NSTX-U

NSTX-U is sponsored by the
U.S. Department of Energy Office of Science
Fusion Energy Sciences

δn mode structure of CAEs & GAEs in NSTX via a novel reflectometer analysis technique*

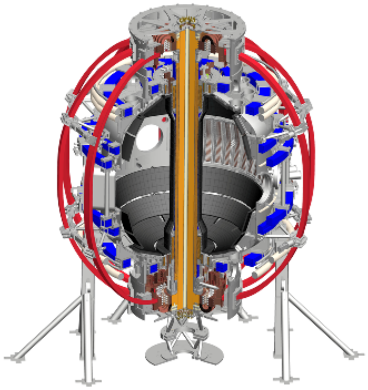
N.A. Crocker, S. Kubota, W.A. Peebles (UCLA), E. D. Fredrickson, E. Belova, A. Diallo, B.P. LeBlanc (PPPL)

NSTX-U Physics Meeting
PPPL
Aug. 7, 2017

*Supported by US DOE Contracts DESC0011810, DE-FG02-99ER54527 & DEAC0209CH11466

UCLA

PPPL

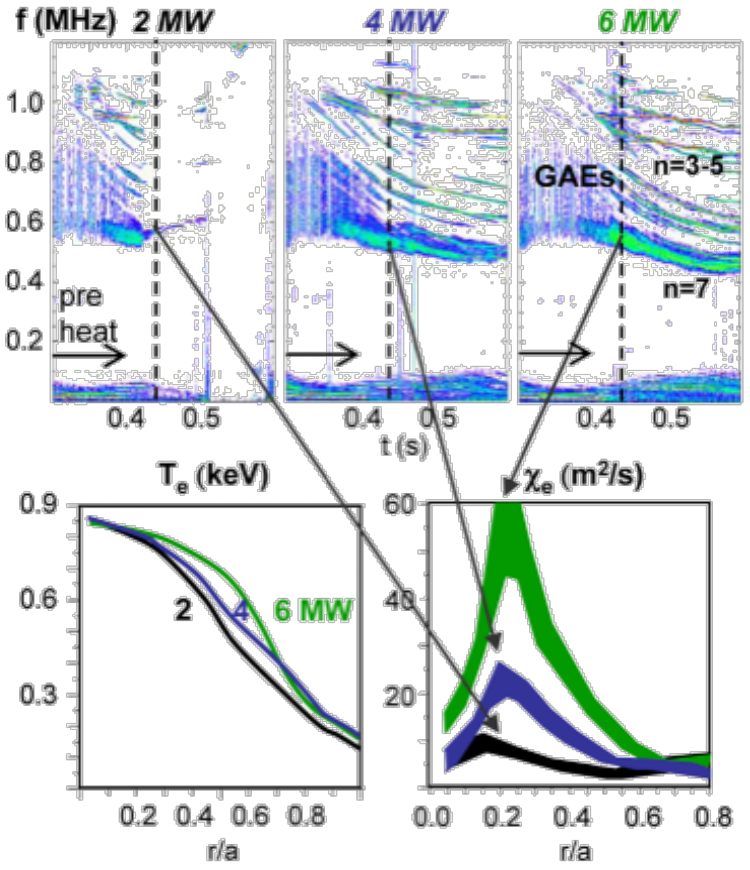


Novel reflectometry analysis shows CAE/GAE δn too small to explain anomalous χ_e

- Compressional (CAE) and Global Alfvén eigenmodes (GAE) proposed to cause high anomalous core χ_e
- New multi-channel reflectometer analysis ⇒ more accurate δn internal amplitude and structure
- Core δn + theory [Gorelenkov NF 2010] for GAE modification of e^- drift orbits ⇒ GAEs too small to explain χ_e from TRANSP
  - Theory uses ORBIT modeling to determine χ_e dependence on amplitude and number of modes
- Measurement compared to HYM simulations
  - Measured and simulated GAE structures show similarities
- New δn + HYM Poynting Flux ⇒ CAE-KAW energy flux small

Motivation: CAEs & GAEs candidates for core energy transport in NSTX

- CAEs & GAEs excited by Doppler-shifted cyclotron resonance with beam ions
  [N. N. Gorelenkov, NF 2003]
- CAE & GAE activity correlates with enhanced χ_e in core
  [D. Stutman, PRL 2009; K. Tritz, APS 2010 Invited Talk; N. A. Crocker, PPCF 2011]
  - χ_e from TRANSP modeling
- Two leading hypotheses:
  - Stochastization of e^- guiding center orbits enhance χ_e
  - CAE-KAW coupling = missing transport channel ⇒ TRANSP gets χ_e wrong

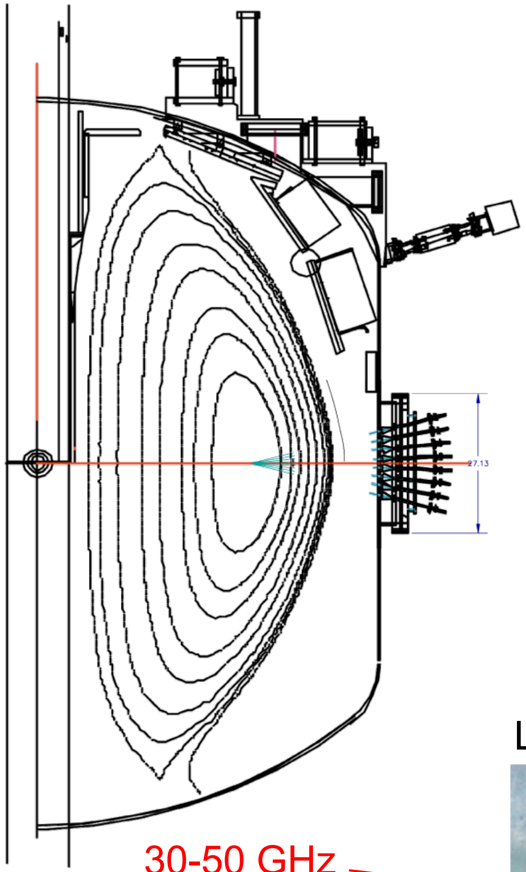


[D. Stutman et al., PRL 102 115002 (2009)]

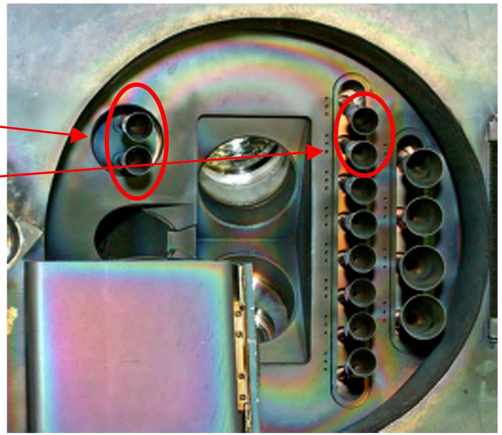
Reflectometers provide radial array of measurements

- 16 channels in two arrays: “Q-band” & “V-band”
  - Q-band: 30, 32.5, 35, 37.5, 42.5, 45, 47.5 & 50 GHz
  - V-band: 55, 57.5, 60, 62.5, 67.5, 70, 72.5 & 75 GHz
- Arrays closely spaced (separated ~ 10° toroidal)
  - Separate launch/receive horn pair for each array
- Horns aimed perpendicular to flux surfaces ⇒ frequency array = radial array
- Cutoffs span large radial range in high density plasmas ($n_0 \sim 1 – 7 \times 10^{19}\ m^{-3}$)

NSTX cross-section



Launch and Receive Horns

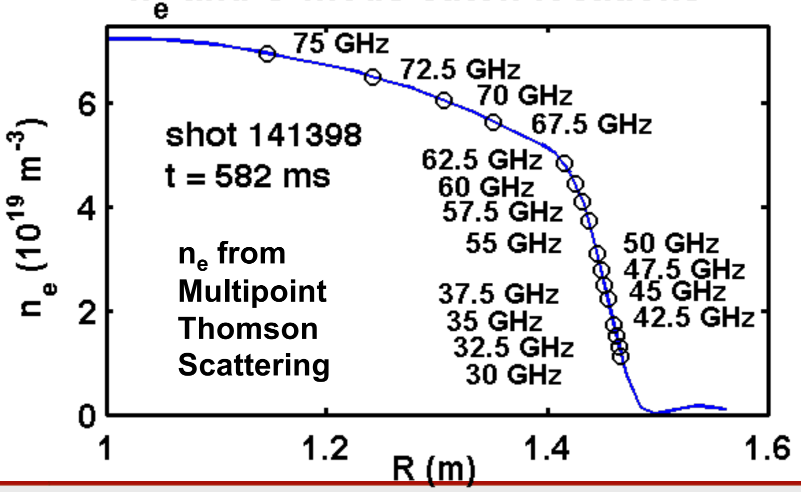


30-50 GHz

55-75 GHz

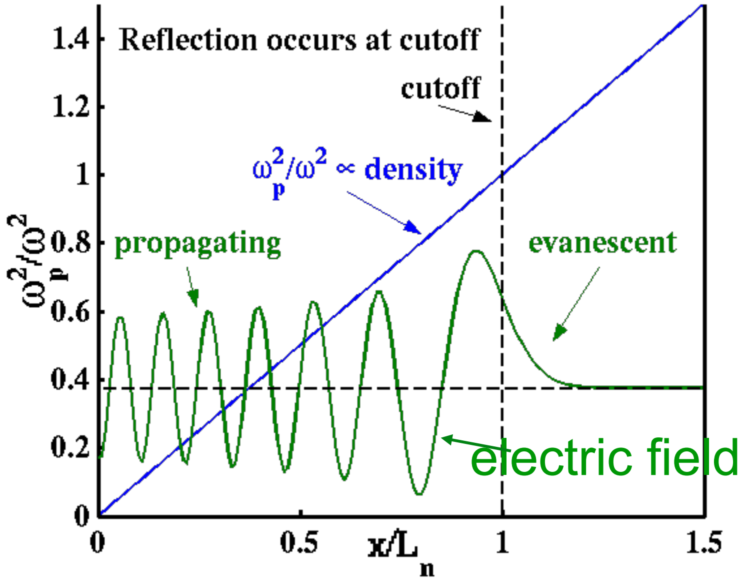
(not shown: horns modified to optimize for frequency range)

n_e and O-mode cutoff locations



Reflectometers measure density fluctuations in plasma

- Microwaves reflect from plasma at "cutoff", where density high enough ($\omega_p = \omega$)
  - O-mode: $\omega^2 = \omega_p{}^2 + c^2k^2$, $\omega_p{}^2 = e^2n_e/\varepsilon_0 m_e$
  - microwaves reflect at $k = 0$



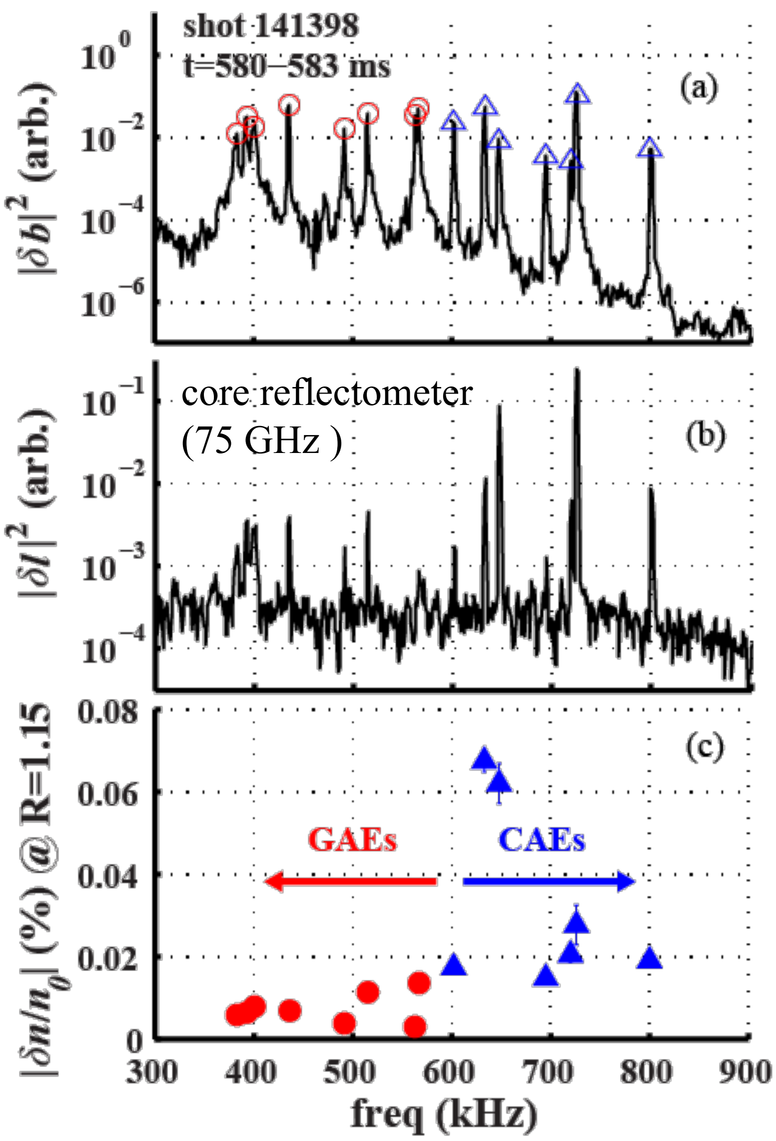
Microwave ("O-mode") propagation

- Reflectometer measures microwave path length fluctuations (δl) caused by δn
- δl sensitive to cutoff motion, but δn along path contributes (a.k.a. "interferometer effect")
  - cutoff motion dominates as $k_r \to 0$ (e.g. external kink mode)

Reflectometer array measures δn of CAEs & GAEs

- Reflectometer array sees global modes identified as CAEs & GAEs

  [N.A. Crocker, PPCF 2011]
- New analysis gives $\delta n/n_0$; in core:
  - CAE: $\delta n/n_0 \sim 10^{-4} - 10^{-3}$
  - GAE: $\delta n/n_0 \sim 10^{-5} - 10^{-4}$
- δn obtained from reflectometer measurements via “synthetic diagnostic”
- Reflectometer “signal-to-noise” improved via correlation with δb



δn determined via synthetic diagnostic

- Synthetic diagnostic used to model path length
  - WKB path length integral:

$$l = l_0 + \delta l = \int_{R_{edge}}^{R_{cutoff}} dR \sqrt{1 - \omega_p^2(R)/\omega^2}$$

$$\omega_p^2\left(R_{cutoff}\right) = \omega^2, \omega_p^2 = \omega_{p0}^2 + \delta\omega_p^2 \propto n_0 + \delta n$$

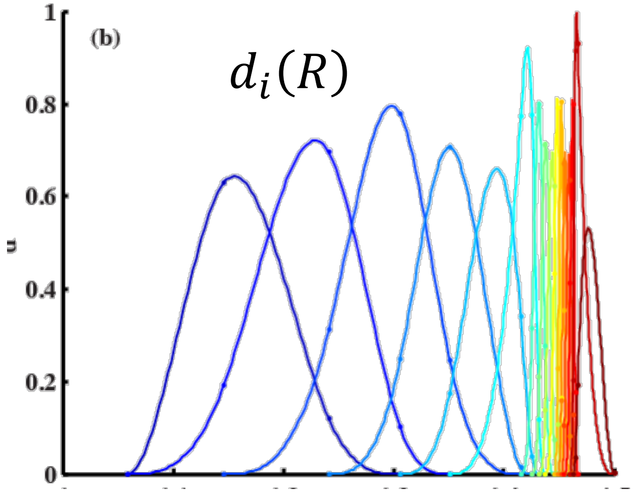
- Perturbation modeled with cutoff displacement (d) basis functions:

$$\delta n(R) = -\nabla n_0(R) \sum_i a_i d_i(R)$$

  - cubic B-splines used for $d_i(R)$
  - set of $a_i \Rightarrow \delta l_{fit}$ for all channels
  - find of set of a_i to minimize

$$\chi^2 = \sum_j \frac{\left(\delta l_{j,meas} - \delta l_{j,fit}\right)^2}{\sigma_{j,meas}^2}$$

Cutoff displacement basis functions (cubic “B-splines”; cutoff locations as knots)

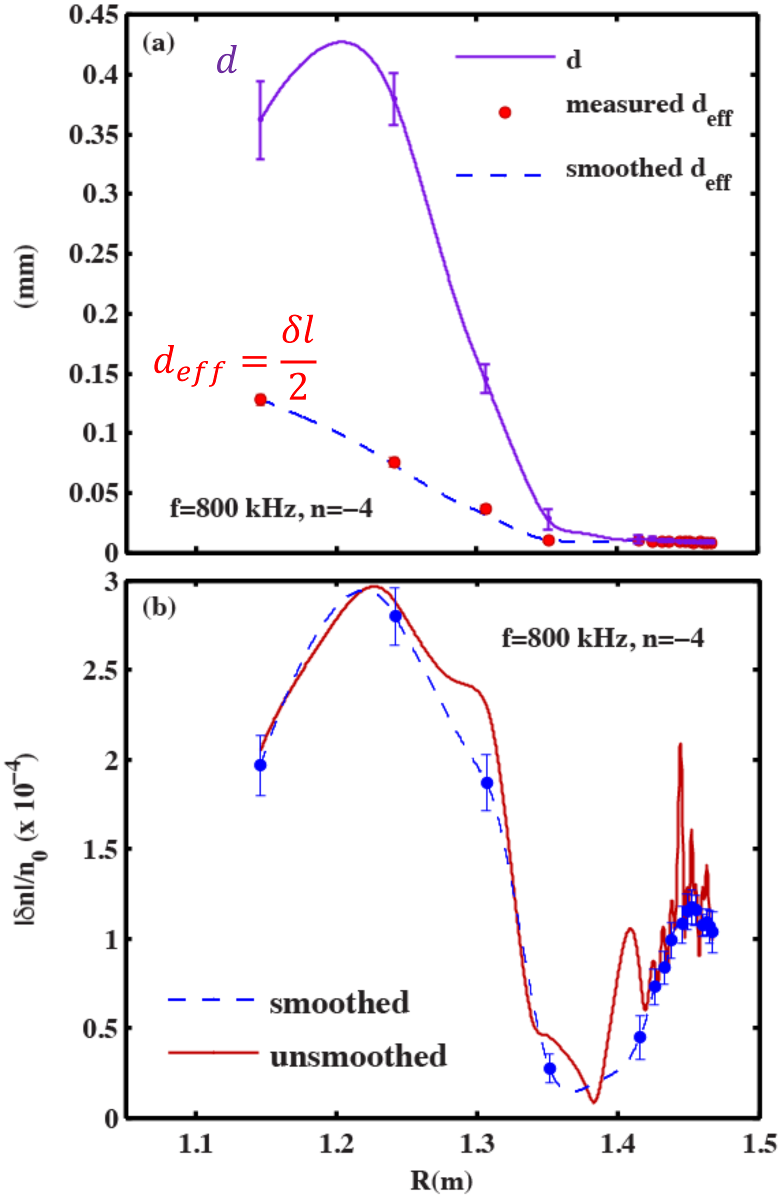


$$\delta n_i(R) = -d_i(R)\nabla n_0(R)$$

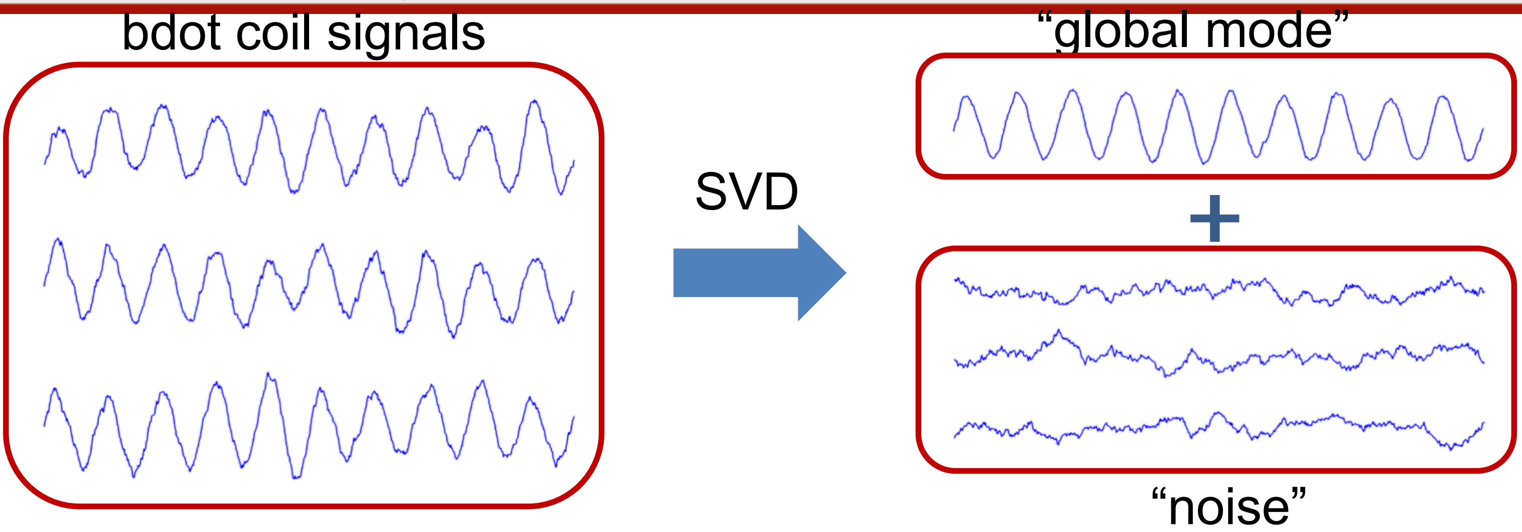
$d_i($

δn determined via synthetic diagnostic

- Fit naturally yields cutoff displacement structure ($d(R)$) along with $\delta n(R)$
- Fit sensitive to noise in δl_{meas} $\Rightarrow$ *use smoothed δl_{meas} for inversion*
  - smoothing is low spatial filter
  - filtering smoothed δl is within uncertainty of δl_{meas}
  - can't know if short scale structure in δn is real, given uncertainties



Singular value decomposition gives better "global mode" δb



bdot coil signals

SVD

"global mode"

+

"noise"

- global mode observed by 10 bdot coils (HN array)
- "filter" coil signals using SVD ⇒ global mode w/reduced noise
  - SVD factors space & time dependence of signal matrix:

$$b_{jk} = \tilde{b}_j(t_k) \rightarrow \tilde{b}_{0\,j}\tilde{b}_{global}(t_k) + \epsilon_j(t_k)$$

- Steps *before* SVD …

1) bandpass filter coil signals to isolate mode
2) make signals complex ⇒ spatial phase (e.g. $n\phi_j$) factors out automatically:

$$\tilde{b}_j(t) = A(t)\cos\left(\theta(t) + \theta_{0j}\right) \rightarrow \hat{\tilde{b}}_j(t) = \frac{1}{\sqrt{2}}A(t)e^{i\left(\left(\theta(t)+\theta_{0j}\right)\right)} = \frac{1}{\sqrt{2}}\int_0^\infty d\omega e^{i\omega t}\int_{-\infty}^{\infty} dt'\tilde{b}(t')e^{-i\omega t'}$$

SVD finds global mode from eigenvector of signal correlation matrix

- SVD solves factoring problem

$$\hat{\tilde{b}}_j(t_k) = \hat{\tilde{b}}_{0\,j}\hat{\tilde{b}}_{global}(t_k) + \hat{\epsilon}_j(t_k)$$

- by minimizing χ^2:

$$\chi^2 = \sum_{j,k}\left|\hat{\tilde{b}}_j(t) - \hat{\tilde{b}}_{0\,j}\hat{\tilde{b}}_{global}(t_k)\right|^2$$

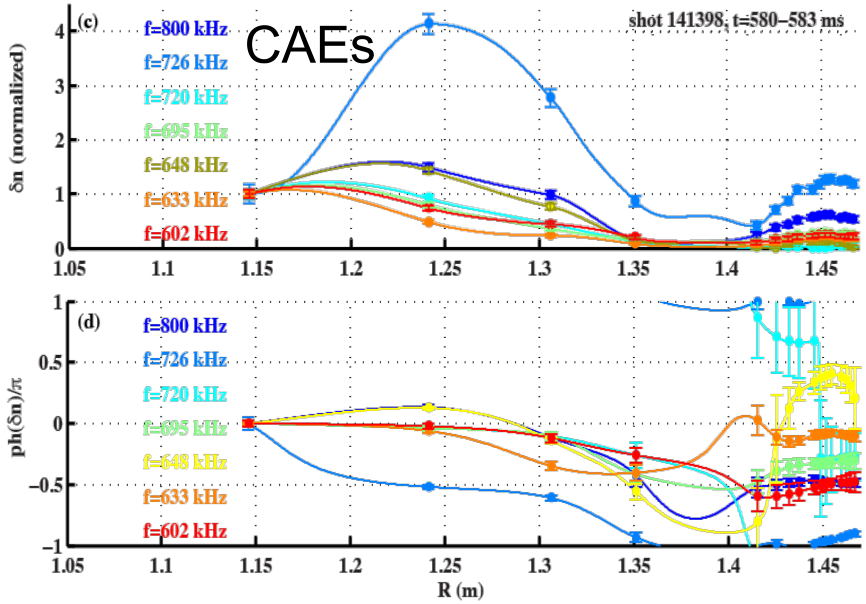
- $\Rightarrow$ spatial coefficients ($\hat{\tilde{b}}_{0\,j}$) of global mode from ***eigenvector*** of correlation matrix ***with largest eigenvalue***:

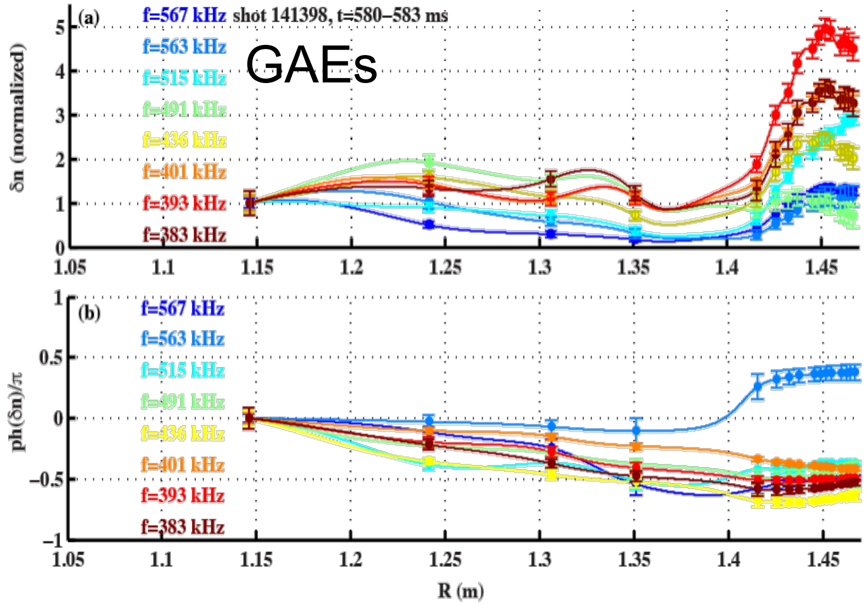
$$\mathbf{C}\hat{\tilde{\mathbf{b}}}_0 = \lambda\hat{\tilde{\mathbf{b}}}_0$$

$$[\mathbf{C}]_{ij} = \left\langle \hat{\tilde{b}}_i(t)\hat{\tilde{b}}_j^*(t)\right\rangle, \left[\hat{\tilde{\mathbf{b}}}_0\right]_j = \hat{\tilde{b}}_{0\,j}$$

CAEs and GAEs have different δn structure

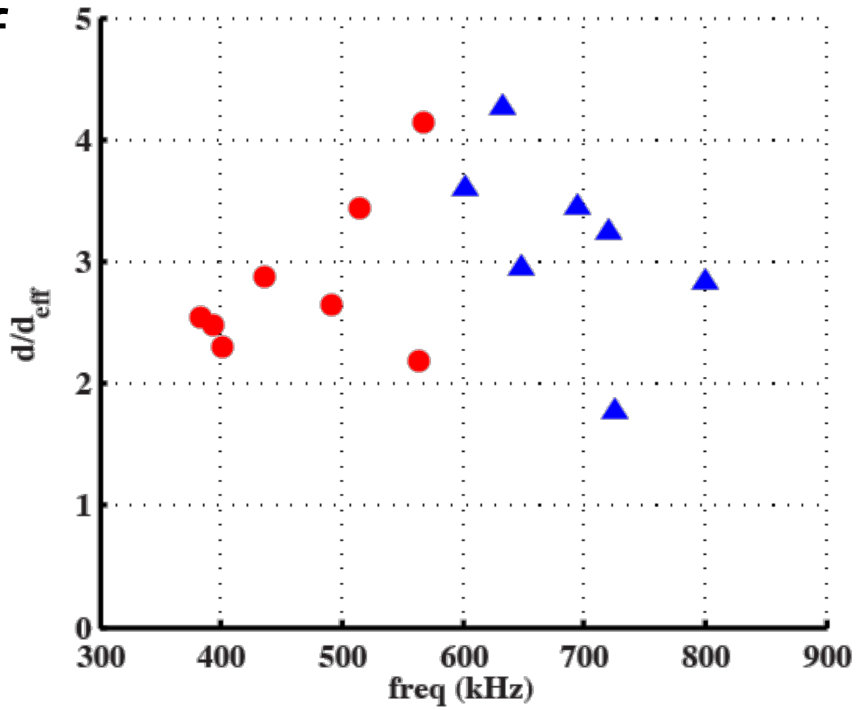
- CAEs have large, broad core peaks & small edge amplitude
- GAEs have low amplitude, broad structure in core & large edge peaks
- Note: large edge peaks can be caused by small edge radial displacements





New analysis gives 2–4 x larger cutoff displacement (d)

- Old analysis: δl attributed to cutoff displacement using "mirror approximation" $\Rightarrow$ $d_{eff} = \delta l/2$
  - "d_{eff}" means "effective displacement"



- GAEs: for shear modes,

cutoff displacement (d) ≈ plasma displacement (ξ_R)

- CAEs: for compressional modes (if $k \gg L_n^{-1}$),

$$\nabla \cdot \boldsymbol{\xi} \approx \delta n / n_0$$

cutoff displacement ≠ plasma displacement

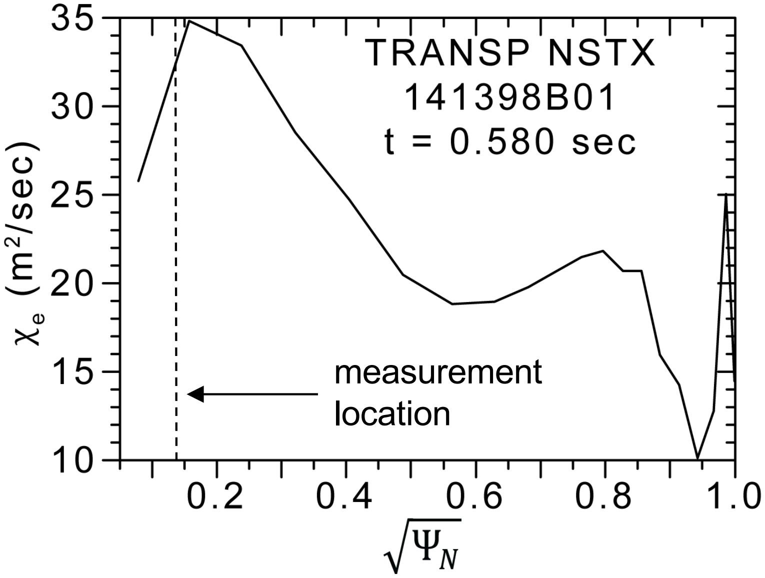
$$\delta n / n_0 = -d \nabla n_0 \Longrightarrow \frac{d}{\xi} \sim k L_n \gg 1$$

Measurements + theory ⇒ GAE amplitude & number too small to explain anomalous χ_e

- Anomalous core χ_e (~ 35 m^2/s) in
- 6 MW H-mode (141398, t = 0.58 sec)
  - TRANSP experimental transport analysis
- Theory for GAE-induced χ_e ⇒ modes needed explain high χ_e
  [Gorelenkov NF 2010]
  - average amplitude: α ~ 4x10^{-4} ($\alpha = {}^{A_\parallel}/_{B_0 R_0}$)
  - strong nonlinear depenence: $\chi_e \propto \alpha^c, c = 3\text{–}6$
  - threshold at ~ 16 modes
- Expt.: 8 GAEs; α = (0.2 – 4.2) x 10^{-4}
  - $\alpha \approx \frac{\xi}{R_0}\frac{k_\parallel}{m/r}$, $\xi \sim d, r \sim R - R_0$
  - $m, k_\parallel$ from measurement:

$$m = q(k_\parallel + n), k_\parallel \approx \frac{2\pi(f - n f_{ROT0})}{V_{A0}}$$

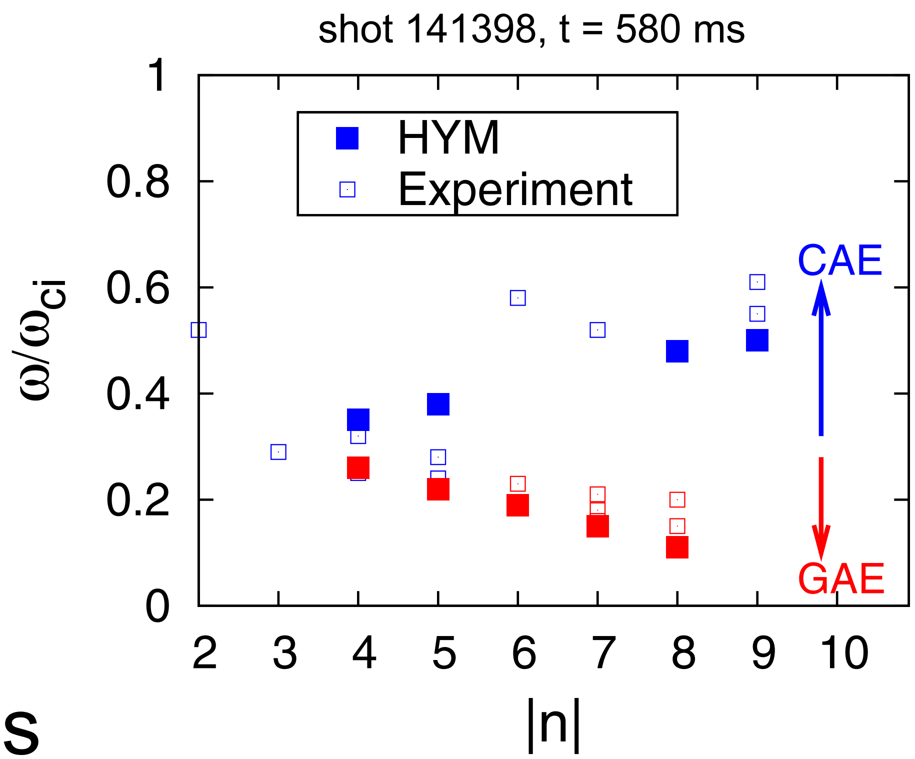
- GAE amplitude & number too small



f (kHz)	*n*	*m*	d(mm)	*α*(x10^{-4})
383	-8	-2.2	0.11	0.3
393	-7	-1.1	0.12	0.7
401	-8	-2.0	0.15	0.5
436	-7	-0.4	0.13	2.0
491	-8	-0.6	0.07	0.9
515	-7	0.7	0.22	2.2
563	-6	2.4	0.06	0.2
567	-8	0.5	0.26	4.2

Initial comparison of HYM simulation & measurement promising

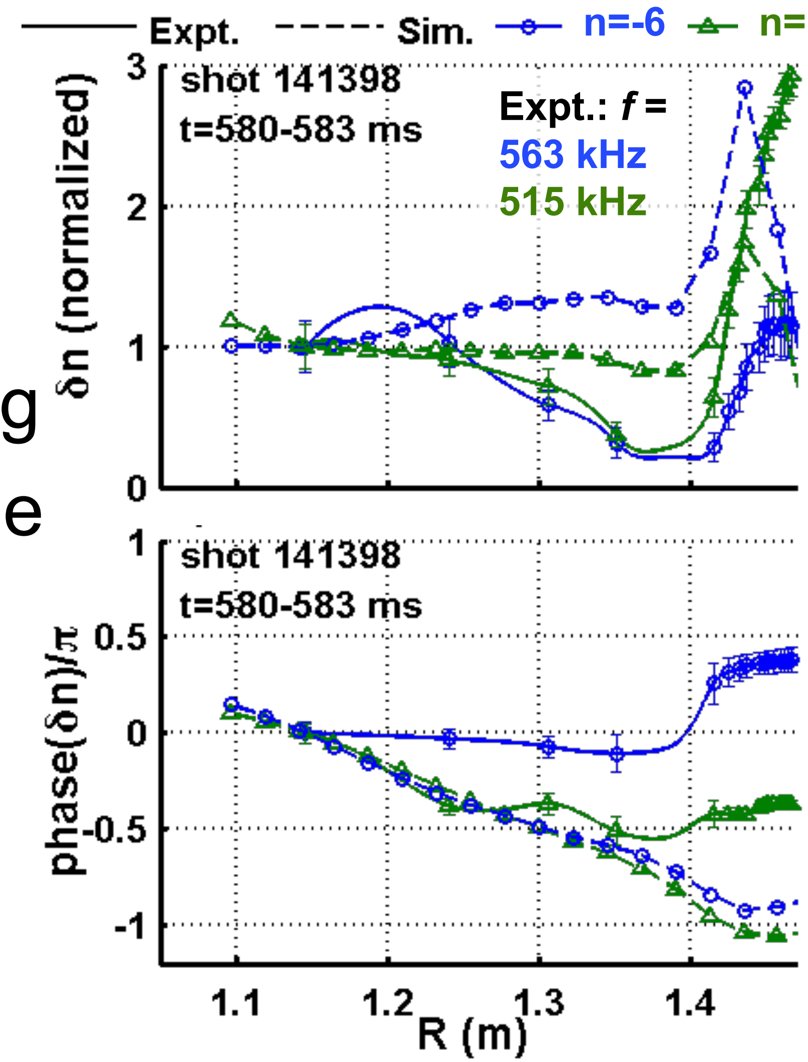
- Hybrid MHD (HYM) code simulates CAE structure & stability
  - 3D, coupled MHD fluid & fully kinetic fast-ions
  - realistic equilibrium
- Simulation & experiment compared for beam heated H-mode plasma
- Most-unstable CAEs & GAEs have *f* & *n* similar to observed experimental spectrum [E Belova PoP 2017]



[E Belova PoP 2017]

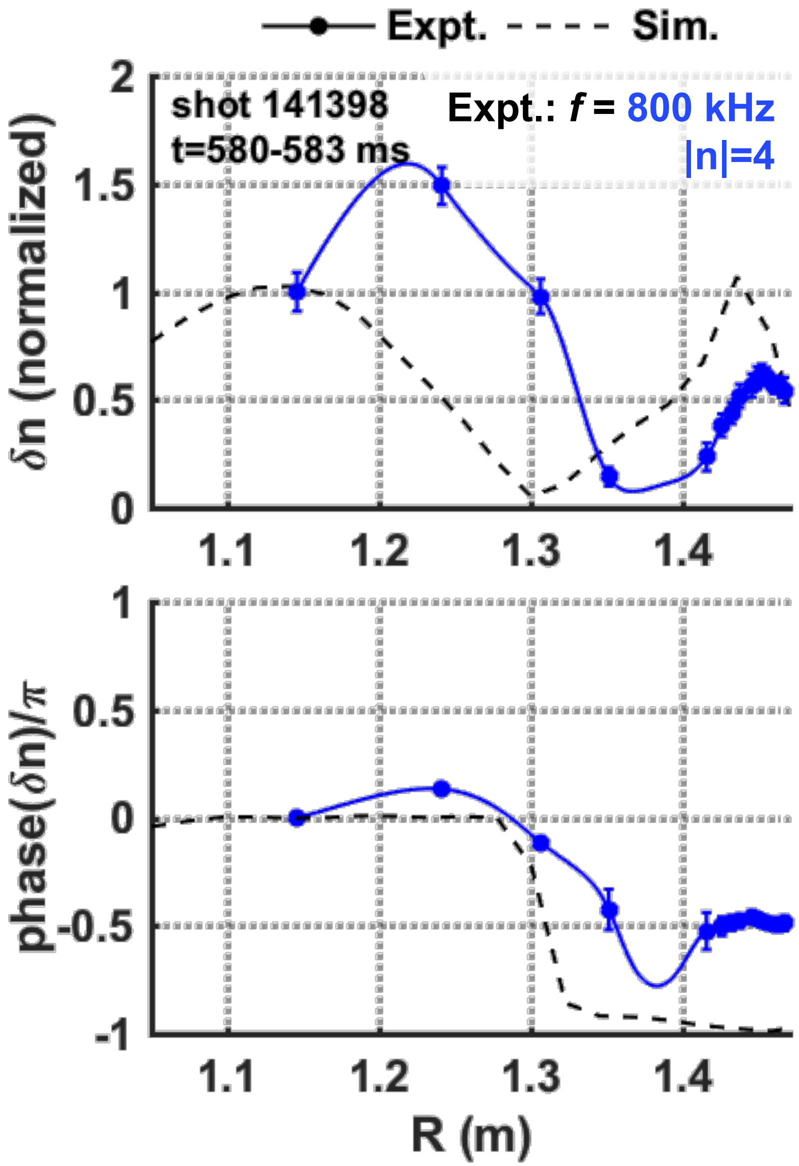
Initial comparison of HYM simulation & measurement promising

- HYM simulation shows most unstable modes at *n* = 6 & 7 as counter propagating GAEs
- Similar (roughly) broad core structures & strong edge peaking
- Simulation shows stronger phase change across minor radius
- Further work needed: expect GAE structure to be sensitive to
  - B_0 structure – included in HYM
  - Hall effect (finite ω/ω_{ci}) & toroidal rotation – under development for HYM



Initial comparison of HYM simulation & measurement promising

- CAEs show similar (roughly) structure: broad core peaks & small edge amplitude
- CAE in simulation is co-propagating. In experiment, counter-propagating.
  - Further work needed to understand
- Further work needed: expect CAE structure to be sensitive to
  - mod(B_0) structure – included in HYM
  - Hall effect (finite ω/ω_{ci}) & toroidal rotation – under development for HYM



Measurements + Simulation ⇒ Small CAE-KAW energy transport

- HYM: n = 4 CAE with $\frac{\delta b_\parallel}{B_0} \sim 6.6 \times 10^{-3}$ ⇒ CAE-KAW coupling transport: $P_{\text{CAE-KAW}}$ = 1.2 MW [E Belova PoP 2017]
- Assuming $\frac{\delta n}{n} \approx \frac{\delta b_\parallel}{B_0}$ in core ⇒ $2 \times 10^{-4} < \frac{\delta b_\parallel}{B_0} < 7 \times 10^{-4}$
  - In core, $k \gg L_n^{-1}$
  - $\frac{\delta n}{n_0}$ measured @ R=1.15 m; $\frac{n_0}{n(\text{R=1.15m})}$=1.05
- CAE-KAW coupling transport $\propto \delta b_\parallel^2$ ⇒ $P_{\text{CAE-KAW}}$ = 0.03 MW total for all modes
  - Assume $P_{\text{CAE-KAW}} / \delta b_\parallel^2$ same for all modes
- In [E Belova PoP 2017], meas. + HYM ⇒ $\frac{\delta b_\parallel}{B_0} \sim 0.9 - 3.4 \times 10^{-3}$ by using d_{eff} to scale simulation ξ, but $d_{eff} \neq \xi$.

Conclusions

- New multi-channel reflectometer analysis ⇒ more accurate δn internal amplitude and structure
  - cutoff displacement larger than previous analysis
- Core δn + theory [Gorelenkov NF 2010] for GAE modification of e^- drift orbits ⇒ GAEs too small to explain χ_e from TRANSP
- Measurement compared to HYM simulations
  - Measured and simulated GAE structures show rough similarities
  - Motivates HYM development current under way
- New δn + HYM Poynting Flux ⇒ CAE-KAW energy flux small

Future work

- ORBIT modeling with experimental mode spectrum of GAEs & CAEs
  - GAEs modeling done. Not reported here. Doesn't change conclusion
  - ORBIT modified for better treatment of CAEs. Requires verifiction…
- Move synthetic diagnostic beyond 1D path integral ⇒ raytracing
  - system uses separate and receive antennae
- Better alternatives to SVD "filtering"?
  - independent component analysis
  - blind source determination
  - …
- Investigate role of compressional/shear coupling on δn (Hall effect, B_0 structure)
  - understand differences in simulation and measured structure
  - improve determination of electromagnetic amplitude (α & $\delta b_{\|}$) from δn