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Ideal Perturbed Equilibrium Code and Arbitrary Jump Conditions at Rational Surfaces

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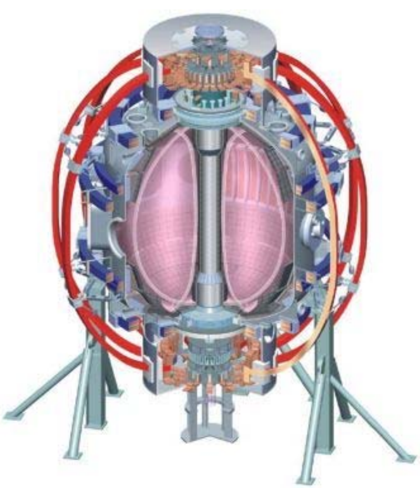

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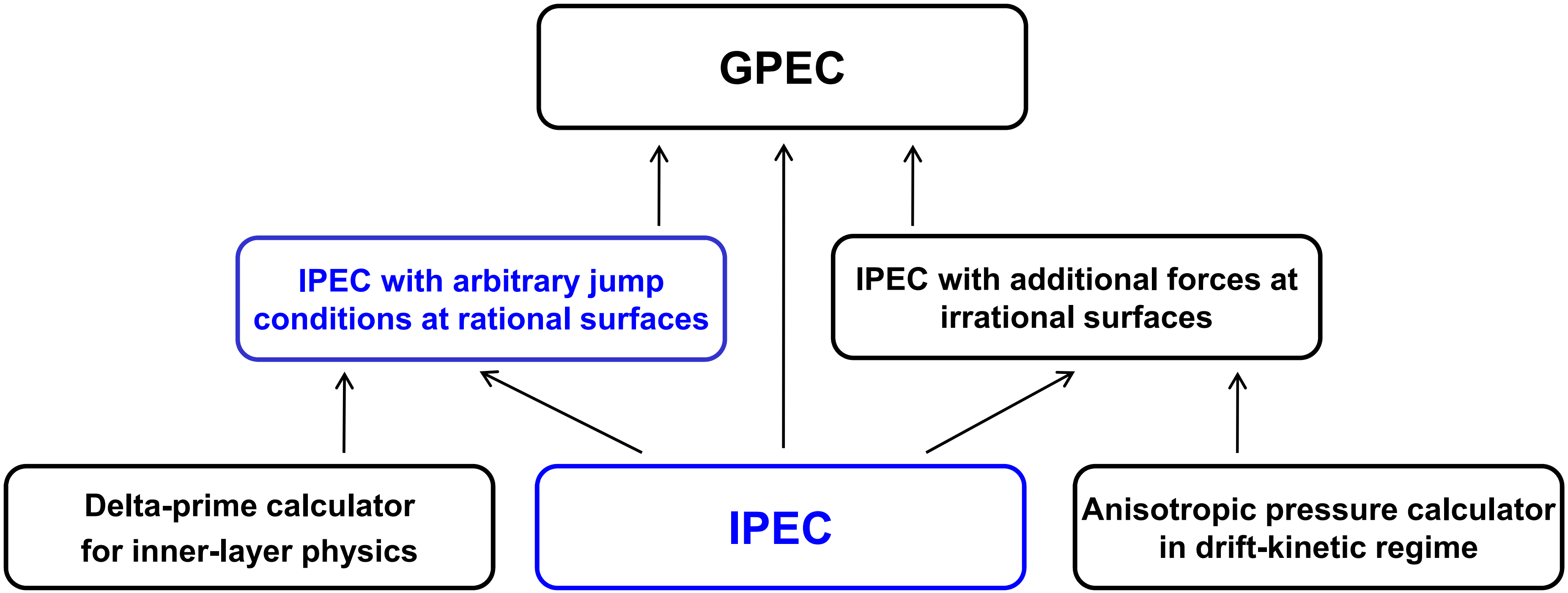
Motivation to perturbed equilibrium codes

- Perturbed equilibrium codes are efficient to study 3D field physics in tokamaks
  - IPEC solves ideal case
  - Future codes will include non-ideal effects



Outline

- Features in Ideal Perturbed Equilibrium Code (IPEC)
  - How to use IPEC
  - Features in inputs and outputs
  - Important concepts
- Progress on IPEC with arbitrary jump conditions
  - Numerical implementations
  - Studies in simple cases

How to run IPEC

- Copy
  - /p/nstxusr/nstx-users/jpark/ipec/ipec_1.00/---, or
  - /p/nstxusr/nstx-users/jpark/ipec/ipec_1.00/rundir/LinuxLahey64/---
- Run
  - /dcon, and then
  - /ipec
- Take a look
  - ipec_singfld_n#.out,
  - ipec_xbnormal_n#.out, etc
- IPEC is as easy as DCON to run

How to make input

- Only two inputs : 2D equilibrium and 3D error-field
- Open and modify
  - equil.in for geqdsk

```
Eq_filename="./g132633.00608"
```

  - ipec.in for error-field on the plasma boundary surface

```
harmonic_flag=t
cosmn(2)=1.2e-4
sinmn(3)=5.7e-5,…

jac_in="hamada"
jsurf_in=0
tmag_in=1
```

```
data_flag=t
data_type="vac3d1" ("surfmn1")
infile="./rwm_1kAt.dat"

jac_in="pest"
jsurf_in=2
tmag_in=0
```

  - ipec.in for fixed boundary solution

```
fixed_boundary_flag=t
```

Issues on input and output coordinates (I)

- IPEC supports many different coordinates
  - Magnetic coordinates are defined

$$\vec{B}\cdot\vec{\nabla}\theta \propto \frac{R^{\alpha} r^{\beta}}{B^{\gamma} B_P^{\delta}}$$

power_rin, power_cin, power_bin, power_bpin,…

  - Surface weighted spectrum is supported

$$b_{mn} = \frac{\oint \left(\delta\vec{B}\cdot\hat{n}\right)(\vartheta,\varphi) e^{-i(m\vartheta - n\varphi)} da}{\oint da}$$

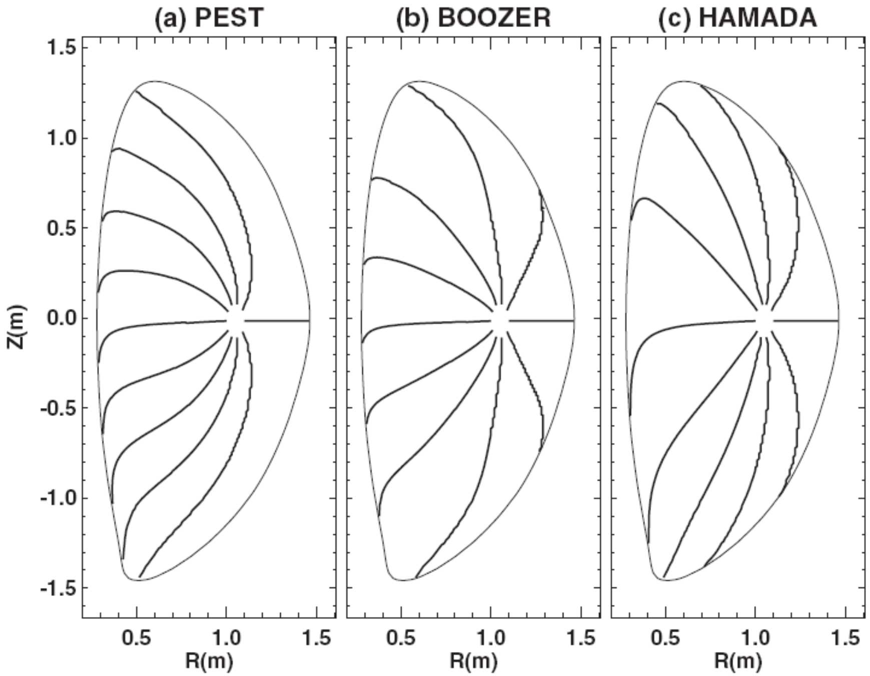
jsurf_in=1

  - Ordinary toroidal angle is supported **tmag_in=0**

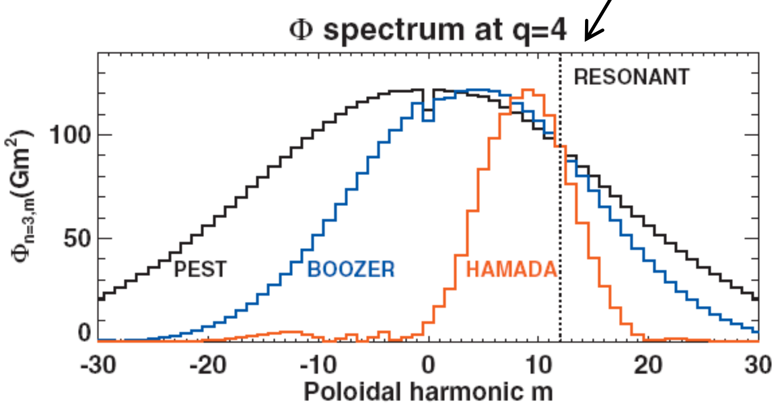
- Inputs and outputs can have different coordinates
- Working coordinates are different, and set by DCON

Issues on input and output coordinates (II)

- Each coordinate has different strength
  - Hamada : best convergence, poor for inboard side
  - PEST and Equal arc : user-friendly, poor for outboard side
  - Boozer : mild convergence and accuracy everywhere
- Physics are independent on coordinates



Resonant field for islands



Output structures in IPEC (I)

- Output ascii files :
  - ipec_response_n#.out : Plasma inductance, permeability, reluctance matrix, each eigenvalues and eigenvectors (resp_flag)
  - ipec_singcoup_matrix_n#.out : Coupling matrix between external fields and total resonant fields, singular currents, islands (singcoup_flag)
  - ipec_singcoup_svd_n#.out : The i^{th} important field for resonant fields, etc, and each singular values, coupling matrix (singcoup_flag)

  When error-field is provided,

  - ipec_control_n#.out : Boundary response, amplification
  - ipec_singfld_n#.out : Resonant fields, induced islands, Chirikovs (singfld_flag)
  - ipec_pmod_n#.out : Eulerian and Lagrangian |b| profiles (pmodb_flag)
  - ipec_xbnormal_n#.out : Normal xi and b field profiles (xbnormal_flag)
  - ipec_brzphi_n#.out : (r,z,phi) components of perturbed b fields in (r,z,phi) coordinates (brzphi_flag, xrzphi_flag, vbrzphi_flag, vpbrzphi_flag, etc)

Output structures in IPEC (II)

- Output binary files
  - xbnormal.bin, ipec_pmodb.bin : profiles for 1d xdraw (bin_flag)
  - xbnormal_2d.bin, etc : profiles for 2d xdraw (bin_2d_flag)
  - pflux_re_2d.bin : perturbed flux surfaces (bin_2d_flag)
  - bnormal_spectrum.bin : surfmn type plots for b normal fields

- Run
  - /xdraw xbnormal, etc

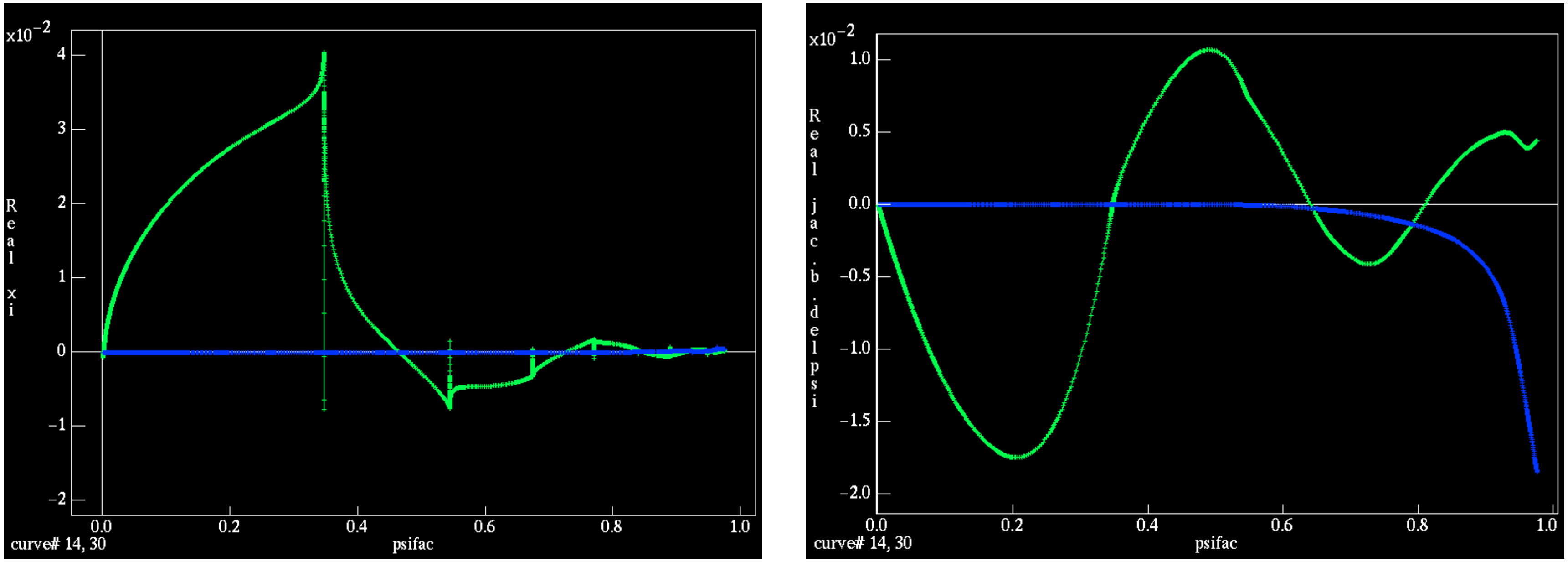
- Xdraw is a powerful tool to examine results promptly, by various interactive features, but
- Other tools (such as IDL) for ascii files are needed to make journal-quality figures

Output : normal displacements and fields

- See **ipec_xbnormal_n#.out** for Fourier harmonic profiles
- **/xdraw xbnormal**

g132633.00608 with n=1 EF currents

Normal displacement **m=2, m=18** **Normal b field**

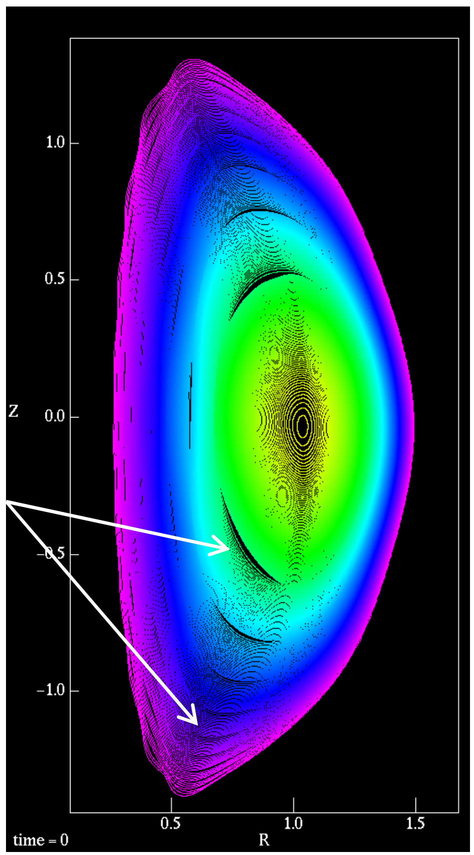


Output : perturbed flux surfaces and fields

- See **ipec_xbnormal_n#.out** for fourier harmonic profiles
- **/xdraw xbnormal_2d, or /xdraw pflux_re_2d**

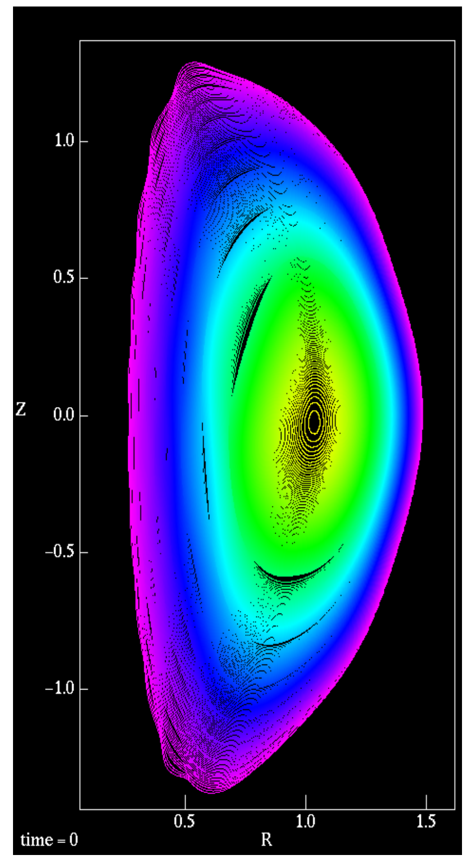
g132633.00608 with n=1 EF currents

φ=0



Rational surfaces

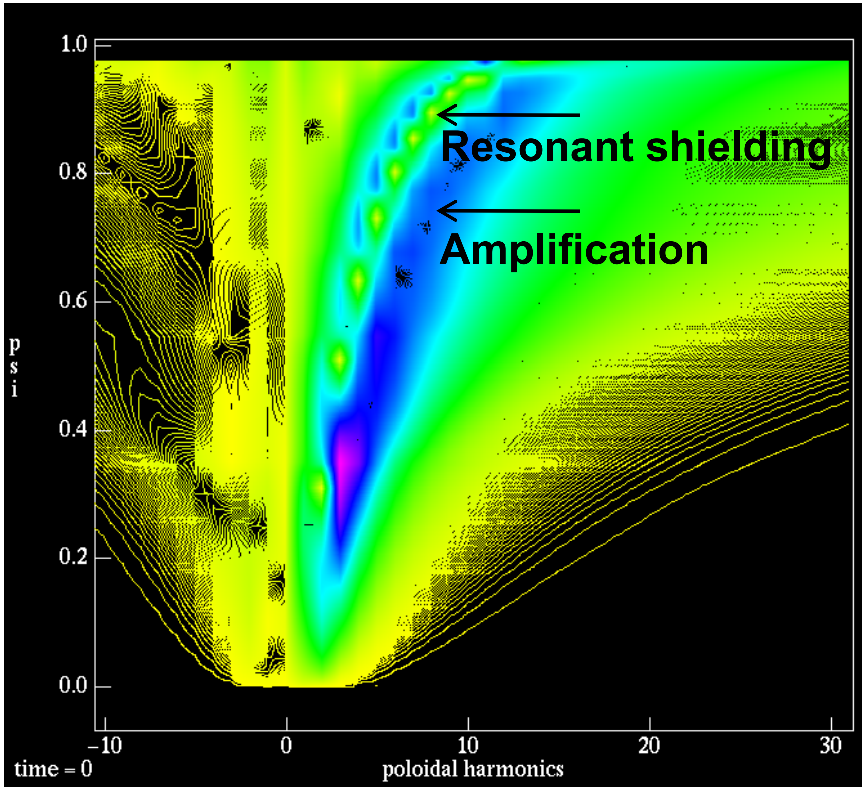
φ=90



Output: surface weighted b field spectrum

- See **ipec_xbnormal_n#.out** for fourier harmonic profiles
- **/xdraw bnormal_spectrum**

g132633.00608 with n=1 EF currents

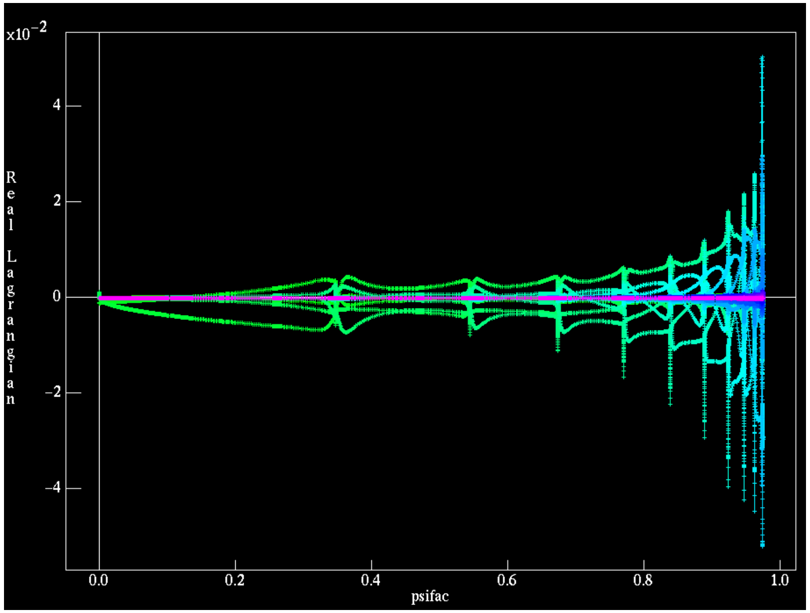


Output: perturbed |b| profiles

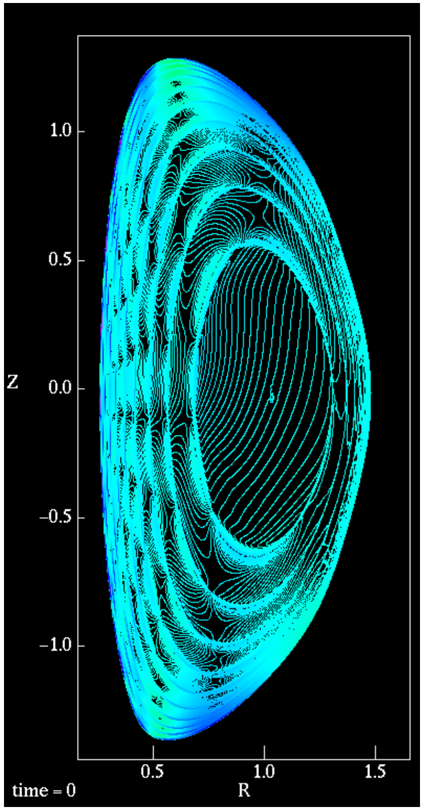
- See **ipec_pmodb_n#.out** for fourier harmonic profiles
- **/xdraw pmodb, or /xdraw pmodb_2d**

g132633.00608 with n=1 EF currents

Each harmonic profiles



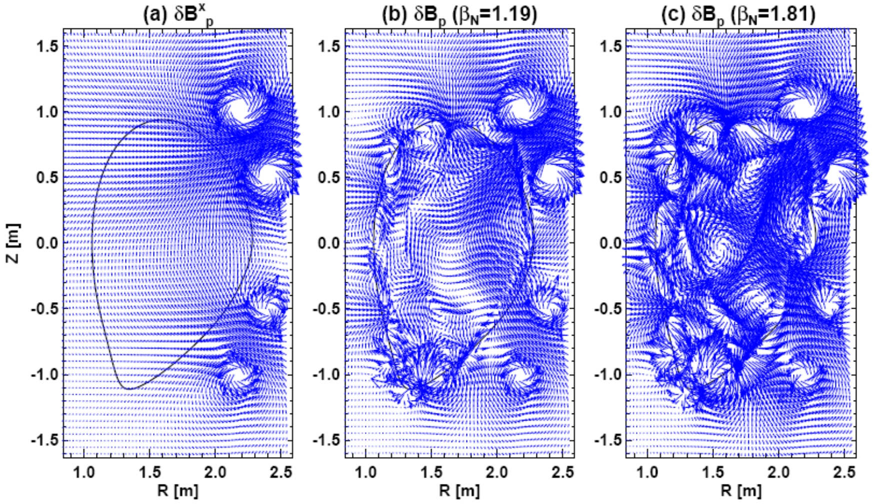
Contour



- NTV routines are separate and not yet implemented in IPEC

Output : perturbed ξ and b vectors in (r,z,phi)

- See **ipec_*rzphi_n#.out** for data
  - Vacuum fields outside the plasma is not provided from IPEC
  - Use **ipec_pbrzphi_n#.out** and add your vacuum fields outside (**l=0**)
  - See **ipec_pbrzphi (or vpbrzphi)_n#.out** to have plasma response
  - See **ipec_xrzphi_n#.out** to have plasma displacements



- Poincare tracing code (S. Hudson) is separate

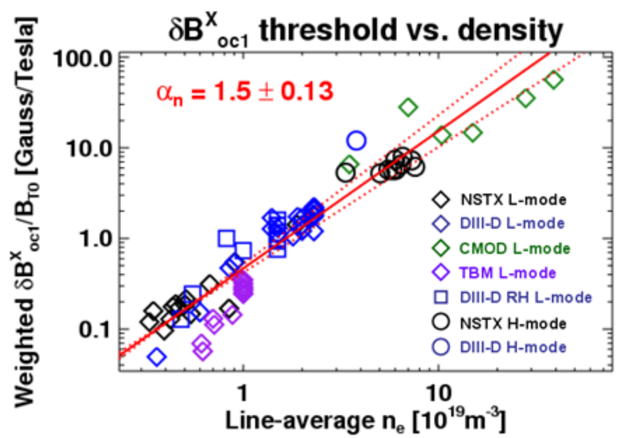
Output : resonant fields, islands, Chirikovs

- See **ipec_singfld_n#.out** for resonant fields, islands, etc

q	psi	real(singflx)	imag(singflx)	real(singcur)	imag(singcur)	islandhwidth	chirikov
2.000	3.46826953E-01	4.69650517E-04	-5.88121248E-04	1.17667469E+03	-1.47349436E+03	7.61589337E-02	7.72369004E-01
3.000	5.44035627E-01	1.82758806E-04	-2.42367945E-04	4.43245509E+02	-5.87815740E+02	3.85164222E-02	5.93357690E-01
4.000	6.73860934E-01	6.10155924E-05	-7.92386151E-05	1.41461753E+02	-1.83710966E+02	2.03607632E-02	4.21800633E-01
5.000	7.70403052E-01	4.94533020E-05	-6.13473827E-05	1.10785858E+02	-1.37431115E+02	1.59441126E-02	4.63529414E-01
6.000	8.39197447E-01	6.91010435E-05	-7.94839799E-05	1.49261365E+02	-1.71688974E+02	1.58277653E-02	6.43480303E-01
7.000	8.88391696E-01	6.79358284E-05	-7.34817892E-05	1.41171858E+02	-1.52696464E+02	1.33011570E-02	7.48965610E-01
8.000	9.23910435E-01	1.57830604E-04	-1.96006561E-04	3.14671694E+02	-3.90784265E+02	1.76860405E-02	1.52374141E+00
9.000	9.47124401E-01	2.88304755E-04	-3.85648019E-04	5.46143270E+02	-7.30543172E+02	1.97119845E-02	2.52142928E+00
10.000	9.62759965E-01	4.93641101E-04	-5.63773598E-04	8.84543509E+02	-1.01021223E+03	2.07356092E-02	3.66874048E+00
11.000	9.74063904E-01	2.23222979E-04	-1.45122837E-04	3.77801042E+02	-2.45617899E+02	1.06256207E-02	1.87998552E+00

- When **singcoup_flag=t**, overlap fields will also be provided
  - This is being used for locking thresholds

mode	real(ovf)	imag(ovf)	overlap(%)
1	-1.59566880E-04	2.09106744E-04	3.03126685E+01
2	-1.12919170E-04	1.25480940E-04	1.94538134E+01
3	2.22724911E-05	-4.13769485E-05	5.41529532E+00
4	2.69295167E-06	-2.98598016E-05	3.45507274E+00
5	7.35413851E-06	-5.78315646E-05	6.71830237E+00
6	-6.01999767E-05	7.27180964E-05	1.08792214E+01
7	5.92764183E-05	1.23037725E-06	6.83261137E+00
8	-5.60090623E-06	6.11843237E-05	7.08049261E+00
9	-3.00285513E-05	-3.81663356E-05	5.59652444E+00
10	2.09479454E-05	-1.07832051E-04	1.26591081E+01



Output : response characteristics

- See **ipec_response_n#.out** for response matrices
  - **Surface and plasma inductance** : Virtual casing currents to fields
  - **Plasma permeability** : External fields to total fields
  - **Eigenvalues and eigenvectors for each matrix** show intrinsic properties of plasma responseseach matrix
- See **ipec_singcoup_matrix_n#.out** for resonant coupling
  - **The i^{th} important field distributions** are complete set of distributions affecting resonant fields
  - **Singular values** show how important they are in error field correction problems
- Coupling for NTV is being planned

Important notes for IPEC run

- IPEC results are robust enough for low-β plasmas, but sensitivity exists in high-β, H-mode plasmas, as DCON
- Boundary cutting issue :
  - If p' ≠ 0 in edge, results depend on the relative locations of rational surfaces
  - Put boundary at the first half in a q-interval (**sas_flag=t, dmlim=0.1-0.5 in dcon.in**) to have robust answer
- Toroidicity issue:
  - NSTX is the most difficult, n=1 is ok, but n≥2 is often unreliable
  - Make sure plasma is far from stability limit for n≥2
- Internal instability issue:
  - Refine equilibrium when DCON finds internal instability, or
  - Avoid q=1 surface by setting **psi_low>psi (q=1) in dcon.in**

Summary and future work (I)

- IPEC is easy to run (**/dcon, /ipec, /xdraw with equil.in, dcon.in, ipec.in**)

  I am still testing some features and preparing manuals, and these jobs will be done **by the end of June**

- Vacuum (Bio-Savart) part will be implemented **by the end of July**

- NTV and Poincare tracing routines are presently separate, but planned for implements **by this year**

- These new features will be version-controlled

- IPEC can be sensitive as DCON, but issues will be constantly studied

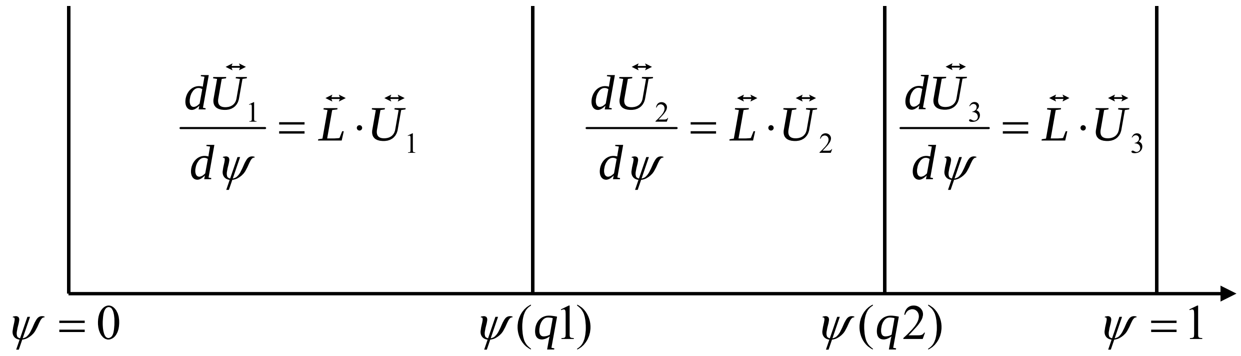
Jump conditions are always necessary to solve force-balance equations

- DCON solves Euler-Lagrange equation for displacements

$$\vec{\Xi} = \left\{ \left(\vec{\xi} \cdot \vec{\nabla} \psi \right)_{mn} \middle| m_{\min} \le m \le m_{\max},\; m_{\max} - m_{\min} + 1 = M \right\}$$

$$\begin{bmatrix} \vec{\Xi} \\ \overleftrightarrow{F} \cdot \vec{\Xi}' + \overleftrightarrow{K} \cdot \vec{\Xi} \end{bmatrix}' = \begin{bmatrix} -\overleftrightarrow{F}^{-1} \cdot \overleftrightarrow{K} & \overleftrightarrow{F}^{-1} \\ \overleftrightarrow{G} - \overleftrightarrow{K}^{+} \cdot \overleftrightarrow{F}^{-1} \cdot \overleftrightarrow{K} & \overleftrightarrow{K}^{+} \cdot \overleftrightarrow{F}^{-1} \end{bmatrix} \begin{bmatrix} \vec{\Xi} \\ \overleftrightarrow{F} \cdot \vec{\Xi}' + \overleftrightarrow{K} \cdot \vec{\Xi} \end{bmatrix} \longrightarrow \frac{d\vec{u}}{d\psi} = \overleftrightarrow{L} \cdot \vec{u}$$

- 2M independent solutions exist depending on initial conditions
- F operator is singular at rational surfaces, and thus 2M independent solutions should be solved between rational surfaces

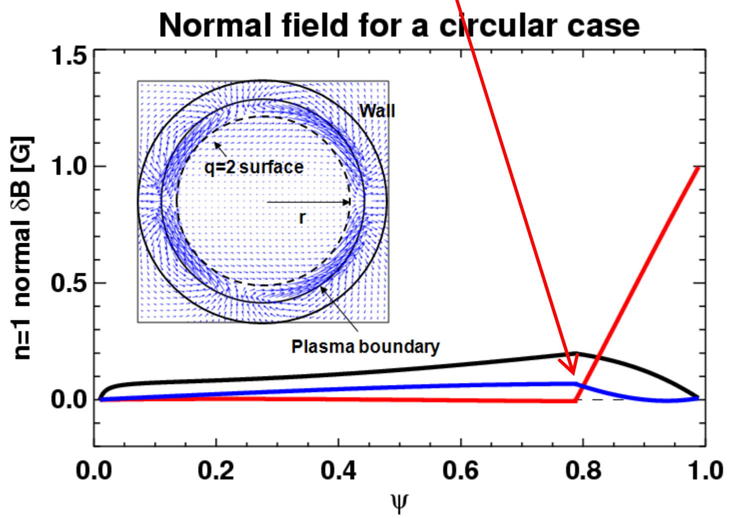


Ideal DCON and IPEC uses only M independent solutions to match boundary

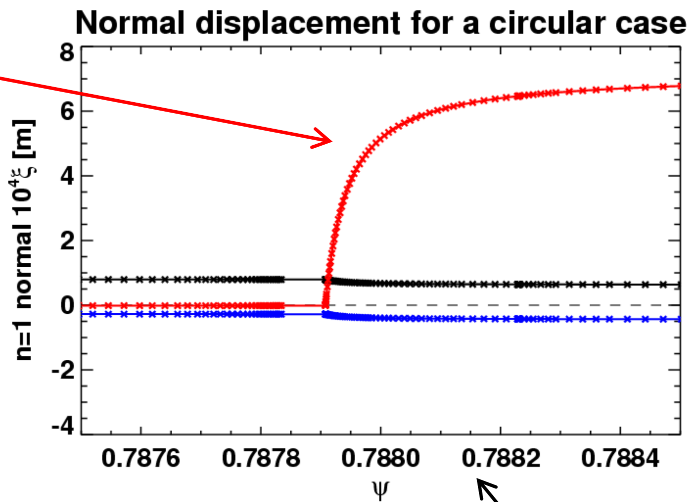
- DCON integrates equations from the magnetic axis – where only regular solutions are allowed $\vec{\Xi} = \vec{0}$
- Only the half part starts from the axis $\vec{U} = \begin{pmatrix} \overleftrightarrow{I} & 0 \\ 0 & \overleftrightarrow{I} \end{pmatrix}$
- At rational surfaces
  - **Non-resonant solutions continuously cross**
  - **Large resonant solutions are forced to be zero**
  - **Small resonant solutions are reinitialized**
  - **Gaussian eliminations (mixing initial conditions) are used to sort out resonant solutions**
- At the boundary, M independent solutions reach and can be determined by boundary conditions on the plasma boundary

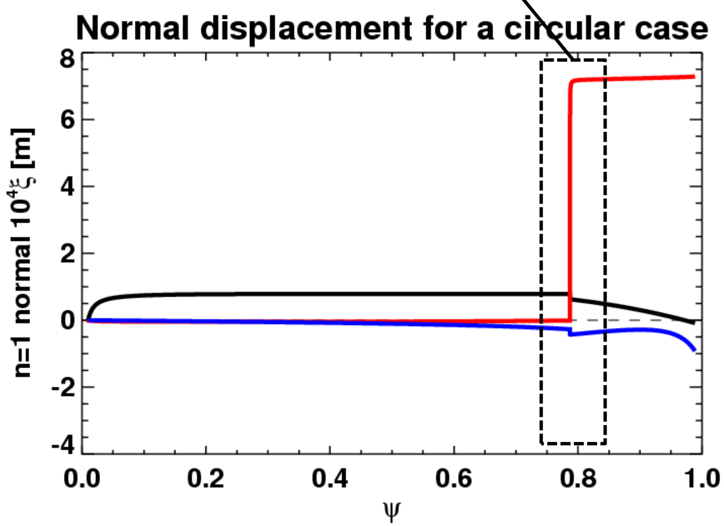
Ideal solutions have zero resonant fields by shielding currents suppressing islands

- Small resonant solution guarantees no resonant field at the rational surface



- Ideal DCON is very precise by preserving small solutions – which are easy to handle in the frame of displacements





Outer-layer solutions can be determined differently with arbitrary jump conditions

- Outer-layer is more ideal
- Outer-layer equations are still E-L equations, and they can be differently determined if inner-layer gives different jump conditions
  - Full relaxation (smooth)
  - Magnetic island size (resonant field)
  - Delta-prime, etc
- Large solutions are now necessary, but their stronger singularity must be more carefully treated

Stepped pressures may be necessary when large resonant solutions are retained

- Resonant solutions are

Small **Large**

$$\begin{bmatrix} \xi \\ f\xi' \end{bmatrix} = c_s \begin{bmatrix} z^{-1/2+\sqrt{-D_I}} \\ z^{1/2+\sqrt{-D_I}}\left(\sqrt{-D_I}-1/2\right)f_0 \end{bmatrix} + c_l \begin{bmatrix} z^{-1/2-\sqrt{-D_I}} \\ z^{1/2-\sqrt{-D_I}}\left(-\sqrt{-D_I}-1/2\right)f_0 \end{bmatrix}$$

$$where$$

$$z = \psi - \psi_R$$

$$D_I = -\frac{1}{4} + \frac{2\pi f P'}{q'\chi'^3}\left\langle \frac{1}{|\nabla V|^2} \right\rangle \left[1 - \frac{2\pi f P'}{q'\chi'^3}\left\langle \frac{1}{|\nabla V|^2} \right\rangle \right] + \left\langle \frac{B^2}{|\nabla V|^2} \right\rangle \frac{P'}{q'^2\chi'^4}\left\{ P'\left[\left\langle \frac{1}{B^2} \right\rangle + \left(\frac{2\pi f}{\chi'} \right)^2 \left\langle \frac{1}{B^2|\nabla V|^2} \right\rangle \right] + \frac{\chi''}{\chi'} \right\}$$

$$\delta B \propto z\xi$$

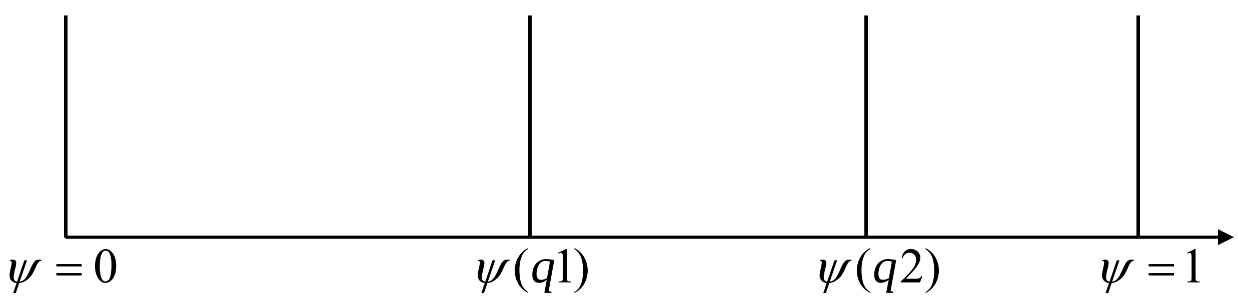
- Large solutions make resonant field singular unless $D_I \geq -\frac{1}{4}$
- Stepped pressure $P' = 0$ ensures $D_I = -\frac{1}{4}$ and resonant solutions can reach a constant

Arbitrary jump conditions have been implemented in IPEC

- Large solutions are also reinitialized and preserved as well as small solutions at each rational surface
  - Asymptotic power series are used to identify small and large solutions

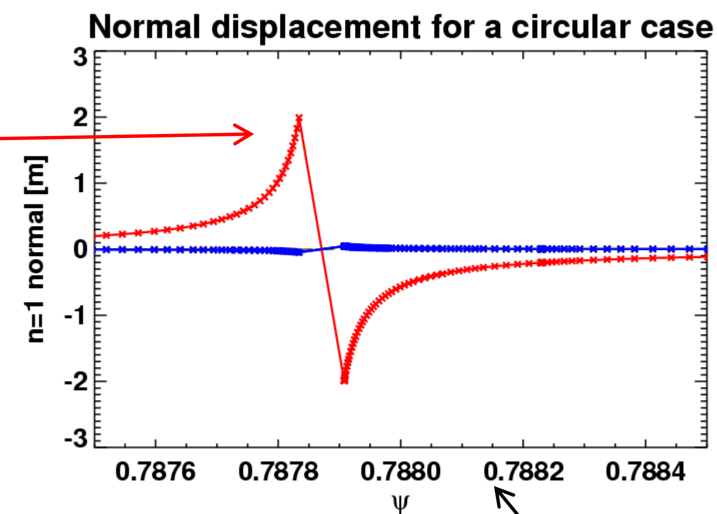
- In each domain, one can determine M+1 solutions using M edge boundary conditions and 1 jump conditions (Backward matching)
  - Displacement should change parity when crossing rational surfaces

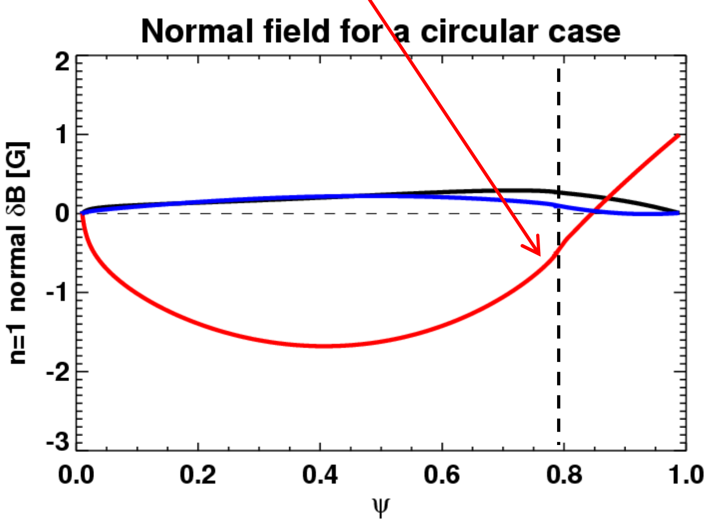
$$\begin{bmatrix} u_{11}(\psi_R) & \cdots & u_{1M}(\psi_R) & l_{1M}(\psi_R) \\ \vdots & \iddots & \vdots & \vdots \\ u_{M1}(\psi_R) & \cdots & u_{MM}(\psi_R) & l_{MM}(\psi_R) \\ u_{L1}(\psi_L) & \cdots & u_{LM}(\psi_L) & l_{MM}(\psi_L) \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_M \\ c_l \end{bmatrix} = \begin{bmatrix} \xi_1(\psi_R) \\ \vdots \\ \xi_M(\psi_R) \\ \xi_l(\psi_L) \end{bmatrix}$$

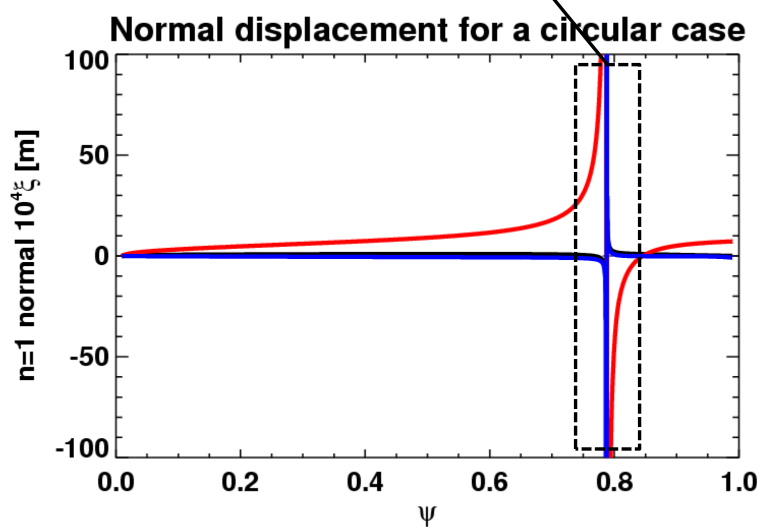


Relaxed solutions have finite resonant fields with finite shielding currents

- Large solutions are preserved and cross the rational surface with jump conditions
- Singular currents can be removed and resonant fields are finite

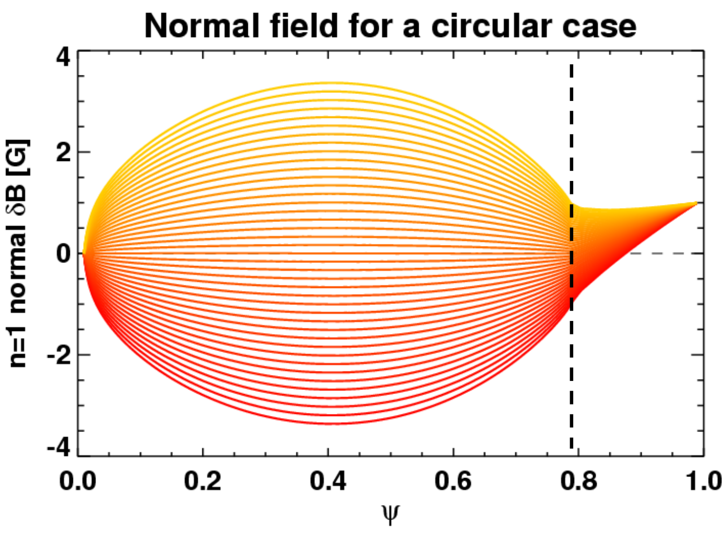


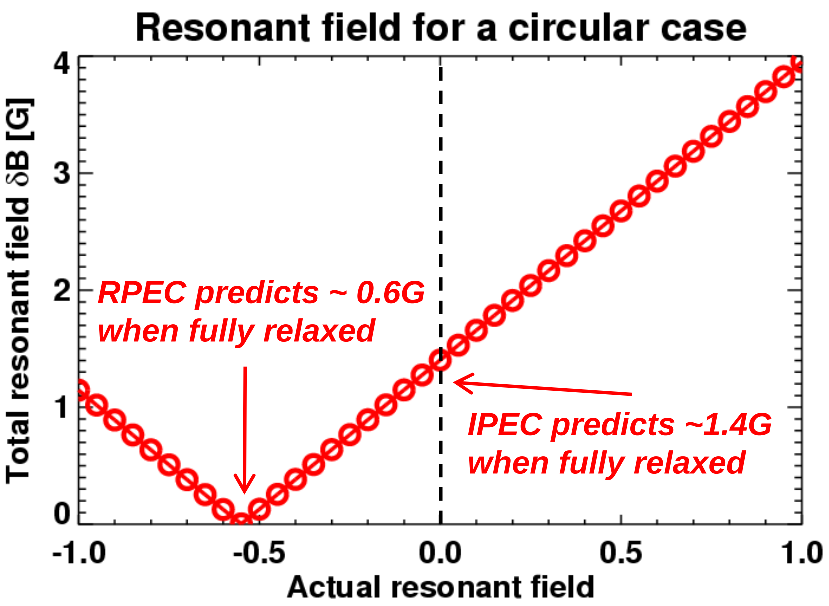




Solutions with islands can be unstable

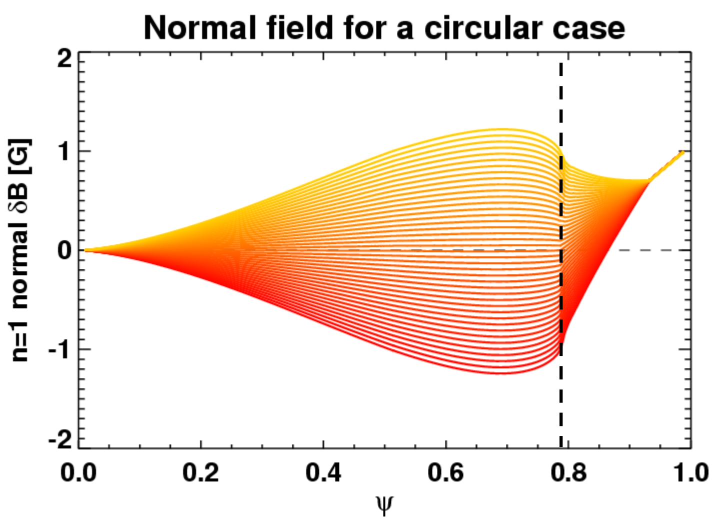
- Fixed boundary cylinder analysis with m=2, n=1
  - **Islands open with more singular currents unless crossing zero**
  - **This is unstable and dW<0 mode exists**

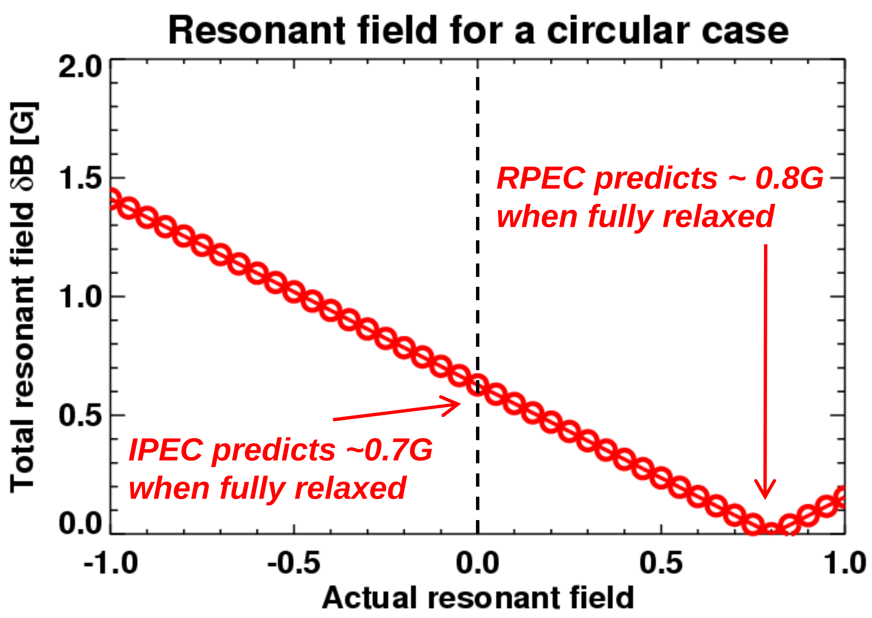




Solutions with islands can be stable

- Fixed boundary cylinder analysis with m=4, n=2
  - **Islands open with less singular currents**
  - **This is stable and all modes dW>0**





Summary and future work (II)

- IPEC has been modified towards arbitrary jump conditions
- Initial test shows that islands can make plasmas unstable even in a very stable case like a cylinder
- Physics may be richer in strong toroidal and high pressure cases
- Thus, different options will be tested
  - **Full 2M matrix matching**
  - **E-L equations for normal b fields**
- Modified IPEC will be able to describe evolutions of outer-layer solutions and be coupled with inner-layer calculators