

δn mode structure of CAEs & GAEs in NSTX via a novel reflectometer analysis technique*

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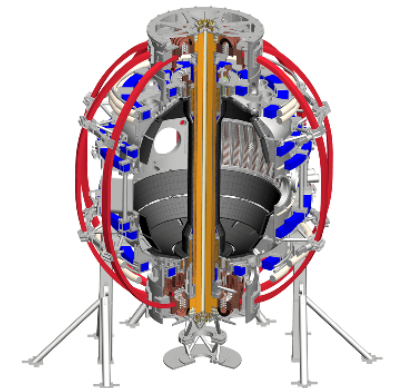
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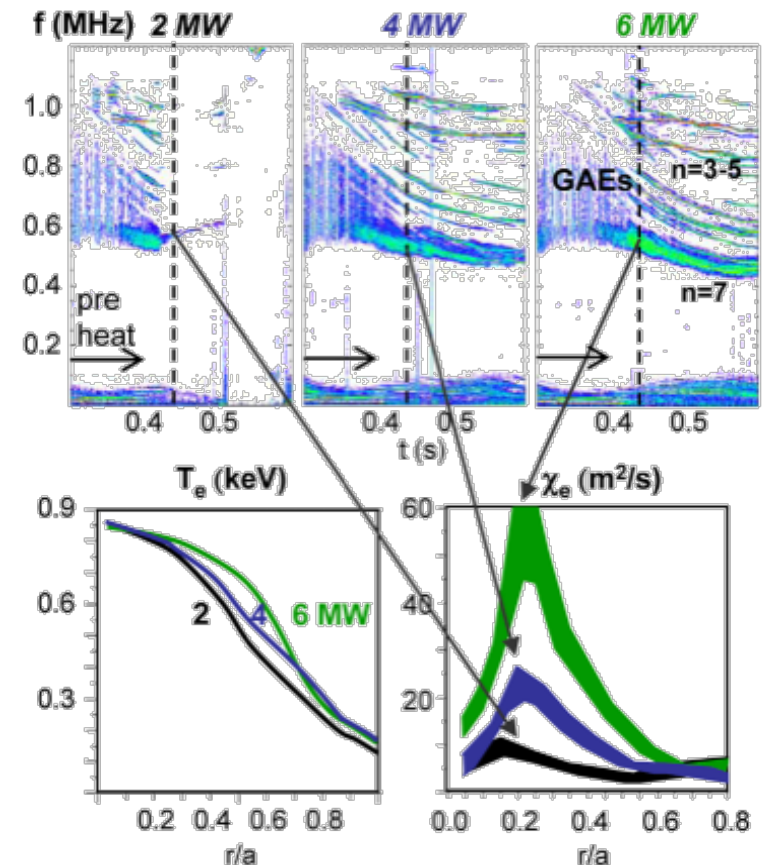


Novel reflectometry analysis shows CAE/GAE δn too small to explain anomalous χ_e

- Compressional (CAE) and Global Alfvén eigenmodes (GAE) proposed to cause high anomalous core χ_e
- New multi-channel reflectometer analysis \Rightarrow more accurate δn internal amplitude and structure
- Core δn + theory [Gorelenkov NF 2010] for GAE modification of e^- drift orbits \Rightarrow GAEs too small to explain χ_e from TRANSP
 - Theory uses ORBIT modeling to determine χ_e dependence on amplitude and number of modes
- Measurement compared to HYM simulations
 - Measured and simulated GAE structures show similarities
- New δn + HYM Poynting Flux \Rightarrow CAE-KAW energy flux small

Motivation: CAEs & GAEs candidates for core energy transport in NSTX

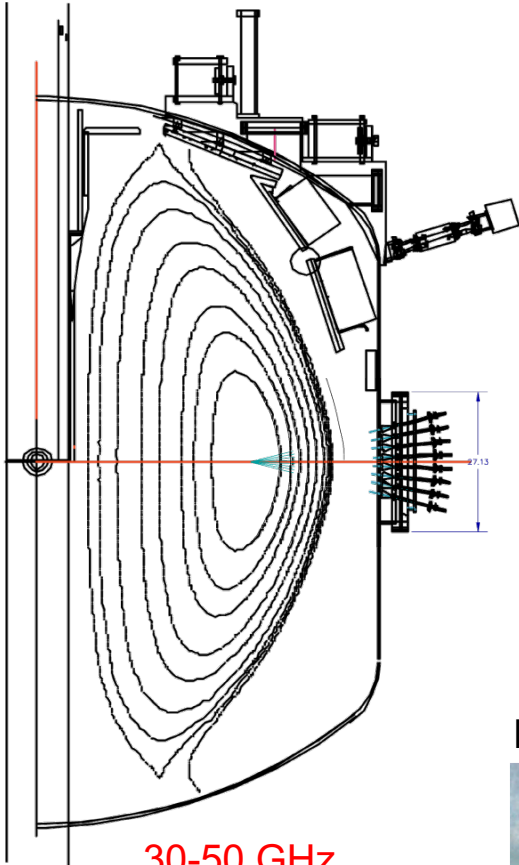
- CAEs & GAEs excited by Doppler-shifted cyclotron resonance with beam ions
[N. N. Gorelenkov, NF 2003]
- CAE & GAE activity correlates with enhanced χ_e in core
[D. Stutman, PRL 2009; K. Tritz, APS 2010 Invited Talk; N. A. Crocker, PPCF 2011]
 - χ_e from TRANSP modeling
- Two leading hypotheses:
 - Stochastization of e^- guiding center orbits enhance χ_e
 - CAE-KAW coupling = missing transport channel \Rightarrow TRANSP gets χ_e wrong



[D. Stutman et al., PRL 102 115002 (2009)]

Reflectometers provide radial array of measurements

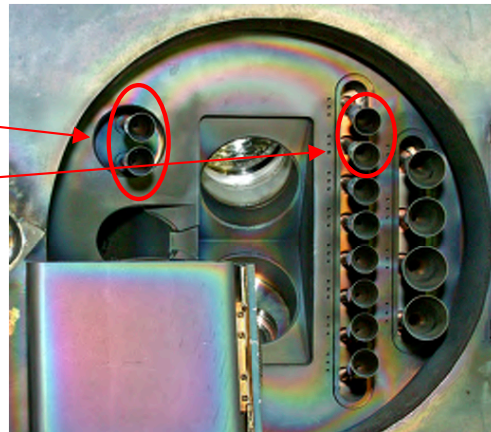
NSTX cross-section



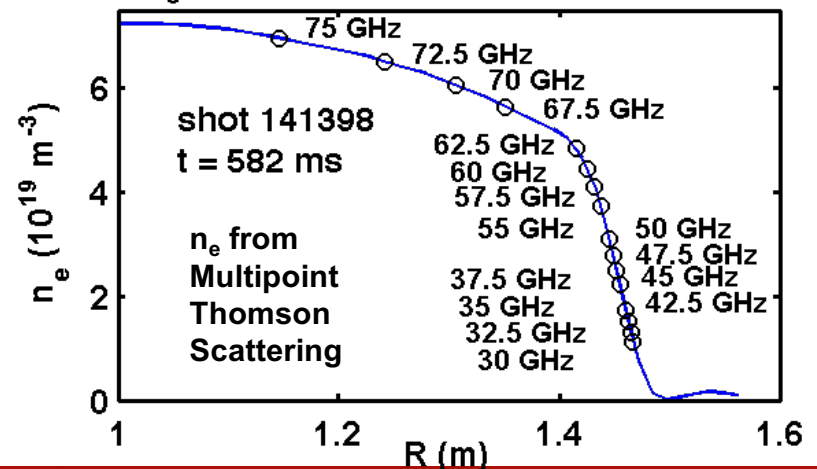
- 16 channels in two arrays: “Q-band” & “V-band”
 - Q-band: 30, 32.5, 35, 37.5, 42.5, 45, 47.5 & 50 GHz
 - V-band: 55, 57.5, 60, 62.5, 67.5, 70, 72.5 & 75 GHz
- Arrays closely spaced (separated $\sim 10^\circ$ toroidal)
 - Separate launch/receive horn pair for each array
- Horns aimed perpendicular to flux surfaces \Rightarrow frequency array = radial array
- Cutoffs span large radial range in high density plasmas ($n_0 \sim 1 - 7 \times 10^{19} \text{ m}^{-3}$)

30-50 GHz
 55-75 GHz
 (not shown: horns modified to optimize for frequency range)

Launch and Receive Horns



n_e and O-mode cutoff locations



Reflectometers measure density fluctuations in plasma

- Microwaves reflect from plasma at “cutoff”, where density high enough ($\omega_p = \omega$)

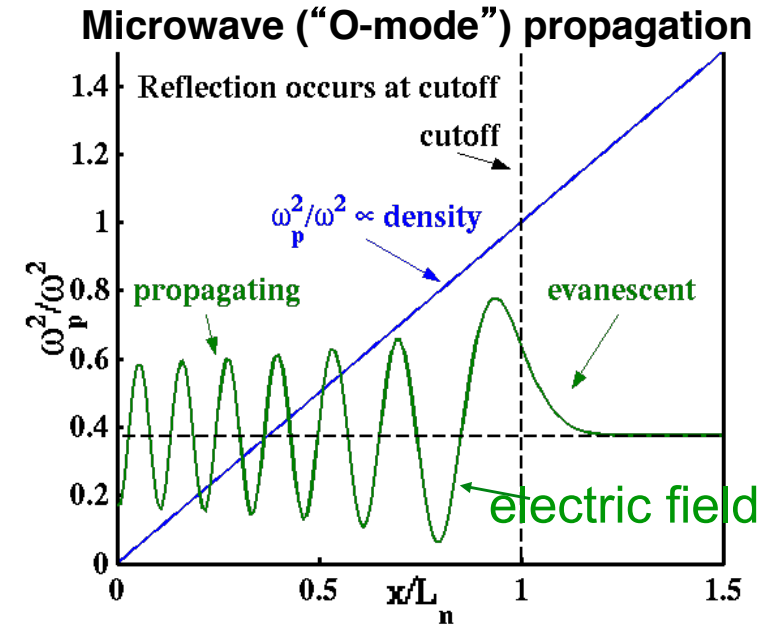
- O-mode: $\omega^2 = \omega_p^2 + c^2k^2$,
 $\omega_p^2 = e^2n_e/\varepsilon_0m_e$

- microwaves reflect at $k = 0$

- Reflectometer measures microwave path length fluctuations (δl) caused by δn

- δl sensitive to cutoff motion, but δn along path contributes (a.k.a. “interferometer effect”)

- cutoff motion dominates as $k_r \rightarrow 0$ (e.g. external kink mode)

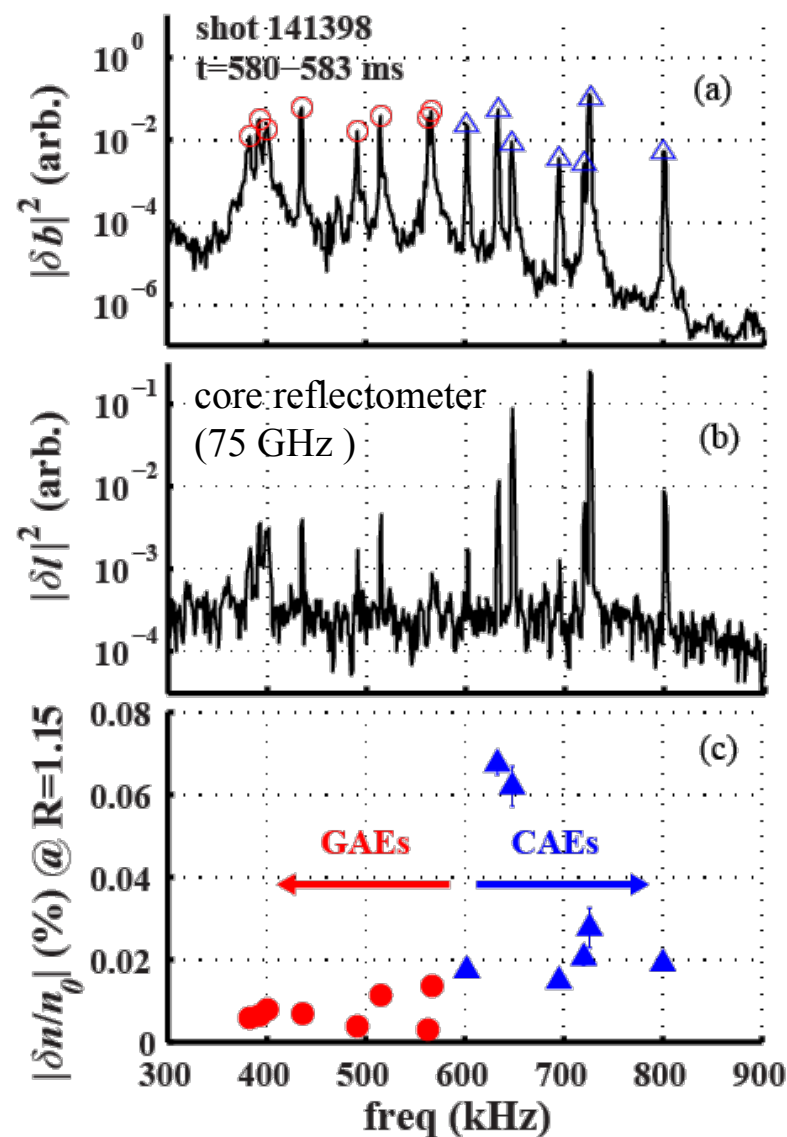


Reflectometer array measures δn of CAEs & GAEs

- Reflectometer array sees global modes identified as CAEs & GAEs

[N.A. Crocker, PPCF 2011]

- New analysis gives $\delta n/n_0$ in core:
 - CAE: $\delta n/n_0 \sim 10^{-4} - 10^{-3}$
 - GAE: $\delta n/n_0 \sim 10^{-5} - 10^{-4}$
- δn obtained from reflectometer measurements “via synthetic diagnostic
- Reflectometer “signal-to-noise” improved via correlation with δb



δn determined via synthetic diagnostic

- Synthetic diagnostic used to model path length

- WKB path length integral:

$$l = l_0 + \delta l = \int_{R_{edge}}^{R_{cutoff}} dR \sqrt{1 - \omega_p^2(R)/\omega^2}$$

$$\omega_p^2(R_{cutoff}) = \omega^2, \omega_p^2 = \omega_{p0}^2 + \delta\omega_p^2 \propto n_0 + \delta n$$

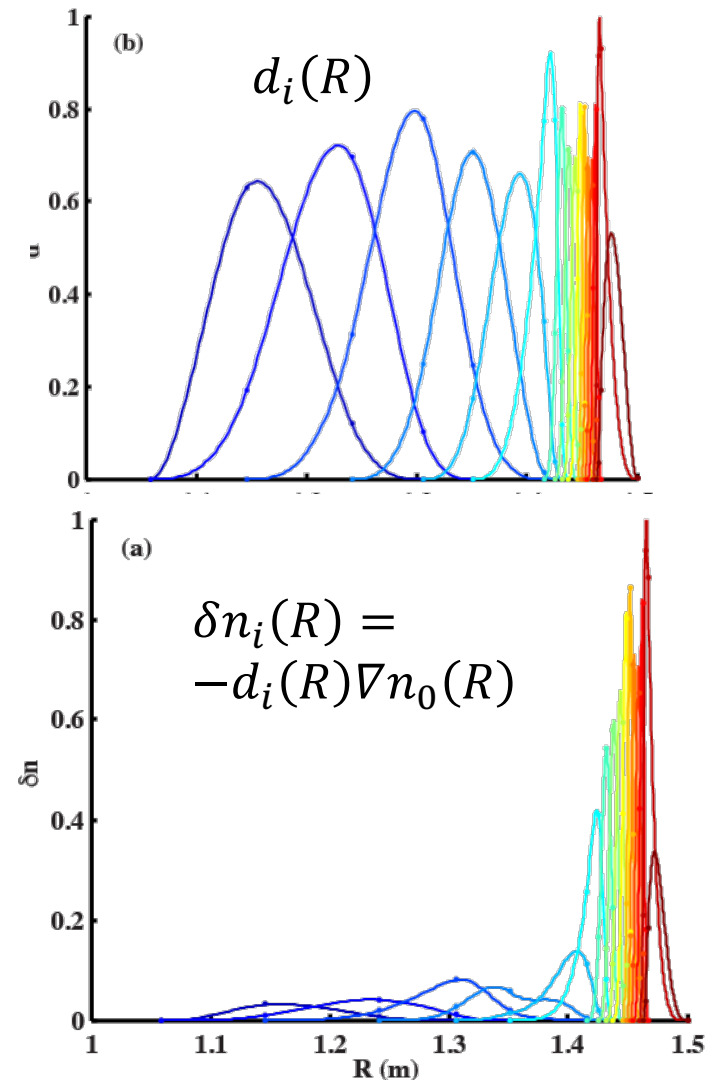
- Perturbation modeled with cutoff displacement (d) basis functions:

$$\delta n(R) = -\nabla n_0(R) \sum_i a_i d_i(R)$$

- cubic B-splines used for $d_i(R)$
- set of $a_i \Rightarrow \delta l_{fit}$ for all channels
- find of set of a_i to minimize

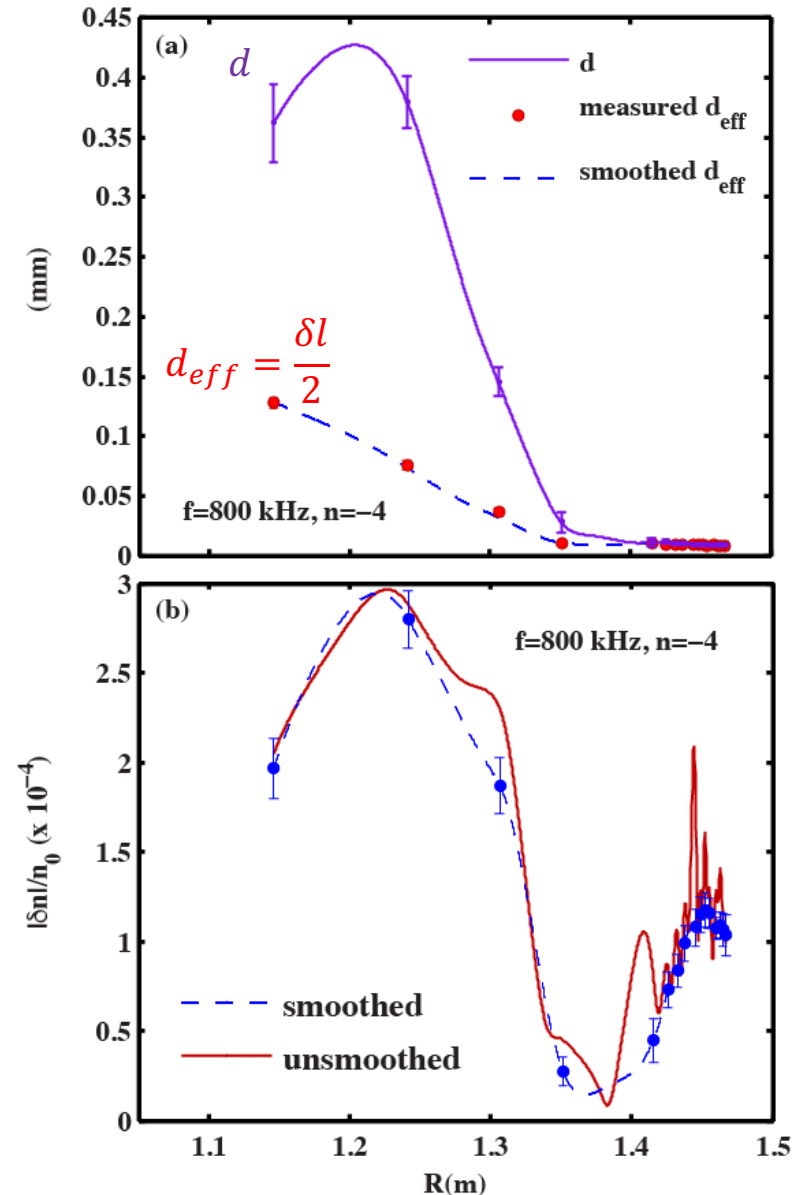
$$\chi^2 = \sum_j \frac{(\delta l_{j,meas} - \delta l_{j,fit})^2}{\sigma_{j,meas}^2}$$

Cutoff displacement basis functions (cubic “B-splines”; cutoff locations as knots)

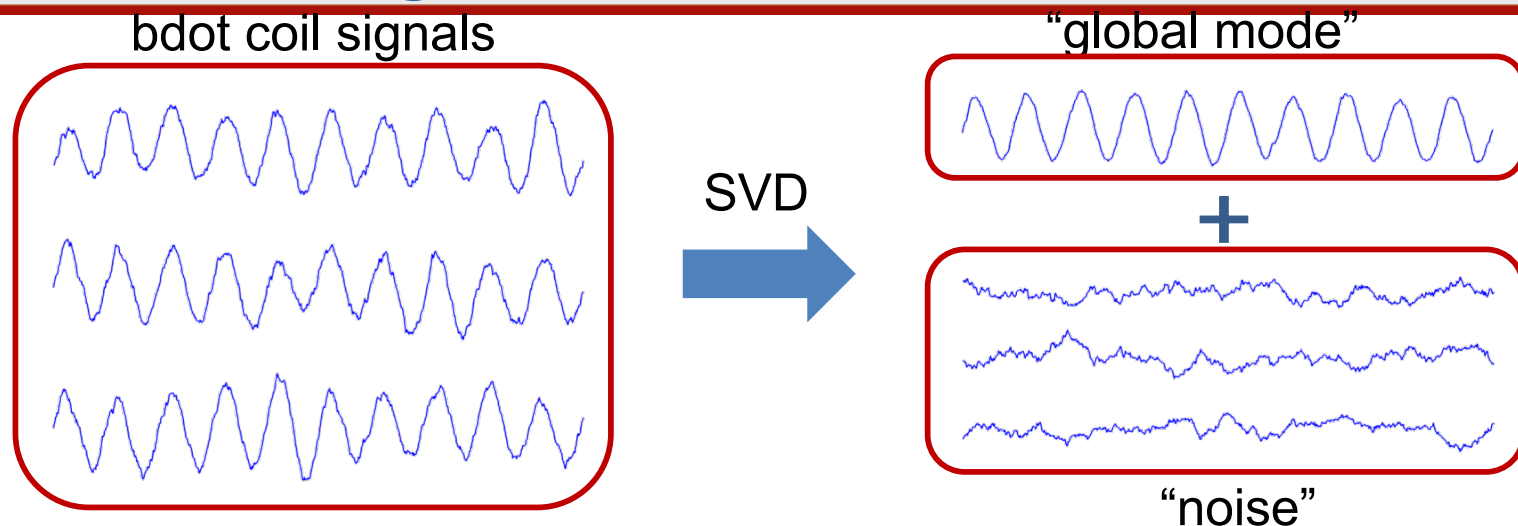


δn determined via synthetic diagnostic

- Fit naturally yields cutoff displacement structure ($d(R)$) along with $\delta n(R)$
- Fit sensitive to noise in δl_{meas}
 \Rightarrow *use smoothed δl_{meas} for inversion*
 - smoothing is low spatial filter
 - filtering smoothed δl is within uncertainty of δl_{meas}
 - can't know if short scale structure in δn is real, given uncertainties



Singular value decomposition gives better “global mode” δb



- global mode observed by 10 bdot coils (HN array)
- “filter” coil signals using SVD \Rightarrow global mode w/reduced noise
 - SVD factors space & time dependence of signal matrix:

$$b_{jk} = \tilde{b}_j(t_k) \rightarrow \hat{b}_{0j} b_{global}(t_k) + \epsilon_j(t_k)$$

- Steps *before* SVD ...

1) bandpass filter coil signals to isolate mode

2) make signals complex \Rightarrow spatial phase (e.g. $n\phi_j$) factors out automatically:

$$\tilde{b}_j(t) = A(t) \cos(\theta(t) + \theta_{0j}) \rightarrow \hat{\tilde{b}}_j(t) = \frac{1}{\sqrt{2}} A(t) e^{i((\theta(t) + \theta_{0j}))} = \frac{1}{\sqrt{2}} \int_0^\infty d\omega e^{i\omega t} \int_{-\infty}^\infty dt' \tilde{b}(t') e^{-i\omega t'}$$

SVD finds global mode from eigenvector of signal correlation matrix

- SVD solves factoring problem

$$\hat{b}_j(t_k) = \hat{b}_{0j} \hat{b}_{global}(t_k) + \hat{\epsilon}_j(t_k)$$

- by minimizing χ^2 :

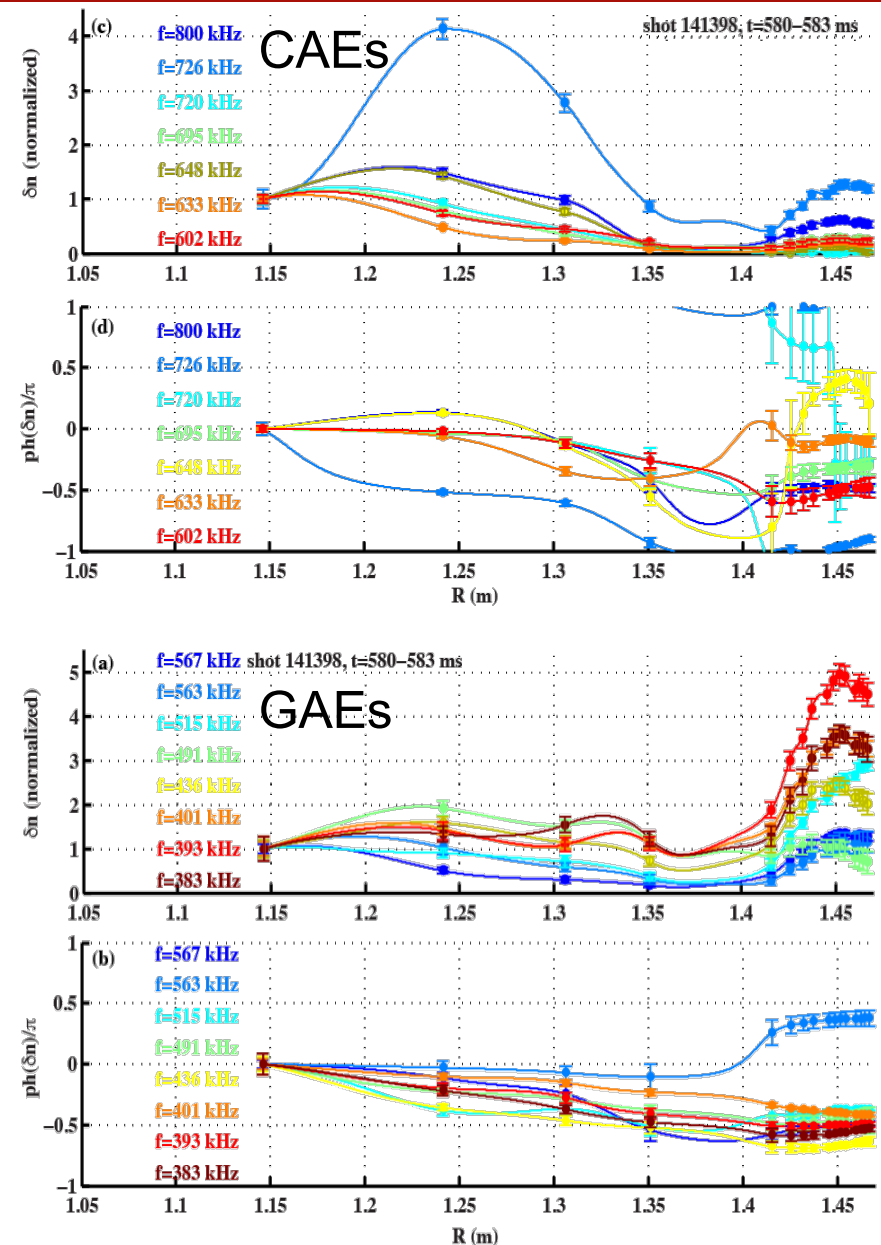
$$\chi^2 = \sum_{j,k} \left| \hat{b}_j(t) - \hat{b}_{0j} \hat{b}_{global}(t_k) \right|^2$$

- \Rightarrow spatial coefficients (\hat{b}_{0j}) of global mode from ***eigenvector*** of correlation matrix ***with largest eigenvalue***:

$$\begin{aligned} \mathbf{C} \hat{\mathbf{b}}_0 &= \lambda \hat{\mathbf{b}}_0 \\ [\mathbf{C}]_{ij} &= \left\langle \hat{b}_i(t) \hat{b}_j^*(t) \right\rangle, [\hat{\mathbf{b}}_0]_j = \hat{b}_{0j} \end{aligned}$$

CAEs and GAEs have different δn structure

- CAEs have large, broad core peaks & small edge amplitude
- GAEs have low amplitude, broad structure in core & large edge peaks
- Note: large edge peaks can be caused by small edge radial displacements



New analysis gives 2–4 x larger cutoff displacement (d)

- Old analysis: δl attributed to cutoff displacement using “mirror approximation” \Rightarrow

$$d_{eff} = \delta l / 2$$

– “ d_{eff} ” means “effective displacement”

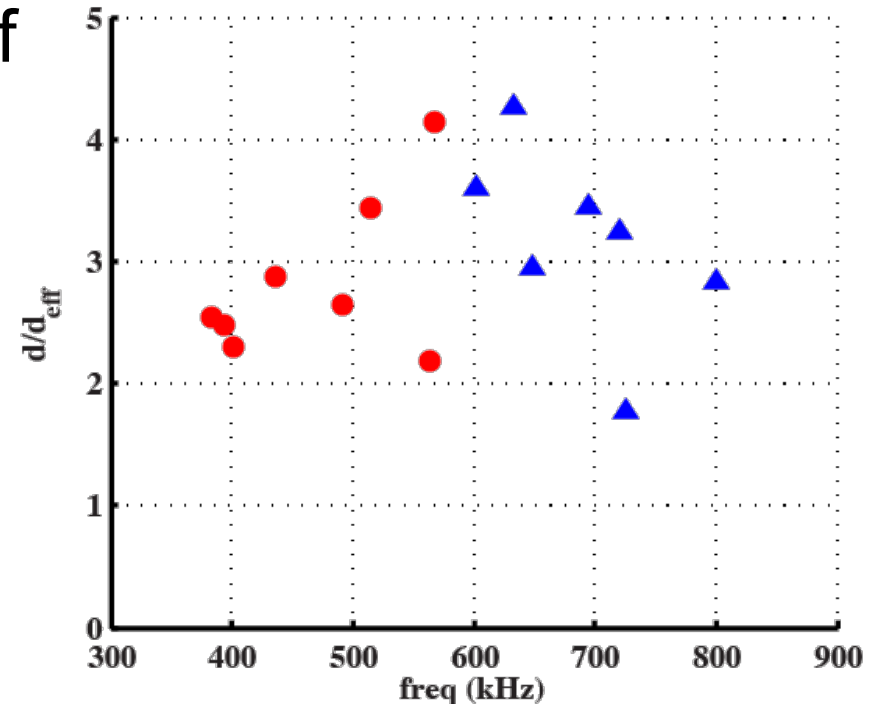
- GAEs: for shear modes, cutoff displacement (d) \approx plasma displacement (ξ_R)

- CAEs: for compressional modes (if $k \gg L_n^{-1}$),

$$\nabla \cdot \xi \approx \delta n / n_0$$

cutoff displacement \neq plasma displacement

$$\delta n / n_0 = -d \nabla n_0 \Rightarrow \frac{d}{\xi} \sim k L_n \gg 1$$



Measurements + theory \Rightarrow GAE amplitude & number too small to explain anomalous χ_e

- Anomalous core χ_e (~ 35 m²/s) in
- 6 MW H-mode (141398, $t = 0.58$ sec)
 - TRANSP experimental transport analysis
- Theory for GAE-induced $\chi_e \Rightarrow$ modes needed to explain high χ_e
 - [Gorelenkov NF 2010]

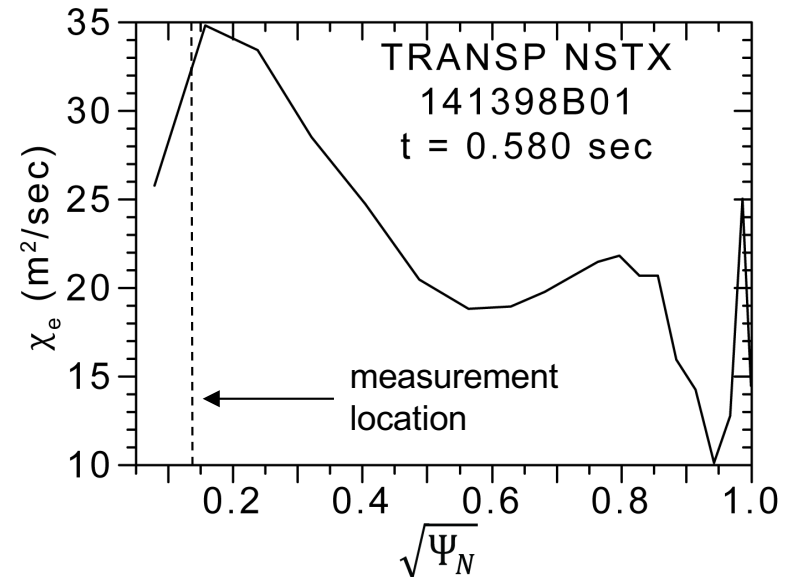
- average amplitude: $\alpha \sim 4 \times 10^{-4}$ ($\alpha = A_{\parallel} / R_0 R_0$)
- Strong nonlinear dependence: $\chi_e \propto \alpha^c, c = 3-6$
- threshold at ~ 16 modes

- Expt.: 8 GAEs; $\alpha = (0.2 - 4.2) \times 10^{-4}$

- m from measurement:

$$m = q(k_{\parallel} + n), k_{\parallel} \approx \frac{2\pi(f - nf_{ROT0})}{V_{A0}}$$

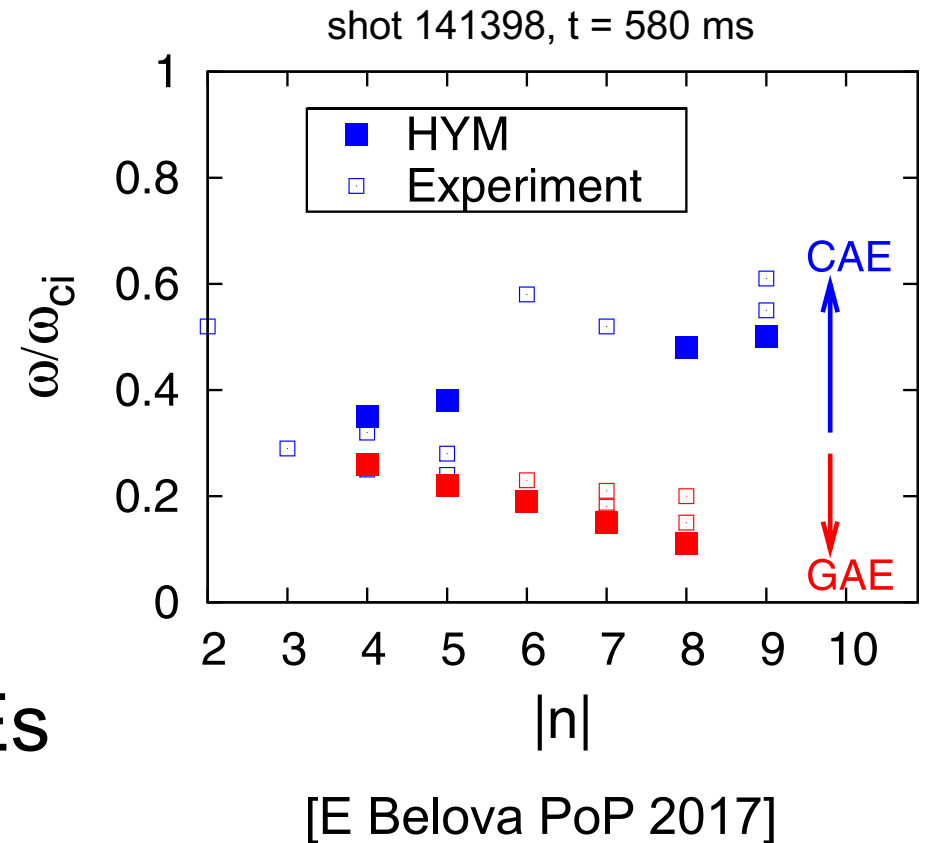
- GAE amplitude & number too small



f (kHz)	n	m	d (mm)	α ($\times 10^{-4}$)
383	-8	-2.2	0.11	0.3
393	-7	-1.1	0.12	0.7
401	-8	-2.0	0.15	0.5
436	-7	-0.4	0.13	2.0
491	-8	-0.6	0.07	0.9
515	-7	0.7	0.22	2.2
563	-6	2.4	0.06	0.2
567	-8	0.5	0.26	4.2

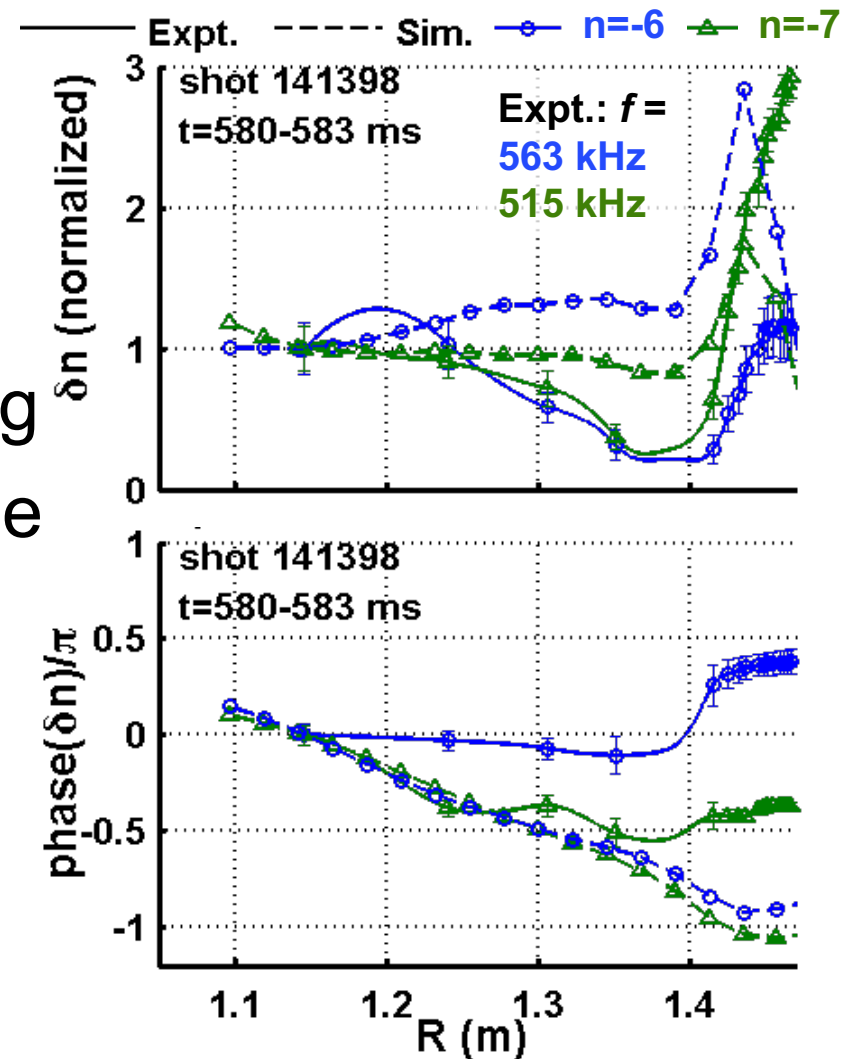
Initial comparison of HYM simulation & measurement promising

- Hybrid MHD (HYM) code simulates CAE structure & stability
 - 3D, coupled MHD fluid & fully kinetic fast-ions
 - realistic equilibrium
- Simulation & experiment compared for beam heated H-mode plasma
- Most-unstable CAEs & GAEs have f & n similar to observed experimental spectrum [E Belova PoP 2017]



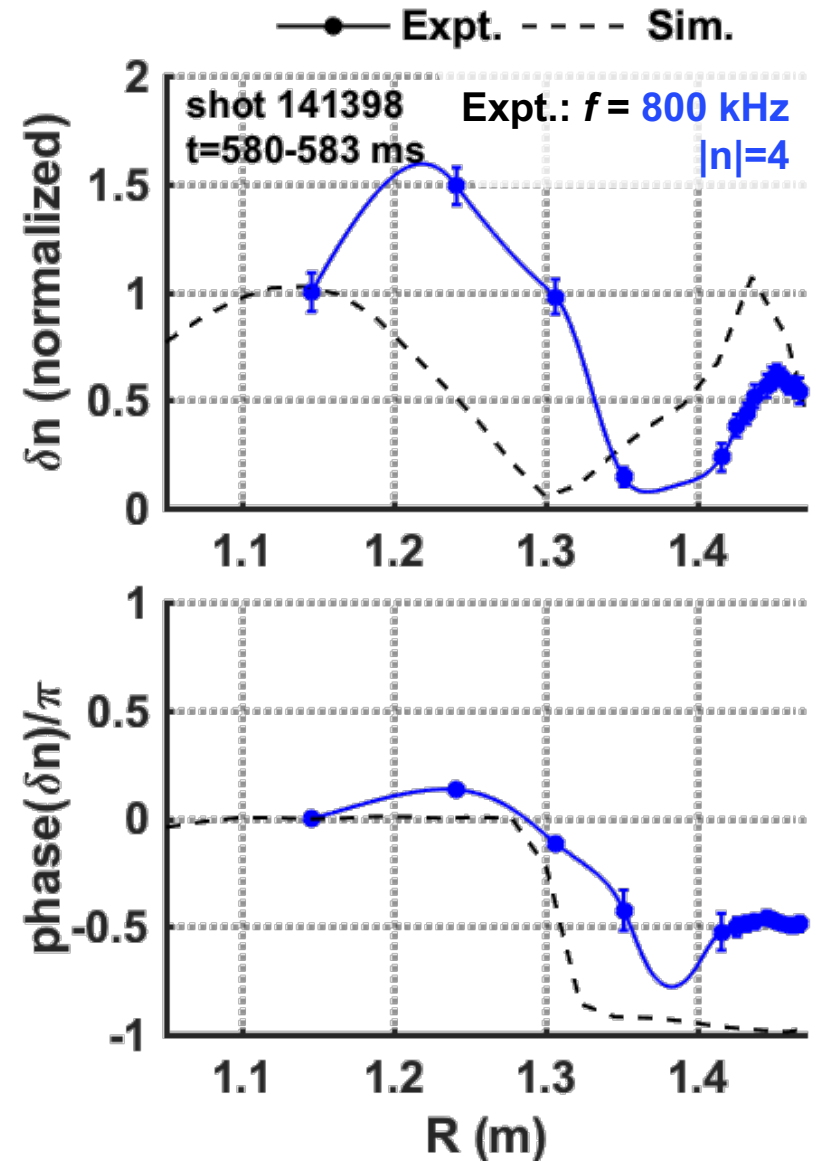
Initial comparison of HYM simulation & measurement promising

- HYM simulation shows most unstable modes at $n = 6$ & 7 as counter propagating GAEs
- Similar (roughly) broad core structures & strong edge peaking
- Simulation shows stronger phase change across minor radius
- Further work needed: expect GAE structure to be sensitive to
 - B_0 structure – included in HYM
 - Hall effect (finite ω/ω_{ci}) & toroidal rotation – under development for HYM



Initial comparison of HYM simulation & measurement promising

- CAEs show similar (roughly) structure: broad core peaks & small edge amplitude
- CAE in simulation is co-propagating. In experiment, counter-propagating.
 - Further work needed to understand
- Further work needed: expect CAE structure to be sensitive to
 - $\text{mod}(B_0)$ structure – included in HYM
 - Hall effect (finite ω/ω_{ci}) & toroidal rotation – under development for HYM



Measurements + Simulation \Rightarrow Small CAE-KAW energy transport

- HYM: $n = 4$ CAE with $\frac{\delta b_{\parallel}}{B_0} \sim 6.6 \times 10^{-3} \Rightarrow$ CAE-KAW coupling transport: $P_{\text{CAE-KAW}} = 1.2 \text{ MW}$ [E Belova PoP 2017]
- Assuming $\frac{\delta n}{n} \approx \frac{\delta b_{\parallel}}{B_0}$ in core $\Rightarrow 2 \times 10^{-4} < \frac{\delta b_{\parallel}}{B_0} < 7 \times 10^{-4}$
 - In core, $k \gg L_n^{-1}$
 - $\frac{\delta n}{n_0}$ measured @ $R=1.15 \text{ m}$; $\frac{n_0}{n(R=1.15\text{m})} = 1.05$
- CAE-KAW coupling transport $\propto \delta b_{\parallel}^2 \Rightarrow P_{\text{CAE-KAW}} = 0.03 \text{ MW}$ total for all modes
 - Assume $P_{\text{CAE-KAW}} / \delta b_{\parallel}^2$ same for all modes
- In [E Belova PoP 2017], meas. + HYM $\Rightarrow \frac{\delta b_{\parallel}}{B_0} \sim 0.9 - 3.4 \times 10^{-3}$ by using d_{eff} to scale simulation ξ , but $d_{\text{eff}} \neq \xi$.

Conclusions

- New multi-channel reflectometer analysis \Rightarrow more accurate δn internal amplitude and structure
 - cutoff displacement larger than previous analysis
- Core δn + theory [Gorelenkov NF 2010] for GAE modification of e^- drift orbits \Rightarrow GAEs too small to explain χ_e from TRANSP
- Measurement compared to HYM simulations
 - Measured and simulated GAE structures show rough similarities
 - Motivates HYM development current under way
- New δn + HYM Poynting Flux \Rightarrow CAE-KAW energy flux small

Future work

- ORBIT modeling with experimental mode spectrum of GAEs & CAEs
 - GAEs modeling done. Not reported here. Doesn't change conclusion
 - ORBIT modified for better treatment of CAEs. Requires verification...
- Move synthetic diagnostic beyond 1D path integral \Rightarrow raytracing
 - system uses separate and receive antennae
- Better alternatives to SVD “filtering”?
 - independent component analysis
 - blind source determination
 - ...
- Investigate role of compressional/shear coupling on δn (Hall effect, B_0 structure)
 - understand differences in simulation and measured structure
 - improve determination of electromagnetic amplitude (α & δb_{\parallel}) from δn