NSTX-U

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Development of a reduced model for resonant fast ion transport in TRANSP

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and many other contributors

NSTX-U Monday Meeting

B238, PPPL

Feb. 10, 2013

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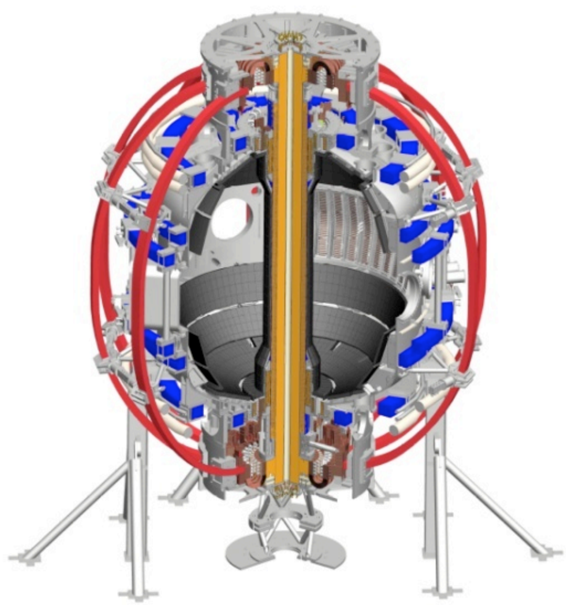

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Four models are presently implemented in TRANSP for fast ion diffusion/convection coefficients, D_b & v_b

All have diffusive/convective nature in radial coordinate; little/no phase space selectivity:

[from http://w3.pppl.gov/~pshare/help/transp.htm]

1) $$D_b = k_{ADIFB} \times D_e$$

k_{ADIFB} : multiplier
$D_e(x,t)$: electron particle diffusivity

2) $$D_b = k_{ADIFB} \times D_e^{WP}$$

$D_e^{WP}(x,t)$: electron particle diffusivity from Ware-pinch corrected flux

3) $$\Gamma_{fi} = -D_b \nabla n_b + v_b n_b$$

diffusion/convection model

4) $$D_b(E,t,x) = \Sigma_k \alpha_k D_k$$

$D_k(E,x,t)$: diffusivity for deeply trapped, barely trapped, co-barely passing, ...

Proposed model introduces selectivity in phase space, generalizes "diffusive transport" *ad-hoc* models

- Info on phase space dynamics is of paramount importance for Verification&Validation of codes, theory-experiment comparison
- "Fluid" (integral) quantities do not provide all information we need
  - Re-computed ("inverted") solutions for F_{nb} based on measured integral quantities (f.i. density, neutrons, E_r/rotation, NB-driven J_{nb}, ...) **are not unique**
- ***Need details on fast ion energy, pitch and their (consistent) evolution***
  - E.g. *AE bursts transiently (and **selectively**) modify phase space; effects propagate during slowing-down

The new model must be "simple enough" to be included in TRANSP/NUBEAM for routine use ⟶ ***reduced model***

Outline

- New ‘kick model’: basic ideas
- Implemented algorithm
- Initial validation against full ORBIT simulations
  – Example from NSTX case w/ TAE avalanche
- Additional remarks and summary

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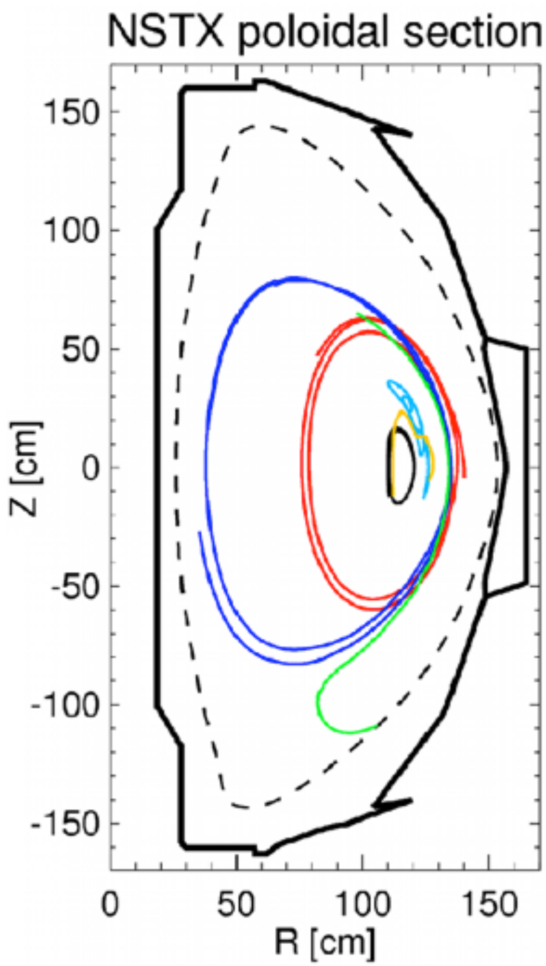
‘Constants of motion’ are the natural variables to describe resonant wave-particle interaction

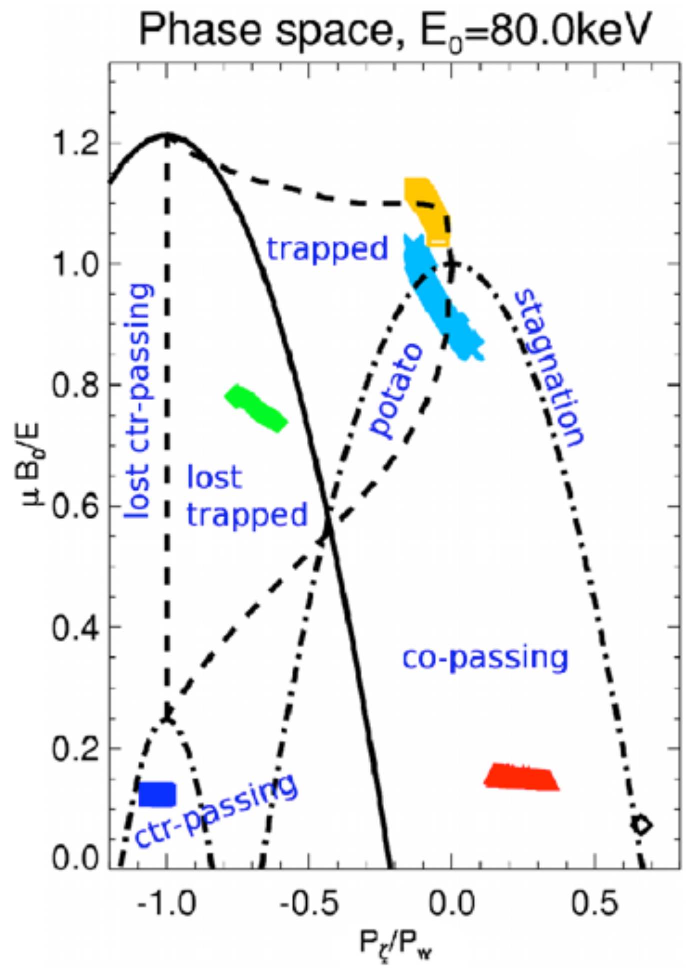
Each orbit fully characterized by:

- E, energy
- $P_\zeta \sim mRv_{par} - \Psi$, canonical angular momentum
- $\mu \sim v_{perp}^2/(2B)$, magnetic moment

Energy exchange rate:

$$\gamma \propto \omega \frac{\partial F_{nb}}{\partial E} + n \frac{\partial F_{nb}}{\partial P_\zeta}$$





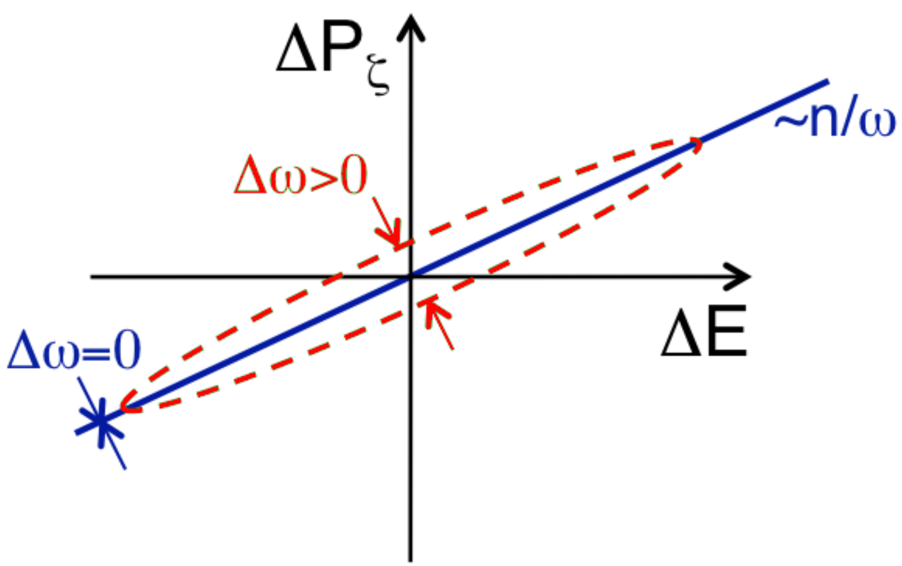
R. B. White, Theory of toroidally confined plasmas, Imperial College Press (2001)

'Kick Model' is based on a *probability distribution function* for particle transport

- Resonances introduce fundamental constraints on particle's trajectory in (E,P_ζ,μ)
- From Hamiltonian formulation – single resonance:

$$\omega P_\zeta - nE = const. \Longrightarrow \Delta P_\zeta / \Delta E = n/\omega$$

$\omega=2\pi f$, mode frequency $\quad$ n, toroidal mode number



For each bin in (E,P_ζ,μ), steps in $\Delta E,\Delta P_\zeta$ are described by

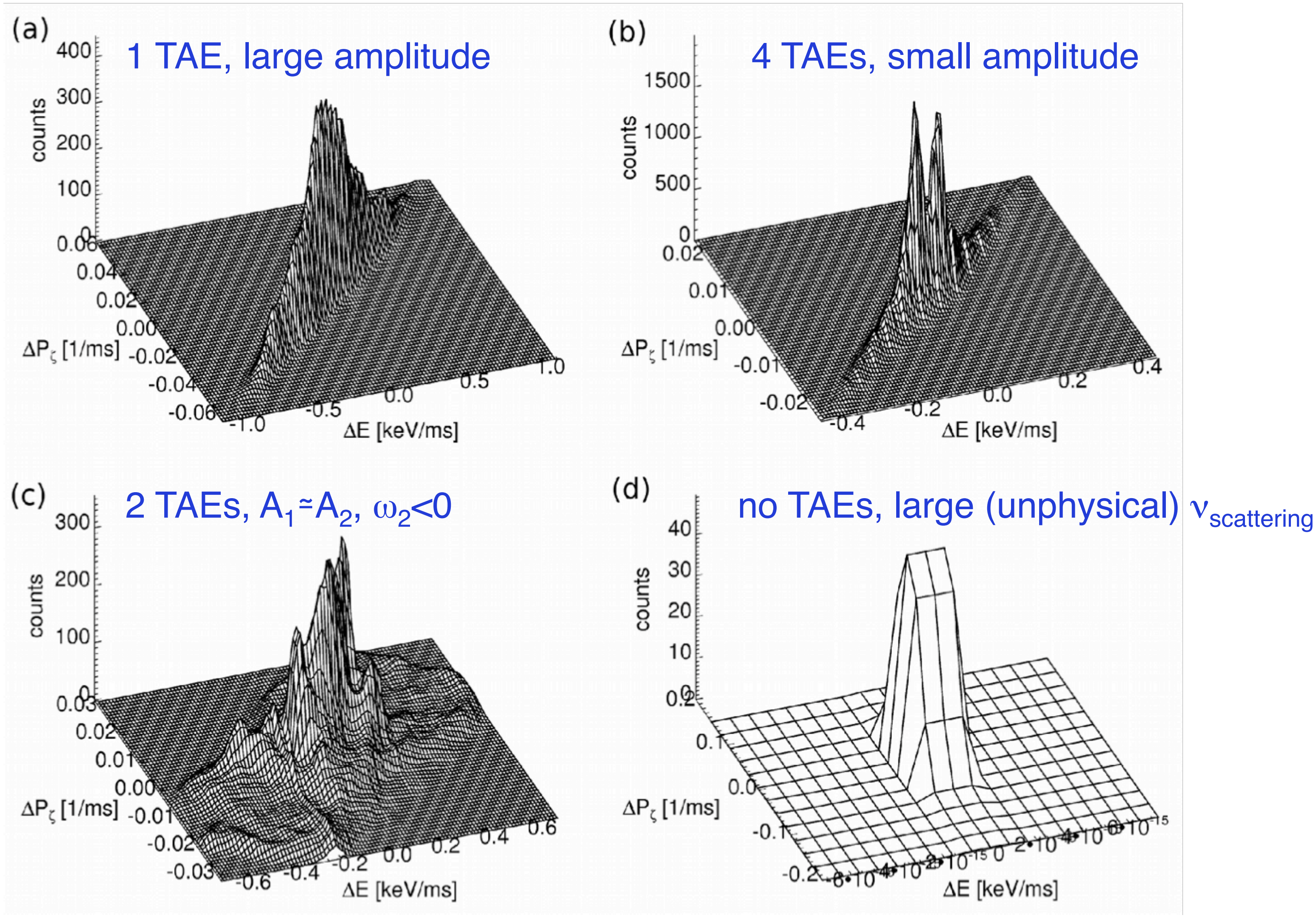
$$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A)$$

which can incorporate the effects of <u>multiple</u> modes & resonances:

Correlated random walk

An analytical formulation for p(ΔE,ΔP$_\zeta$|P$_\zeta$,E,μ) *could* be developed – but it would be quite unpractical

E.g., use: $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu) = \Sigma_{i=1}^{N} w_i\, p_i(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$



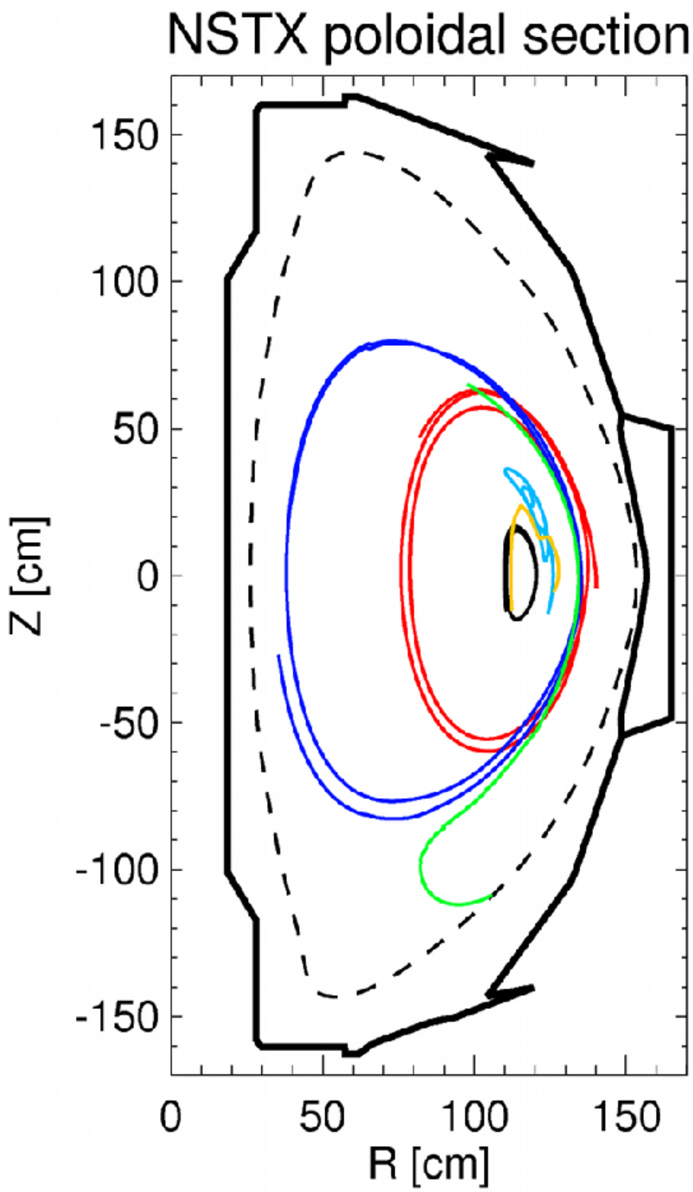
In practice, will use the full 5D matrix for p(ΔE,ΔP$_\zeta$|P$_\zeta$,E,μ)

A quick summary

Two main ingredients for new model:

- Probability distribution function for particle's "kicks" in E, P_ζ
- Mode amplitude scaling factor $A_{mode}(t)$

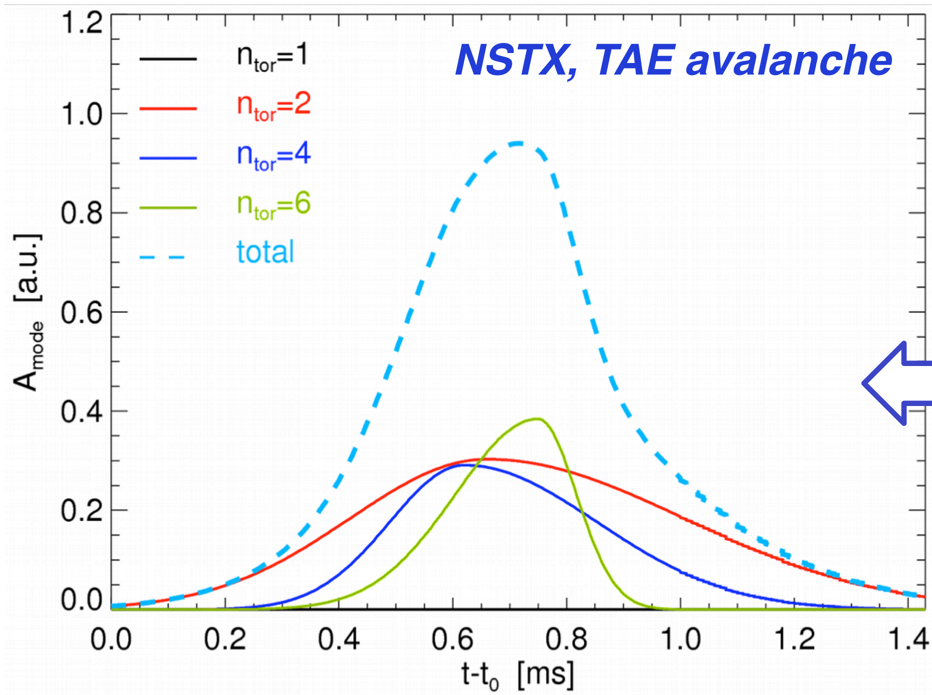
‘Transport probability’ $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ can be computed through numerical codes (e.g. ORBIT) or theory



- Run ORBIT with A_{mode}=1, constant
- Simulation time long enough (~1ms, many toroidal transit times) to capture *AE effects
- Track (P_ζ,E,μ) in time for each particle, steps δt_{sim}~25-50μs
- Compute $\Delta E,\Delta P_\zeta,\Delta\mu$
- Re-bin over (P_ζ,E,μ) space

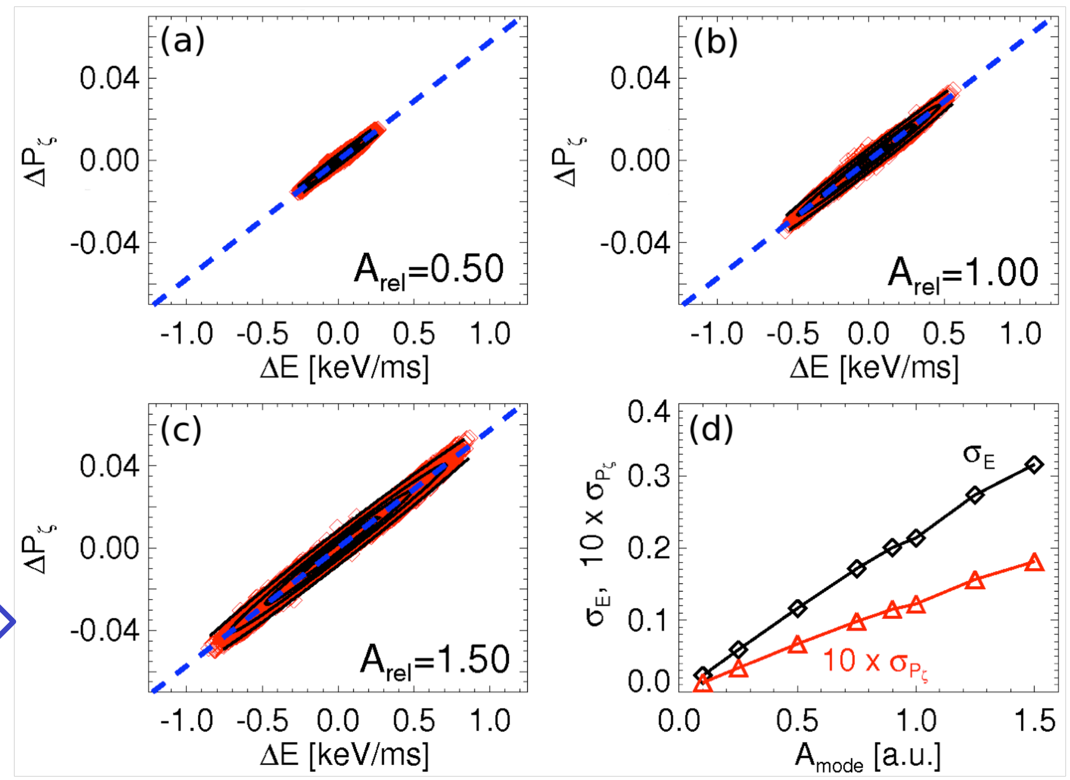
> *Get $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ for each bin*

Mode amplitude can evolve on time-scales shorter than typical TRANSP/NUBEAM steps of ~5-10 ms



F_{nb} evolution must be computed as a sequence of sub-steps

- Duration δt_{step} sufficiently shorter than time-scale of mode evolution
- Examples here have δt_{step}~25-50 μs

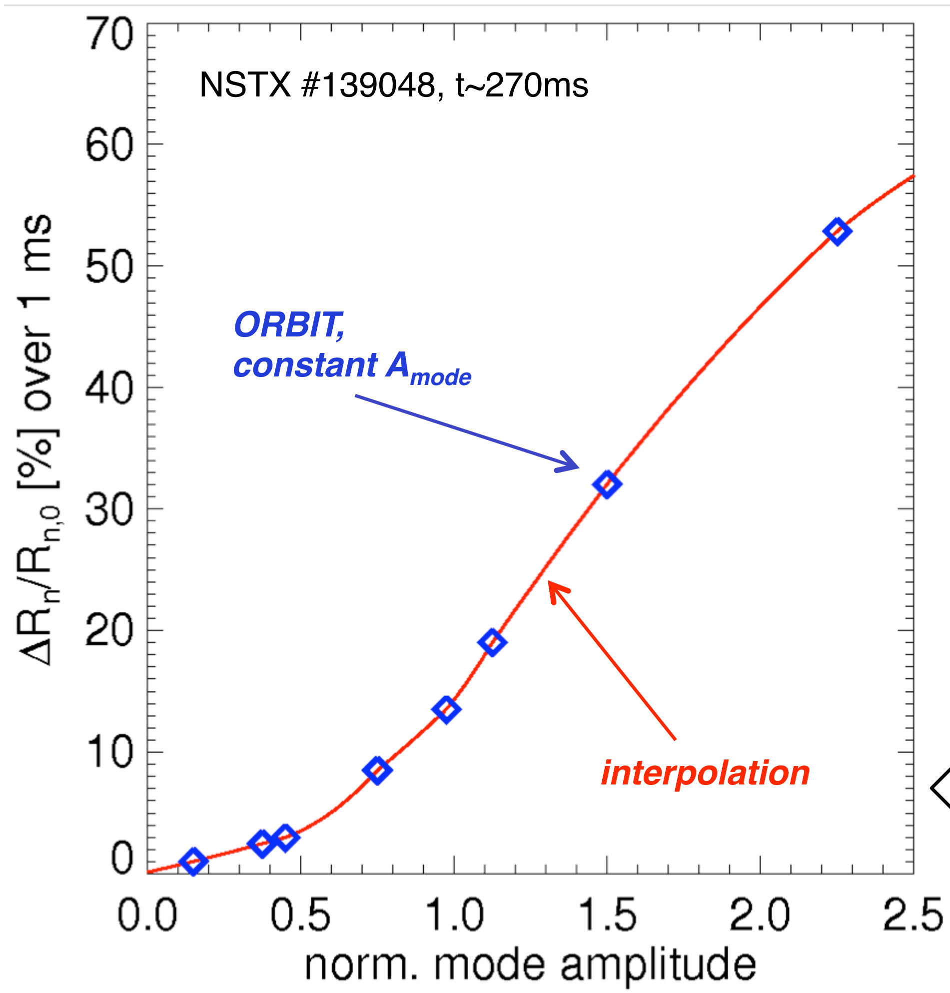


Energy and P_ζ steps assumed to scale linearly with mode amplitude

- Roughly consistent with ORBIT simulations

Scaling factor $A_{mode}(t)$ is obtained from measurements, *observables* such as neutron rate + modeling

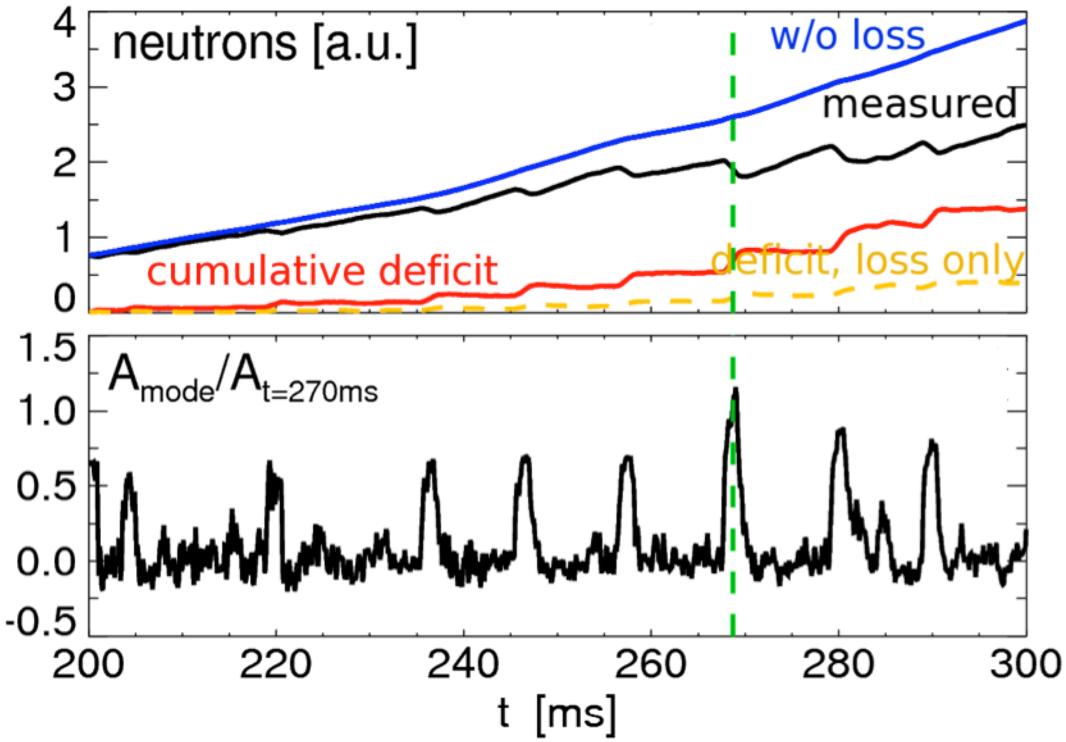
- Best option: use experimental data (e.g. reflectometers, ECE)
- If no mode data available, A_{mode} can be estimated based on other measured quantities



- **Example: use measured neutron rate**
  - Compute ideal modes through NOVA
  - Rescale relative amplitudes from NOVA according to magnetics
  - Rescale total amplitude based on computed neutron drop from ORBIT
  - Scan mode amplitude w.r.t. experimental one, $A_{mode}=1$: get table
  - Build $A_{mode}(t)$ from neutrons vs. time, table look-up

Example: $A_{mode}(t)$ computed from measured neutron rate or Mirnov coils' signal

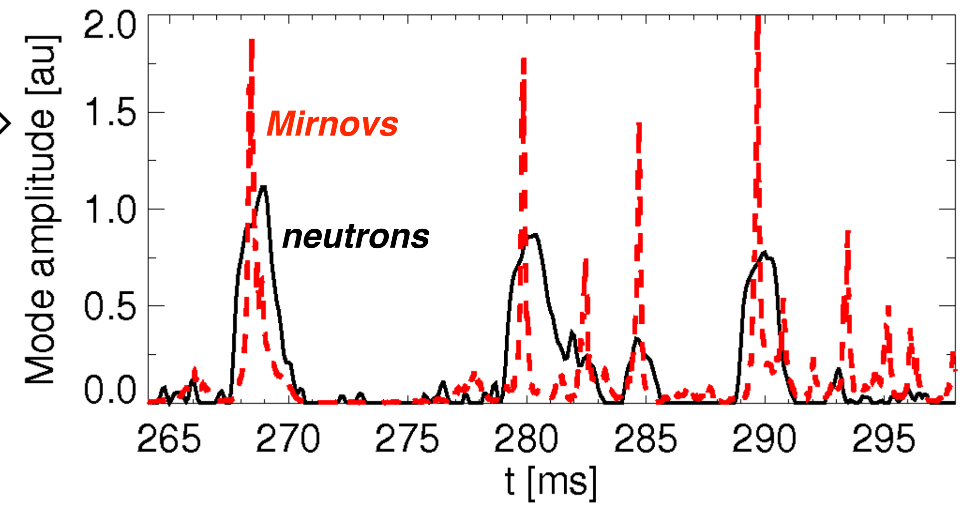
Get $A(t)$ from measured neutrons+table look-up:



– Compute fractional R_n drops vs. time

– Use table R_n vs A_{mode} to find corresponding (normalized) mode amplitude

–Comparison w/ $A_{mode}(t)$ from Mirnov coils



–Do different waveforms lead to differences in fast ion evolution?

Not on relevant time scales > 1ms

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- New ‘kick model’: basic ideas
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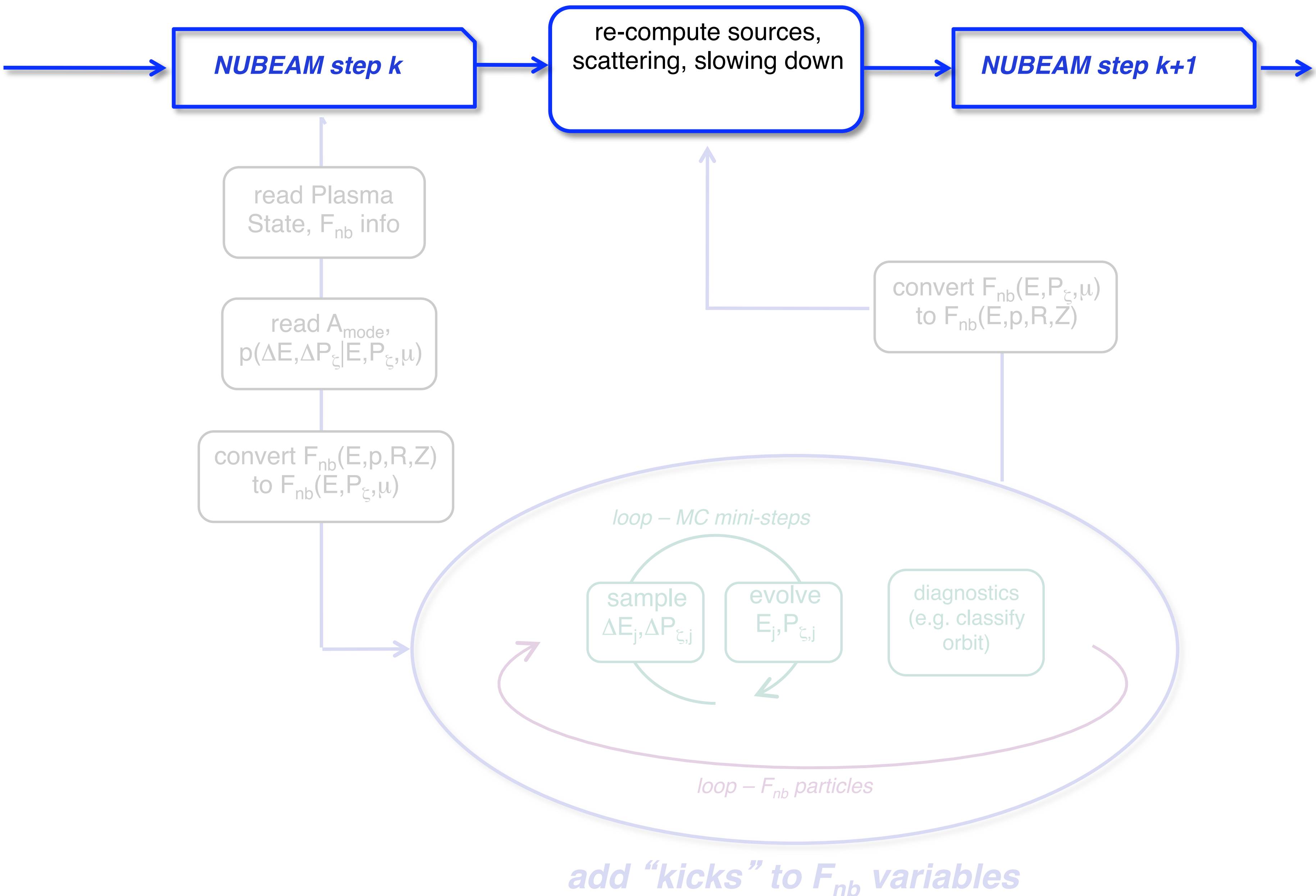
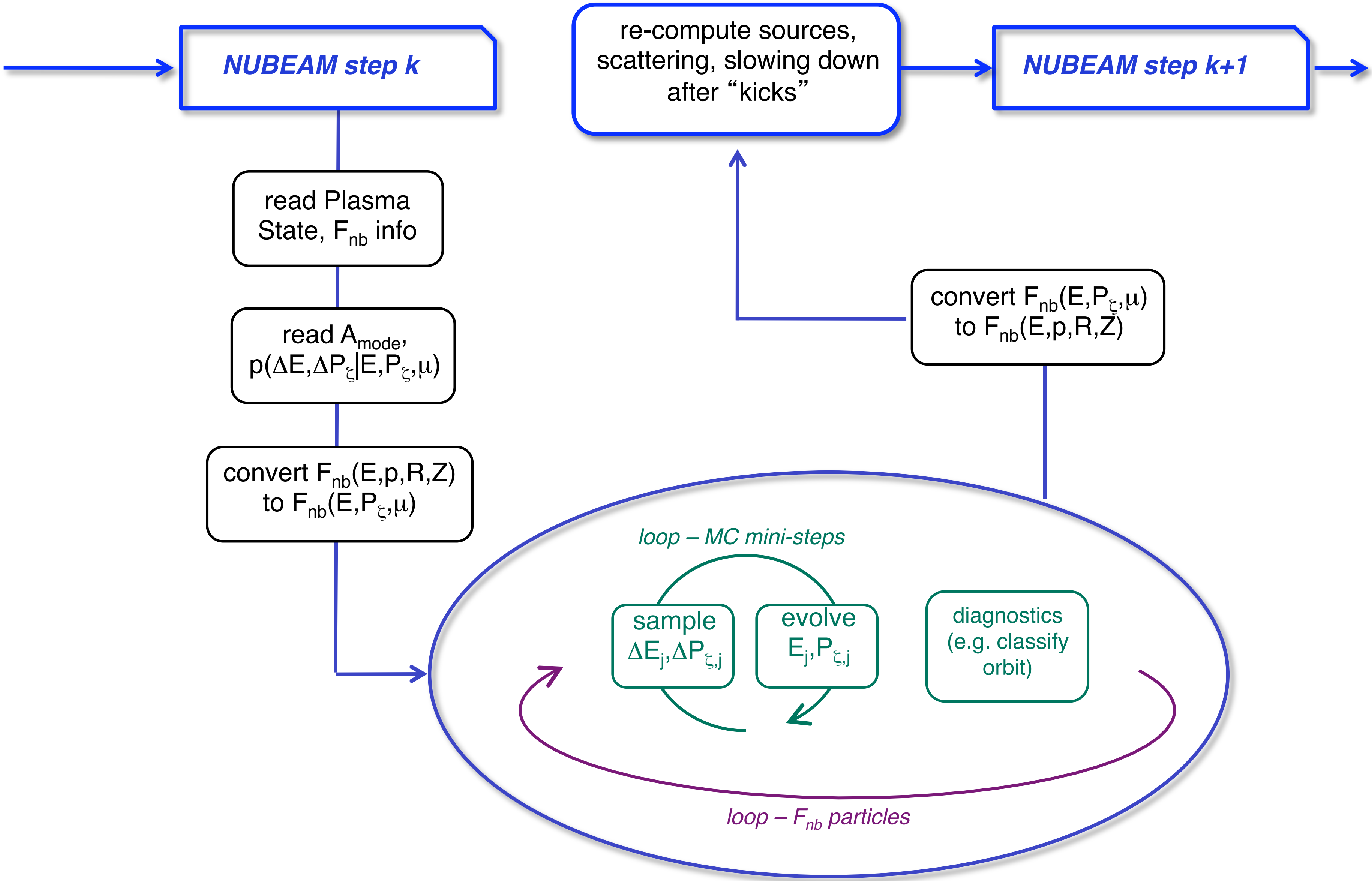
Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP



Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP



add "kicks" to F_{nb} variables

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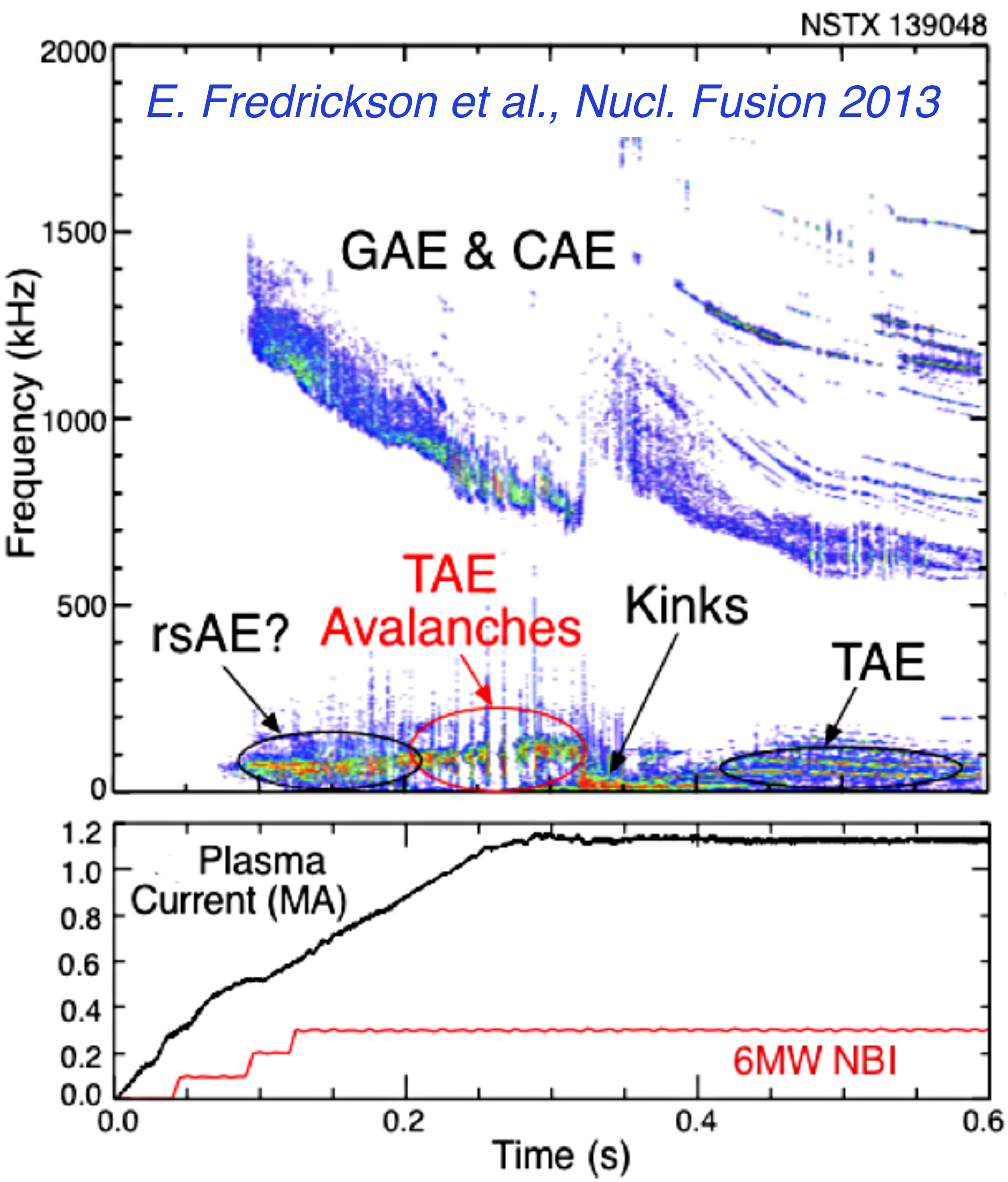
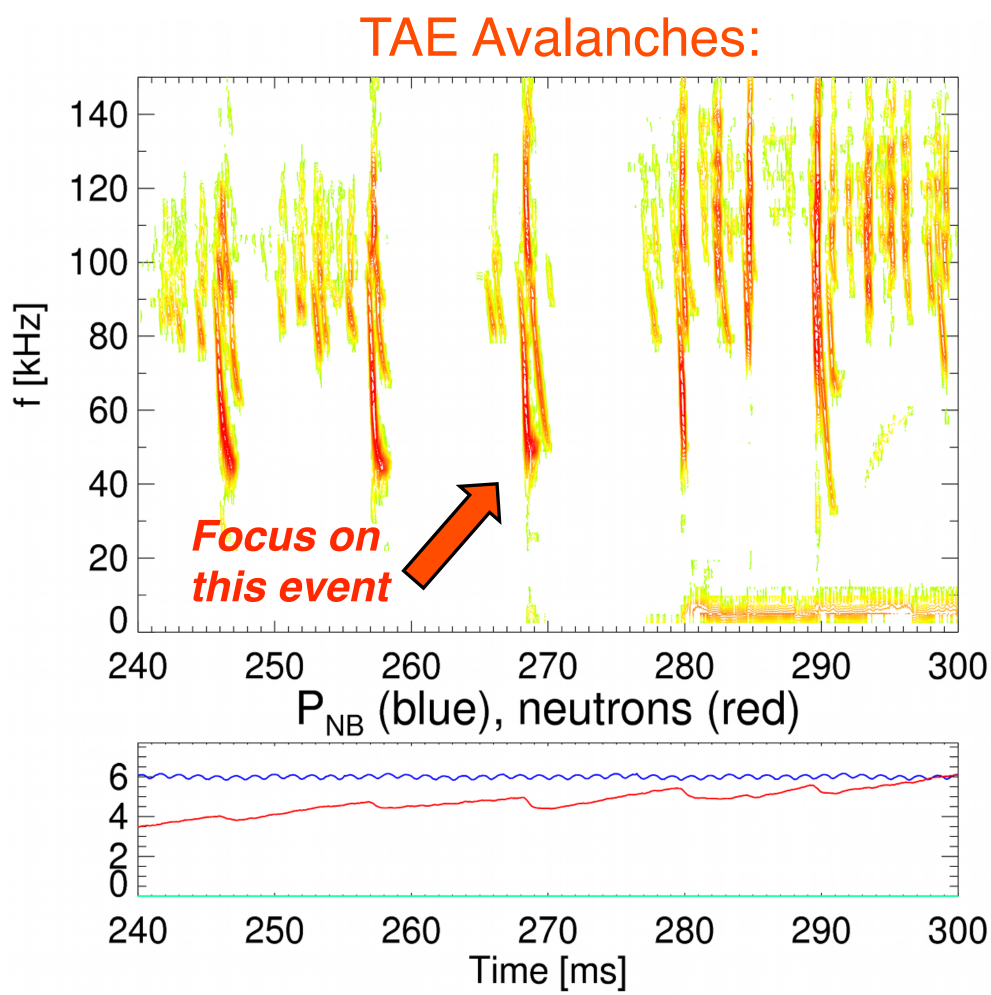
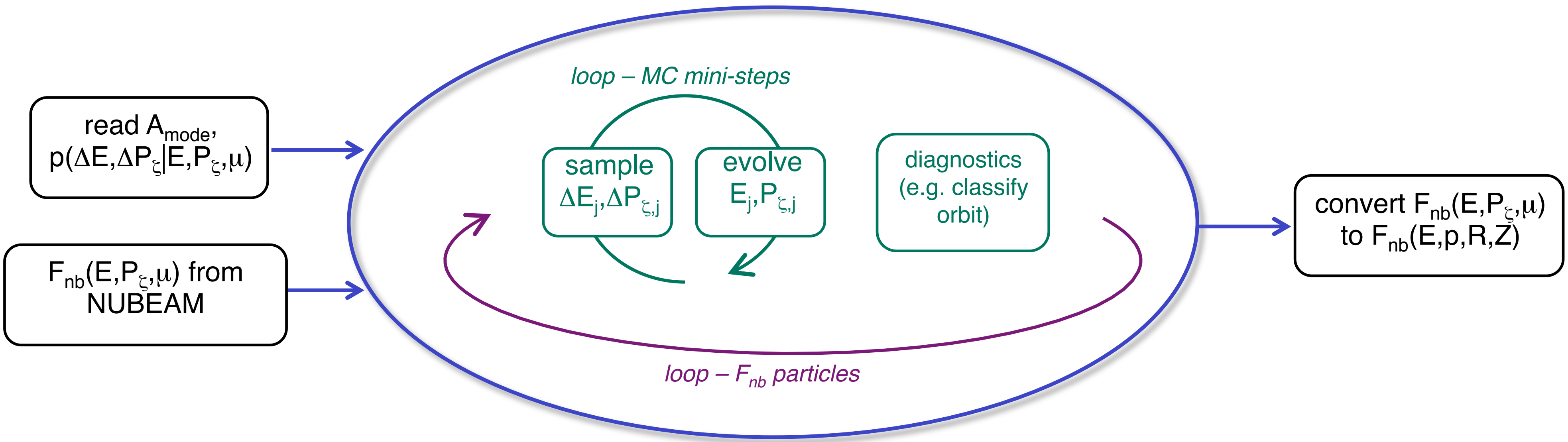
Experimental scenario for initial validation: NSTX H-mode plasma with bursts of TAE activity



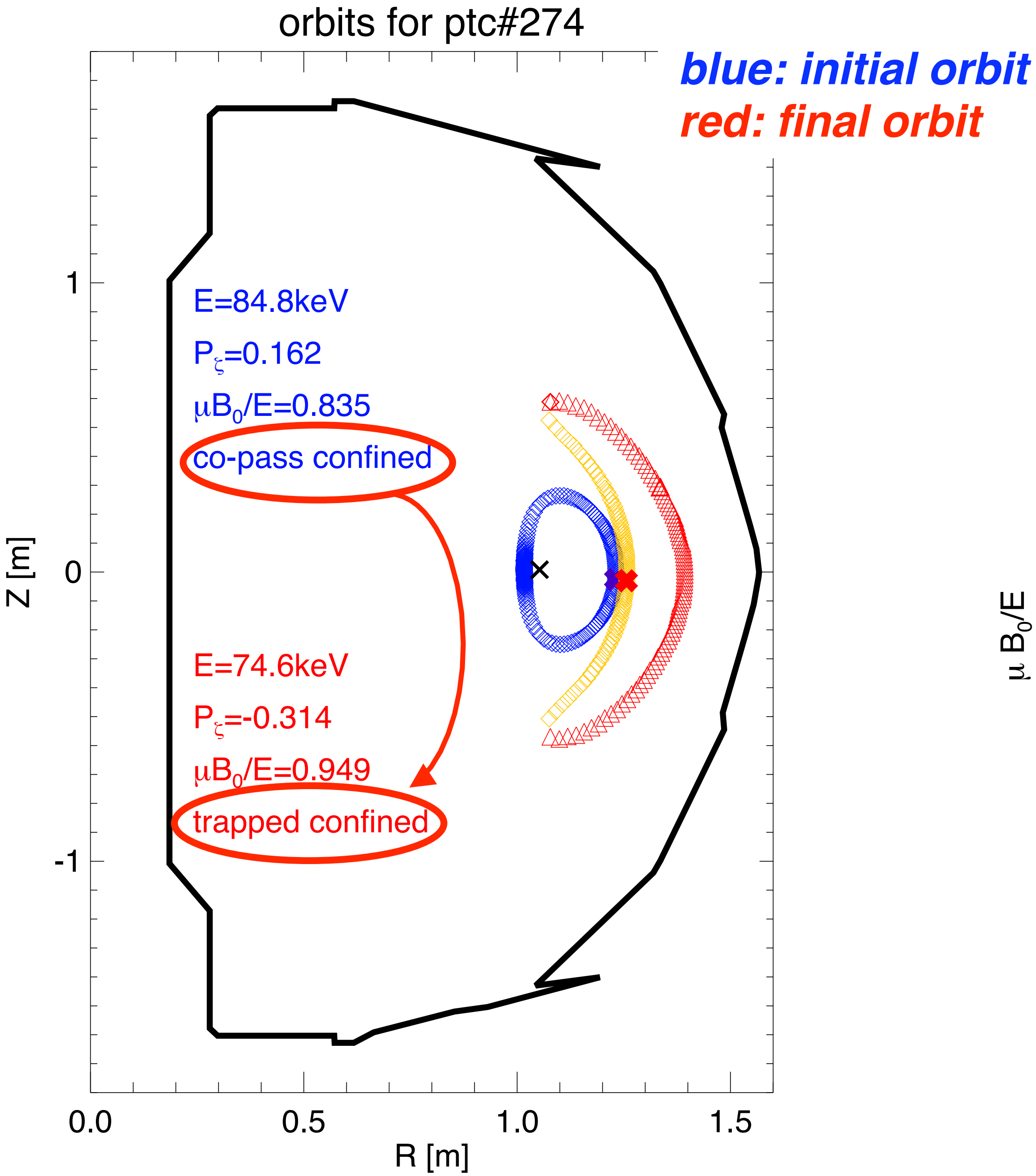


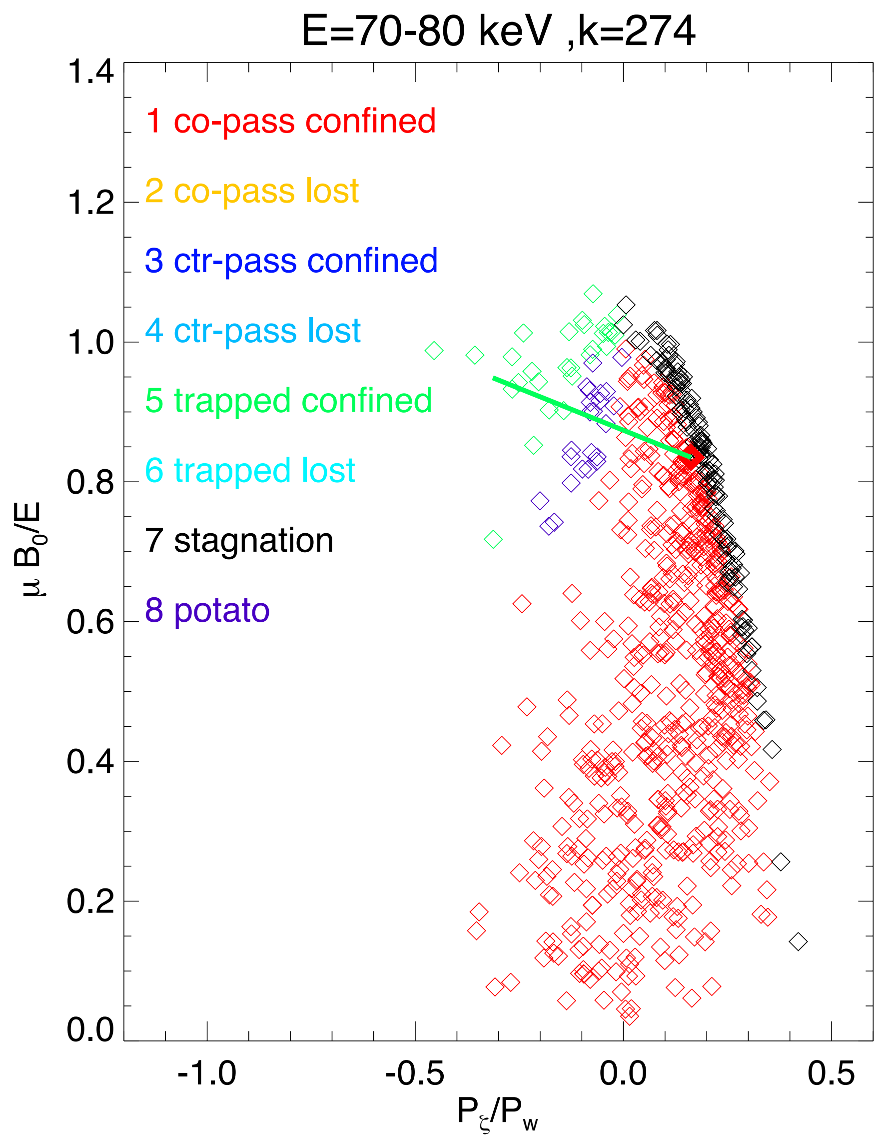
First tests: evolve F_{nb} from NUBEAM assuming fixed background, compare with ORBIT



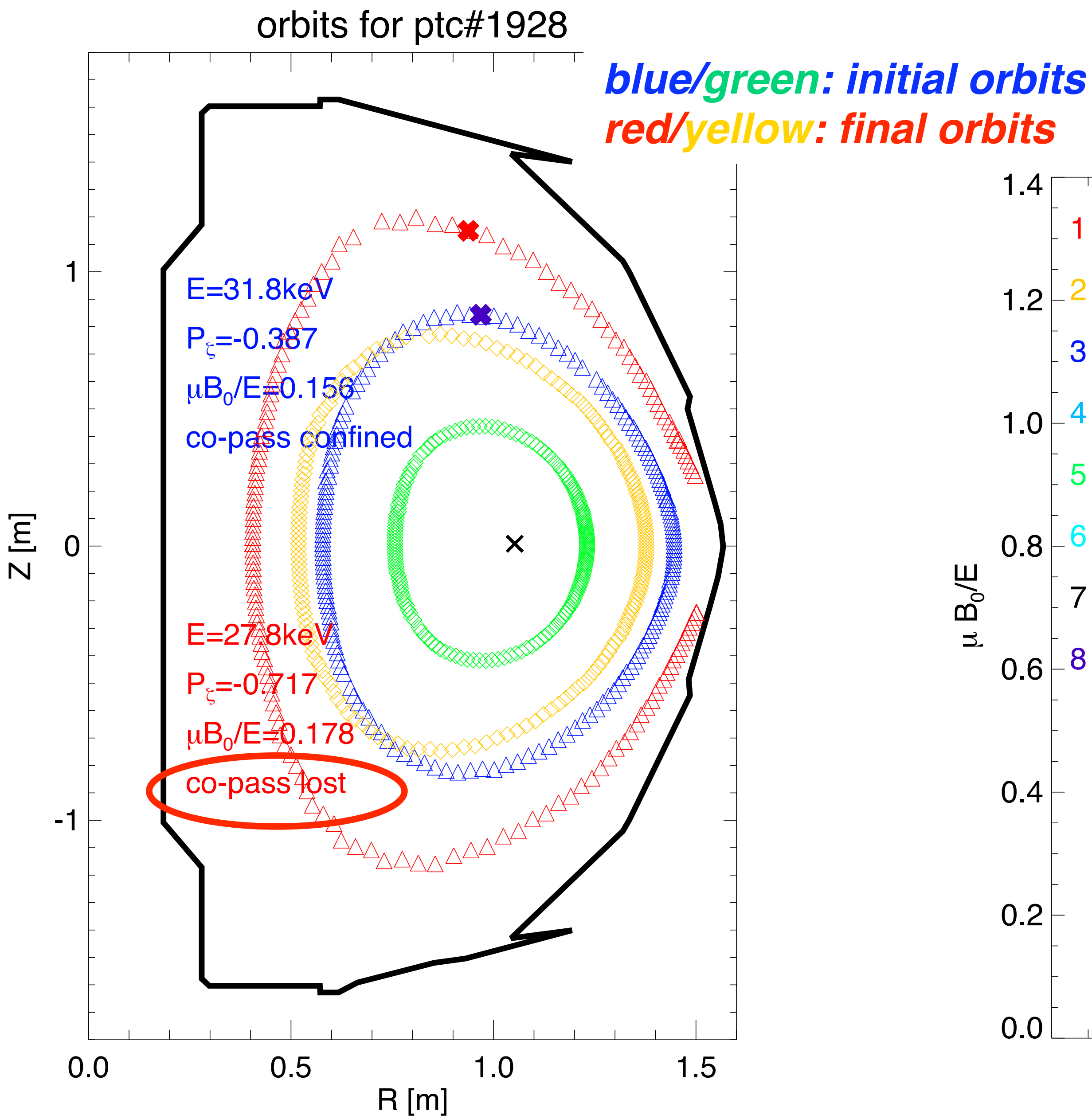
add "kicks" to F_{nb} variables

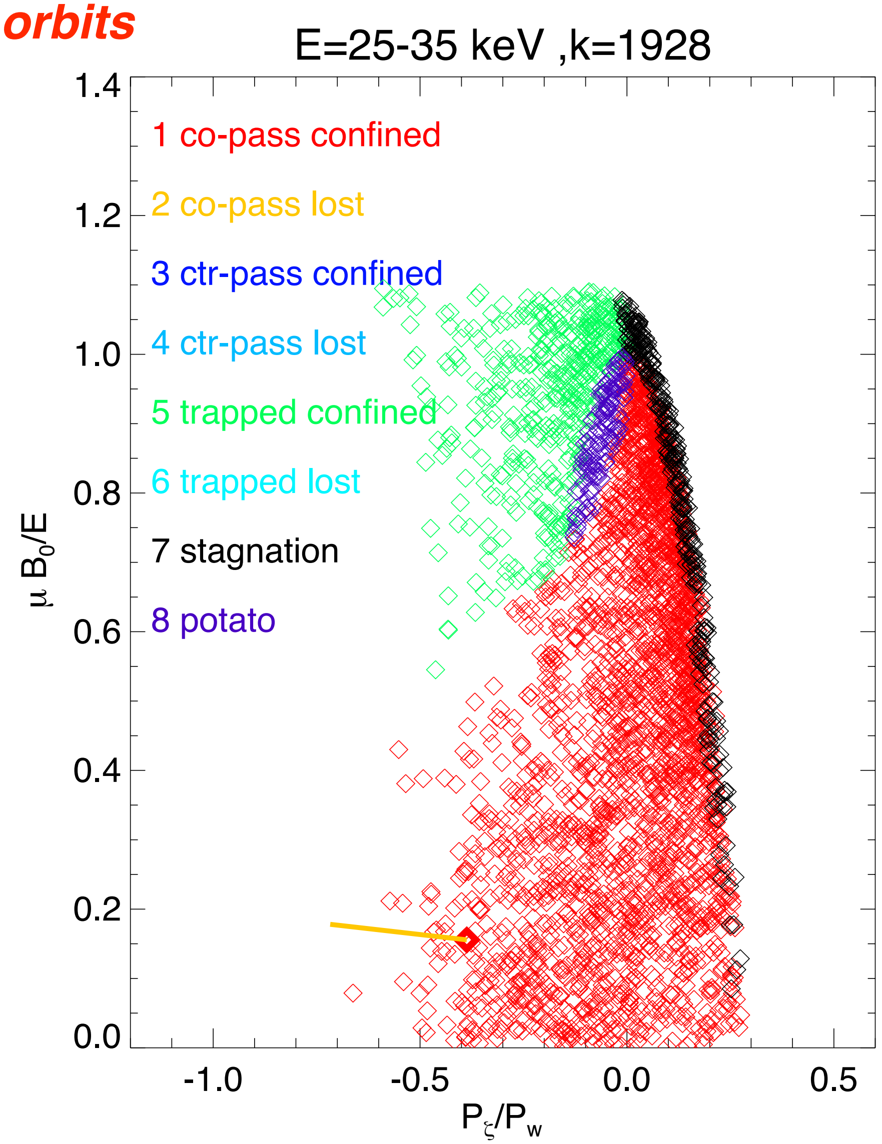
Change in orbit type observed for some particles: fast ions *are* kicked around in phase space by TAEs



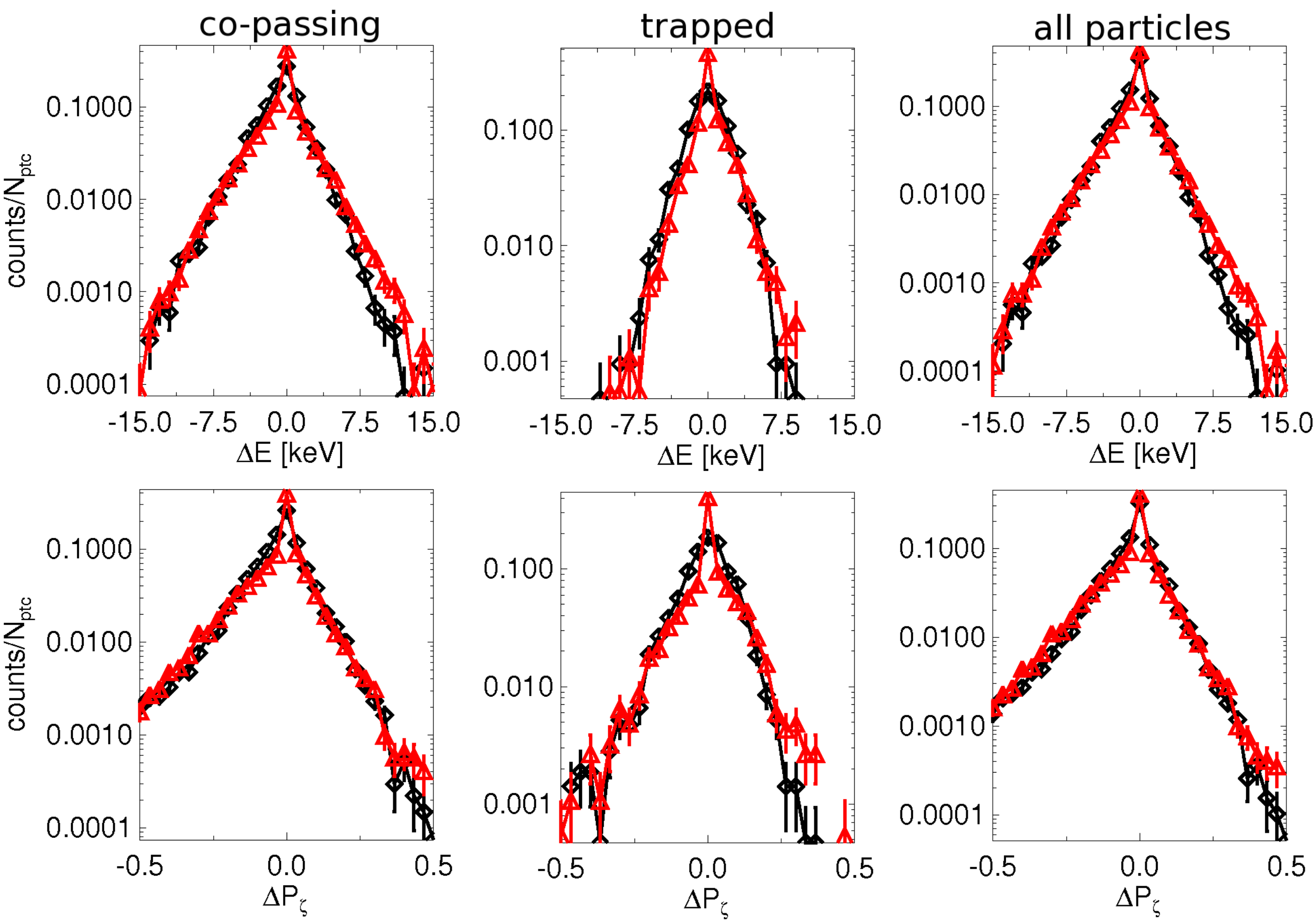


Only a few particles (~0.1%) are actually lost for the cases examined here; redistribution dominates





Good agreement with ORBIT is preserved when evolving F_{nb} over 5 ms, typical macro-step of NUBEAM



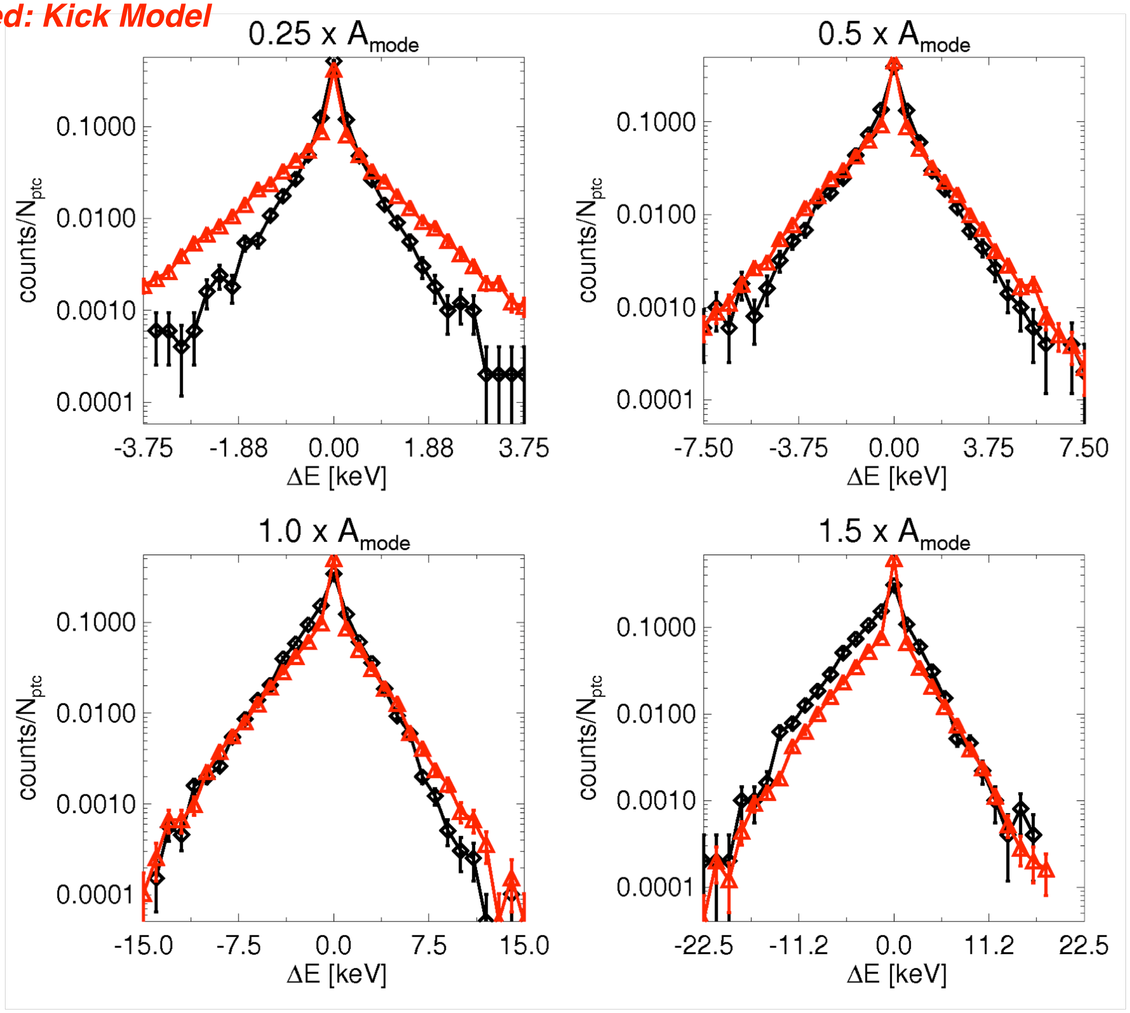
black: ORBIT

red: Kick Model

Reconstruction is satisfactory for different classes:
co-, trapped (, counter- not present in *this* input F_{nb})

Tests assuming different mode amplitudes are satisfactory, though not perfect...

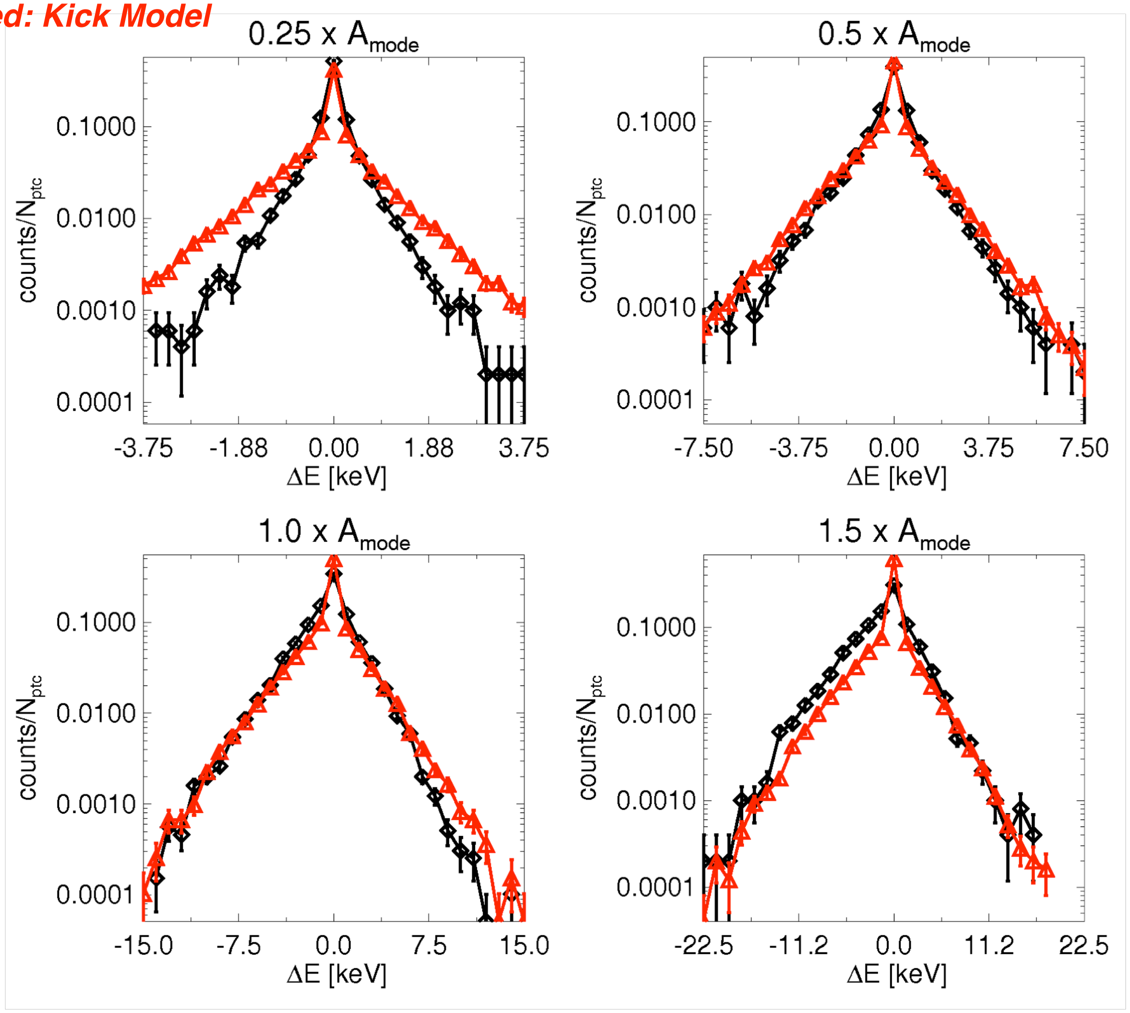
black: ORBIT
red: Kick Model

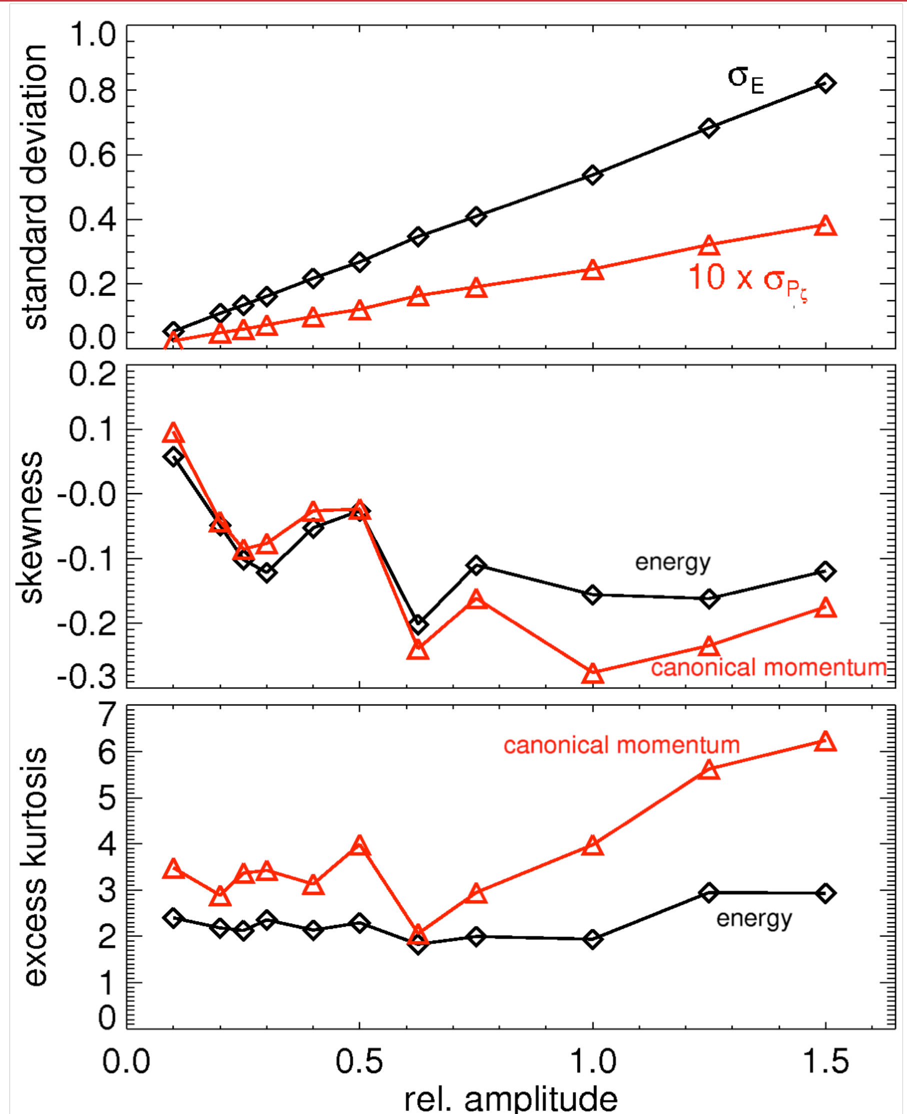


- Reconstruction vs. amplitude is satisfactory

Tests assuming different mode amplitudes are satisfactory, though not perfect...

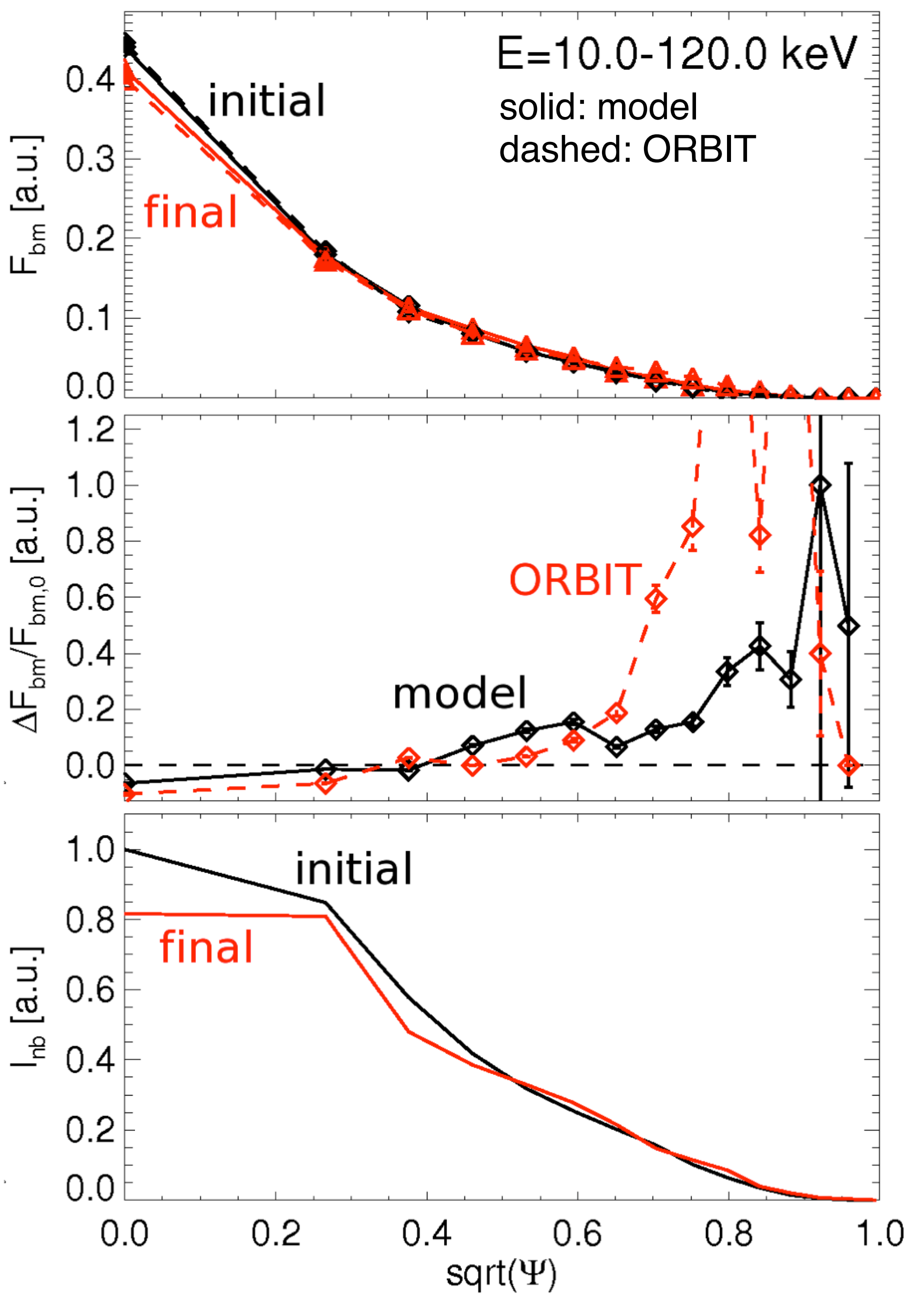
black: ORBIT
red: Kick Model





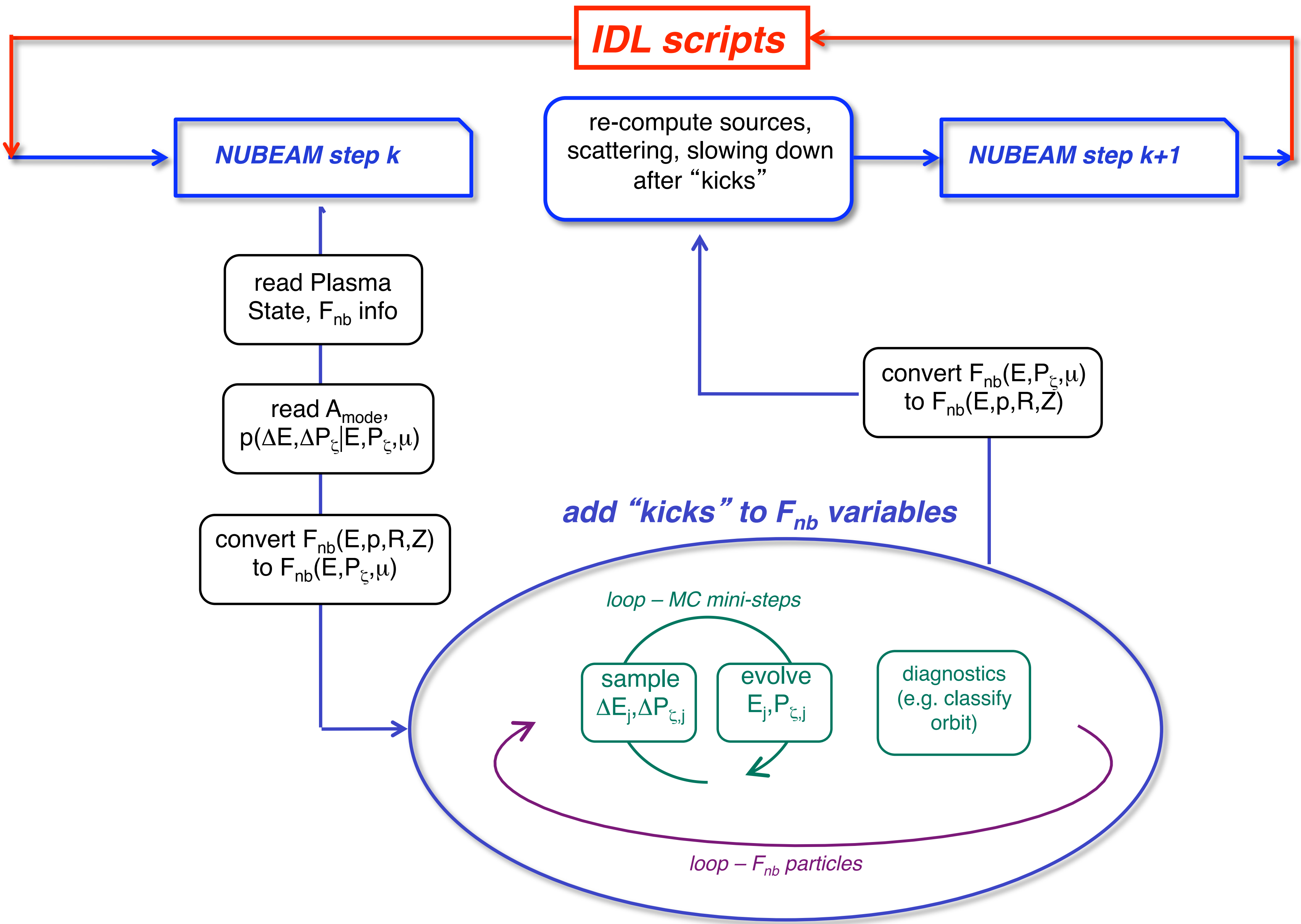
- Reconstruction vs. amplitude is satisfactory
- **Differences ascribed to simplifications in $p(\Delta E,\Delta P_\zeta)$ scaling with A_{mode}: shape *does* change**

F_{nb} after 5ms shows fast ion redistribution, only modest losses; NB driven "current" also affected.

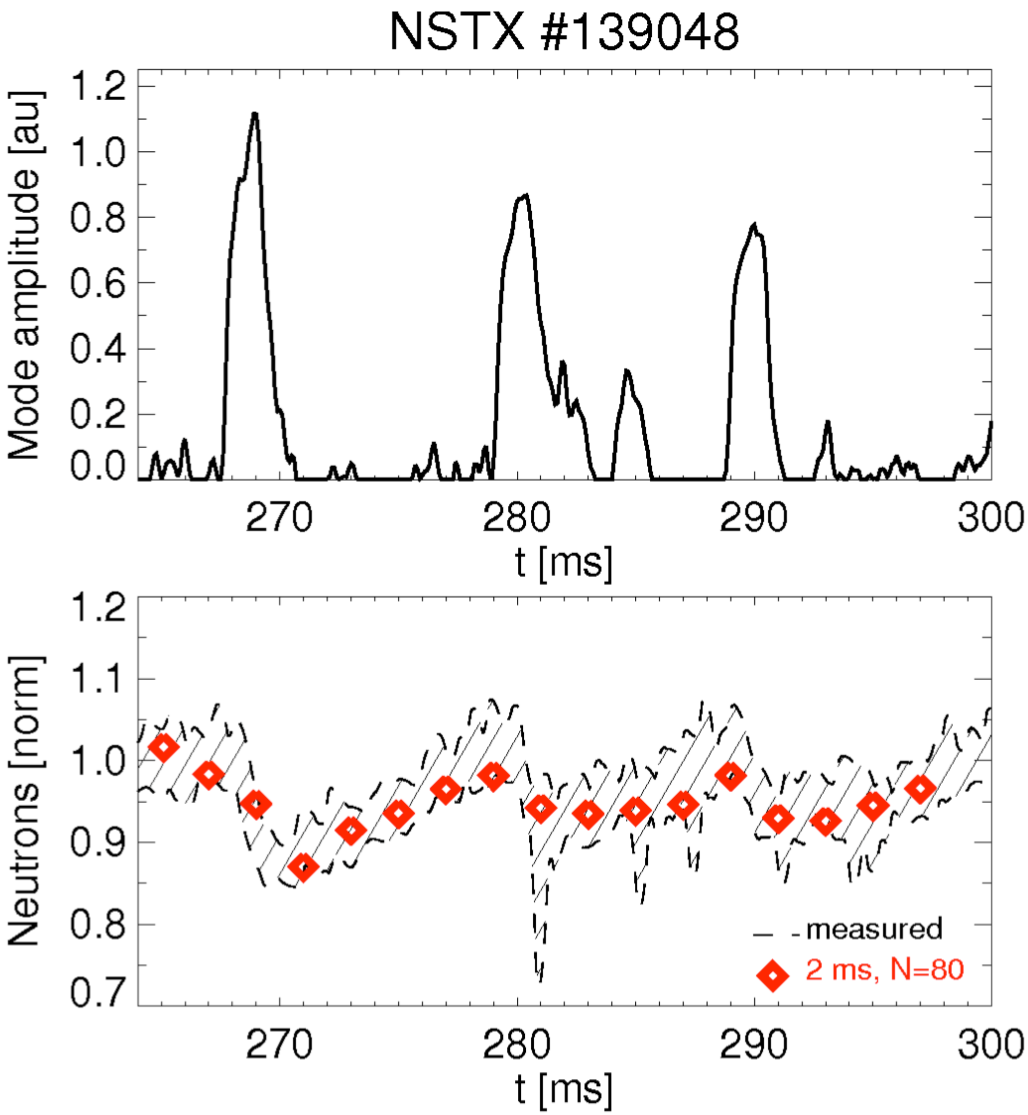


- Compare full ORBIT simulation with reconstructions from reduced model
- F_{nb} drops in the core, fast ions redistributed to larger radii
- Define rough proxy for NB-driven (parallel) current, $I_{nb} \sim F_{nb}$ p v
  - Larger (relative) variations for I_{nb} than for F_{nb} in the core
  - > **Need NUBEAM/TRANSP for more quantitative calculations of I_{nb}**

Next step: use stand-alone NUBEAM and IDL scripts to simulate F_{nb} evolution for >>5 ms



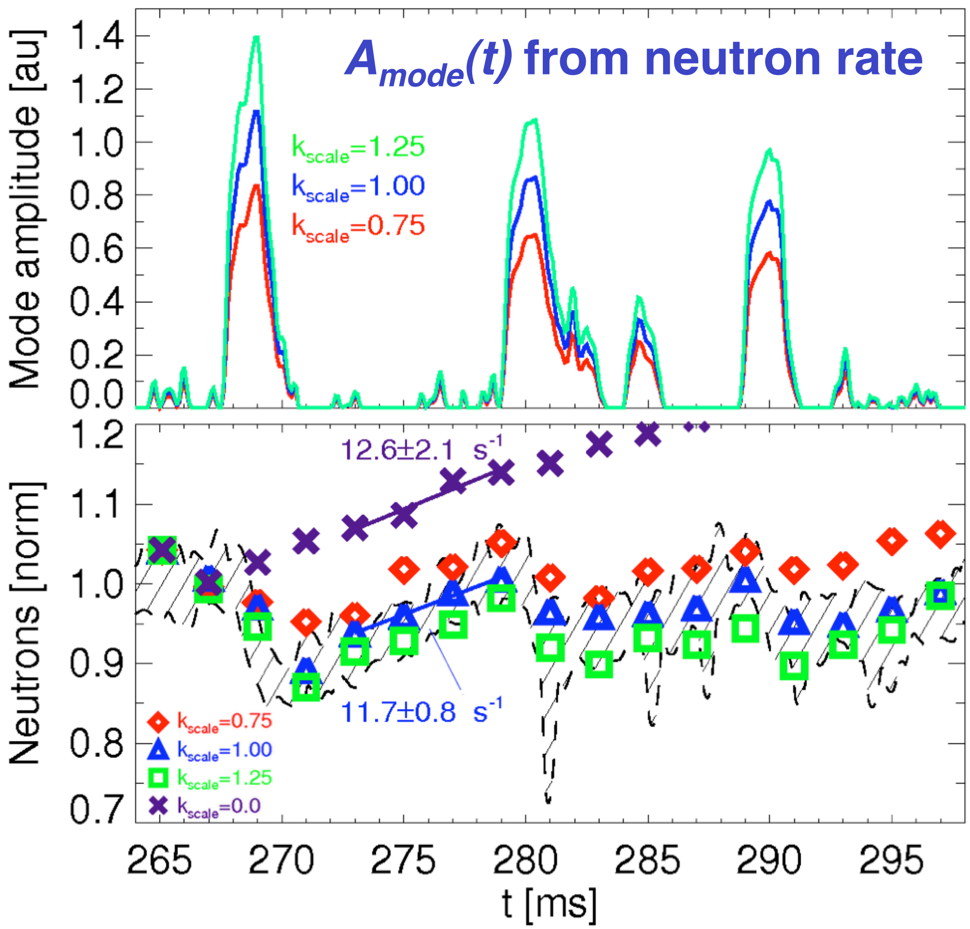
Reduced model reproduces neutron evolution for nominal $A_{mode}(t)$ from neutrons, Mirnovs



- Initial comparison of model predictions w/ experimental neutron rate
- Use stand-alone version of NUBEAM, iterate with reduced model (IDL scripts)
- Background plasma is fixed
  - Normalize exp. neutron rate to central ion density vs. time
  - Normalize all neutron rates at t=267 ms (before first burst)

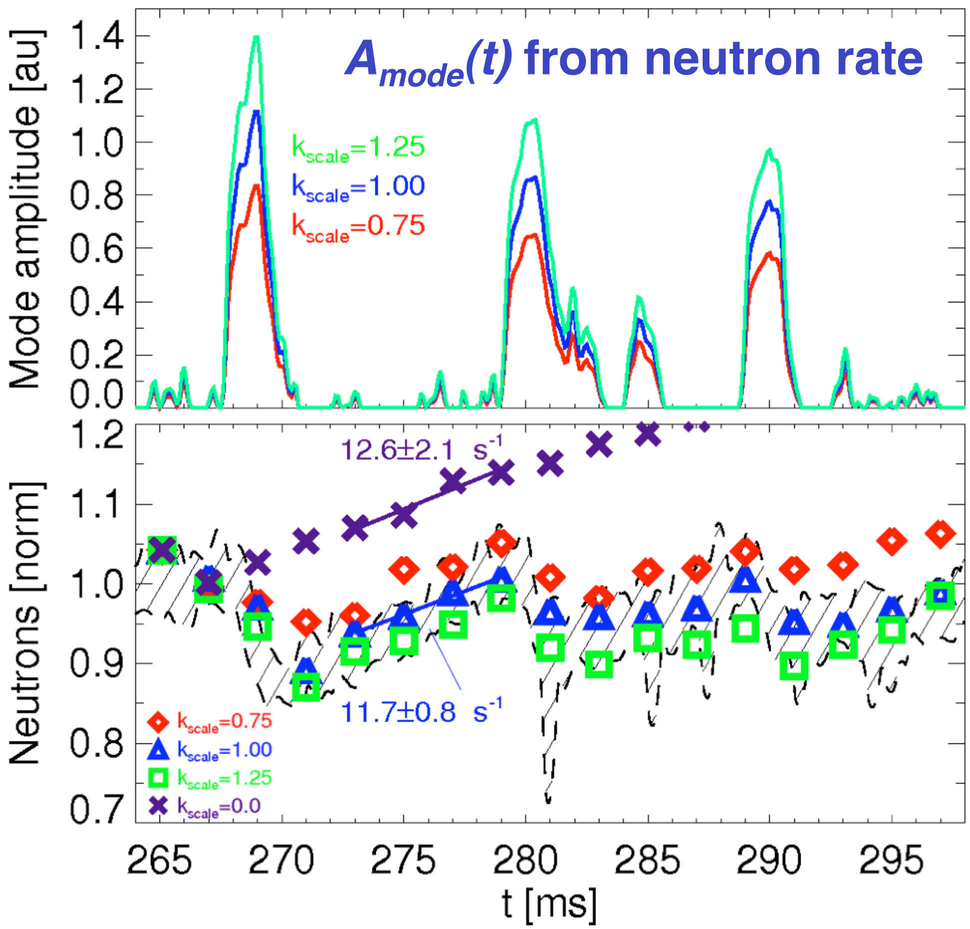
Satisfactory agreement for $A_{mode}(t)$ from neutron rate, Mirnovs

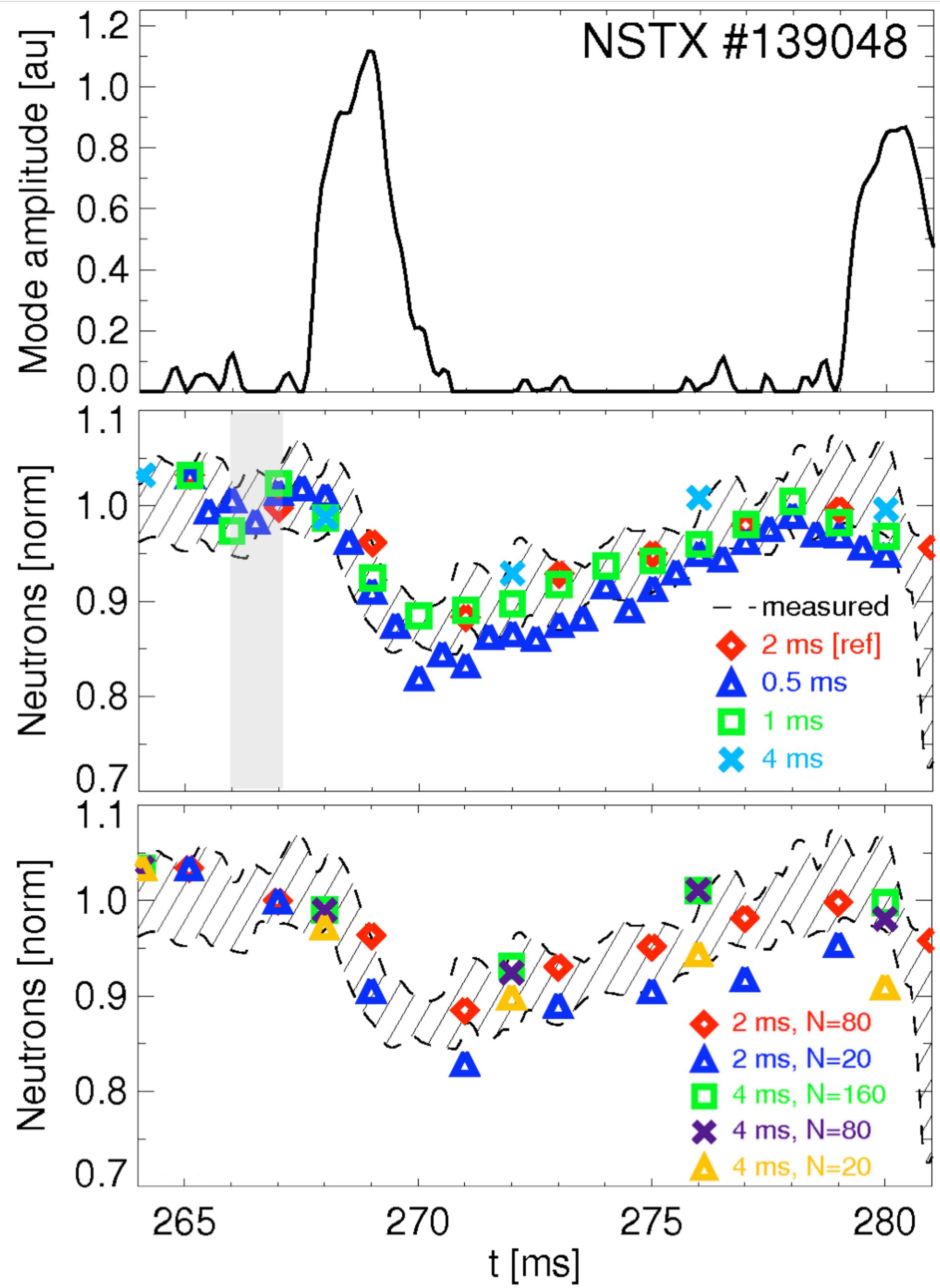
Results are sensitive to input mode amplitude; time steps must be chosen to satisfy "statistical" approach



- Typical time scales:
  - Slowing down: 15-30ms
  - Collisions: 2-5ms
  - AEs: 0.1-2ms

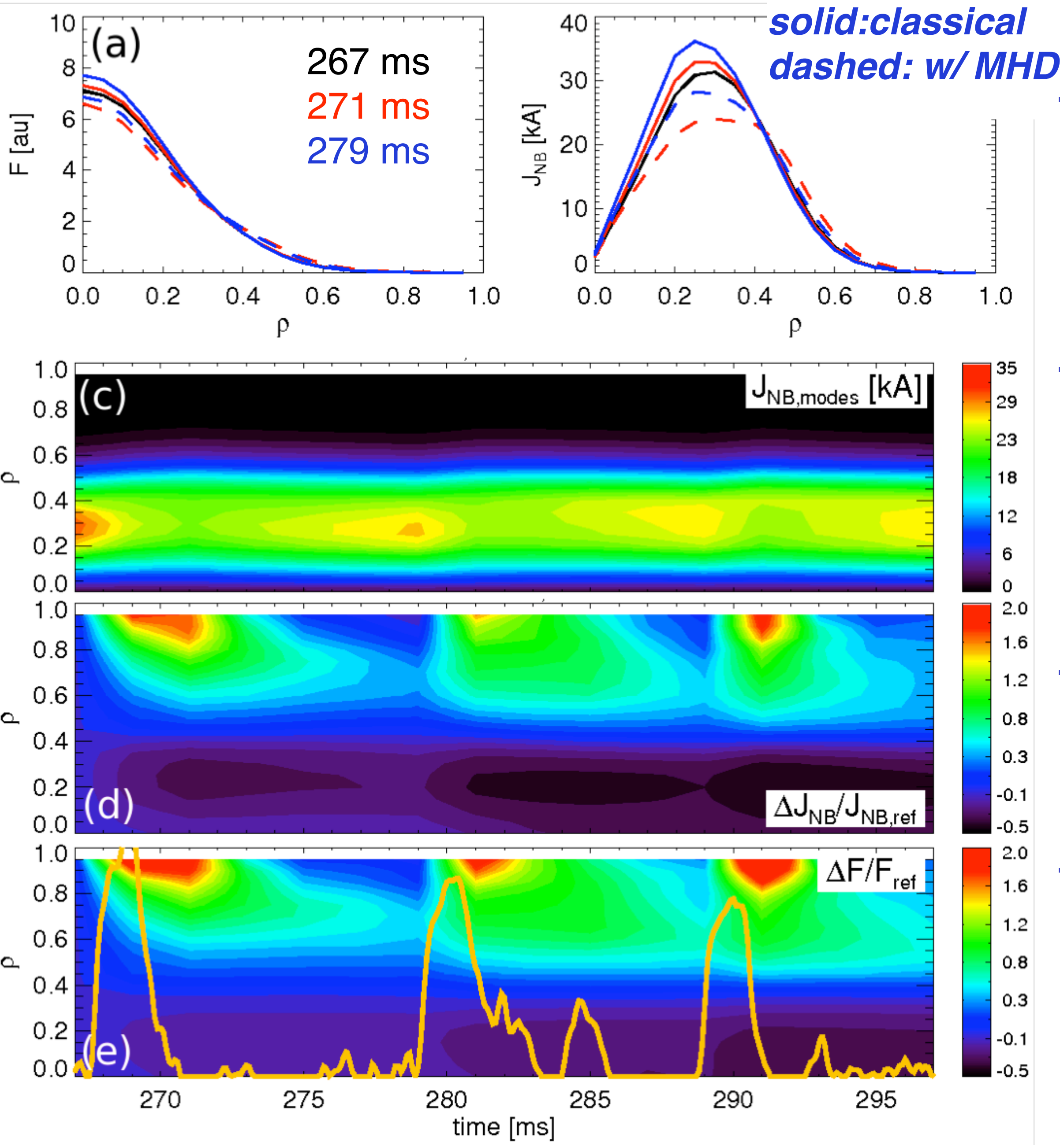
Results are sensitive to input mode amplitude; time steps must be chosen to satisfy "statistical" approach





- Typical time scales:
  - Slowing down: 15-30ms
  - Collisions: 2-5ms
  - AEs: 0.1-2ms
- N: number of micro-steps for MC particle evolution, duration 25μs

Reduced model + NUBEAM computes "measurable" fast ion redistribution; NB-driven current strongly affected



- Initial tests show fast ion redistribution induced by each TAE avalanche
- Clear effects on NB-driven current, J_{nb}
  - Stronger effect than on F_{nb}
- AE effects persist on slowing down time scales
- Constant NB injection counteracts F_{nb}, J_{nb} depletion

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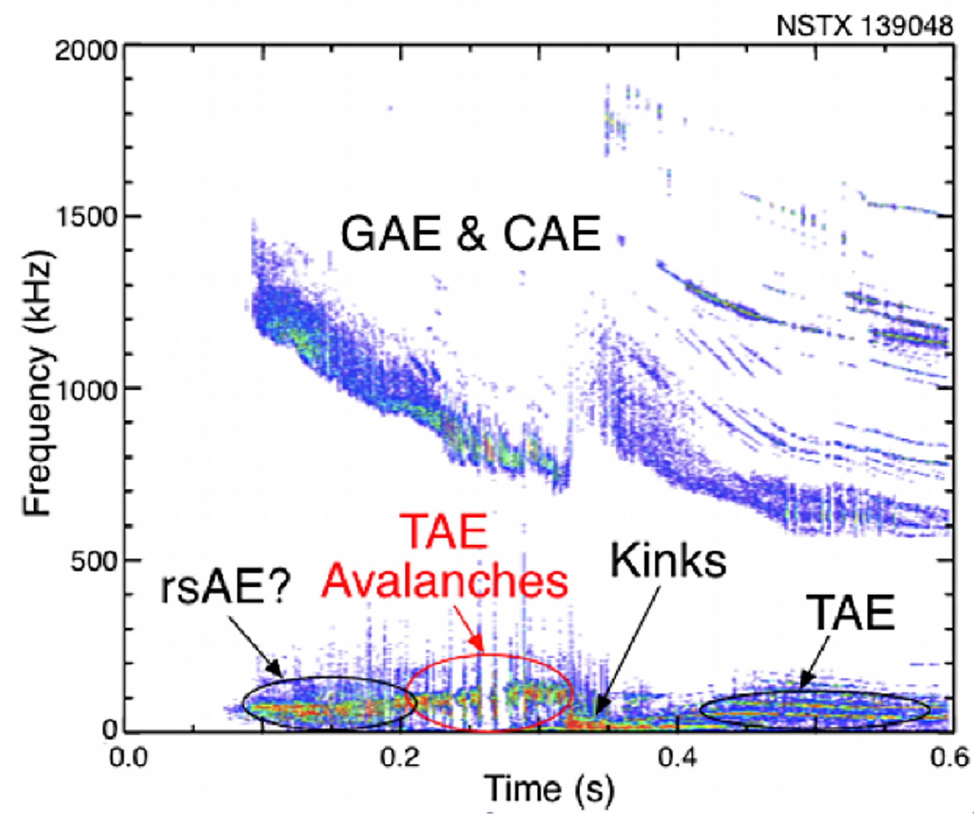
The model has been recently improved to account for multiple "classes" of instabilities at a given time

Scenarios with more than one type of modes are quite common:



- This general case is of great practical interest
  - More realistic F_{nb} evolution when modes have comparable amplitude
  - Can account for different equilibria at different stages of the discharge
  - Can mock-up scenarios such as 'fast ion channeling'

> Each "class" is modeled by its $p_k(\Delta E, \Delta P_\zeta)$, weighted by $A_{mode,k}$
  – Instantaneous $p(\Delta E, \Delta P_\zeta) = \Sigma_k A_{mode,k} \times p_k(\Delta E, \Delta P_\zeta)$

Can the model be used in 'predictive' mode?

- As it is, the model is OK to analyze 'real' discharges
  - Need mode structure to calculate $p(\Delta E, \Delta P_\zeta)$, e.g. from NOVA-K and reflectometers' data + ORBIT
  - Need data (Mirnovs, neutrons, etc.) to infer $A_{modes}(t)$
- Two possibilities for 'predictive' runs:
  - Find reasonable guess for unstable modes (e.g. NOVA-K)
  - Explore different scenarios w/ scan of $A_{modes}(t)$: weak *AE activity, bursts/avalanches, etc.

  or:

  - Have an additional module to compute $A_{modes}(t)$ self-consistently?
  - Still requires mode structure, probably estimates for γ_{drive}, γ_{damp}
  - Let $A_{modes}(t)$ evolve according to F_{nb} evolution – but how?
  - *Couple to other reduced models, e.g. 1.5D Quasi-Linear?*

Summary

- Test algorithm for reduced fast ion transport model developed, being verified
  - Results compared with full ORBIT runs
  - Confirmed validity of approach for practical case

- Successful preliminary tests for NSTX case with TAE modes (*avalanches*)
  - Shot#139048, t~265-300 ms:H-mode avalanches
  - Strong redistribution of fast ions observed
  - Modest losses, consistent with previous detailed modeling
  - NB-driven current J_{nb} is affected, too

> **Implementation in NUBEAM under way**

  > **Extensive Verification&Validation planned (multi-machine)**

  > **Identify issues, possible improvements to the model**

Backup

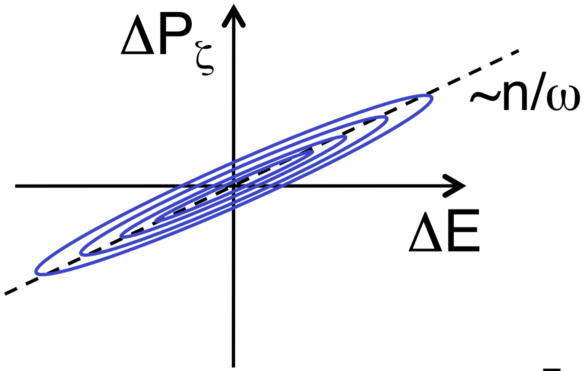
For low-frequency *AEs with $\omega \ll \omega_{ci}$ such as TAEs, magnetic moment μ is conserved (...but maybe it's not)

- *In this presentation, it is assumed that $\Delta\mu$=0*

- <u>However</u>: $\Delta\mu$=0 hypothesis can break down if
  - ρ_f ~ radial width of the modes
  - ρ_f ~ scale-length of equilibrium profiles
  - ➪ *Both conditions are likely to be met in spherical tokamaks (e.g. NSTX)*

➪ **Proposed model can be generalized to cases where μ is *not* conserved**
  - Also important for ω_{ci}-range instabilities: GAE/CAEs

Example for single-resonance case: analytical *probability distribution function*



For each *bin* in (E,P_ζ,μ), steps in ΔE,ΔP_ζ can be approximated by a bivariate $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A)$

$$p = p_0\, e^{-\frac{1}{2(1-\rho)}\left[\frac{(\Delta E - \Delta E_0)^2}{\sigma_E^2} + \frac{(\Delta P_\zeta - \Delta P_{\zeta 0})^2}{\sigma_{P_\zeta}^2} - 2\rho \frac{(\Delta E - \Delta E_0)(\Delta P_\zeta - \Delta P_{\zeta 0})}{\sigma_E\, \sigma_{P_\zeta}}\right]}$$

$$p_0 = \frac{1}{2\pi\, \sigma_E\, \sigma_{P_\zeta}\, \sqrt{1-\rho^2}}$$ *normalization*

$$\rho = \frac{< (\Delta E - \Delta E_0)(\Delta P_\zeta - \Delta P_\zeta 0) >}{\sigma_E\, \sigma_{P_\zeta}}$$ *correlation parameter,*

such that $$\Delta P_\zeta(\Delta E) = \Delta P_{\zeta 0} + sign(\rho) \times \frac{\sigma_{P_\zeta}}{\sigma_E} (\Delta E - \Delta E_0)$$

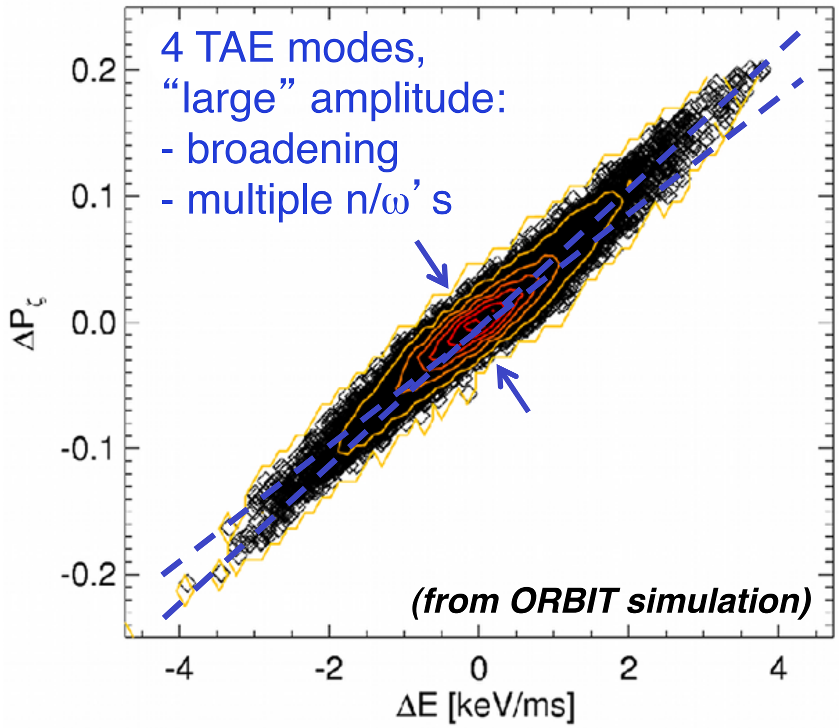
...and all parameters depend, in principle, on the mode amplitude A=A(t)

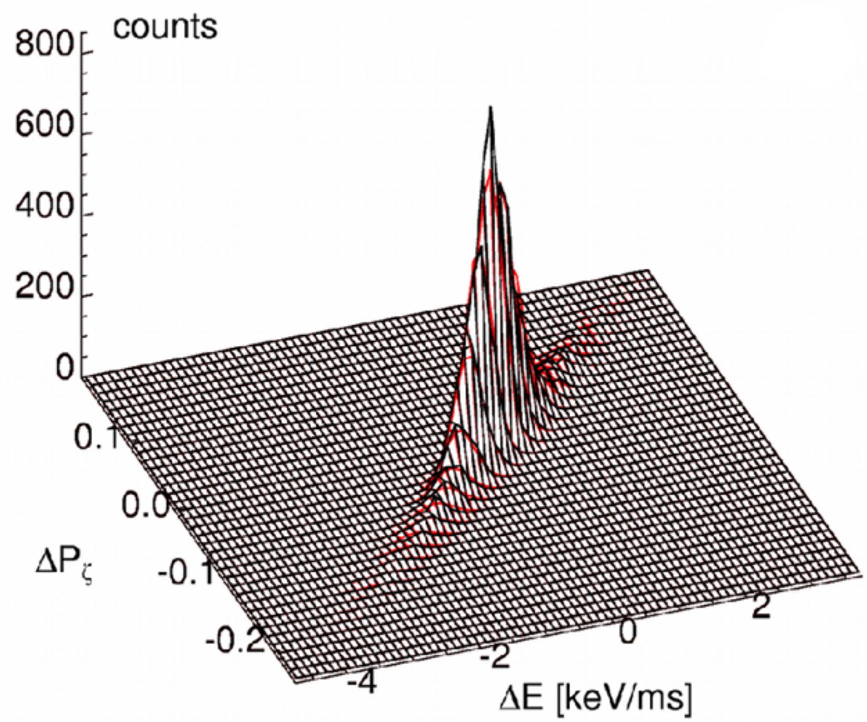
Single (isolated) resonances introduce fundamental constraints on particle's trajectory in (E, P_ζ, μ)

- From Hamiltonian formulation:

$$\omega P_\zeta - nE = const. \Longrightarrow \Delta P_\zeta / \Delta E = n/\omega$$

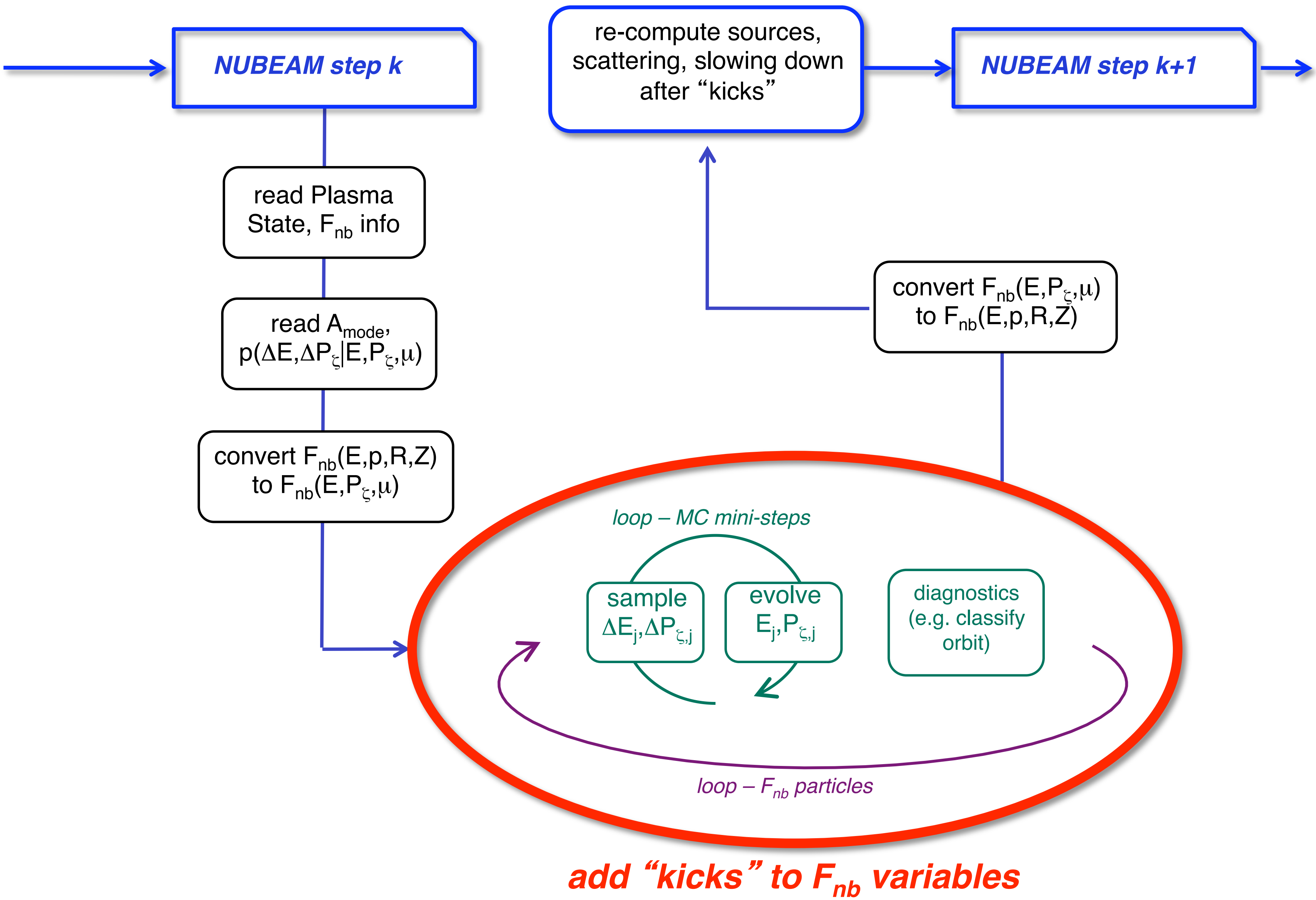
$\omega = 2\pi f$, mode frequency $\quad$ n, toroidal mode number





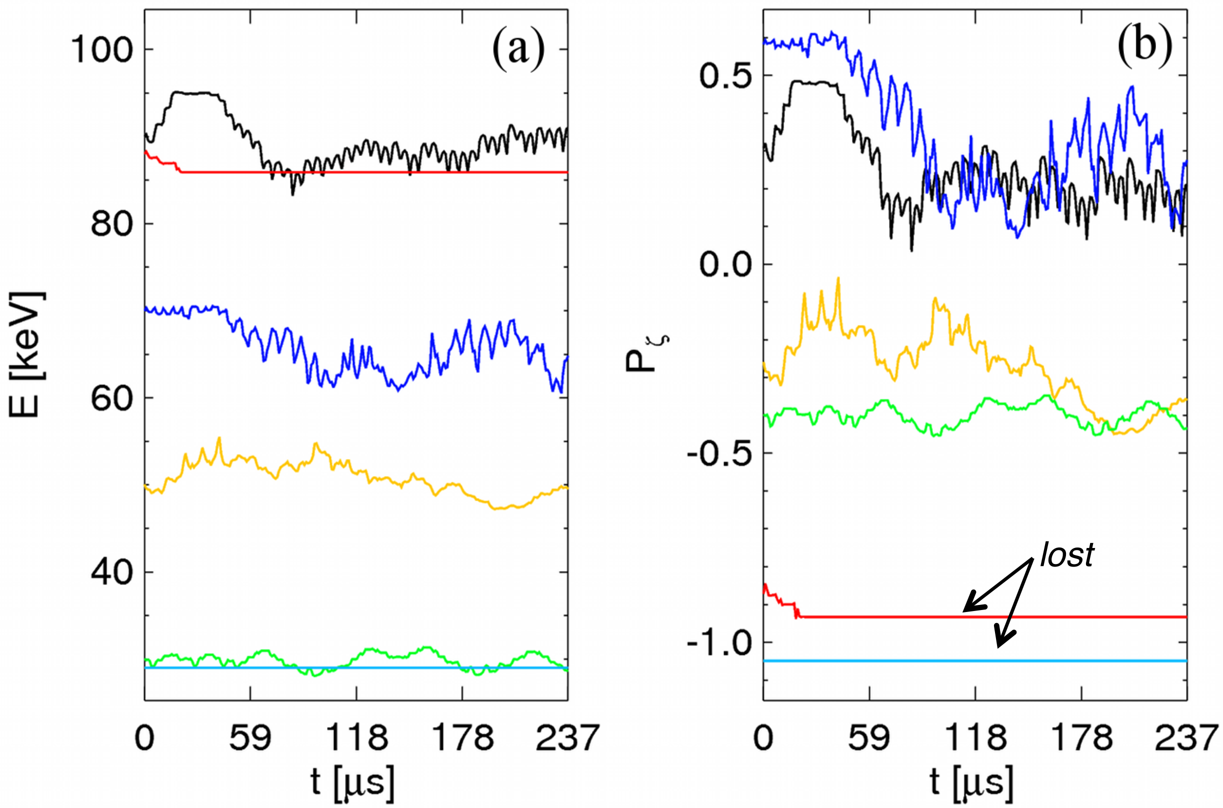
Presence of multiple modes/resonances distorts the 'ideal' (linear) relationship

Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP



Scheme to evolve F_{nb} takes into account (in a semi-empirical way) constraints of resonant interaction

- Particle's motion is characterized by different time-scales:
  - Fast oscillation in wave field – *neglected*
  - 'Jumps' $\Delta E, \Delta P_\zeta$ around instantaneous energy, P_ζ
  - Slow (secular) drift from initial energy, P_ζ



Putting all together

At each 'macroscopic' NUBEAM step:

Loop over particles (steps II–VII)

I. Re-normalize bins (P_ζ,E,μ) based on q-profile, fields, ...

II. Identify 'bin' in (P_ζ,E,μ) for current 'particle' (i.e. orbit)

III. Extract steps σ_E,$\sigma_{P\zeta}$ (σ_μ) from multivariate p(ΔE,ΔP_ζ,$\Delta\mu$)

IV. Compute sign $S_{r,i}$ from p(ΔE,ΔP_ζ): positive or negative kicks

V. Rescale steps based on A_{modes}(t)

VI. Advance E, P_ζ (μ):

$$\begin{cases} \overline{\Delta E}_i = S_{r,i} \times A_{mode}(t = \bar{t}) \times \sigma_{E,i} \\ \Longrightarrow E_i = E_i + \overline{\Delta E}_i \end{cases}$$

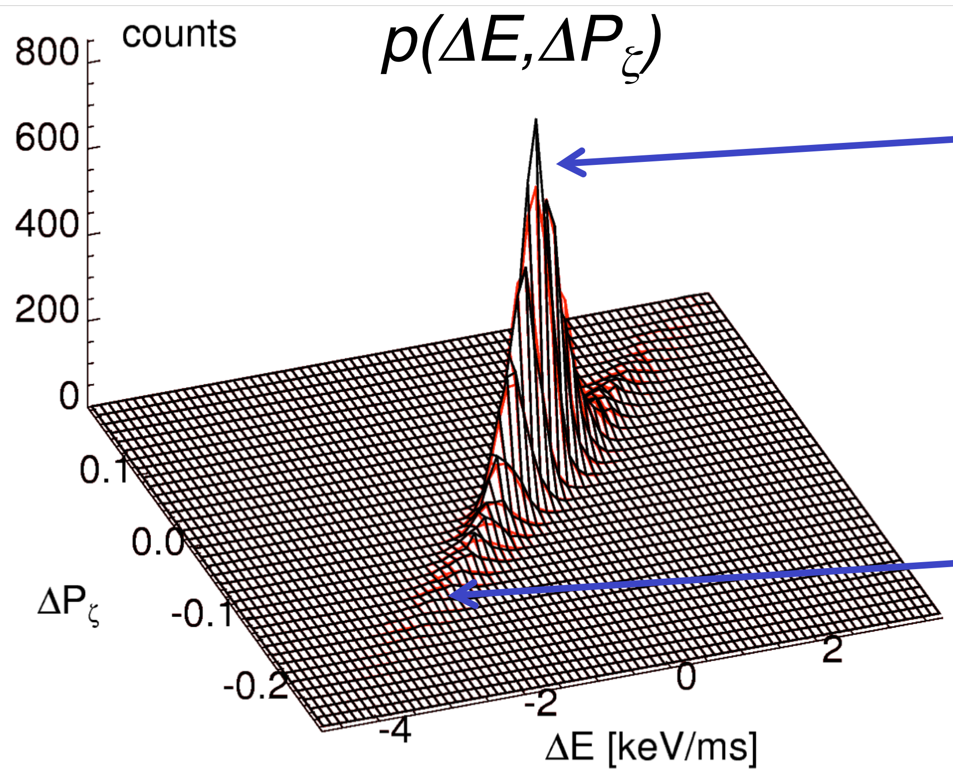
VII. Advance particle's trajectory in phase space for next step

VIII. Compute slowing down, scattering (NUBEAM)

where steps II-VII are divided in sub-steps for each particle

Required input, e.g. through 'Ufiles':
scaling factor ("mode amplitude") A_{mode}, probability p(ΔE,ΔP_ζ)

Discrete bins in (P_ζ,E,μ) can contain both *resonant* and *non-resonant* particles



- 'Non-resonant' particles have small fluctuations around initial (E, P_ζ)
- 'Resonant' particles can experience large ΔE, ΔP_ζ variations

- To keep track of particle's class:
  - Sample steps σ_E, $\sigma_{P\zeta}$ at first step only
  - This mimic the "correlated random walk" experienced by the particles
  - Exception: particle move to a different bin -> re-sample

$p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ can be skewed to positive/negative $\Delta E,\Delta P_\zeta$, causing overall "drift" of $F_{nb}(P_\zeta,E,\mu)$

- Introduce 'random sign' for *i*-th step in MC procedure, $S_{r,i}$
- For each particle (e.g. pair of correlated steps σ_E, $\sigma_{P\zeta}$), calculate $S_{r,i}$ from probability of positive vs. negative steps

  – From $p(\Delta E,\Delta P_\zeta)$ compute

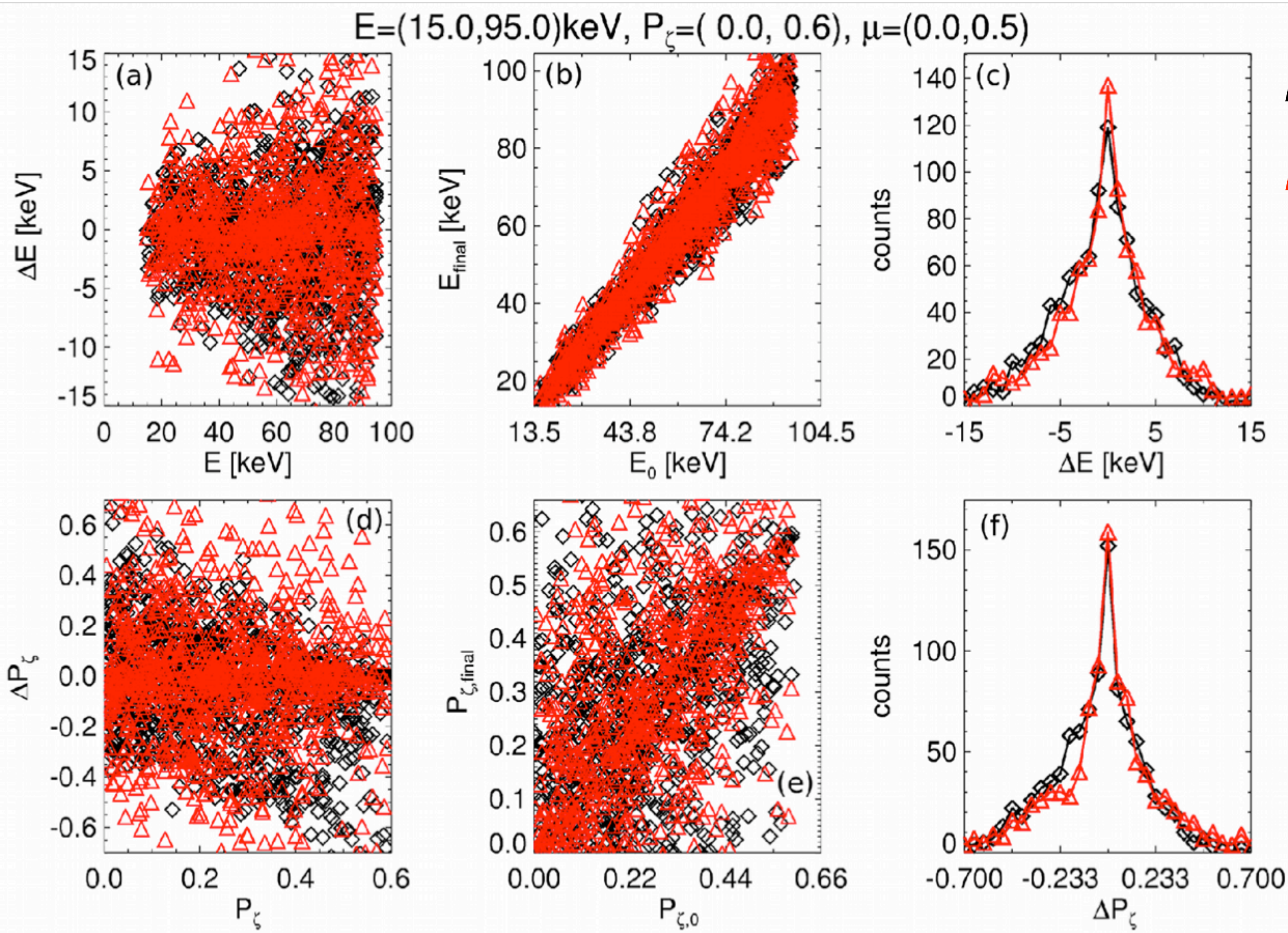
$$p_+ \doteq p(\sigma_{E,i},\sigma_{P_\zeta,i}) \quad ; \quad p_- \doteq p(-\sigma_{E,i},-\sigma_{P_\zeta,i})$$

  – Then define f_{sign}:

$$f_{sign} = \frac{p_+}{p_+ + p_-}$$

  – Finally, use $0<f_{sign}<1$ to bias random extraction of $S_{r,i}=+1,-1$

Example: evolving F_{nb} over 270 μs in 5 sub-steps

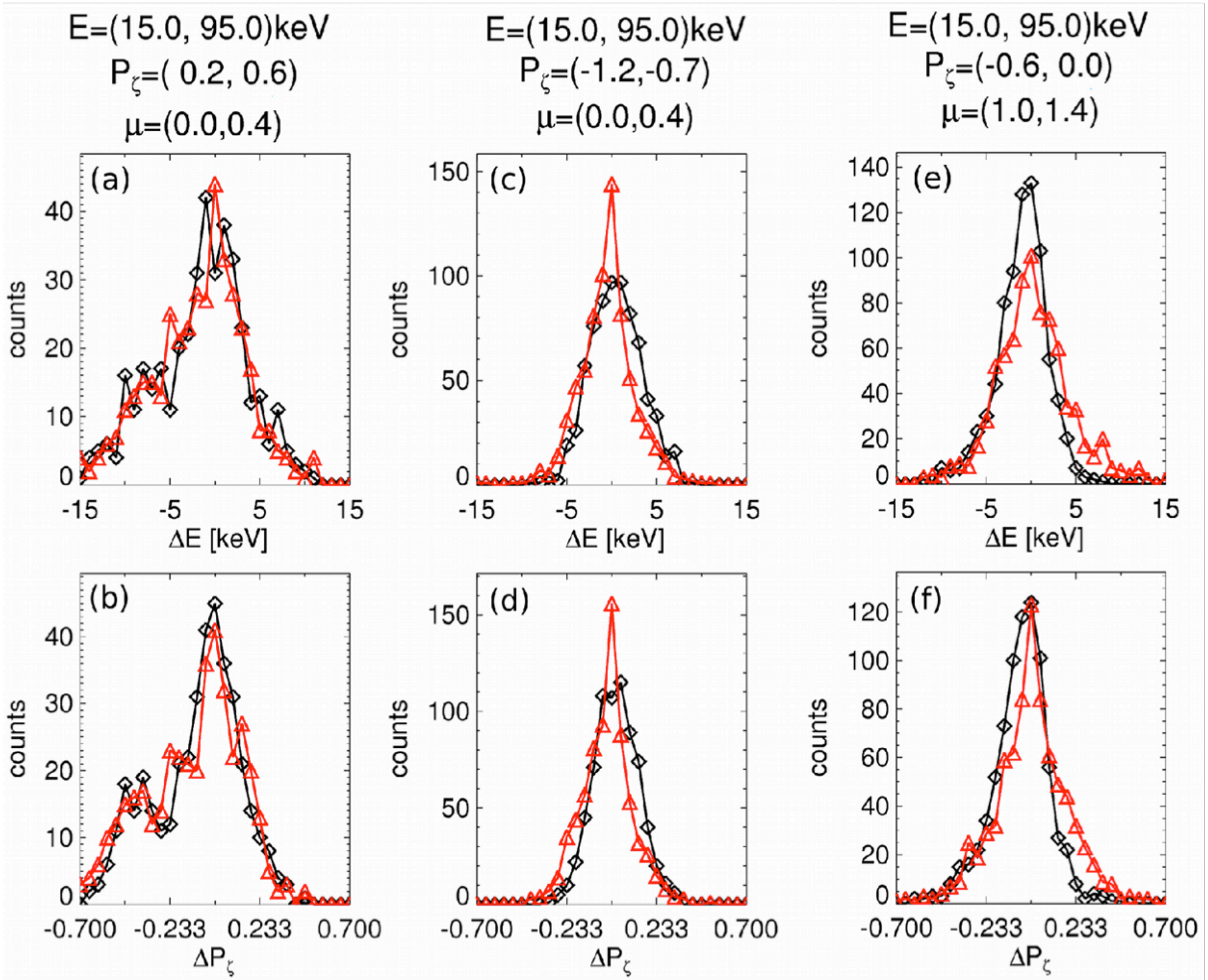


black: ORBIT

red: Model

Reconstruction works for different classes: co-, counter-, trapped

co-passing *counter-passing* *trapped*



black: ORBIT

red: Model

Reconstruction works at different sub-steps

black: ORBIT ***red: Model***