial z} \right)(\nabla I)^2$$

$$\left|\nabla I(P'_{j,k})\right| = \left| \frac{1}{\sin\gamma(P_{j,k})} \frac{\partial I(P'_{j,k})}{\partial s'} \right| \qquad \frac{\partial I(P'_{j,k})}{\partial s'} = \frac{\Delta s}{\Delta s'} \frac{\partial I(P_{j,k})}{\partial s}$$

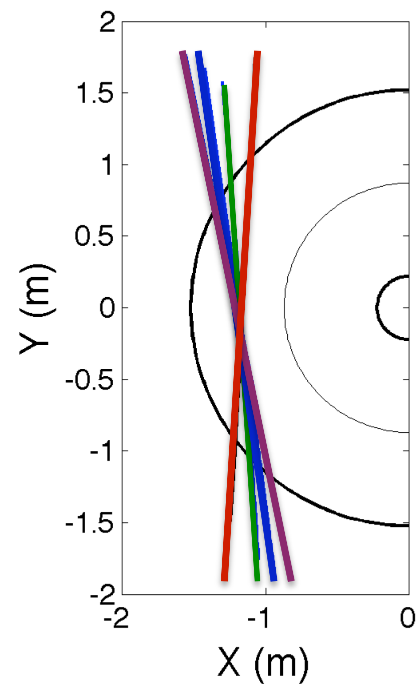
Taking into account the scattering volume DOES matter

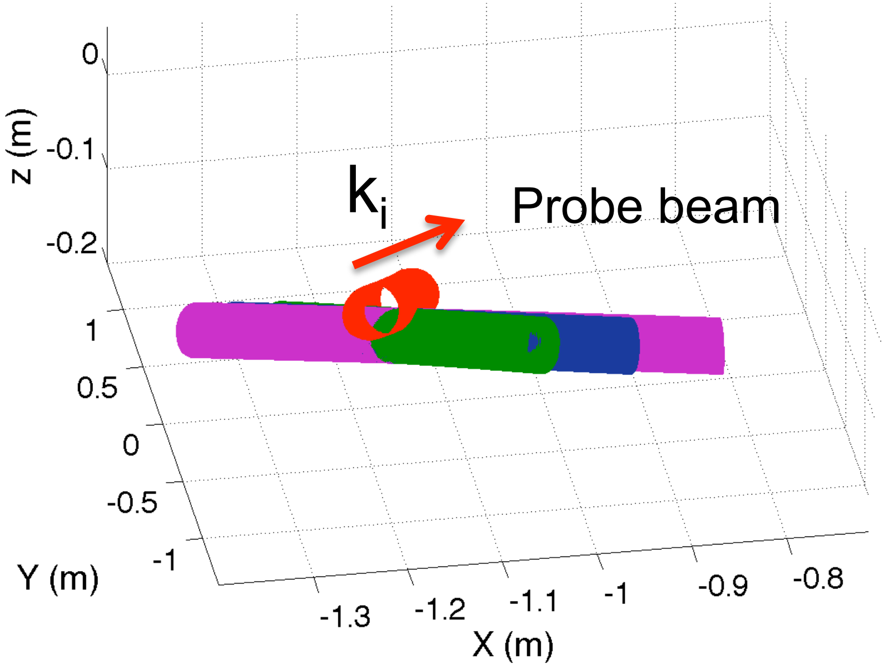
- the spectral amplitude changes
- the slope changes

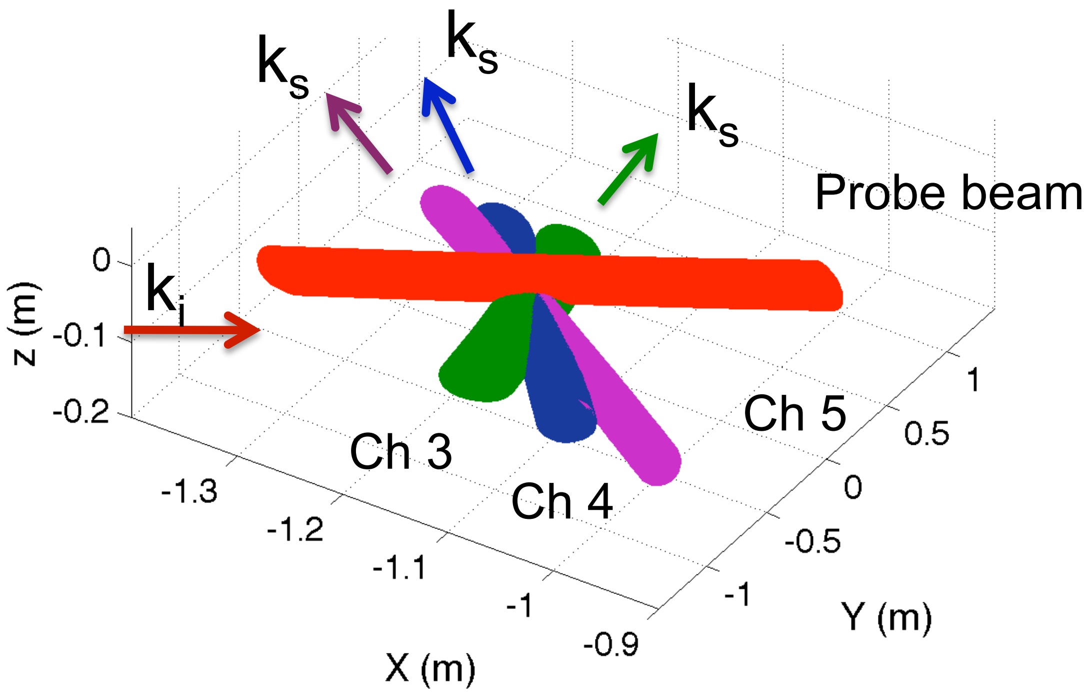
$\Rightarrow$ Different spectral indices are extracted in the two cases



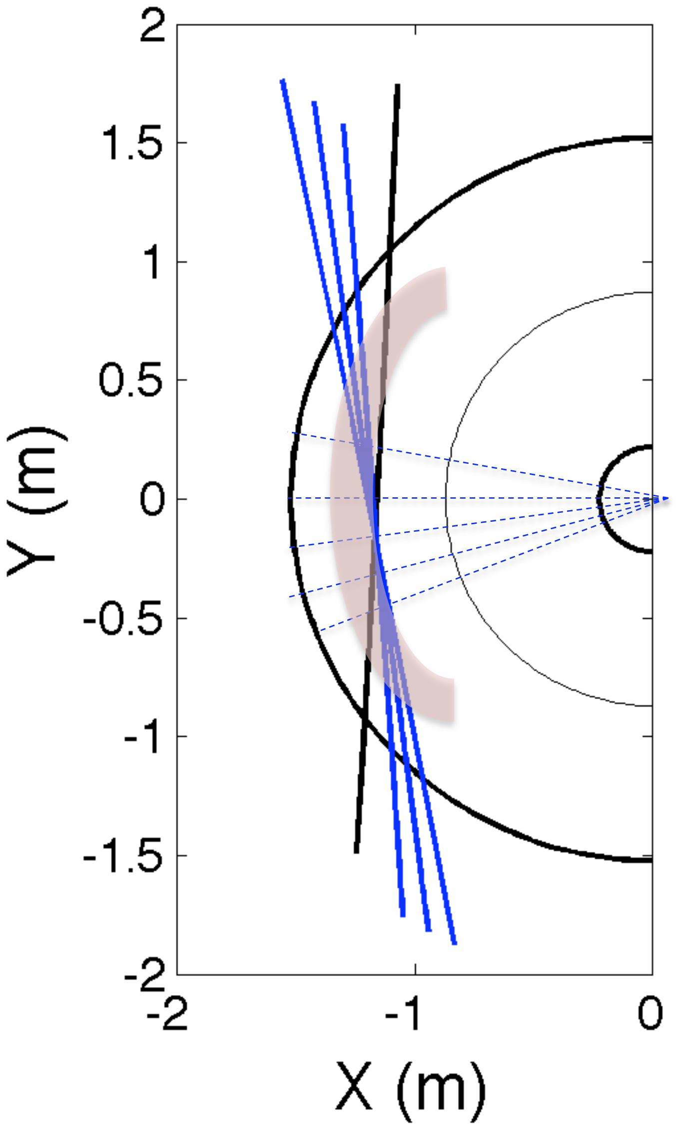
The misalignment between the receiving window and the beam axis should be taken into account

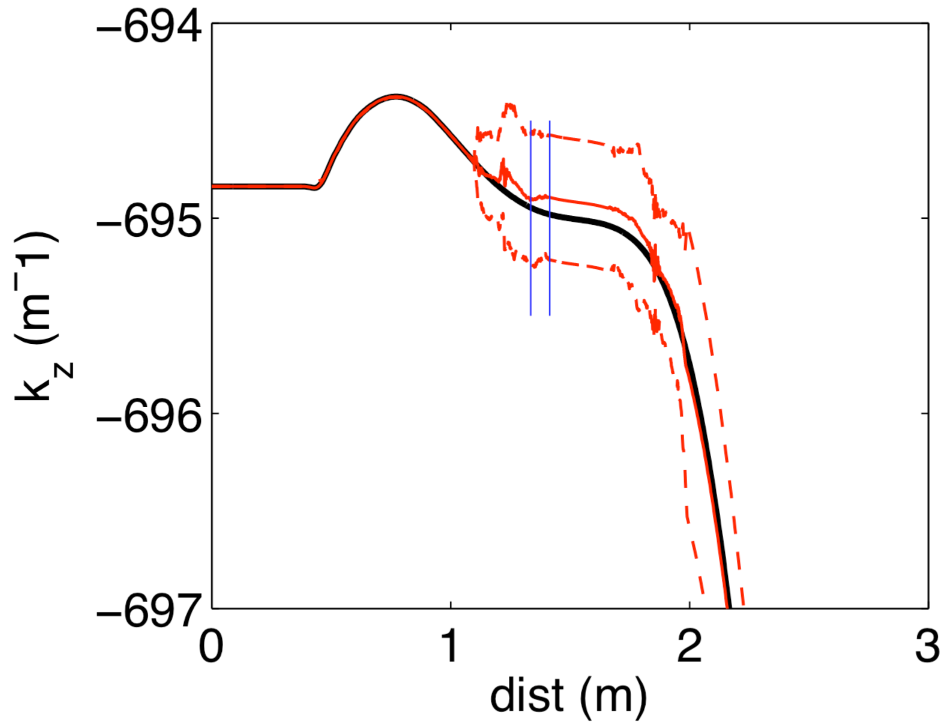






Effect of anisotropic density fluctuations





Δk_R ~10%

$\Delta k^{i,s}_z$ ~1%

High-k system measures density fluctuations in a limited k-range

$$I(t) \ = A(t)\cos[\alpha(t)]$$ In-phase

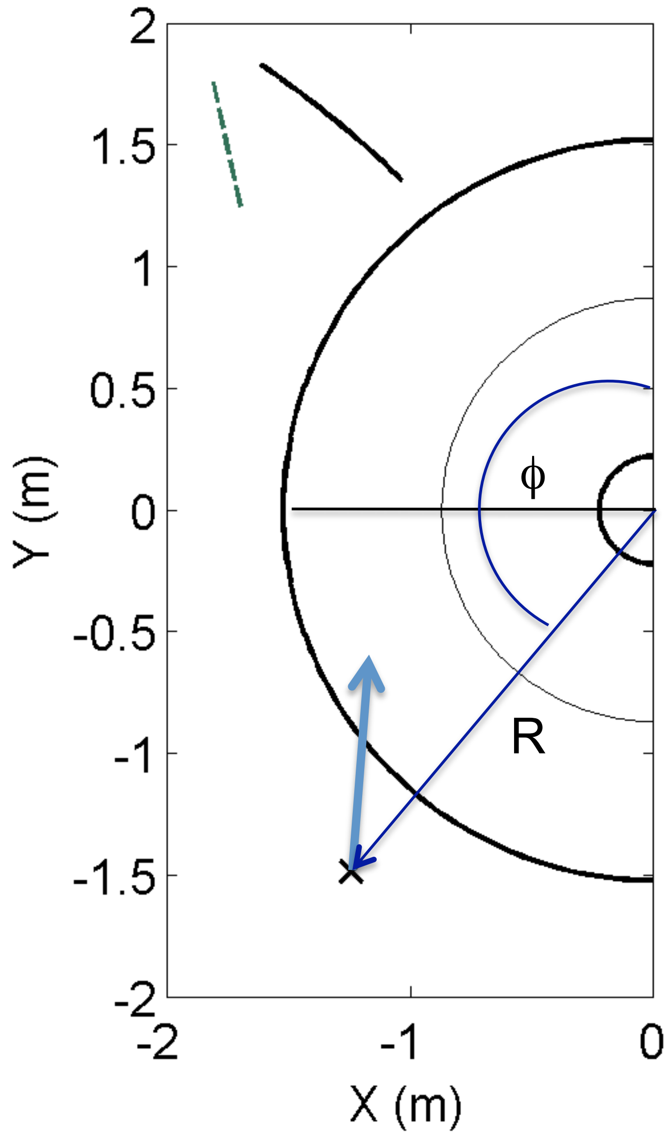
$$Q(t) = A(t)\sin[\alpha(t)]$$ Quadrature

$$I(t) + iQ(t) = A(t)e^{i\alpha(t)} \propto \tilde{n}(\vec{k},t)$$

$$\vec{k} \equiv (k_R, k_\phi, k_z) \quad \Rightarrow \quad P(k_\perp^j, \omega)$$

Use the equilibrium reconstruction to convert into:

$$\vec{k}_\perp^j \equiv (k_r^j, k_\theta^j)$$

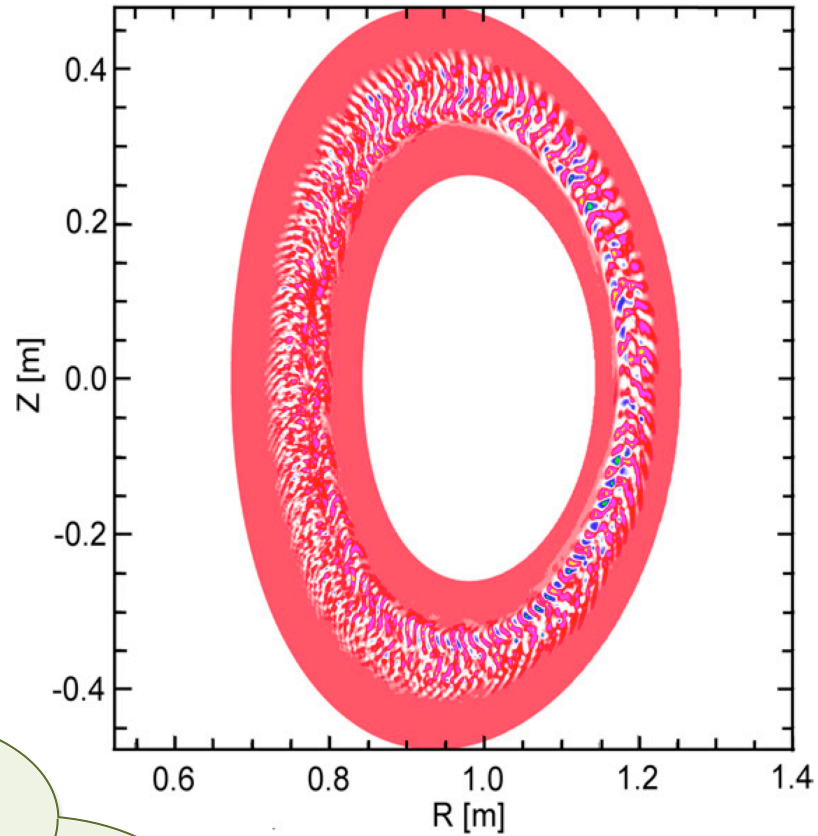


GTS simulates fluctuations in the real domain

$$r = \sqrt{\psi / \psi_e}$$ Use magnetic flux coordinates

$$\Delta r = \sqrt{T_i / T_c}$$ non-uniform grid along r

$$\Delta\theta(r)$$ Uniform grid along θ on each flux surface



$$\tilde{n}(R,z,\phi,t)$$

2xFT[k]

Requirements:

- uniform grid for FFT => interpolation
- preserve magnetic geometry => (r,θ)
- minimize number of operations
- minimize the errors due to interpolation

$$P(k_r, k_\theta, t)$$

64 planes
toroidally distributed

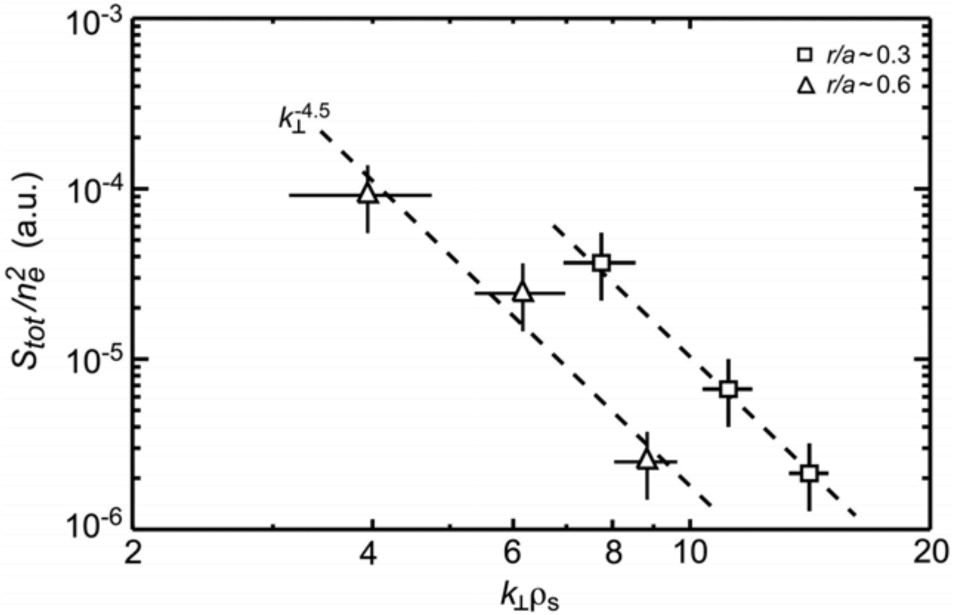
Ways of comparing and related issues/1

$P_{HK}(k_\perp^j, \omega)$ Discrete in $k_\perp$

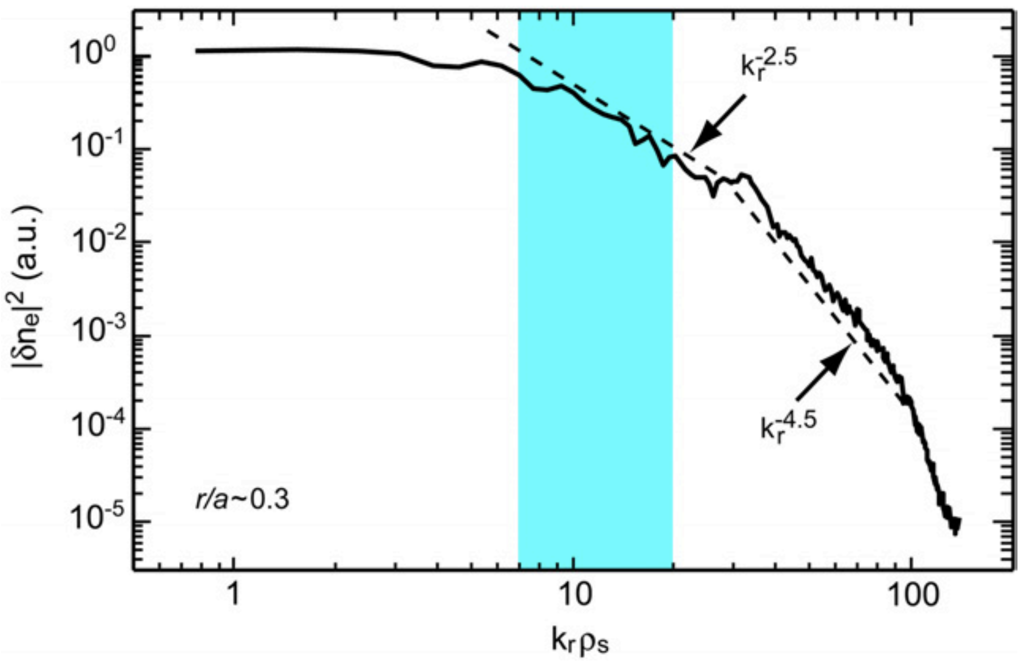
Good statistics in ω

$P_{SIM}(k_r, k_\theta, \omega)$ Wide range in (k_r, k_θ)

A selection is needed in (k_r, k_θ)



[Fig.15, NF **49** (2009) 055001]

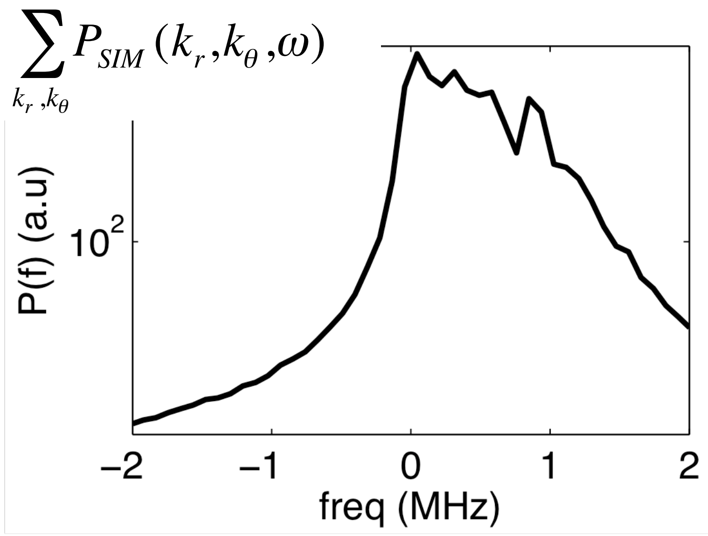


[Fig.23, NF **49** (2009) 055001]

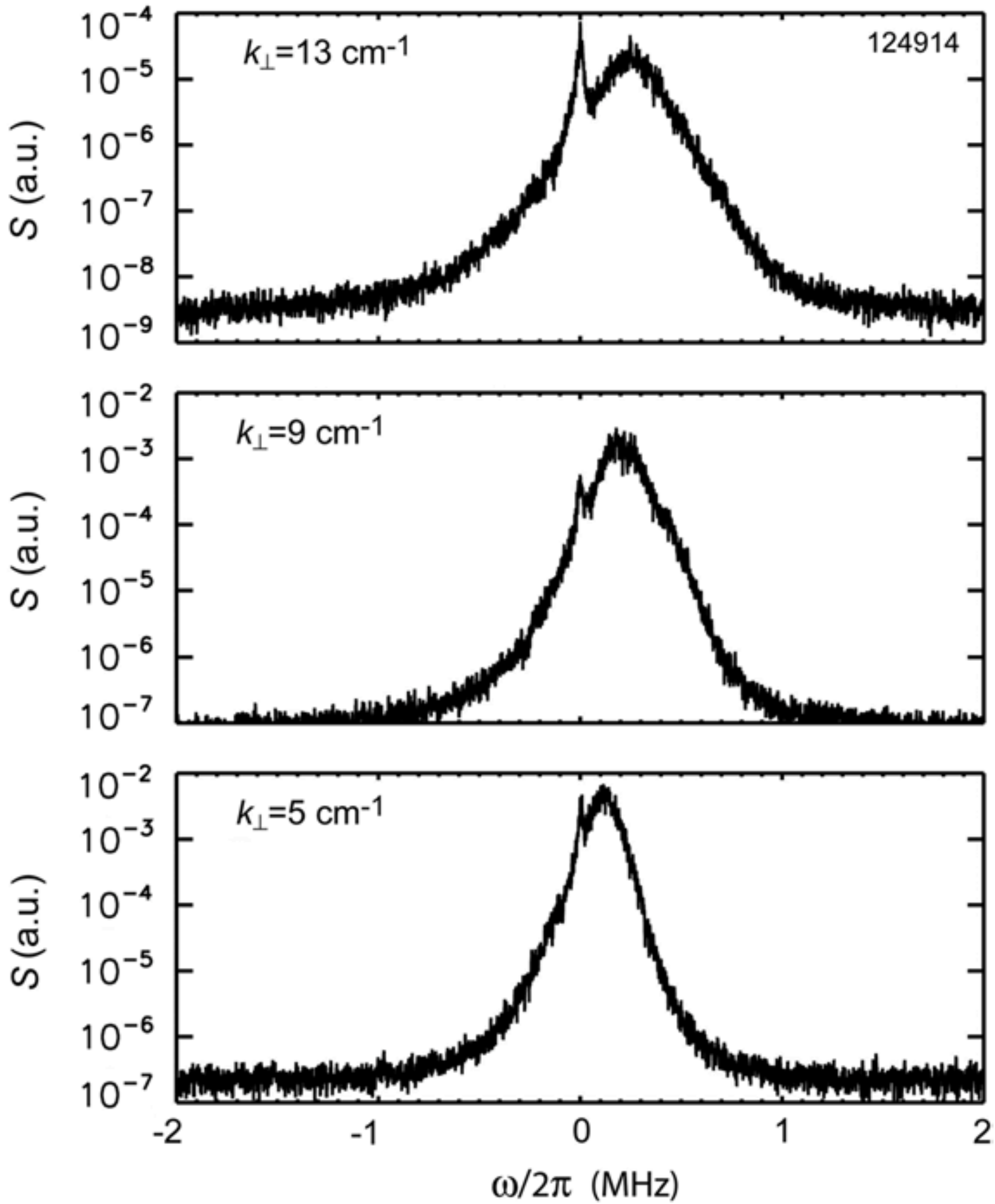
Comparison based on P(k)

- Limited range of exp. $k_\perp$
- Identify (k_r, k_θ)
- Calibration needed for quantitative comparison

Ways of comparing and related issues/2



δt => f_N ~2.3 MHz < $f_{N,HK}$



[Fig.11, NF **49** (2009) 055001]

Comparison based on P(ω)

- short time series in simulations
(Δω small enough to resolve low-ω)

- Doppler shift due to **ExB**

- Select (k_r, k_θ)

- **Comparison still possible when data are not calibrated**