The development and validation of the Resonance line Broadened Quasi-linear (RBQ) model for the Alfvénic mode driven fast ion relaxation

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PPPL, 13 November, 2017

This work is supported in part by DoE contract No. DE-AC02-09CH11466

Why reduced (QL) models? And why now?

- *Linear theory of Energetic Particle (EP) Alfvénic Instabilities is well developed and understood.*
- *Diagnostic tools are maturing for detailed measurements of EP distribution (V&V).*
- *Burning plasma XPs are nearing but expensive, multi billion dollars devices.*
- *Need to make confident predictions on the operational regimes.*
    - PIC/continuum codes (MEGA, Todo et al.'16,'17) provide good resolution but expensive to run (recent DIIID experiment takes 1-5 months to run) ⇒ need efficient **reduced models (not even BP problems).**
    - Initial value (legacy) codes exist within PPPL, capable of addressing EP transport with AE present: M3D-K, M3D-C1, HYM, GTS. **Our estimates show that RBQ2D will require *several petaflop computers.***
    - Efficient realistic RBQ (this talk) formalism is targeted by RBQ1D + NOVA-K codes for TRANSP/WDM simulations.

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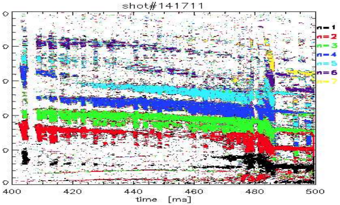
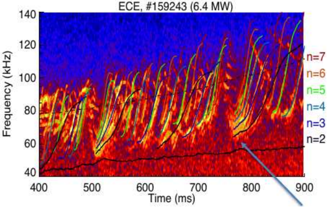
Both regimes need to be addressed in present experiments

chirping NSTX & *steady state DIII-D*





N.N. Gorelenkov, PoP'05, IAEA'15 (or R. Waltz stiff transport, APS'14) CGM (crit.gradient model) does not provide phase space resolution!!

1. Linear perturbative AE simulations are the basis for RBQ
   - Hybrid NOVA/NOVA-K suite of codes, ...
   - Realistic resonance broadened QL (RBQ) formalism is available: Dupree'66, Kaufman'72, Berk et al.,'95, Ghantous, PhD'14, Duarte, PhD'17, Gorelenkov et al., IAEA'17.

Steady-state (QL compatible) and (marginally expected) **chirping** frequency, **regimes** are predicted in BPs, ITER!

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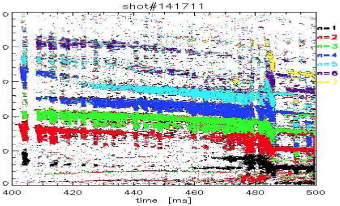
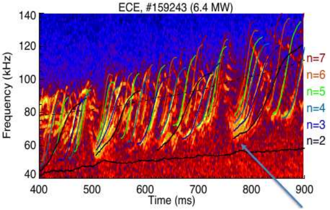
chirping NSTX & *steady state DIII-D*





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Outline

Flux coordinate NOVA-K formalism helps address WPI (wave-particle interaction)

3 constants of motion (COM):

Canonical toroidal momentum

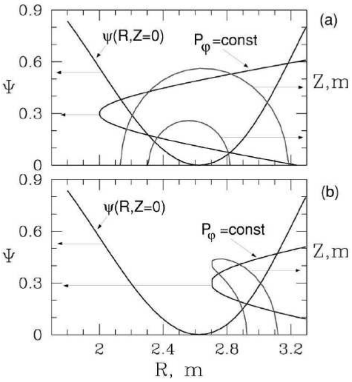
$$P_\varphi = \frac{e\psi}{mc} - \sigma_\| v \sqrt{1-\lambda B/B_0} R \frac{B_\varphi}{B},$$

magnetic moment

$$\mu \equiv \frac{\mathscr{E}_\perp^2}{B} \equiv \frac{\lambda \mathscr{E}}{B_0},$$

kinetic energy

$$\mathscr{E} = \frac{v^2}{2}.$$



TFTR #103101 NOVA orbits (Gorelenkov, PoP'99)

Realistic representation of EP guiding center orbits for RBQ is employed:

FOW effects are included via drift orbit numerically & FLR effects analytically, $J_0(k_\perp \rho_{h\perp})$. Orbit averaging is straightforward to represent WPI matrices $g \sim \langle \boldsymbol{E} \cdot \boldsymbol{v}_d \rangle$.

RBQ1D diffusion operator through pitch angle scattering

Action-angle formalism (Kaufman'72) through flux variables, ψ, v, $\lambda \equiv \mu B_0$, where $\mu = v_\perp^2/2B$. Set of equations for particle DF follows from Vlasov kinetic equation as per *H.L.Berk, B.N.Breizman et al. NF'95, Fitzpatrick PhD'97*

$$\frac{\partial f}{\partial t} = \pi \sum_{l,M} \frac{\partial}{\partial P_\varphi} C_M^2 \mathscr{F}_{lM} \frac{\partial}{\partial P_\varphi} f_{lM} + \nu_{eff}^3 \sum_{l,M} \frac{\partial^2}{\partial P_\varphi^2} \left(f_{lM} - f_0\right),$$

where EP distribution is evolved due to scattering terms on RHS amended by the Krook "source" operator. AE amplitude satisfies

$$C_M(t) \sim e^{(\gamma_L - \gamma_d)t} \Rightarrow \frac{dC_M^2}{dt} = 2\left(\gamma_L - \gamma_d\right) C_M^2.$$

RBQ maintains perturbative *AE structure, but DF is evolved *(Duarte, PhD'17 on RBQ model).*

RBQ1D future goal: 2D diffusion resolution in $\mathscr{E}, P_\varphi$. Benefits are:

- Realistic computations of CD, loss distribution over the first wall.
- Time efficient.

WPI resonances broadened in P_φ: ω_{bWPI}(amplitude) + γ_L(growth) + ν_{scatt}(scat)

Inherited from Dupree'66 - broadening is the platform for momentum and energy exchange between particles and waves:

$$\delta\left(\Omega = \omega - n\dot{\varphi} - m\dot{\theta} - l\omega_b\right) \to \mathscr{F}\left(\Delta P_\varphi\right) - window\ \ function,$$

where $\Delta P_\varphi \Omega'_{P_\varphi} = c_\omega \omega_{bWPI} + c_\gamma \gamma_L + c_\nu \nu_{scatt}$, $c_{\omega,\gamma,\nu}$ are chosen for 1D following, *K.Ghantous, et al. PoP'14.*

RBQ1D assumptions for ion diffusion due to waves for broadened terms, $\omega_{bWPI}\&\gamma_L\&\nu_{scatt}$:

$$\Delta\mathscr{E} = \frac{\omega}{n}\Delta P_\varphi, \quad \omega \to 0.$$

But could be different depending on the process. For RBQ1D we have in mind the following recipes for tests if appropriate:

- ω_{bWPI}: $\Delta\mathscr{E} = \frac{\omega}{n}\Delta P_\varphi$; $\Delta\mu = 0$.
- γ_L: $\Delta\mathscr{E} = \frac{\omega}{n}\Delta P_\varphi$; $\Delta\mu = 0$ (small, hard to demonstrate with ORBIT).
- Collisional ν_{scat}: $\Delta\mathscr{E} = 0$; $\Delta P_\varphi \neq 0$; $\Delta\mu \neq / = 0$.
    - anomalous turbulent diffusion ν_{asc}: $\Delta\mathscr{E} = \frac{\omega_{asc}}{n_{asc}}\Delta P_\varphi$; $\Delta\mu = 0$, what are ω_{asc}, n_{asc}.

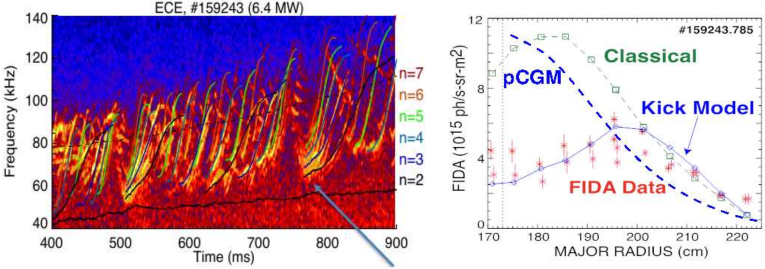
Engage ORBIT in quantitative estimates of EP single particle dynamics *(G.Meng, IAEA'17).*

Building blocks of RBQ1D - most recent progress (IAEA TCM'17)

- Preprocessing for RBQ: ideal MHD (NOVA) + kinetic (NOVA-K) to compute WPI matrices
  - Use built-in **chirping criterion** to determine the QL model (non-)applicability. ITER is marginally QL-compatible in the steady state regime.
- RBQ simultaneous solution of multiple AE amplitude evolution system.
  - RBQ in predictive runs finds self-consistent AE amplitudes and evolves them.
- Evolve EP distribution function.
  - Employ diffusion solver for EP distribution advance.
- Compute EP diffusion for WDM.
  - Employ PDF (probability distribution function) - make use of the kick model to TRANSP interface.
- Process EP diffusion within WDM (TRANSP).

Apply to DIII-D case studied in CGM experiments

Detailed analysis of DIII-D CGM XP has been done, *AEs, #159243, 6.4*MW* *(Collins et al, PRL'16, Heidbrink et al., PoP'17)*.



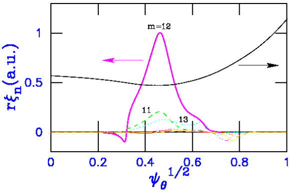
805msec is chosen near rational q_{min} for detailed study

Earlier developed pCGM does not reproduce hollow EP profiles in radius and underestimate the neutron deficit by the factor of 2.

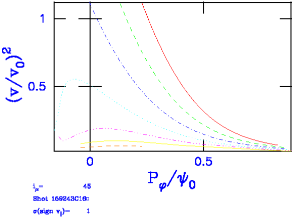
⇒ need velocity space resolution such as in the Kick Model *(Podesta et al., PPCF'14)*

Initial RBQ application is for one $n = 4$ RSAE (at q_{min})

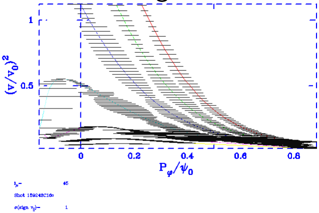
Structure from NOVA



Resonances from NOVA-K



Broadening from RBQ



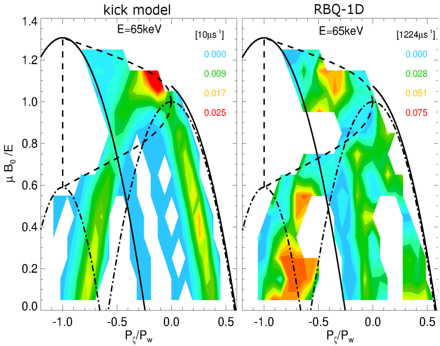
EP ion location $\Longleftrightarrow$ one point on $(v/v_0)^2$ vs P_φ COM (const.-of-motion) plane. Broadening is defined by RSAE amplitude $\delta B_\theta/B = 7 \times 10^{-3}$, $f = 84kHz$, *Collins'16 PRL*.
The choice of this RSAE is critical to understand the EP transport leading to hollow EP pressure profiles in DIIID.

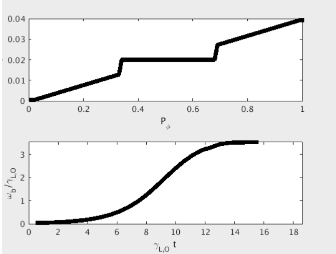
On the output the results are written in terms of PDFs of the kick model

Resonances exist throughout the plasma even though TAE is at $0.4 < \sqrt{\psi_\theta \left\{P_\varphi\right\}} < 0.6$.

PDF COM contour map

selected resonance broadening & amplitude evolution



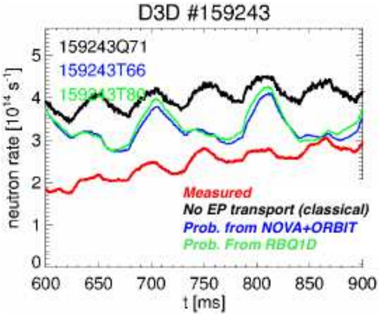


In RBQ 7 dominant bounce frequency harmonics are kept, out of 40.

RBQ growth rates are computed at each time step to and compared with NOVA-K.

EP driven γ/ω is computed decreasing at each step after broadening.

RBQ and kick model can operate in interpretive regimes for TRANSP/WDM



- both models give similar EP confinement, the neutron rate
- study most unstable region near q_{min}, $s=0$ to:
  - address scientific quest on the reasons for hollow EP pressure profile - could RSAE be the reason for that?
  - both codes demonstrate that RSAEs open up a channel for fast EP transport through q_{min} region

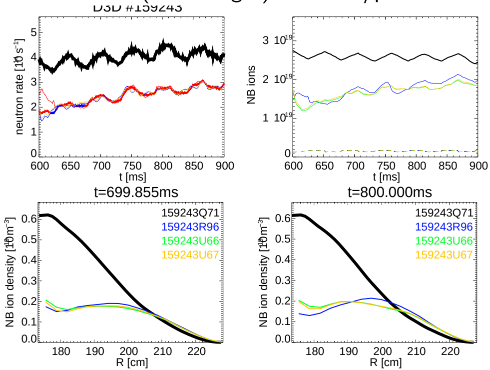
The broadened resonances overlap because the amplitude is large $\delta B/B \simeq 1\%$. .

The diffusion is almost 1D in P_φ, $\Delta\mathscr{E} \sim 0$; $\Delta P_\varphi \neq 0$ will be assumed.

Near zero derivative $\partial\Omega/\partial P_\varphi$ requires regularization of Taylor-like expansion.

11 modes are chosen for RBQ1D in the interpretive version as per kick model

Color-coded (on the right) evolution/profiles



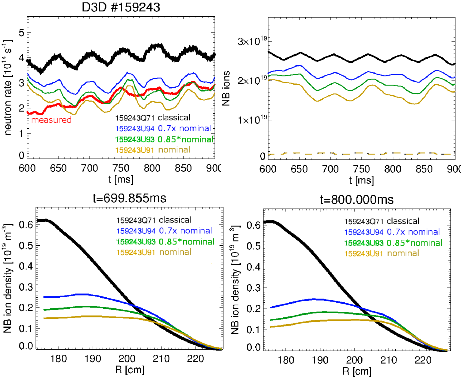
- Classical NUBEAM calculations no AE effects
- Kick model + with neutron flux constraint
- RBQ with neutron flux constraint
- measured (if shown) beam ion profile

First RBQ applications in the interpretive mode agree with measurements.

PDFs need to be reworked to a better representation of EP diffusion in the COM phase space.

Apply predictive RBQ1D analysis with fixed amplitudes

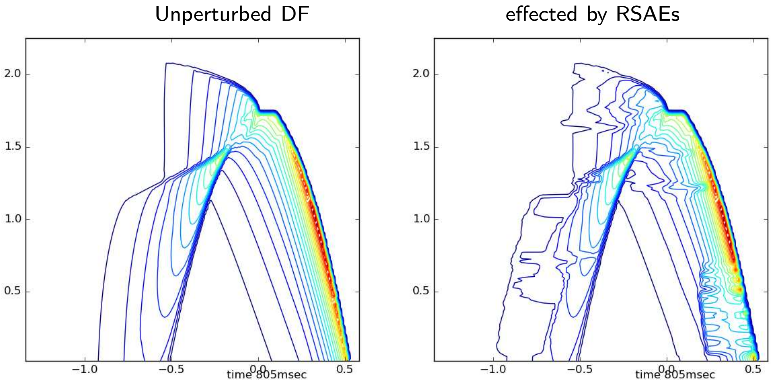
Color-coded (in figures) evolution/profiles



- Assumptions: distribution is evolved by RBQ to compute amplitudes at $\gamma_L = \gamma_d$.
- AE amplitudes are computed in RBQ1D at $t = 805 msec$, - time of measurements.
- Amplitudes are kept the same throughout observed times.
- Neutron rate is consistent with measurements.
- Hollow EP density is due COM location sensitive diffusion.

- First RBQ application in predictive mode agrees with neutron deficit from measurements if amplitude is reduced by $\sim 15\%$.
- Need to substitute PDF approach to EP diffusion in the COM phase space for better resolution/intermittency.

Distribution evolves near resonances such as for this $n = 4$ RSAE



Strong redistribution is seen for well passing ions.
Shown are injected and slowed down ions, $\mathscr{E} \simeq 60 - 70 keV$.

Status and plans for RBQ

- RBQ model is being built based on QL approach *(Dupree, PhFl'66, Kaufman, PhFl'72, Fried et al., UCLA'72, Berk, et al.,NF'95,PoP'96)*
- Broadening technique is included for non-slanted 1D mesh.
- RBQ employs full diffusion solver module prepared for single and multiple resonance problem.
- RBQ1D is applied to DIII-D cases for V&V. (Gorelenkov, et al., IAEA TCM'17, NF'17 to be published)
  Can we apply RBQ1D to NSTX-U with chirping frequency AEs?
- 2D extension will be developed within ISEP SciDac.
  - 1D problem requires direct knowledge of diffusion coefficients in WDM
  - & significant computational resources:
    - conservative estimates for RBQ approach imply 2.5 *PFlops* computer power for BP.
- ISEP SciDac deliverable could be at risk without full support: full postdoc position.