## Validation of Kinetic Turbulent-Neoclassical Theory for Edge Intrinsic Rotation in DIII-D Plasmas

## by

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## Toroidal Rotation is Beneficial to the Performance of Tokamaks

- Rotation and shear improve confinement and stability

C Chrystal, PoP (2017)
 strongly influenced by intrinsic sources ${ }^{\mathbf{1 , 2}}$.
${ }^{1}$ BA Grierson IAEA (2016)
${ }^{2}$ C Chrystal PoP, 24, 056113 (2017)

# Capturing the Physics of Edge Momentum Transport has been a Challenge for Predicting the Rotation Profile 

## Previous Related Theory Models:

- Neoclassical models : drastically under-predict momentum transport i.e. $\chi_{\varphi} \ll \chi_{i}$. Experiments show $\chi_{\varphi} \sim \chi_{i}$, suggesting turbulence dominated transport of momentum.
- Turbulent models : primarily focus on core, use quasilinear formulation to identify residual stress, momentum pinch.
- Orbit loss models : neglect turbulent transport which can strongly modify the loss-cone-predicted rotation.


## Stoltzfus-Dueck Theory Model:

- Predicts the value and direction for intrinsic rotation at the pedestal top.
- Prediction can be used as BC for predictive simulations.


## Main Results

## A) Intrinsic rot. Theory (T. Stoltzfus-Dueck):

- for moderate $\beta_{\mathrm{N}}$ theory prediction in reasonably good agreement with exp. pedestal top rotation
- At high $\beta_{\mathrm{N}}$ exp. rotation degrades with increasing $\beta_{\mathrm{N}}$, in contrast with theory prediction

Main-ion Ped. Top Rotation



## Intrinsic Rotation Theory Model

# Stoltzfus-Dueck Model¹ Predicts Value, Direction of Velocity at Core-Edge Boundary 



How is the intrinsic rotation generated?

- For typical outboard ballooning turbulence:
ctr-lp passing ions experience a stronger drift-orbit-averaged turbulent diffusion than co-Ip ions.
$\rightarrow$ ctr-lp ions are depleted at a higher rate
$\rightarrow$ co-lp intrinsic rotation of the plasma


# A Central Result: Steady State Momentum Balance Equation at the Pedestal Top 

Density flux
Angular momentum flux

Co-lp torque applied to plasma core


Outward momentum flux through pedestal

- $\Pi$ (Rotation drive) represents a "residual stress".
- Intrinsic rotation is a special case:

$$
\tau=0 \longrightarrow v_{\mathrm{int}}=-\frac{\Pi}{\Gamma_{p}}
$$

## Model Yields An Explicit Formula for Flux-Surf.-Avg. Main-ion Toroidal Velocity at Pedestal Top

Passing fraction $f_{\text {pass }} \sim 29 \%$ for typical DIII-D ped. top

Theory Prediction edge safety factor

$$
v_{\mathrm{int}} \approx f_{\mathrm{pass}} \times 0.104\left(0.5 d_{c}-\bar{R}_{X}\right) \frac{q}{L_{\widetilde{\phi}}(\mathrm{cm})} \frac{\left.T_{i}\right|_{\mathrm{pt}(\mathrm{eV})}}{B_{T}(\mathrm{~T})} \mathrm{km} / \mathrm{s}
$$

Ballooning in/out asymmetry
normalized X-pt major radius:

$$
\bar{R}_{X}=\frac{2 R_{X}-\left(R_{\text {out }}+R_{\text {in }}\right)}{R_{\text {out }}-R_{\text {in }}}
$$

radial decay length of potential fluctuations

- Strong dependence on X-pt location (Validated on TCV for Ohmic L-modes ${ }^{1}$ )
- $v_{i n t}$ captures observed features of edge intrinsic rotation:
- Reproduces Rice scaling ( $q / B_{T} \sim 1 / l p$ )
- Predicts co-lp vel. for typical inboard X-pt


## Developing the Rotation Database

## Exp. Main-ion Rotation is Inferred From Impurity Measurements

- $\Omega_{c}<\Omega_{D} \rightarrow$ Impurity rotation not suitable for theory validation.
- Main-ion rot. ( $\Omega_{D}$ ) inferred from measured impurity rot. on LFS $\left(\Omega_{c}\right)$ by analytically calculating their Neoclassical offset. ${ }^{1}$

- $\Omega_{c}$ corrected for PS flow to obtain rigid rot.
- $\Omega_{D}$ validated with direct CER measurements for a limited number of discharges



# Validation of Model is Performed Across Wide Range of Plasma Conditions 

## Database

Main-ion, flux surface averaged ped. top rot.:
44 low-torque discharges ( $\mid$ T|<0.5 N.m), ~14000 CER measurements, L and H mode, $\beta_{N}$ up to 2.5, USN and LSN, Forward and Reverse Ip, ECH and balanced NBI heating

- Potential fluctuations measurements not available ; $L_{T e}$ used as a substitute for $L_{\phi}$. (from experiments $L_{\phi} \sim 1-2 L_{T e}$ )
- Pedestal top radial location identified from fit to $T_{e}$
- X-pt poloidal location is approximately constant for entire database, with X-pt on the inboard side.


## Theory Predicts Monotonic Increase in Rotation with $\beta_{\mathrm{N}}$



- Theory prediction is independent from rotation measurements.
- $d_{c}=1$ for moderate out-board ballooning used for whole database.
- Co-lp spin-up at L-H transition is captured by the increase in Ti| $\left.\right|_{p t}$ and decrease of $\mathrm{L}_{\mathrm{Te}}$ resulting from steepening of gradients.


## for Moderate $\beta_{N}$, Theory in Reasonably Good Agreement with Experiments



- For L-mode further fine-tuning can be made by fitting for the optimum ballooning parameter $d_{c},\left(d_{c} \sim 1.6\right.$, stronger ballooning for L-mode than $H$ mode).
- For ITER baseline, theory predicts a ped. top intrinsic rotation of $\sim 4 \mathrm{kRad} / \mathrm{s}$. (Chrystal PoP 2017 predicted ~ 7 kRad/s).


## at High $\beta_{N}$, Intrinsic Theory Does Not Predict the Observed Rotation Degradation




- Disagreement at high $\beta_{N} \rightarrow$ problematic if model will be used for high performance conditions.
D)PPPL


## Why is there a disagreement at high $\beta_{N}$ ??

## Parameter Range with Large Disagreement with Theory is Isolated at High $\mathrm{P}_{\mathrm{NBI}}$



- $P_{\text {NBII }}>4$ MW rotation velocities are lower than $\mathrm{P}_{\mathrm{NBI}}<4 \mathrm{MW}$.


## This Database Connects with Previous Mystery of Main-Ion Rotation Degradation at High $\mathrm{P}_{\mathrm{NBI}}$ in DIII-D

- $D^{+}$rotation degradation at high $\beta_{N}$ is consistent with previous reports in DIII-D for $\mathrm{He}^{2+}$.



## For High $\mathrm{P}_{\mathrm{NBI}}$, Large Ctr-Ip Torque in the Edge (>1 N.m) Leads to Rot. Degradation

$$
\begin{gathered}
\mathbf{j}_{\mathbf{f}}=-\mathbf{j}_{\mathbf{i}} \longrightarrow \begin{array}{l}
\mathbf{j}_{\mathbf{i}} \times \mathbf{B} \text { torque } \\
\text { on plasma }
\end{array} \\
\frac{d}{d t} \int \rho R V_{\varphi} d V=-\dot{N} e \Delta \psi \quad \begin{array}{l}
\text { - Transport and } \\
\text { friction losses }
\end{array}
\end{gathered}
$$

- In the absence of collisions canonical angular momentum is conserved

$$
p_{\varphi}=m_{i} R v_{\varphi}-e \psi
$$

- Radial displacement:

$$
\begin{aligned}
& \text { co-lp } \quad e \Delta \psi=-m_{i} R_{0} v_{\varphi 0}<0 \quad \text { (banana orbit) } \\
& \text { ctr-Ip } e \Delta \psi=m_{i} R_{s} v_{\varphi s}-m_{i} R_{0} v_{\varphi 0}>0 \text { (lost orbit) }
\end{aligned}
$$



When both directions of NBI are applied and are "nominally balanced", the total torque absorbed by the plasma can be net ctr-lp, due to fast ion prompt loss ${ }^{1}$

## TRANSP/NUBEAM Calculation Performed for Entire Database

Discharge 157557:
$\beta_{\mathrm{N}} \sim 1.2 \quad$ (moderate)
$\mathrm{P}_{\mathrm{NBI}} \sim$ 3.25 MW
Discharge 157906:
$\beta_{\mathrm{N}} \sim 2 \quad$ (high) $\rightarrow$ degraded rot. $\mathrm{P}_{\mathrm{NBI}} \sim$ 6.5 MW

- Torque deposited in core(edge) is co-lp (ctr-lp)
- Note: NBI alternates between co\&ctr; at CER measurement times prompt edge torque can be larger or smaller than average.

Vol. integrated absorbed torque (N.m), calculated by TRANSP


# Calculating the Rotation With Finite Torque is Required for Both Validation and Future Predictions 

- Simplified momentum evolution equation for the edge:

$$
\frac{d \omega_{\tau}}{d t}=-\frac{\omega_{\tau}}{t_{\phi}^{\text {edge }}}+\frac{\tau_{\text {edge }}}{I_{\text {edge }}} \quad \Longrightarrow \omega_{\tau}(t)=\int_{-\infty}^{t} \mathrm{e}^{-\left(t-t^{\prime}\right) / t_{\phi}^{\text {edge }}} \frac{\tau_{\text {edge }}\left(t^{\prime}\right)}{I} d t^{\prime}
$$

- Edge momentum transport time: $t_{\phi}^{\text {edge }} \sim \frac{L_{\mathrm{ped}}^{2}}{\chi_{\phi}^{\text {edge }}} \approx \frac{L_{\mathrm{ped}}^{2}}{\operatorname{Pr} \chi_{i}^{\text {edge }}}$
- Theory of turbulent transport predicts $\chi_{\mathrm{i}}$ and $\chi_{\varphi}$ to be comparable: the Prandtl number, $\operatorname{Pr}=\chi_{\phi} / \chi_{i}$ is expected to be close to unity

$$
\chi_{\phi}^{\text {edge }} \approx \operatorname{Pr} \chi_{i}^{\text {edge }}
$$

- Pr estimated analytically ${ }^{1}$; a single value used for DB
- $\chi_{i}^{\text {edge }}$ is estimated using $Q_{i}^{\text {edge }}$ (heat flux through edge) computed by TRANSP.


## For High $\mathrm{P}_{\mathrm{NBI}}$, Calculated Net Torque in the Edge $\left(\tau_{\text {edge }}\right)$ is Ctr-lp and Large (>1 N.m)

Net torque in the edge:
(using TRANSP/NUBEAM)

Ctr-lp torque is collisionless $\mathbf{J} \times \mathrm{B}$, promptly absorbed in edge.

Co-lp torque is deposited in the core, transported to the edge slowly $\mathbf{t} \sim t_{\phi}^{\text {core }}$; smoothed out in the process.

prompt averaged over $t_{\phi}^{\text {core }}$


For $P_{\text {NBI }}>4$ MW, while calculated torque ejected from beam holes is $\mid$ tinj|< 0.5 , net edge torque can be <-1 N.m

## Calculated Ctr-lp Rotation Shift is Large for High $\boldsymbol{\beta}_{\mathrm{N}}$



- Large ctr-lp torque at high $P_{\text {NBI }}(\sim-1 \mathrm{~N} . \mathrm{m}) \Longrightarrow \omega_{\mathrm{T}} \sim-30 \mathrm{kRad} / \mathrm{s}$
- Finite rotation change for moderate- $\beta_{\mathrm{N}} \mathrm{H}$-mode ( $\sim$ - $7 \mathrm{kRad} / \mathrm{s}$ )


## NBI Torque Correction to Intrinsic Theory Improves the Predicted Rotation



- Torque corrected rotation (red) is much closer than intrinsic values (blue) to the experimental rotations ( $\omega_{\tau} \sim-30 \mathrm{kRad} / \mathrm{s}$ ).

- Torque correction brings the theory prediction for H -mode closer to the experimental values ( $\omega_{\mathrm{T}} \sim-7 \mathrm{kRad} / \mathrm{s}$ ).

A New Rotation Model, Tested in Low-Torque DIII-D Plasmas, is Available for Predictive Simulations

- Pedestal top rotation prediction shows good agreement for DIIID low-torque rotation database.
- Model now includes finite NBI torques $\rightarrow$ uses simple inputs available to modeling codes.
- Future Research :
- Using direct measurements of turbulent fluctuation profiles, ballooning (in/out asymmetry) for L- and H-mode, for more accurate predictions.
- Exploring ELMs and 3D fields effect on momentum/rotation.
- Exploring the intrinsic rotation generation in higher collisionality regimes


## Supplemental Slides

Rotation degradation - Current Dependence

Forward Ip discharges 157910 and 164988 have the lowest $\omega_{T}$, comparable to the values in moderate $\beta_{\mathrm{N}}$ range
$>$ Edge CER measurements use the 330 beams.

330 beam is in positive tor. direction; contributes a co-lp(ctr-lp) torque when Ip is directed forward(reverse).

Discharge 164988 has largest Ip ~1.25 MA; smaller radial steps result in less orbit loss.


## Important to Correct For Difference Between Impurity and Main-ion Toroidal Rotation



- Main-ion rotation > carbon rotation, consistent w/ direct CER.
- L-to-H transition at $\beta_{\mathrm{N}} \sim 0.75$.


## Impurity and Main-Ion Both Show Degraded Rotation at High $\beta_{N}$



- Rot. increases in L-to-H transition.
- From medium to high $\beta_{N} \mathrm{H}$-modes (crossing $\beta_{N} \sim 1.7$ ) rot. decreases for both carbon and deuterium (due to finite NBI edge torque)

DPPPL

## Exp. Main-lon Rotation is Independent of Ped. Top Density

Exp. main-ion rotation

$\beta_{N}<0.75($ L-mode $)$
$0.75<\beta_{\mathrm{N}}<1.7\left(\right.$ moderate $\left.\beta_{N}\right)$
$\beta_{\mathrm{N}}>1.7\left(\right.$ high $\left.\beta_{\mathrm{N}}\right)$

- For each $\beta_{N}$ range in database, main-ion rotation is independent of pedestal-top main-ion density.
- Theory prediction is also independent of density.


## Parameter Range with Large Disagreement with Theory is Isolated at High $\mathrm{P}_{\mathrm{NBI}}$



- Good agreement at moderate $\mathrm{P}_{\mathrm{NBI}}$

- For high $\mathrm{P}_{\mathrm{NBI}}$ theory prediction is significantly larger than exp.

DPPPL

