Recent Progress in EM-GTS code

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Abstract

- The Startsev-Lee (S-L) EM scheme has been reformulated for general geometry.

- The scheme shows a lot of promise for the simulation of the micro-tearing modes in ST tokamaks like NSTX.

- We have also implemented the Mishichenko's (MS) general geometry EM scheme into EM-GTS for cross-scheme-verification.

- The EM-GTS is currently being used to study the electromagnetic modes in the pedestal region of the DIIID.

- The benchmarking of the code in the Cyclone and pedestal DIIID geometry against published results is ongoing to better understand the resolution requirements of the implemented EM schemes.

S-L EM scheme description: Vlasov Equations

- The linearized Vlasov equation is

$$\partial_t \delta f + \hat{L}\delta f = -(\delta \hat{L}) F_0$$

- Here $\hat{L} = v_\parallel \partial_\parallel + \mathbf{v}_{dr} \cdot \nabla + \dot{v}_\parallel \partial/\partial v_\parallel + C_{col} \equiv v_\parallel \partial_\parallel + C_{col} + \hat{L}_{dr}$.

- with $\delta f = w F_0$, equation for the weight w becomes

$$\partial_t w + \hat{L} w = -\frac{(\delta \hat{L}) F_0}{F_0} = -\frac{[E_\parallel v_\parallel - (\mathbf{v}_{dr} \cdot \nabla)\phi]}{T} + \kappa \cdot \mathbf{v}_E [\phi - v_\parallel A_\parallel] + O(\rho^*)$$

- Where, $O(\rho^*)$ means all terms which are ρ^* time smaller than the kept terms. Here $\kappa = -\nabla \log n - (v^2/2T - 3/2)\nabla \log T$.

- Defining the new weight $w = \widehat{w} + \kappa \cdot \mathbf{v}_E [\int dt \phi]$, equation for the new weight $\widehat{w}$ becomes

$$\partial_t \widehat{w} + \hat{L}\widehat{w} = -\frac{[E_\parallel v_\parallel - (\mathbf{v}_{dr} \cdot \nabla)\phi]}{T} + \kappa \cdot \mathbf{v}_E \left[\int dt E_\parallel\right] - \hat{L}_{dr} \kappa \cdot \mathbf{v}_E \left[\int dt \phi\right] \equiv RHS$$

- Defining the second weight $\bar{w} = \hat{L}\widehat{w}$, we obtain the equation for $\bar{w}$:

$$\partial_t \bar{w} + \hat{L}\bar{w} = \hat{L}(RHS)$$

S-L EM scheme description: Field Equations

- With the new weights the gyro-kinetic Poisson's equations becomes

$$\nabla^2\phi =< w >=< \widehat{w} > + < \kappa \cdot \mathbf{v}_E \left[\int dt\phi \right] >$$

- Using $E_\parallel = -\partial_t A_\parallel - \partial_\parallel \phi$, the time derivative of the Ampere's equation

$$\nabla^2 A_\parallel = \beta < v_\parallel w >= \beta < v_\parallel \widehat{w} >$$

- becomes:

$$\nabla^2(-E_\parallel - \partial_\parallel\phi) = \beta < v_\parallel \partial_t \widehat{w} >= -\beta < v_\parallel \hat{L}\widehat{w} > +\beta < v_\parallel RHS >= \beta < v_\parallel (RHS - \bar{w}) >$$

- In the single weight formulation

$$< v_\parallel \bar{w} >\equiv< v_\parallel \hat{L}\widehat{w} >= \nabla\cdot < v_\parallel \mathbf{v}\widehat{w} > + < (\nu(v)v_\parallel - \dot{v_\parallel})\widehat{w} >$$

- The double weight formulation avoids calculation of the divergence of the highly fluctuating quantity $< v_\parallel \mathbf{v}\widehat{w} >$.

- The source term for the Vlasov equation for $\bar{w}$ can be calculated analytically or numerically as

$$\hat{L}(RHS) = \frac{RHS(\mathbf{z} + \dot{\mathbf{z}}dt, t) - RHS(\mathbf{z} - \dot{\mathbf{z}}dt, t)}{2dt}$$

Alternative Approach

- Alternatively, define the new weight w_M as $\widehat{w} = w_M - v_\parallel \int dt E_\parallel / T$, which satisfies the equation

$$\begin{aligned} \partial_t w_M + \hat{L} w_M &= \frac{[v_\parallel (\mathbf{v}\cdot\nabla)\int dt E_\parallel + (\dot{v}_\parallel - \nu(v) v_\parallel)\int dt E_\parallel + (\mathbf{v}_{dr}\cdot\nabla)\phi]}{T} \\ &+ \kappa\cdot\mathbf{v}_E\left[\int dt E_\parallel\right] - \hat{L}_{dr}\kappa\cdot\mathbf{v}_E\left[\int dt\phi\right] \end{aligned}$$

- With this weight the gyro-kinetic Poisson's equation becomes

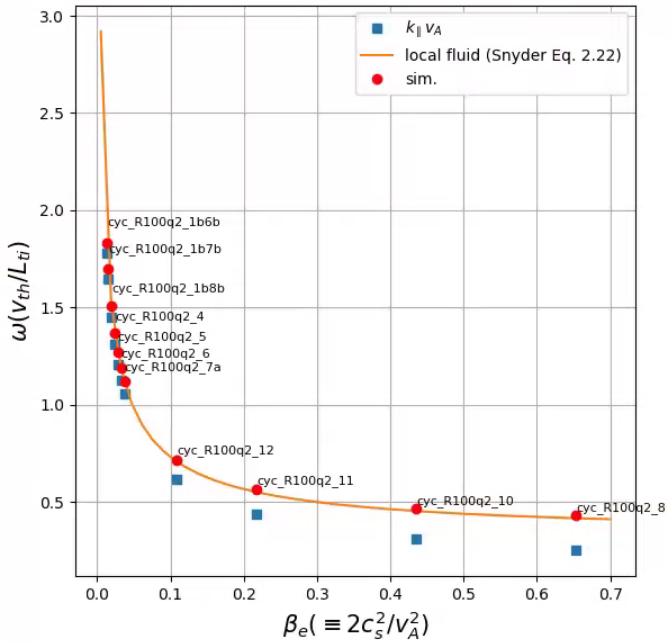
$$\nabla^2\phi = < w_M > + < \kappa\cdot\mathbf{v}_E\left[\int dt\phi\right] >$$

- Using $\int dt E_\parallel = -A_\parallel - \partial_\parallel \int dt\phi$, the Ampere's equation $\nabla^2 A_\parallel = \beta < v_\parallel \widehat{w} >$ becomes:

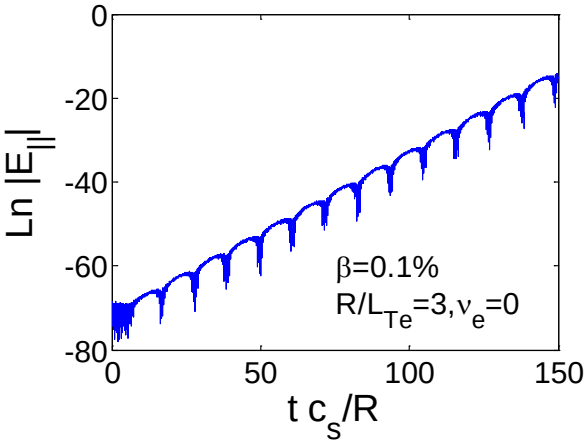
$$(\nabla^2 - \beta < v_\parallel^2 >)\int dt E_\parallel = -\nabla^2\left(\partial_\parallel \int dt\phi\right) - \beta < v_\parallel w_M >$$

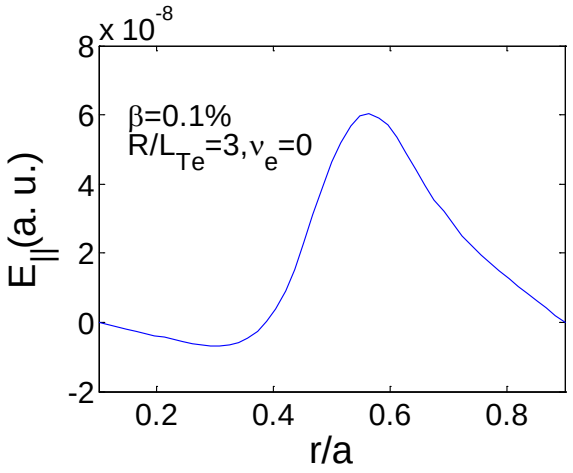
- This approach is similar to the Mishichenko's EM scheme.

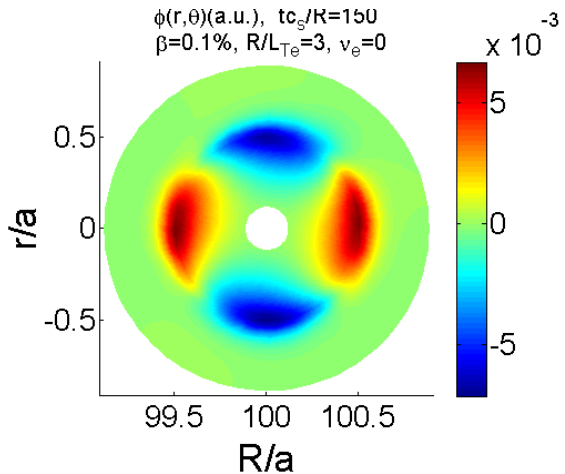
Alfven waves and Drift-tearing modes



- β_e-scan of alfven mode frequency is shown for (m,n)=(2,2) mode in cylindrical geometry with flat q(r)=2.
- Simulations of global drift-tearing (m,n)=(2,1) mode is shown in cylindrical geometry and for electron beta $\beta_e = 0.2\%$ and $R/L_{Te} = 3$ and $\mathrm{q(r)} = 1.7/(1 - r^2/2a^2)$

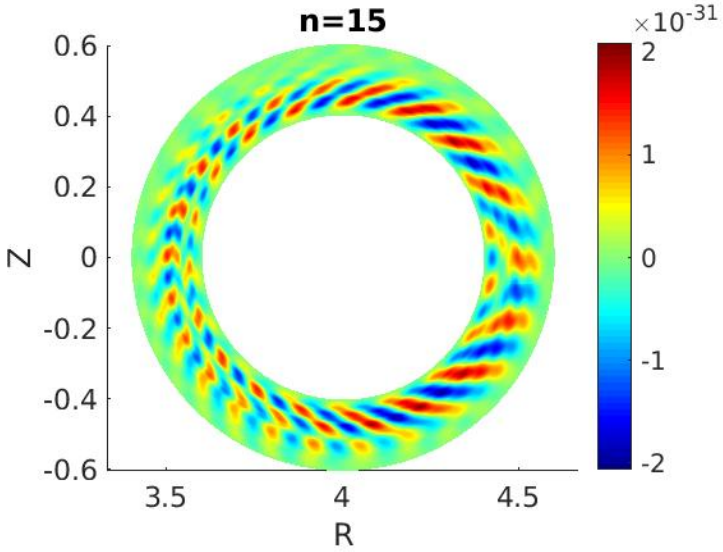






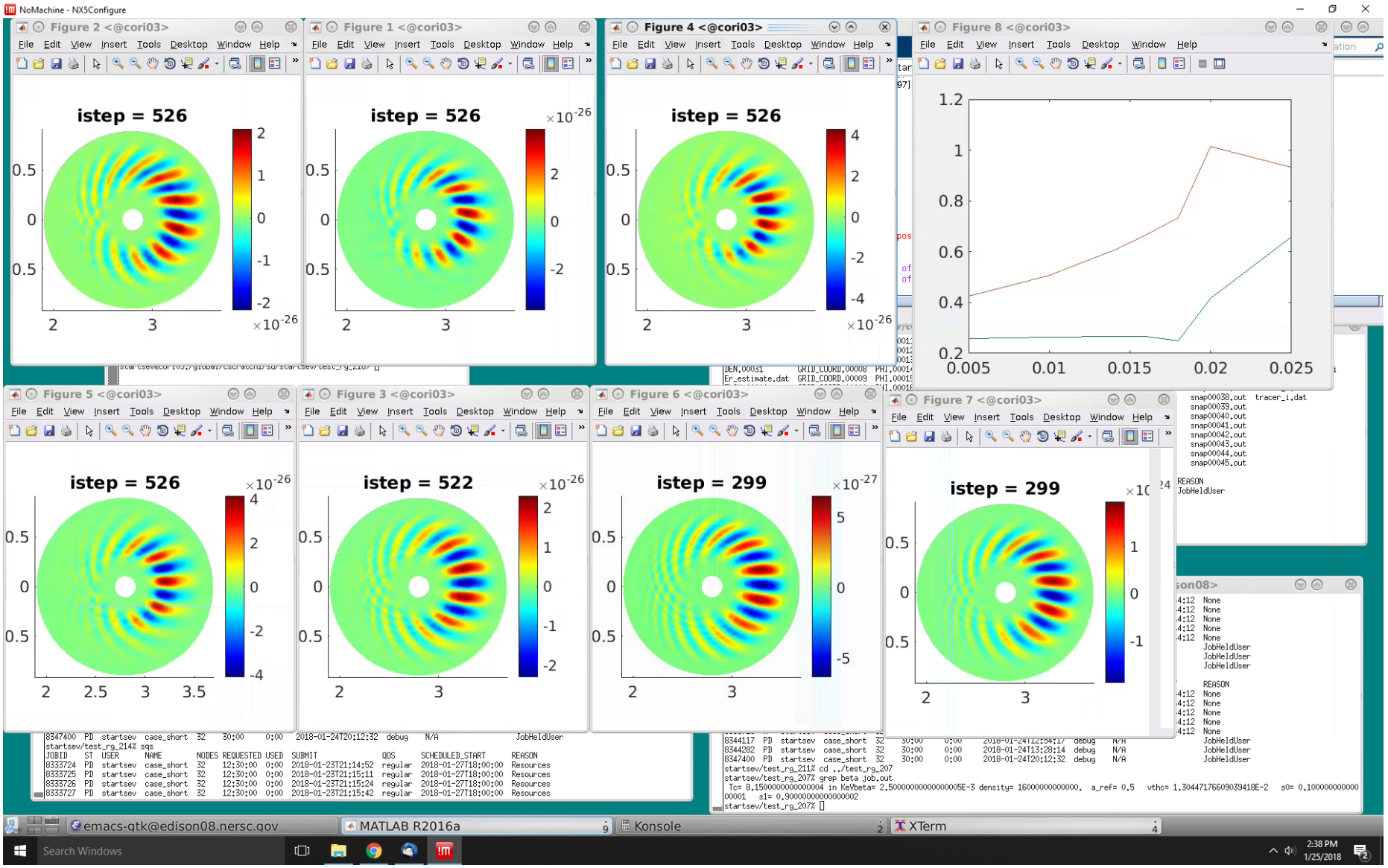
Micro-tearing mode

- Simulation of the semi–collisional micro-tearing mode in the annulus of the circular flux-surfaces tokamak with aspect ratio R/a=4 and for electron beta $\beta_e = 4\%$ and, $a/\varrho_s = 50$, $\nu_{ei} = 0.8\ c_s/a$, $R/L_{Te} = R/L_{Ti} = 7, R/L_n = 0.5$.
- Electrostatic potential ϕ is plotted in the poloidal plane. Classic MTM structure is apparent. The mode rotates in the electron diamagnetic direction (clock-wise).

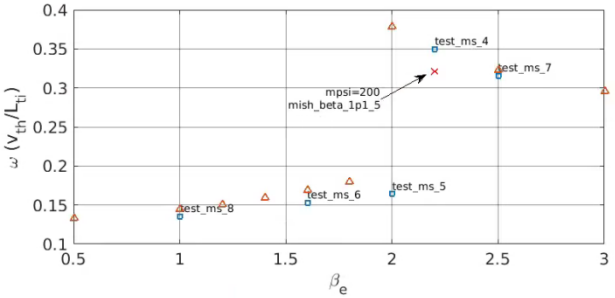


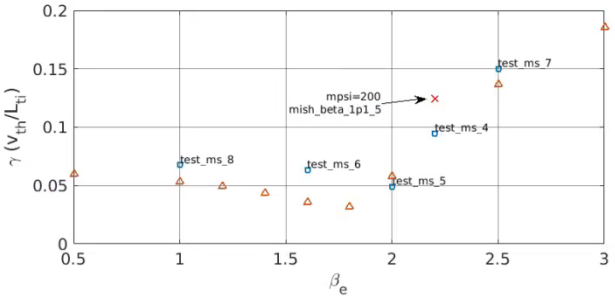
ITG to KBM transition simulation

- β_e-scan of the Cyclone case, $a/\varrho_s = 50\,, n = 6,\ R/L_{Te} = R/L_{Ti} = 6.92, R/L_n = 2.22.$

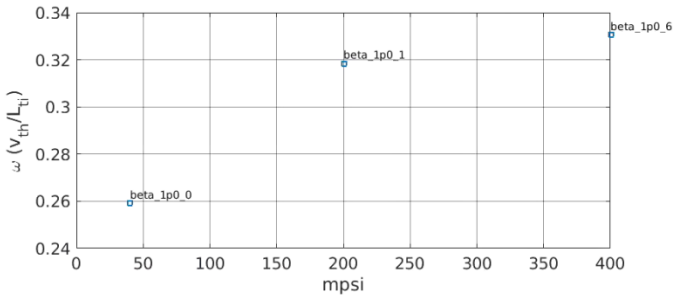


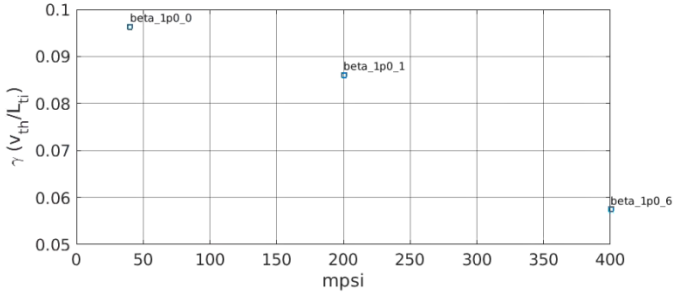
ITG to KBM cross-benchmarking (MS vs. S-L)



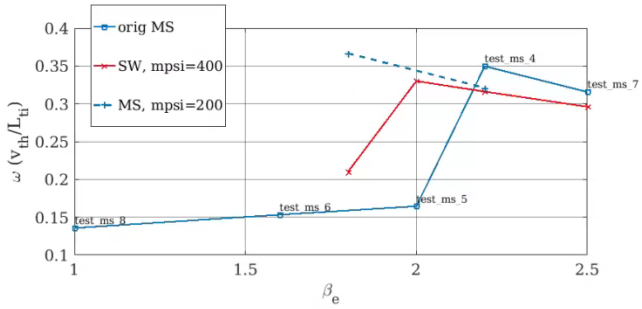


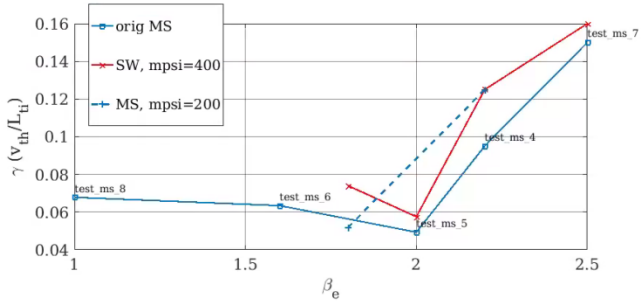
- Red squares are results of MS runs with resolution $\frac{\Delta r}{\varrho_s} = 1.0, (mpsi = 50), \frac{\Delta l_\theta}{\varrho_s} = 0.5.$
- Blue triangles are results of MS runs with resolution $\frac{\Delta r}{\varrho_s} = 0.25, (mpsi = 200), \frac{\Delta l_\theta}{\varrho_s} = 0.5.$





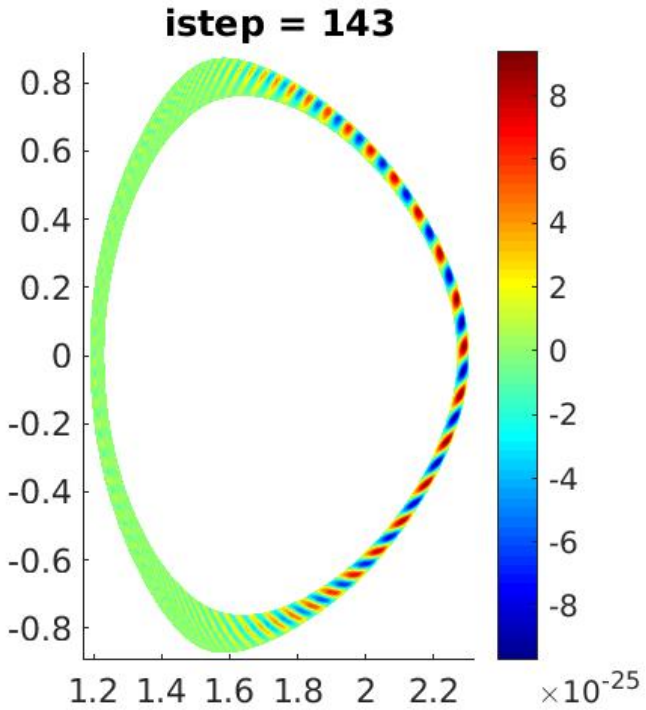
- (left) S-L radial resolution convergence runs.
- (right) Comparison of MS vs. S-L.

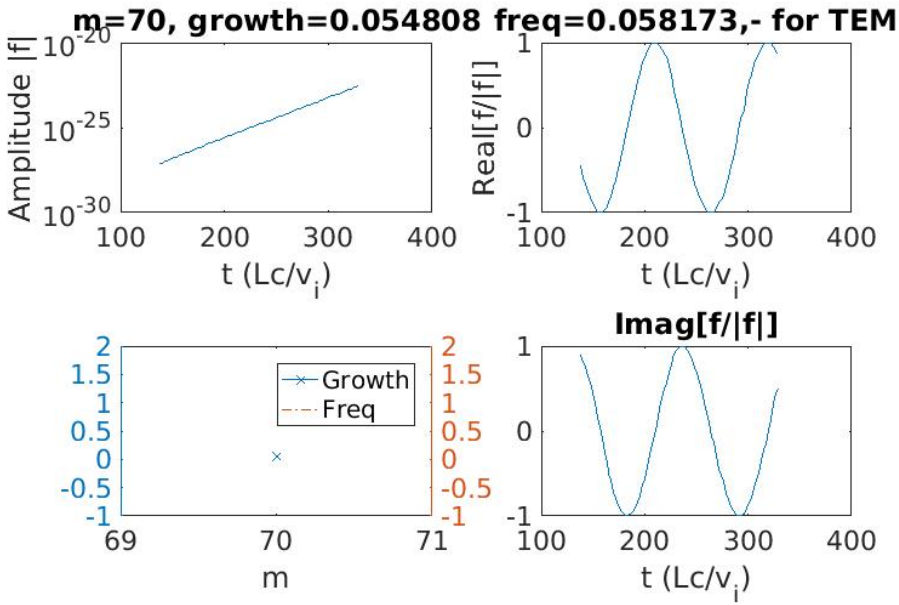




DIII-D pedestal test simulation with EM-GTS

- Simulation of DIII-D test case with EM GTS in pedestal region 0.9<ψ<1. Parameters are the same as in 2015 Holod&Lin paper $\beta_e = 0.37\%$ and R/L_{te}=25, R/L_n=2, R/L_{ti}=30, q(ψ=0.95)=3.5. The mode number n=20.
- The mode is ballooning and rotating in ion diamagnetic direction. The frequency and the growth rate are very close to Holod's result.
- Plotted are time history and 2-d poloidal plot of electrostatic potential, also showing frequency and growth rate in the units of c_s/L_{ti}.





Development of electromagnetic capabilities in GTS relevant for NSTX

- Preliminary simulation of global KBM (Fig.a) and MTM (Fig.b) for NSTX equilibria which is the demonstration that the EM GTS code can operate in NSTX geometry and can produce modes with tearing and ballooning parities.
- Electrostatic potential ϕ is plotted in poloidal plane of NSTX for electron beta increasing from $\beta_e = 0.5\%$. (a), to $\beta_e = 1.6\%$. (b), to $\beta_e = 3.2\%$. (c). As beta increases the unstable low-n micro-tearing mode (MTM) (a) which has its maximum on high-field side switches to high-n MTM (b) on low-field side. As beta is increased further, the kinetic ballooning mode (KBM) is destabilized in low field side of NSTX (c).

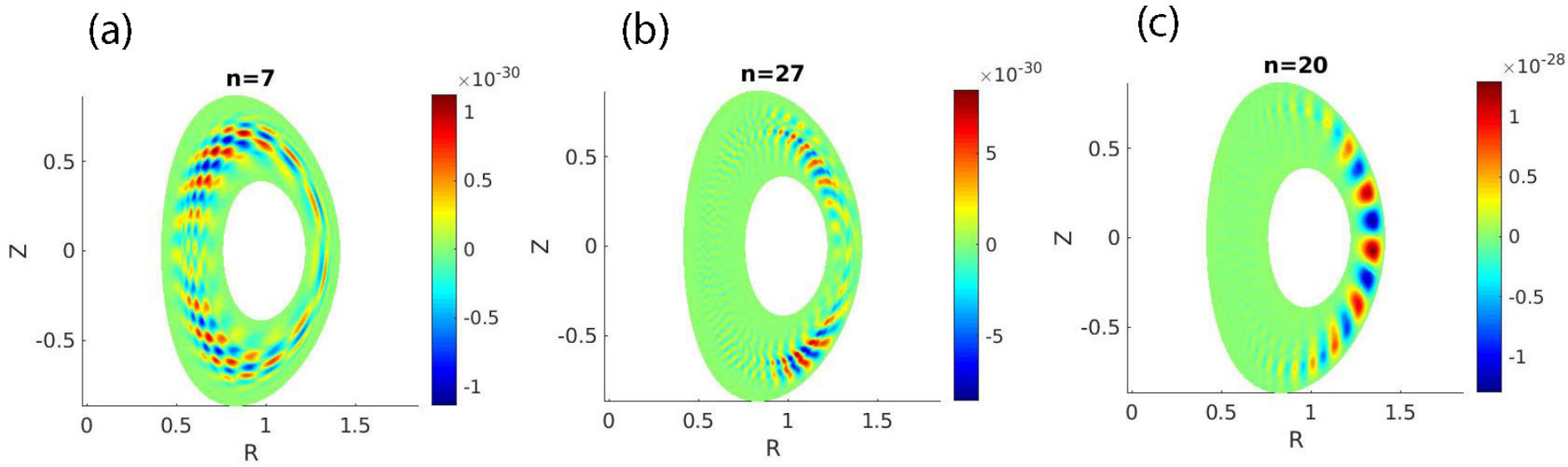


Illustration of Previous Approaches

- In the simplest case with no gradients or collisions and neglecting the ion's gyro-center motion.

- Introducing particle weights $\delta f_e = w_e F_{0e}$, we need to solve the Poisson's and Ampere's law equations

$$\nabla_\perp^2 \phi = \int w_e F_{e0}, \quad \text{and} \quad \nabla_\perp^2 A = \beta \int w_e F_{e0},$$

- where Vlasov equation for electrons is

$$\frac{\partial w_e}{\partial t} + v_\parallel \partial_\parallel w_e = v_\parallel (\partial_\parallel \phi + \partial_t A).$$

- To eliminate $\partial_t A$, introduce new $\bar{w}_e = w_e - v_\parallel A$, so Equation for $\bar{w}_e$ becomes

$$\frac{\partial \bar{w}_e}{\partial t} + v_\parallel \partial_\parallel \bar{w}_e = v_\parallel \partial_\parallel (\phi - v_\parallel A).$$

- Field equations become

$$\nabla_\perp^2 \phi = \int \bar{w}_e F_{e0}, \quad \text{and} \quad (\nabla_\perp^2 - \beta m) A = \beta \int \bar{w}_e F_{e0},$$

- Here $m = m_i/m_e = \int v_\parallel^2 F_{e0}$.

Illustration of Previous Approaches, cont'd

- Solving linear Vlasov equations we obtain set of field equations

$$-k_\perp^2\phi = n_e = -\frac{\omega A - k_\parallel \phi}{k_\parallel}(1+X),$$

$$-(k_\perp^2 + \beta m)A = \beta j_e = \beta\left[-mA - \frac{\omega}{k_\parallel^2}(\omega A - k_\parallel\phi)(1+X)\right],$$

- Here $X = X[\omega/(k_\parallel v_{the})] \approx 0$ for $\omega/k_\parallel \ll v_{the}$.

- To model the errors due to sources in these equations calculated using particles we write

$$-k_\perp^2\phi = n_e = -C_n\frac{\omega A - k_\parallel \phi}{k_\parallel},$$

$$-(k_\perp^2 + C_N\beta m)A = \beta j_e = \beta C_j\left[-mA - \frac{\omega}{k_\parallel^2}(\omega A - k_\parallel\phi)\right],$$

- where C_N is introduced by hand to cancel the error in calculating the sources. All $C \approx 1$.

Illustration of Cancelation Problem

- The dispersion relation

$$\omega^2 = \frac{k_\parallel^2}{\beta}\left(\frac{C_n + k_\perp^2}{C_j}\right)\left[1 + (C_N - C_j)\frac{\beta m}{k_\perp^2}\right].$$

- To get correct alfven wave dispersion we need

$$(C_N - C_j) \ll \frac{k_\perp^2}{\beta m} = (\lambda_e k_\perp)^2.$$

- where $\lambda_e = \rho_i/\sqrt{\beta m} \ll \rho_i$ is electron skin-depth.

- For $k_\perp \sim 1/a$ and $a/\rho_i = 100$, $\beta = 0.01$ and $m = m_i/m_e = 1836$

$$(C_N - C_j) \ll 5 \cdot 10^{-6}.$$

- Therefore, extreme care is required to cancel large terms in Ampere's law to obtain correct low (m,n) shear-alfven wave when $\beta m_i/m_e \gg 1$.

A Better Way

- Instead of two equations for ϕ and A with separately calculated sources

$$-k_\perp^2\phi = n_e = -\frac{\omega A - k_\parallel \phi}{k_\parallel}(1+X),$$

$$-(k_\perp^2 + \beta m)A = \beta j_e = \beta\left[-mA - \frac{\omega}{k_\parallel^2}(\omega A - k_\parallel\phi)(1+X)\right],$$

- Let's combine them into one equation for $E_\parallel = i(\omega A - k_\parallel \phi)$

$$-(k_\perp^2 + \beta m)E_\parallel = Source\left\{\beta\left[-mE_\parallel - \frac{\omega^2}{k_\parallel^2}E_\parallel(1+X)\right] + (1+X)E_\parallel\right\},$$

- Where $Source\{something\}$ means calculated using particles.

- The reason for using $E_\parallel$ and not A and ϕ is because for $\beta m_i/m_e >> 1$

$$k_\parallel\phi \sim \omega A \sim \frac{1}{(\rho_i k_\perp)^2}E_\parallel > E_\parallel, \quad \text{and} \quad v_\parallel(k_\parallel A) \sim \sqrt{\beta\frac{m_i}{m_e}}\frac{1}{(\rho_i k_\perp)^2}E_\parallel >> E_\parallel,$$

A Better Way, cont'd

- For $X \approx 0$, and introducing C factors to model source errors we obtain

$$-(k_\perp^2 + C_N \beta m)E = C_s \left[-\beta m E + \left(1 - \beta \frac{\omega^2}{k_\parallel^2}\right) E \right],$$

- and dispersion relation

$$\omega^2 = \frac{k_\parallel^2}{\beta} \left(\frac{C_s + k_\perp^2}{C_s} \right) \left[1 + (C_N - C_s) \frac{\beta m}{C_s + k_\perp^2} \right].$$

- To get correct alfven wave dispersion we need

$$(C_N - C_s) \ll \frac{C_s}{\beta m} = \left(\frac{\lambda_e}{\rho_i} \right)^2.$$

- For arbitrary $k_\perp$ and $a/\rho_i = 100$, $\beta = 0.01$ and $m = m_i/m_e = 1836$

$$(C_N - C_s) \ll 0.05.$$

- This is easily achieved by using reasonable number of particles per-cell in addition to one-two iterations of field equations.

$$-(k_\perp^2 + \beta m)E_\parallel = \beta Source + \beta \left[Source \left\{ < v_\parallel^2 E_\parallel > \right\} - m E_\parallel \right],$$