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Local compressional and global Alfvén eigenmode structure on NSTX and their effect on core energy transport*

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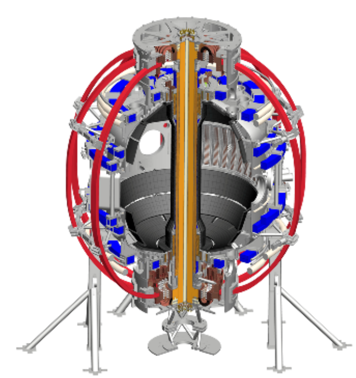
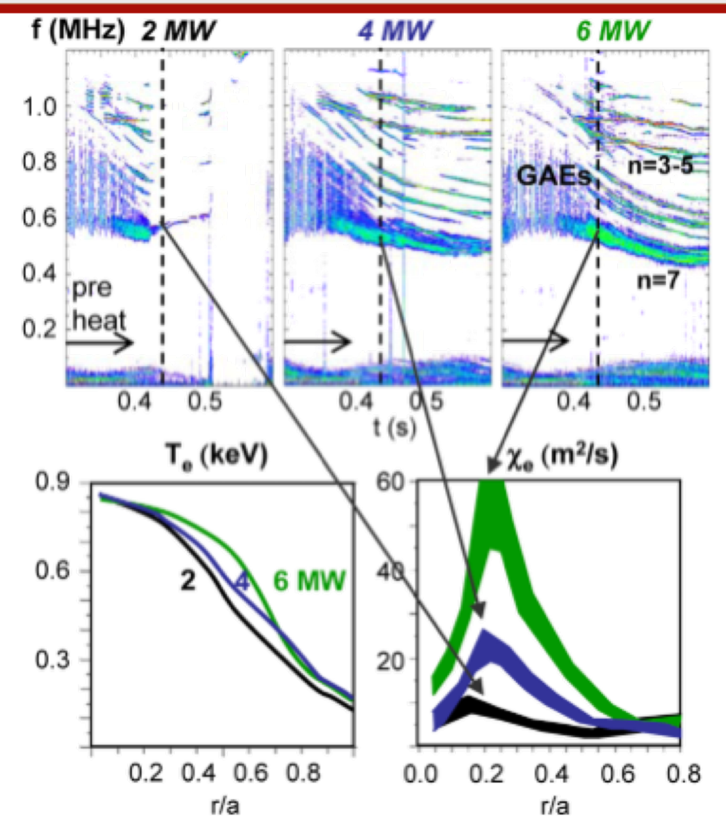


Novel reflectometry analysis improves CAE & GAE measurements and understanding of χ_e

- Motivation – compressional (CAE) and Global Alfvén eigenmodes (GAE) proposed to cause high anomalous core χ_e
- New multi-channel reflectometer analysis $\Rightarrow$ more accurate δn amplitude and structure
- δn used to test hypothesis for anomalous core χ_e
  - simulation of e^- drift orbit modification predicts χ_e – compared to TRANSP
- δn compared to HYM simulations
- δn used with HYM to test alternate hypothesis – CAE-KAW coupling

Motivation: CAEs & GAEs candidates for core energy transport in NSTX

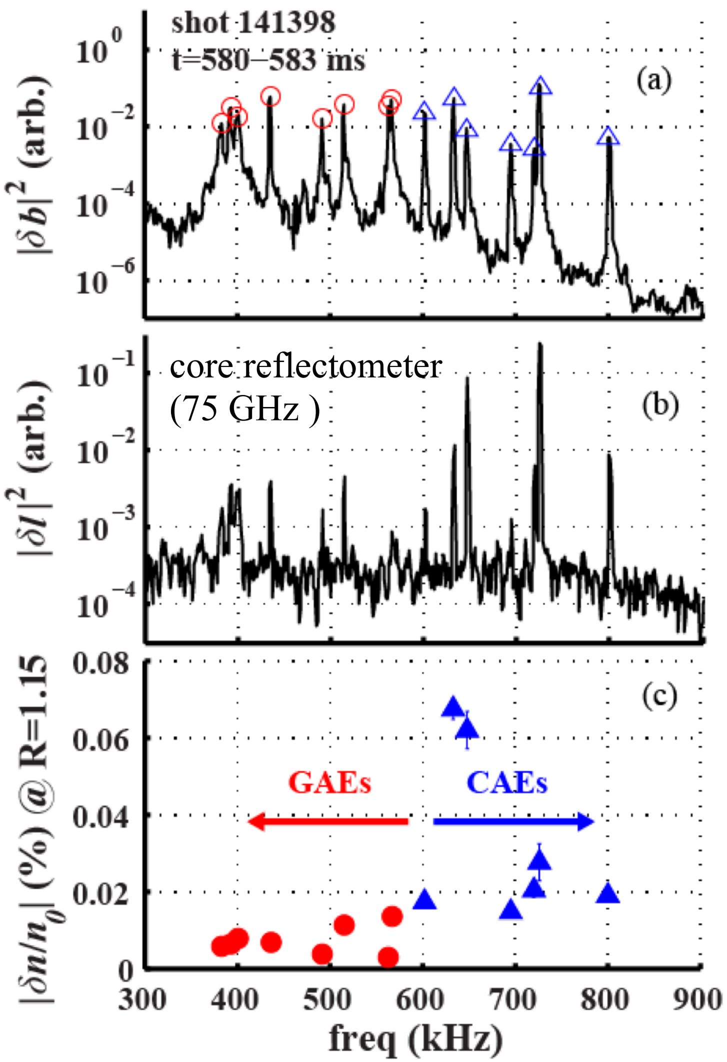
- CAEs & GAEs excited by Doppler-shifted cyclotron resonance with beam ions
  [N. N. Gorelenkov, NF 2003]
- CAE & GAE activity correlates with enhanced χ_e in core
  [D. Stutman, PRL 2009; K. Tritz, APS 2010 Invited Talk; N. A. Crocker, PPCF 2011]
  – T_e profile flattens as P_{NB} increases
  – χ_e from TRANSP modeling
- Two leading hypotheses:
  – Stochastization of e^- guiding center orbits enhance χ_e [NN Gorelenkov, NF 2010]
  – Coupling to KAWs = missing transport channel ⇒ TRANSP gets χ_e wrong [Ya.I. Kolesnichenko, PRL 2010, E.V. Belova, PRL 2015]



[D. Stutman et al., PRL 102 115002 (2009)]

Reflectometer array measures δn of CAEs & GAEs

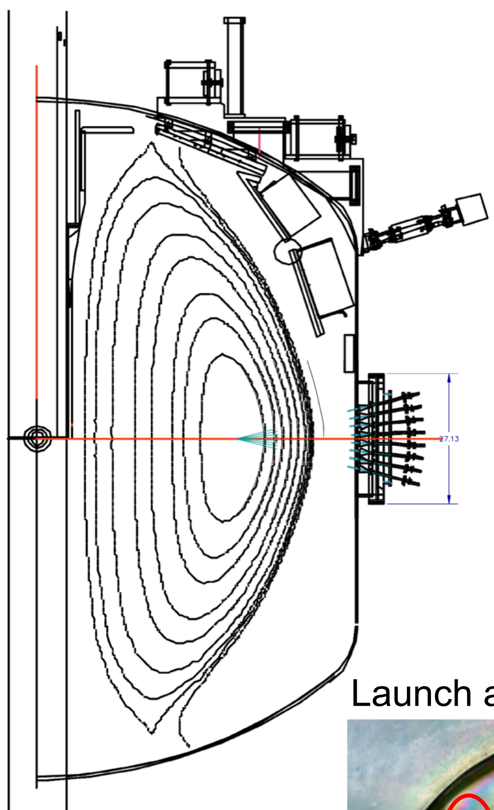
- Reflectometer array sees global modes identified as CAEs & GAEs
  [N.A. Crocker, PPCF 2011]
- New analysis gives $\delta n / n_0$; in core:
  - CAE: $\delta n / n_0 \sim 10^{-4} – 10^{-3}$
  - GAE: $\delta n / n_0 \sim 10^{-5} – 10^{-4}$
- δn from measurements via "synthetic diagnostic"
- Reflectometer "signal-to-noise" improved via correlation with δb



Measurement Technique

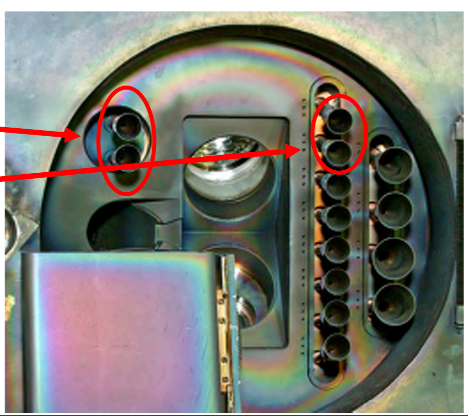
Reflectometers provide radial array of measurements

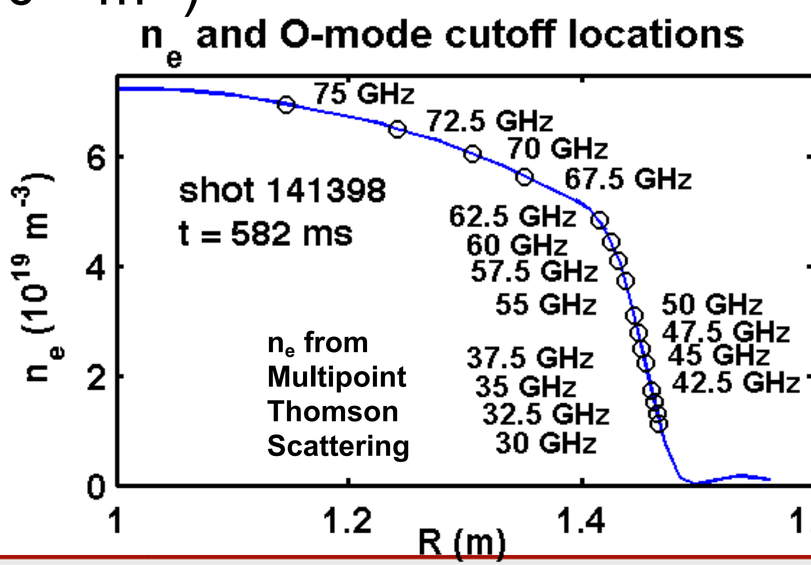
NSTX cross-section



- 16 channels in two arrays: “Q-band” & “V-band”
  - –30 – 50 GHz & 55 – 75 GHz
- Arrays closely spaced
  - – Separate launch/receive horn pair for each array
- Propagation direction: $-\hat{R} \Rightarrow$ frequency array = radial array
- large radial range in high n_e plasmas ($n_0 \sim 1 - 7 \times 10^{19}\ \mathrm{m}^{-3}$)

Launch and Receive Horns

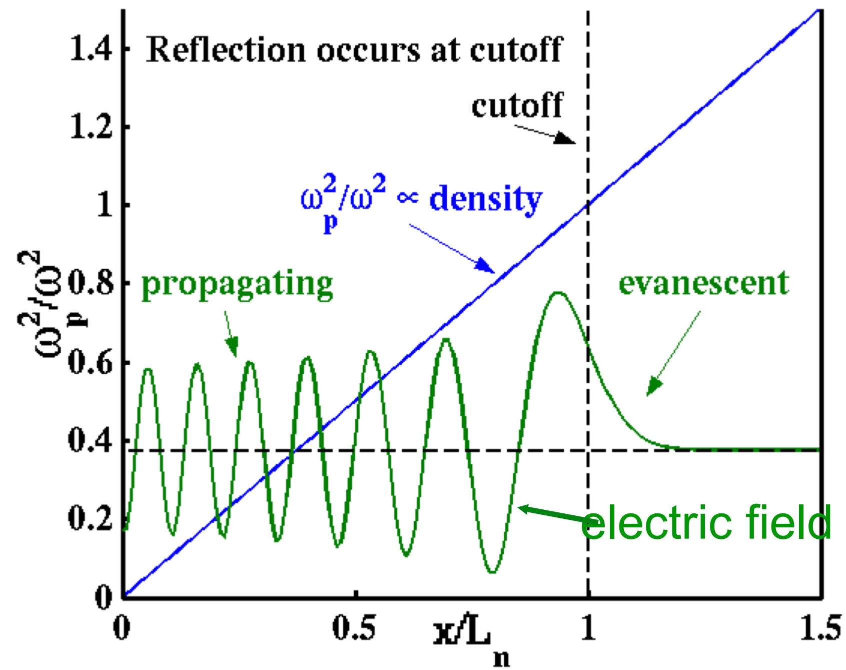




Reflectometers measure density fluctuations in plasma

- Microwaves reflect at "cutoff"
  - O-mode: $\omega^2 = \omega_p{}^2 + c^2k^2$
  - microwaves reflect at $k = 0$ ($\omega_p = \omega$)
- Measurement: path length fluctuations (δl) caused by δn

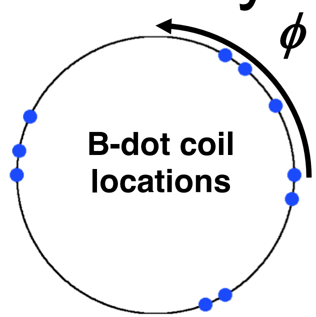
Microwave ("O-mode") propagation



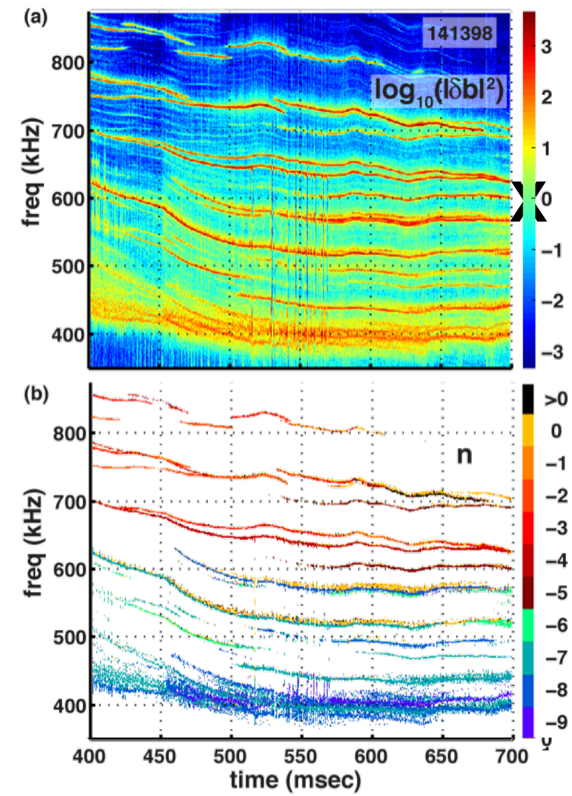
- δl sensitive to cutoff motion, but δn along path contributes (a.k.a. "interferometer effect")
  - cutoff motion dominates as $k_r \to 0$ (rigid displacement)

Toroidal mode numbers and frequencies determined with edge B-dot array

- Modes appear as peaks in δb spectrum
- n determined from δb measured by edge toroidal array of B-dot coils



- Smallest coil spacing is 10° $\Rightarrow$ can distinguish $|n| \leq 18$



N. A. Crocker, et al. Nucl. Fusion 2013

Reflectometer Analysis Technique

δn determined via synthetic diagnostic

- Synthetic diagnostic models path length
  - WKB path length integral:

$$l = l_0 + \delta l = \int_{R_{edge}}^{R_{cutoff}} dR \sqrt{1 - \omega_p^2(R)/\omega^2}$$

$$\omega_p^2(R_{cutoff}) = \omega^2, \omega_p^2 = \omega_{p0}^2 + \delta\omega_p^2 \propto n_0 + \delta n$$

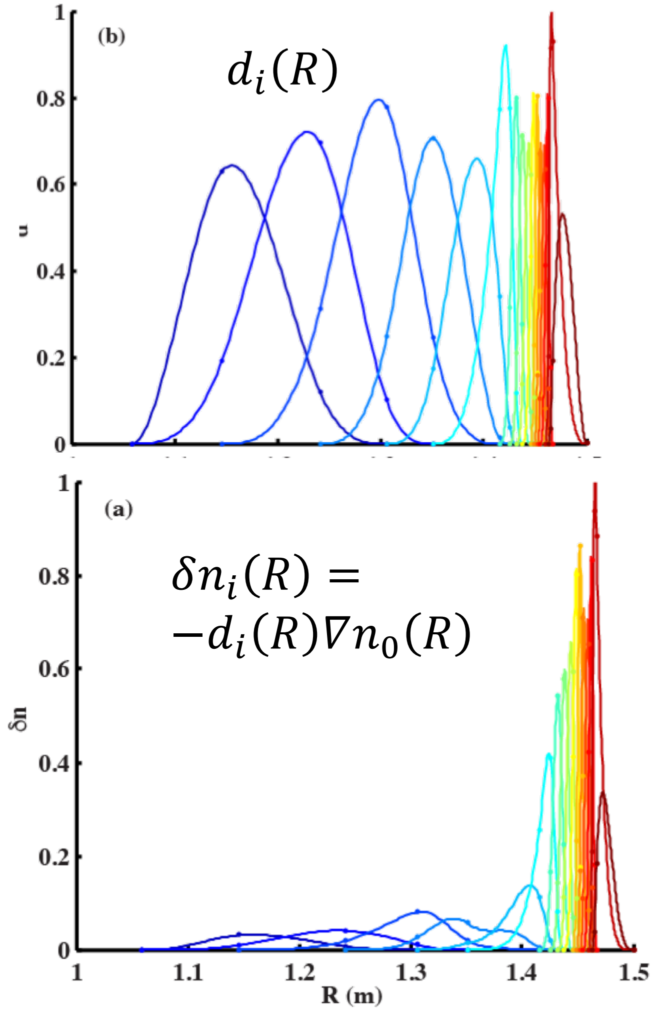
- δn modeled with cutoff displacement (d) basis functions:

$$\delta n(R) = -\nabla n_0(R) \sum_i a_i d_i(R)$$

  - cubic B-splines for $d_i(R)$
  - set of $a_i \Rightarrow \delta l_{fit}$ for all channels
  - find of set of a_i to minimize

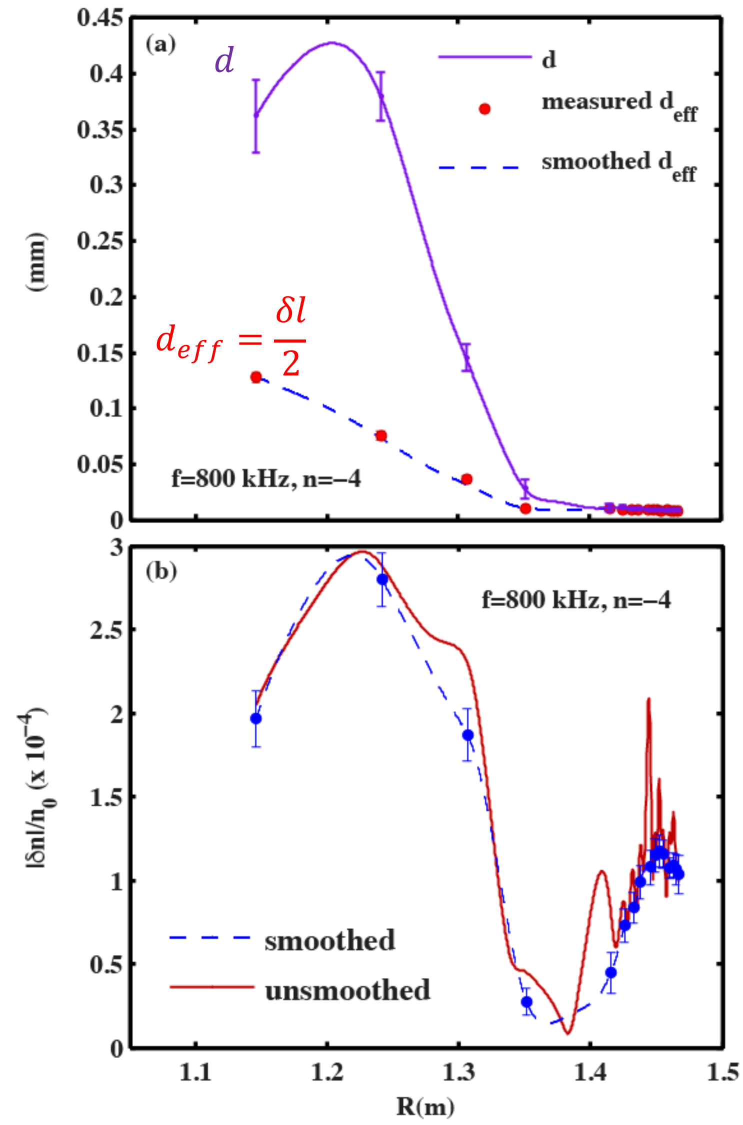
$$\chi^2 = \sum_j \left(\delta l_{j,meas} - \delta l_{j,fit}\right)^2 / \left(\sigma_{j,meas}^2\right)$$

Cutoff displacement basis functions (cubic "B-splines"; cutoff locations as knots)

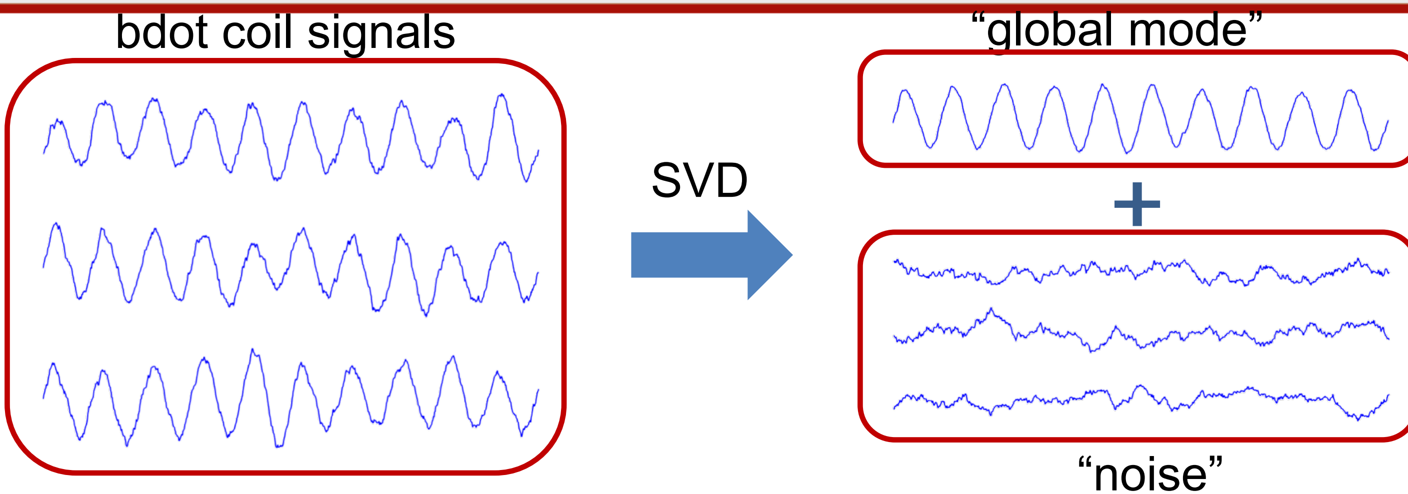


δn determined via synthetic diagnostic

- Fit naturally yields $d(R)$ along with $\delta n(R)$
- Fit sensitive to noise
  $\Rightarrow$ *use smoothed δl_{meas} for inversion*
  - smoothing = low spatial filter
  - smoothed δl within uncertainty of δl_{meas}
  - can't know if short scale structure in δn is real, given uncertainties



Singular value decomposition gives better "global mode" δb



- global mode observed by 10 bdot coils (HN array)
- "filter" signals with SVD ⇒ global mode w/reduced noise
  - SVD factors space & time dependence of signal matrix:

$$b_{jk} = \tilde{b}_j(t_k) \rightarrow \tilde{b}_{0\,j}\tilde{b}_{global}(t_k) + \epsilon_j(t_k)$$

- Steps *before* SVD …

1) bandpass filter coil signals to isolate mode
2) make signals complex ⇒ spatial phase (e.g. $n\phi_j$) factors out automatically:

$$\tilde{b}_j(t) = A(t)\cos\big(\theta(t) + \theta_{0\,j}\big) \rightarrow \hat{\tilde{b}}_j(t) = \frac{1}{\sqrt{2}}A(t)e^{i\left(\left(\theta(t)+\theta_{0\,j}\right)\right)} = \frac{1}{\sqrt{2}}\int_0^{\infty} d\omega e^{i\omega t}\int_{-\infty}^{\infty} dt'\tilde{b}(t')e^{-i\omega t'}$$

SVD finds global mode from eigenvector of signal correlation matrix

- SVD solves factoring problem

$$\hat{\tilde{b}}_j(t_k) = \hat{\tilde{b}}_{0\,j}\hat{\tilde{b}}_{global}(t_k) + \hat{\epsilon}_j(t_k)$$

- by minimizing χ^2:

$$\chi^2 = \sum_{j,k} \left| \hat{\tilde{b}}_j(t_k) - \hat{\tilde{b}}_{0\,j}\hat{\tilde{b}}_{global}(t_k) \right|^2$$

- $\Rightarrow$ spatial coefficients ($\hat{\tilde{b}}_{0\,j}$) of global mode from ***eigenvector*** of correlation matrix ***with largest eigenvalue***:

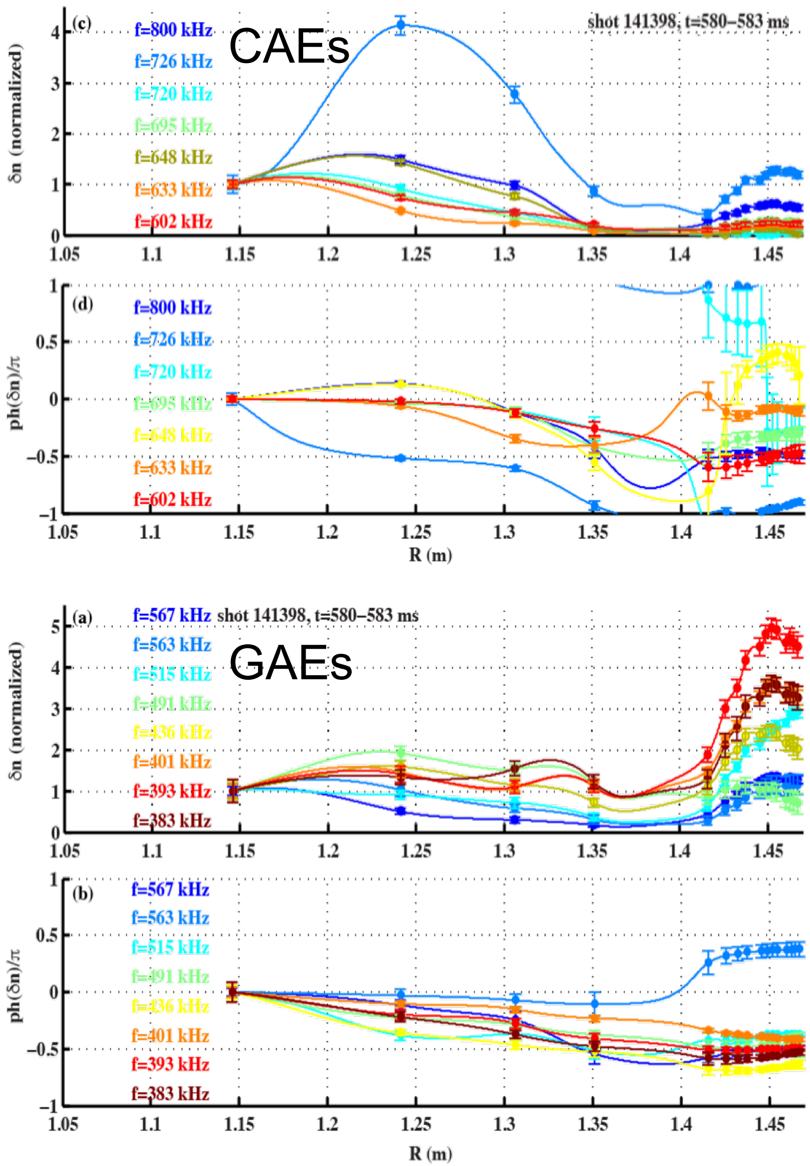
$$\mathbf{C}\hat{\tilde{\mathbf{b}}}_0 = \lambda\hat{\tilde{\mathbf{b}}}_0$$

$$[\mathbf{C}]_{ij} = \left\langle \hat{\tilde{b}}_i(t)\hat{\tilde{b}}_j^*(t) \right\rangle, \left[\hat{\tilde{\mathbf{b}}}_0\right]_j = \hat{\tilde{b}}_{0\,j}$$

Results of Reflectometer Analysis

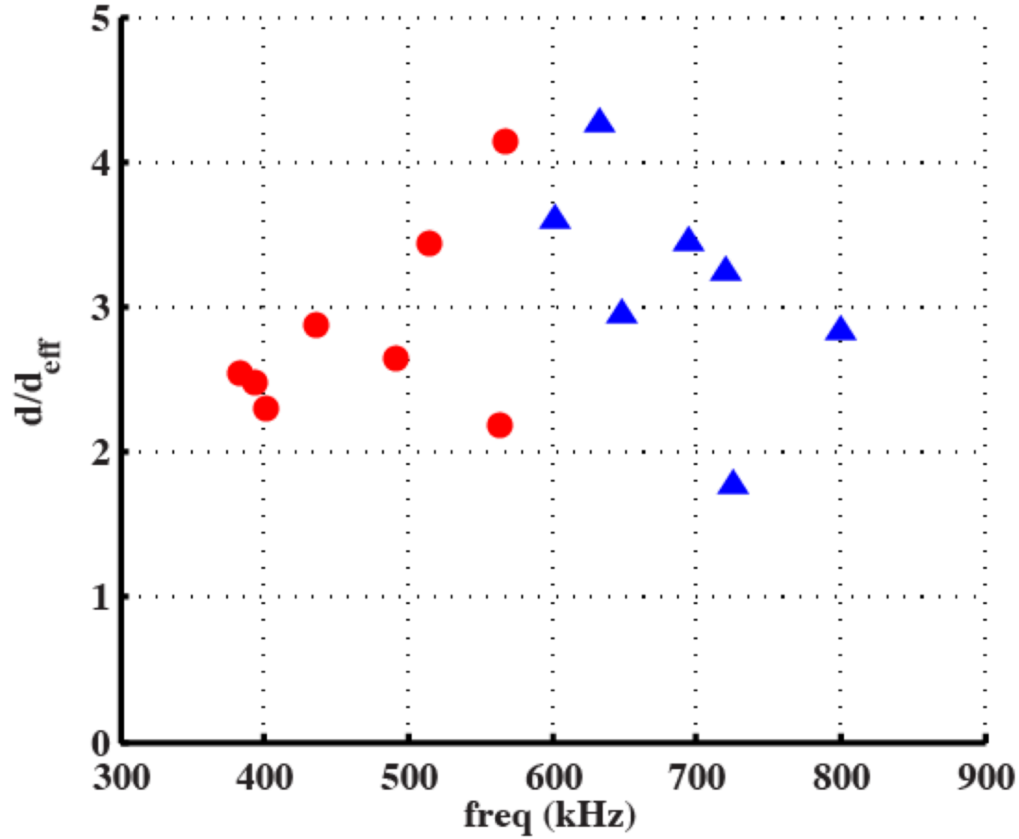
CAEs and GAEs have different δn structure

- CAEs have large, broad core peaks & small edge amplitude
- GAEs have low amplitude, broad structure in core & large edge peaks
- Note: large edge peaks can be caused by small edge radial displacements



New analysis gives 2–4 x larger cutoff displacement (d)

- Old analysis: $d_{eff} = \delta l/2$
  - δl attributed to cutoff displacement ("mirror approximation")
- New d is 2 – 4 x larger $\Longrightarrow$ larger plasma displacement



- Cutoff displacement ≠ plasma displacement
  - Compression also causes cutoff displacement

Plasma displacement (ξ) estimated from δn

- Get ξ from measurement:

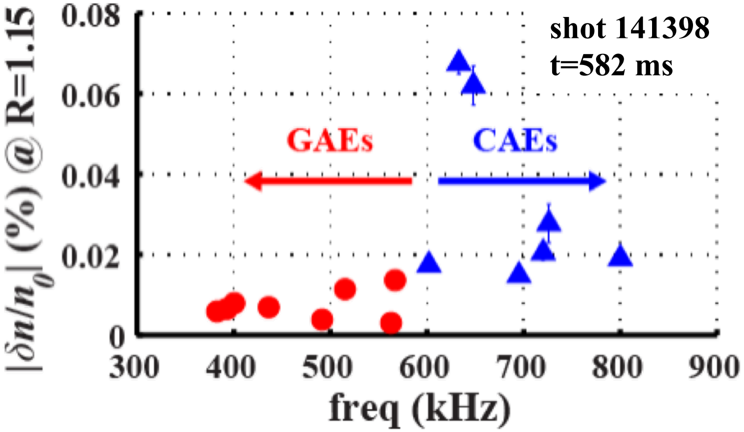
  $\delta n/n0 = -\nabla\cdot\boldsymbol{\xi} - \boldsymbol{\xi}\cdot\nabla\ln(n_0)$

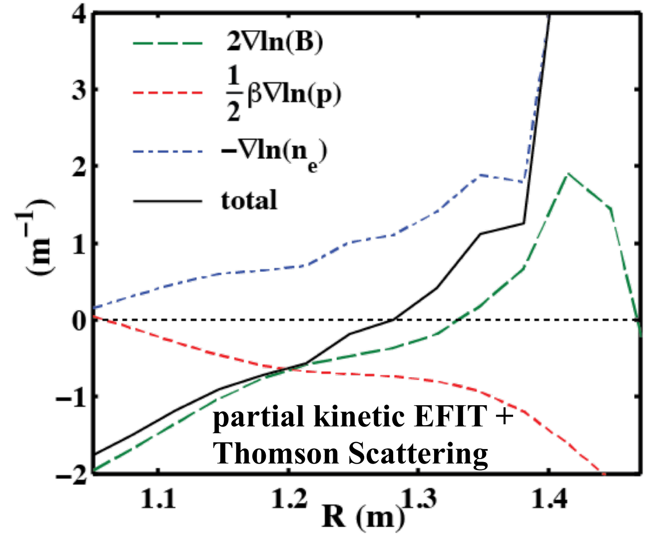
- Neglect finite ω/ω_{ci}, assume $E_{\parallel} = 0$

  $\nabla\cdot\boldsymbol{\xi} = \nabla\cdot((\mathbf{E}\times\mathbf{B})/(-i\omega B^2))$

  $\approx -\delta b_{\parallel}/B_0 - \boldsymbol{\xi}\cdot\left(\frac{1}{2}\beta\nabla\ln(p_0) + 2\nabla\ln(B_0)\right)$

- GAEs: $\xi_R \approx 0.7 L_n\,\delta n/n$ @R = 1.15 m
  - assume $\tilde{b}_{\parallel} = 0$
  - L_n ~ 1.7 m
  - n_0/n ≈1.05
- CAEs: $\delta n/n_0 \approx -\nabla\cdot\boldsymbol{\xi} \approx \delta b_{\parallel}/B_0$

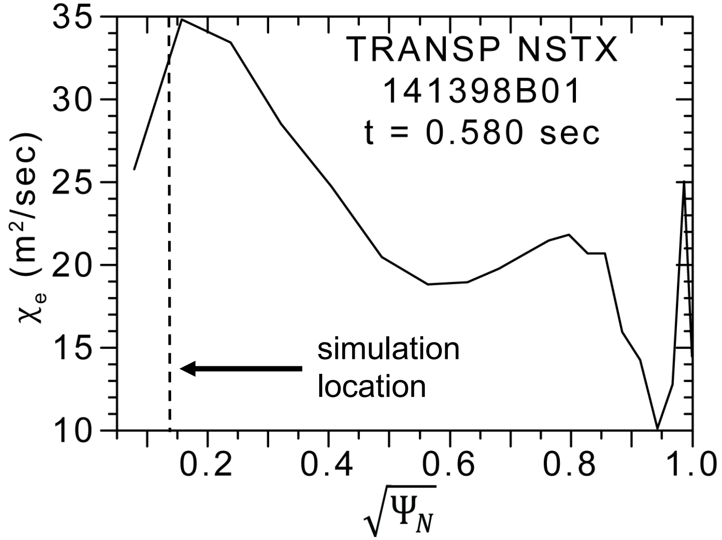




Prediction of χ_e and comparison with TRANSP

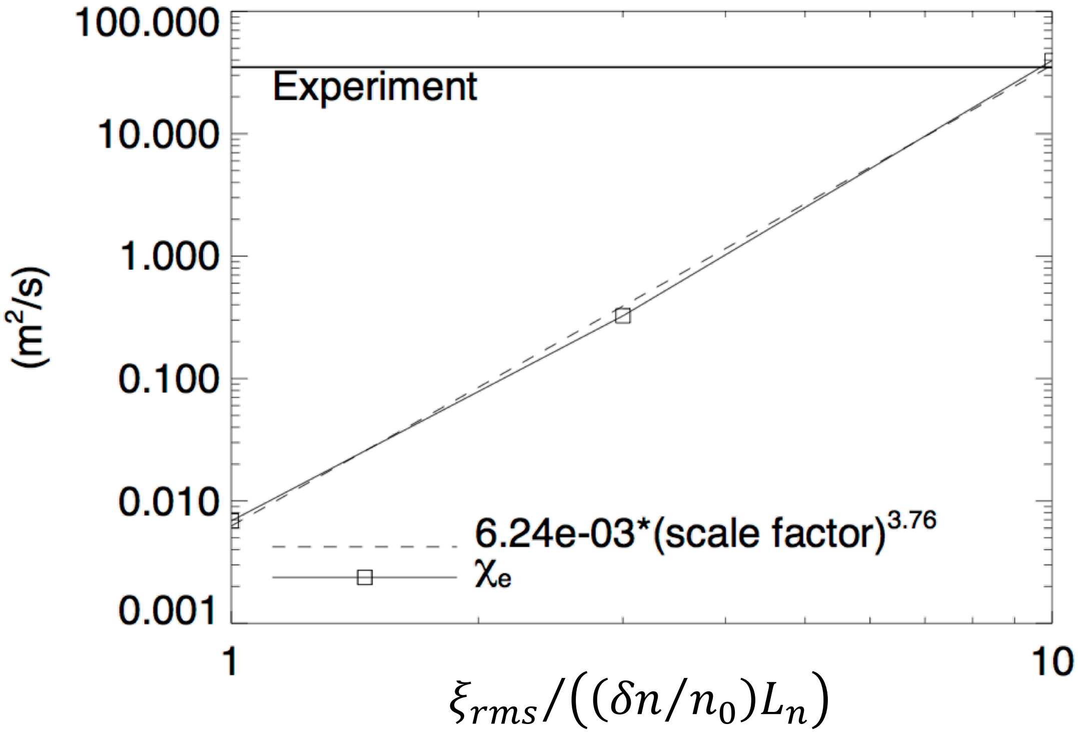
χ_e from GAEs simulated for 6 MW H-mode 141398, t = 0.58 sec

- Anomalous core χ_e (~ 35 m²/s) in 6 MW H-mode
- e^- guiding center orbit spreading simulated by ORBIT => χ_e (see e.g. [NN Gorelenkov NF 2010])
  - B-field from experiment (B_{T0}=0.45 T)
  - at t = 0, isotropic thermal population (T_e = 1 keV), δ-function at $\Psi_N^{1/2}$ = 0.15
  - collisionless
  - population spreads with time => D_e, $\chi_e = \frac{3}{2} D_e$
- 8 GAEs
  - ξ_{rms} ~ 0.4 mm (using $\xi \approx (\delta n/n_0) L_n$)
  - $\omega = k_{||} V_A$ => |m| = 0 – 2
  - poloidal+toroidal Fourier modes used



f (kHz)	n	m	ξ (mm)
383	-8	-2	0.1
393	-7	-1	0.11
401	-8	-2	0.13
436	-7	0	0.12
491	-8	0	0.06
515	-7	1	0.21
563	-6	2	0.05
567	-8	1	0.25

χ_e from GAEs in simulation much less than from TRANSP



- χ_e<< 1 m^2/s for $\xi_{rms} \sim (\delta n/n_0)L_n$
- scaling study ⇒ χ_e sensitive to amplitude ($\chi_e \propto \xi^{3.76}$)
- need ξ = 10*ξ_{rms} for agreement with TRANSP

Inclusion of CAEs as shear modes increases simulated χ_e, but still not enough

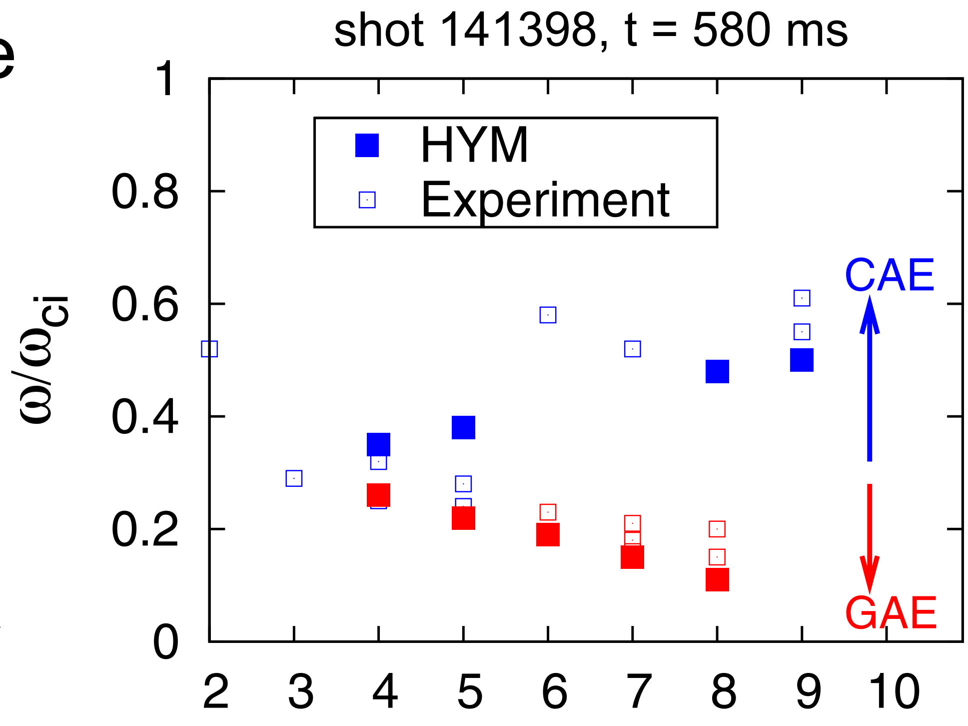
- 7 CAEs (15 modes total)
- If CAEs are shear modes:
  CAE δn >> GAE $\delta n \Rightarrow$ CAE ξ >> GAE ξ
  - ξ_{rms} ~ 1.8 mm for CAEs (1.9 mm all modes)
  - using $\xi \approx (\delta n / n_0) L_n$
- Shear CAEs $\Rightarrow$ large *m*
  - $\omega = k_{||} V_A$ => |m| = 4-10
- χ_e = 8 m^2/s at ξ_{rms} ~ 1.9 mm
  - expect 2 m^2/s from GAE-only simulation scaling
    - more modes = more stochastic?
- Need ξ_{rms} ~ 3*(1.9 mm) for χ_e = 34 m^2/s ~ $\chi_{e,\mathrm{expt}}$

f (kHz)	*n*	*m*	ξ (mm)
602	-5	4	0.31
633	-4	5	1.23
648	-1	8	1.05
695	-5	5	0.26
720	0	10	0.36
726	-3	7	0.57
800	-4	7	0.32

Comparison with HYM Simulation and Test of CAE-KAW Coupling

Initial comparison of HYM simulation & measurement promising

- Hybrid MHD (HYM) code simulates CAE structure & stability
  - 3D, ideal MHD fluid & δf solver full orbit fast-ions
  - realistic equilibrium
- Simulation & experiment compared for beam heated H-mode plasma

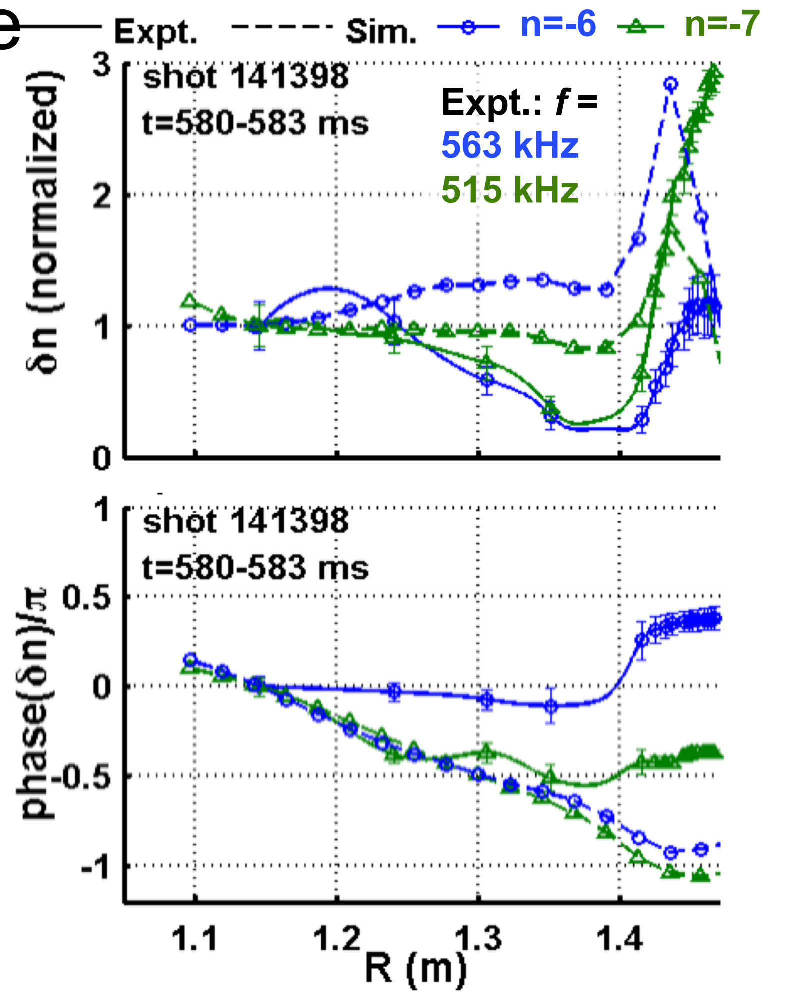


[E Belova PoP 2017]

- Most-unstable modes have *f* & *n* similar to observed experimental spectrum [E Belova PoP 2017]

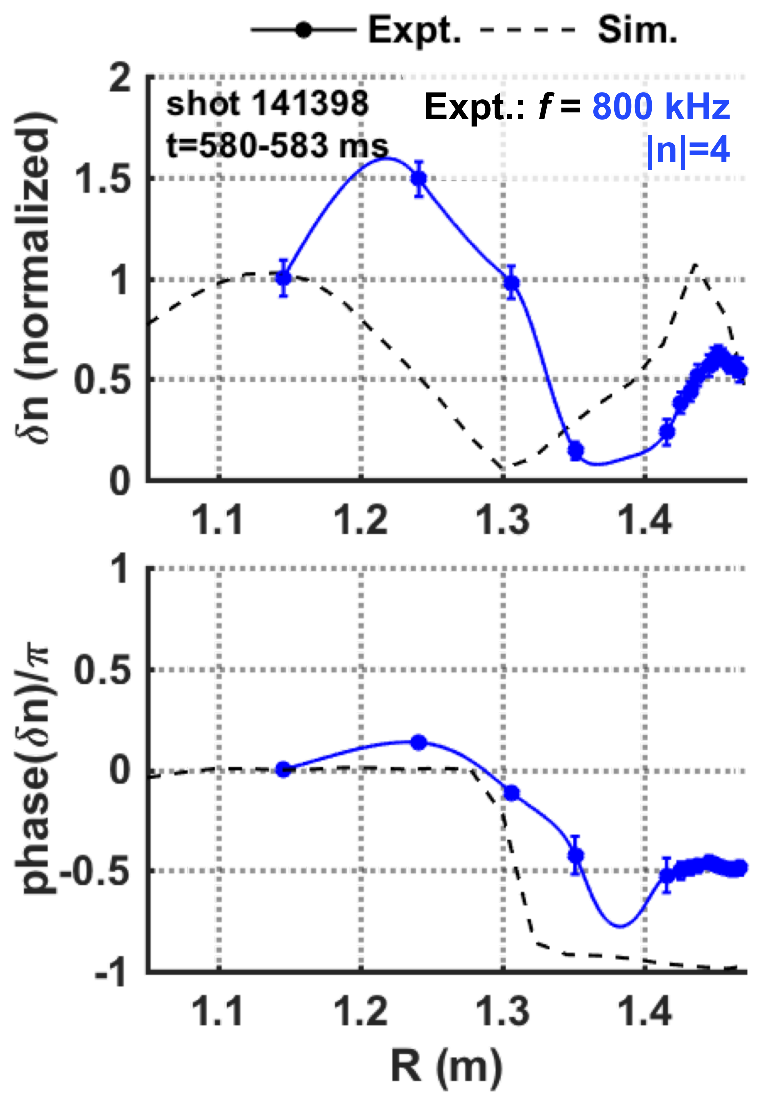
Initial comparison of HYM simulation & measurement promising

- HYM simulation: most unstable n = 6 & 7 modes counter propagating GAEs
- Structure similarities: broad core & strong edge peaking
- Simulation shows stronger phase change across minor radius
- Expect structure sensitive to
  - B_0 structure – included in HYM
  - Hall effect (finite ω/ω_{ci}) & toroidal rotation – under development for HYM



Initial comparison of HYM simulation & measurement promising

- CAE structure similarities: broad core peaks & small edge amplitude
- CAEs co-propagating in simulation; counter-propagating in experiment.
  - further work needed to understand
- Expect structure sensitive to
  - $|B_0|$ structure – included in HYM
  - Hall effect (finite ω/ω_{ci}) & toroidal rotation – under development for HYM



Measurements + Simulation ⇒ Small CAE-KAW energy transport

- HYM: n = 4 CAE, $\frac{\delta b_\parallel}{B_0} \sim 6.6 \times 10^{-3}$ ⇒ P = 1.2 MW CAE-KAW energy coupling transport: [E Belova PoP 2017]
  - simulation ξ scaled to $d_{eff} \Rightarrow \frac{\delta b_\parallel}{B_0} \sim 0.9 - 3.4 \times 10^{-3}$
- New approach: $\delta n/n \approx \delta b_\parallel/B$ in core ⇒ $2 \times 10^{-4} < \delta b_\parallel/B < 7 \times 10^{-4}$
  - $\delta n/n_0$ measured @ R=1.15 m
- P ∝ $\delta b_\parallel^2$ ⇒ P = 0.03 MW total for all modes
  - assume P/$\delta b_\parallel^2$ same for all modes
- Could improve estimate by rescaling HYM modes with measured $\delta n/n$.

Conclusions and Future Work

New δn measurements advance understanding of CAE & GAE effect on core energy transport

- New multi-channel reflectometer analysis ⇒ more accurate δn internal amplitude and structure
  - cutoff displacement larger than previous analysis
- GAE modification of e^{-} drift orbits ⇒ GAEs (or GAE+shear CAEs) too small to explain χ_e from TRANSP
- Measured and simulated GAE structures show rough similarities
  - HYM development currently under way may explain differences
- New δn + HYM Poynting Flux ⇒ CAE-KAW energy flux small

Many avenues for future work

- Improve reflectometer analysis
  - better synthetic diagnostic ⇒ raytracing
  - better alternatives to SVD filtering?
- Improve ORBIT modeling
  - better GAEs (finite ω/ω_{ci}, realistic poloidal structure,…)
  - ORBIT modified for CAEs. Requires verification…
  - simulation with modes from HYM (or other codes)
  - better electromagnetic amplitudes from δn

Many avenues for future work

- Understand simulation & measured structure differences
  - exploit measured phase to understand role of Hall effect & rotation?
- Improve CAE-KAW estimate from HYM: rescale HYM modes with measured $\delta n/n$

Appendix: Plasma displacement (ξ) estimated from δn

- Get ξ from measurement: ${}^{\delta n}/_{n_0} = -\nabla \cdot \boldsymbol{\xi} - \boldsymbol{\xi} \cdot \nabla \ln(n_0)$
- Neglect finite ω/ω_{ci}, Assume $E_\parallel = 0$ & $\mathbf{J_0} \times \mathbf{B_0} - \nabla p_0 = 0$

$$\nabla \cdot \boldsymbol{\xi} = \nabla \cdot \left(\frac{\delta \mathbf{E} \times \mathbf{B_0}}{-i\omega B_0^2} \right) = \frac{\mathbf{B_0}}{-i\omega B_0^2} \cdot \nabla \times \delta \mathbf{E} - \frac{\delta \mathbf{E}}{-i\omega} \cdot \nabla \times \frac{\mathbf{B_0}}{B_0^2}$$

$$= \frac{\mathbf{B_0}}{-i\omega B_0^2} \cdot \nabla \times \delta \mathbf{E} - \frac{\delta \mathbf{E}}{-i\omega B_0^2} \cdot \nabla \times \mathbf{B_0} - \frac{\delta \mathbf{E}}{-i\omega} \cdot (\nabla B_0^{-2} \times \mathbf{B_0})$$

$$= -\frac{\delta \mathbf{B} \cdot \mathbf{B_0}}{B_0^2} - \frac{\mu_0 \delta \mathbf{E}}{-i\omega B_0^2} \cdot \mathbf{J_0} + \frac{2\delta \mathbf{E}}{-i\omega B_0^2} \cdot (\nabla \ln(B_0) \times \mathbf{B_0})$$

$$= -\frac{\delta \mathbf{B} \cdot \mathbf{B_0}}{B_0^2} - \mu_0 \frac{\delta \mathbf{E} \times \mathbf{B_0}}{-i\omega B_0^4} \cdot \nabla p_0 - \frac{2\delta \mathbf{E} \times \mathbf{B_0}}{-i\omega B_0^2} \cdot \nabla \ln(B_0)$$

$$= -\frac{\delta \mathbf{B} \cdot \mathbf{B_0}}{B_0^2} - \frac{1}{2} \beta \boldsymbol{\xi} \cdot \nabla \ln(p_0) - 2\boldsymbol{\xi} \cdot \nabla \ln(B_0)$$