Linear stability analysis of divertor filaments

Derek A Baver, James R Myra
Lodestar Research Corporation
Filippo Scotti
Lawrence Livermore National Laboratory

Introduction

- Filamentary structures (“blobs”) are governed by nonlinear processes.
- However, linear calculations can still provide valuable insight into their behavior.
  - Location of modes
    - Divertor vs. outboard midplane
  - Shape of modes
    - Parallel penetration length
    - Mode width
    - Avoidance of specific locations, i.e. x-point
  - Relative importance of different drive mechanisms
    - Geodesic vs. normal curvature
    - Curvature vs. sheath drive
- ArbiTER provides a flexible tool for doing linear calculations.

Introduction to ArbiTER

- The Arbitrary Topology Equation Reader (ArbiTER) is an eigenvalue code for solving linear PDE's in diverse geometries.
- PDE's are discretized using finite difference methods.
  - Recent upgrade also permits finite element methods.
- Model equations are defined using a specialized equation parser.
  - Inherited from the edge fluid eigenvalue code 2DX*.
- ArbiTER expands these capabilities by adding a topology parser.
  - Permits arbitrary connectivity.
  - Permits variable number of dimensions.
- Resulting discretized equations are then solved using the SLEPc eigensolver package.

*D. A. Baver, J. R. Myra and M.V. Umansky, Comp. Phys. Comm. **182,** 1610, (2011)

Physics model

- Model equations:
  - Resistive ballooning mode.
  - Model contains curvature drive, electromagnetic $A_{||}$, collisional resistivity.
  - Sheath boundary conditions on divertor.
  - Curvature has normal, geodesic parts.

$$\gamma \nabla_\perp^2 \delta\phi = \frac{2B}{n} C_r \delta p - \frac{B^2}{n} \partial_\parallel \nabla_\perp^2 \delta A + v_E \cdot \nabla \delta\phi$$

$$\gamma \delta n = -\delta v_E \cdot \nabla n + v_E \cdot \nabla \delta n$$

$$-\gamma \nabla_\perp^2 \delta A + \gamma \frac{n}{\delta_{er}^2} \delta A = \nu_e \nabla_\perp^2 \delta A - \mu n \nabla_\parallel \delta\phi + v_E \cdot \nabla \nabla_\perp^2 \delta A$$

where

$$C_r \equiv \vec{b} \times \kappa \cdot \nabla$$

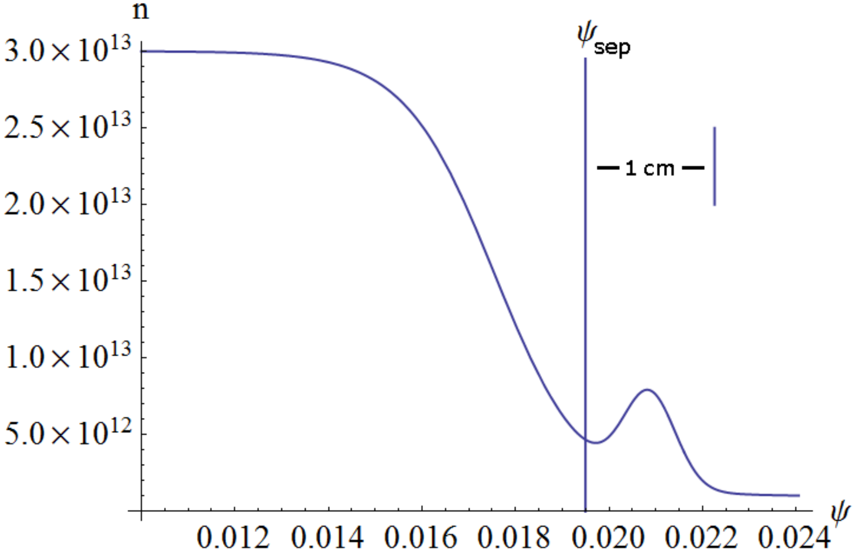
$$\delta v_E \cdot \nabla Q \equiv -i \frac{k_b (\partial_r Q)}{B} \delta\phi$$

$$v_E \cdot \nabla \delta Q \equiv -i \frac{k_b (\partial_r \phi)}{B} \delta Q$$

Physics model

- Perturbed density profile used to model blob.
- Simulates mean effect of blob on radial density profile.
  - Can think of blob as superposition of non-axisymmetric instability with axisymmetric perturbation.
- Controls location of dominant instabilities.
  - Ensures modes near flux surface of interest make it into list of most unstable modes.

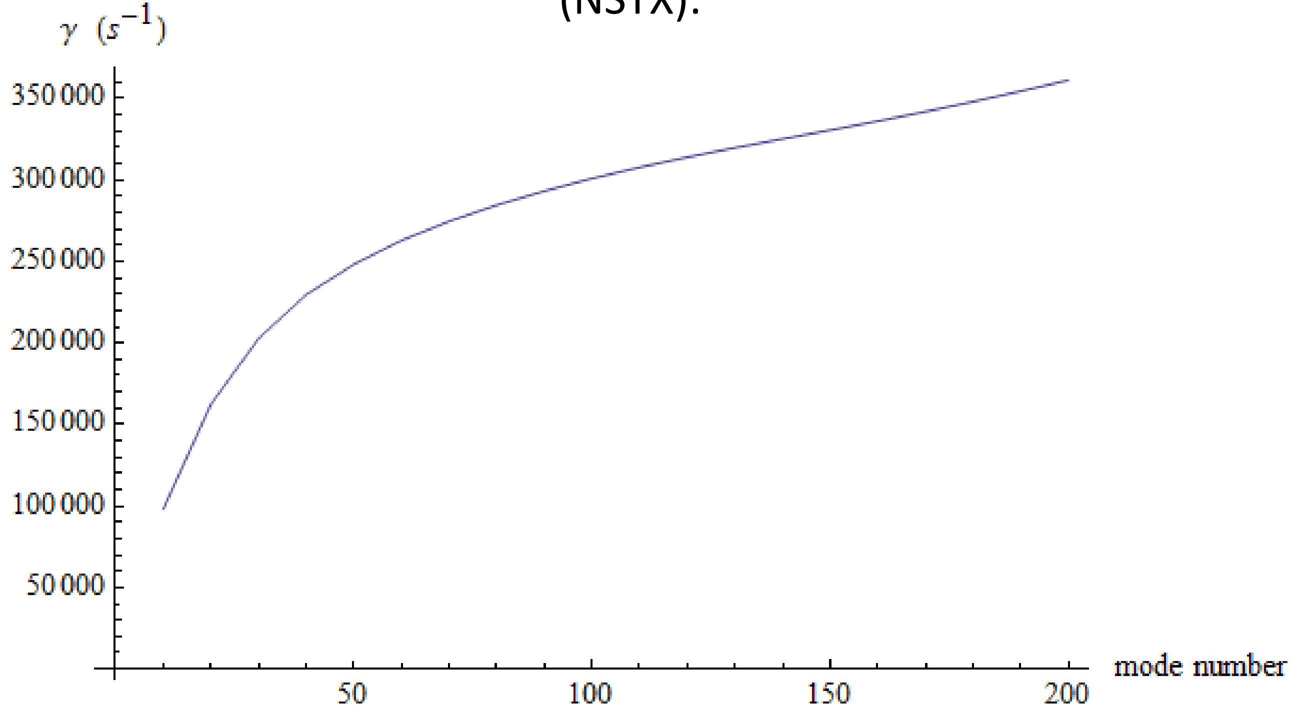
Base case density profile:



Results

- Robust instability at blob-relevant mode numbers in NSTX

Growth rate as a function of toroidal mode number for density at n = .79 x 10^{13} cm^{-3} (NSTX):

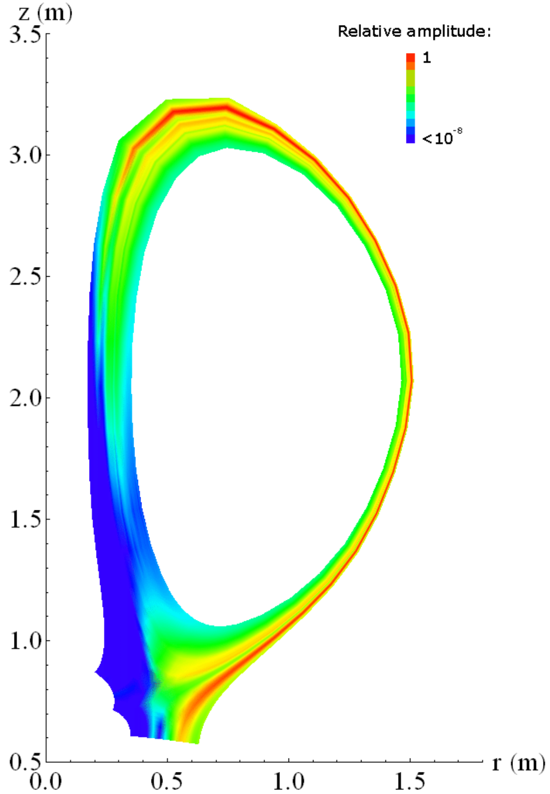


Results

- OBM-localized modes show consistent trends in penetration length

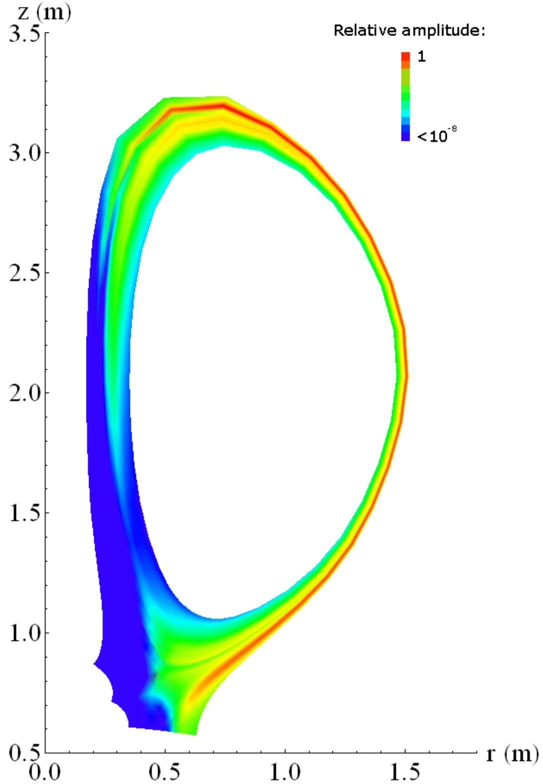
Spatial structure of eigenmode amplitude (ϕ) of fastest growing bump-localized mode for three test cases (NSTX):

A



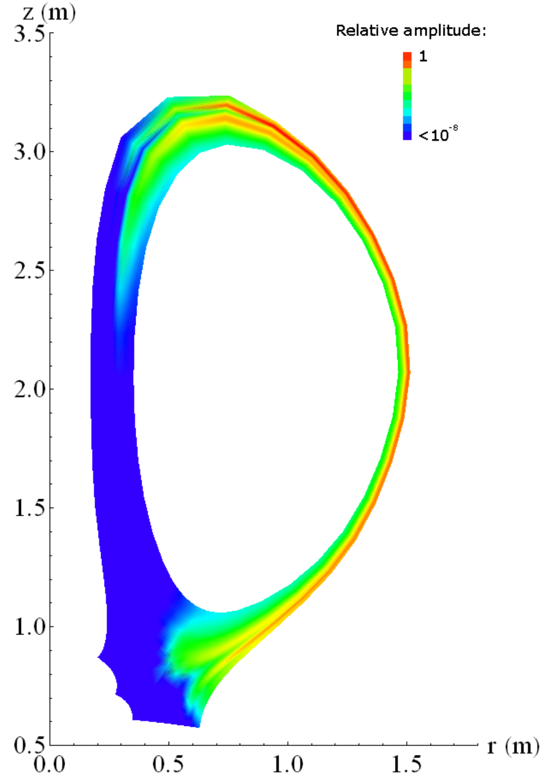
Bump n_e=2.63x10^{12}, T_e=37.8 eV

B



Bump n_e=7.9x10^{12}, T_e=12.6 eV

C

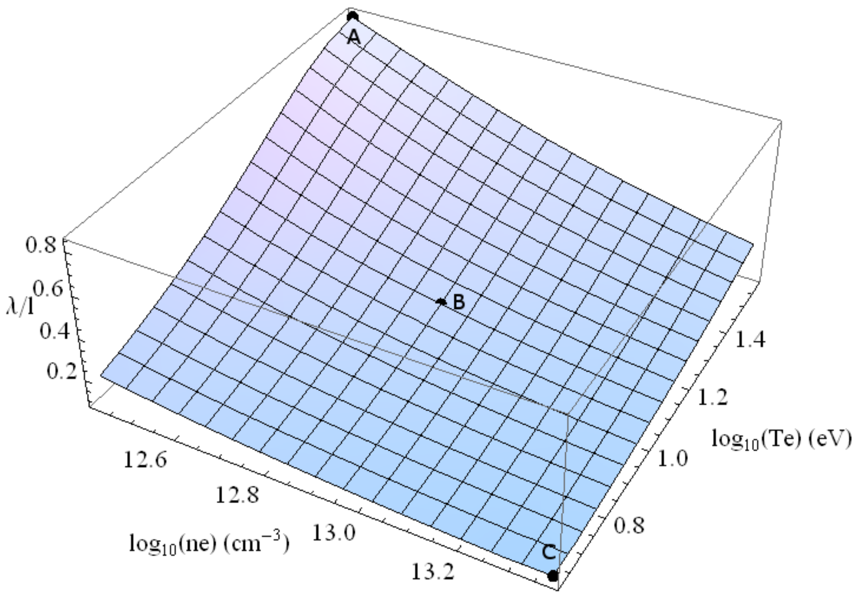


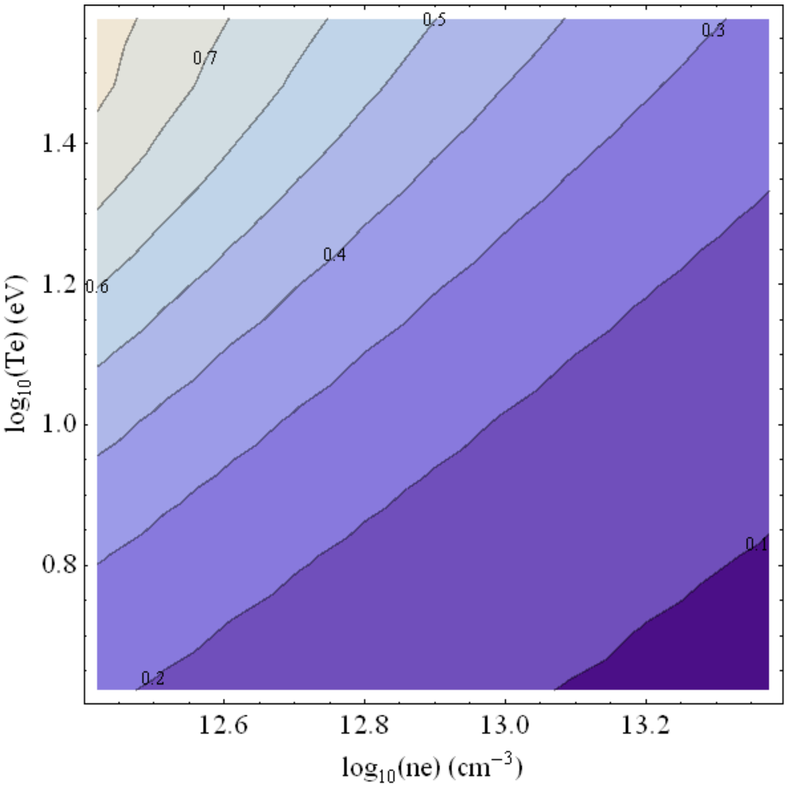
Bump n_e=2.37x10^{13}, T_e=4.2 eV

Results

- Mode penetration to divertor in NSTX scales with collisionality

Penetration ratio $\lambda/l_{X\text{-}D}$* as a function of temperature and density at mode peak for toroidal mode number 100 (NSTX):

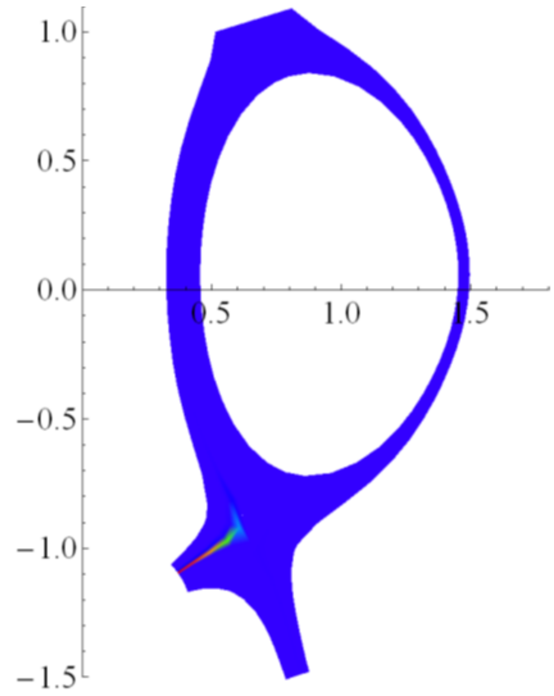




* $\lambda/l_{X\text{-}D}=-1/\mathrm{Ln}(\psi_{xpt}/\psi_{div})$

Results

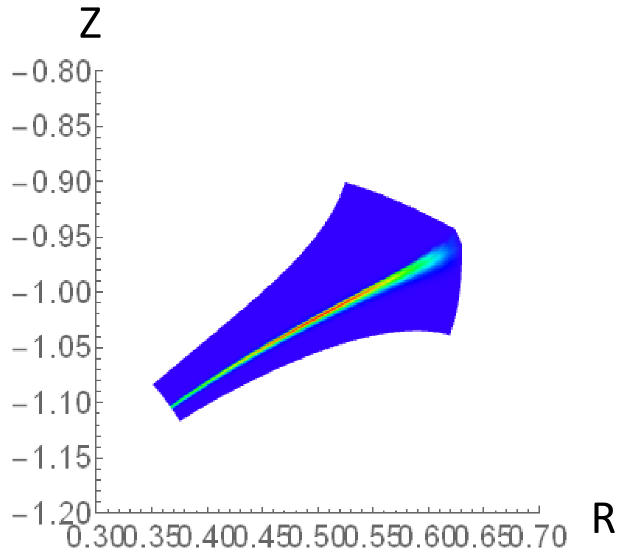
- Eigenmodes localized to divertor plate also occur.
  - Usually found in list of unstable eigenmodes in other studies.
- Studies of their mode structure reveal importance of geodesic curvature.
  - Curvature drive is sensitive to mode structure.
- Fine radial structure observed.
  - High radial resolution is key to making accurate calculations of these instabilities.



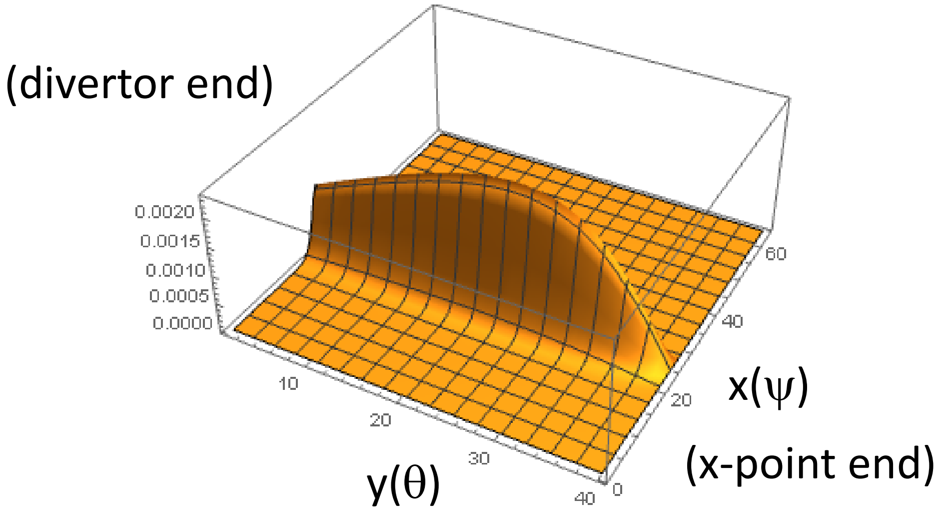
Sample eigenmode in NSTX-U-based grid.

Results

- Use of divertor-localized grids can increase resolution for studies of divertor-localized modes.
- Sample eigenmode for n=4, inner leg, potential fluctuations:
  - In RZ coordinates:

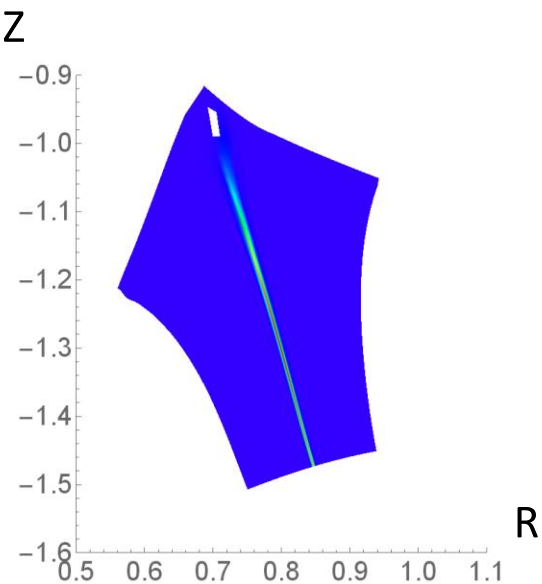


  - In grid coordinates:

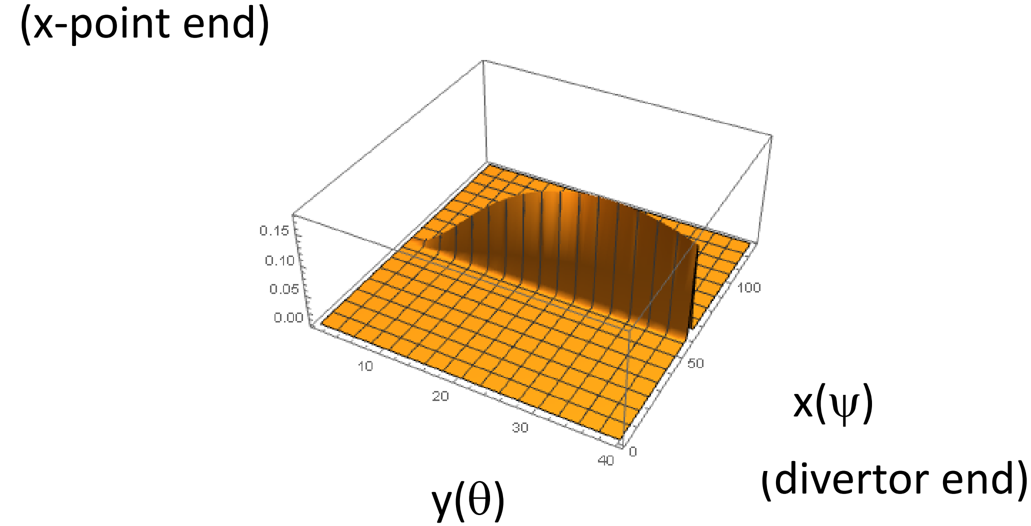


Results

- Sample eigenmode for n=20, outer leg, density fluctuations:
  - In RZ coordinates:



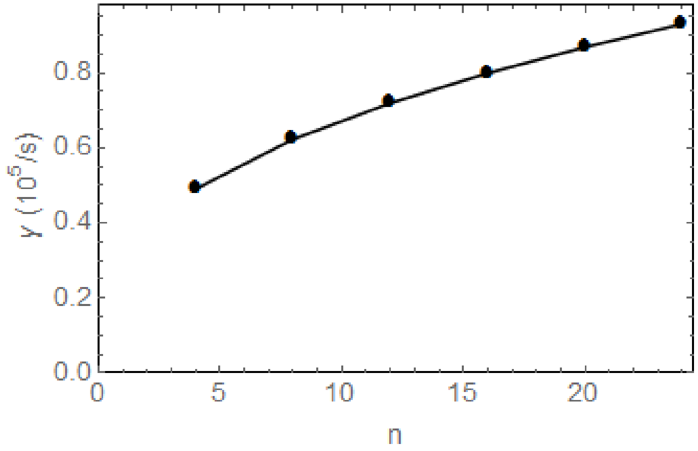
  - In grid coordinates:



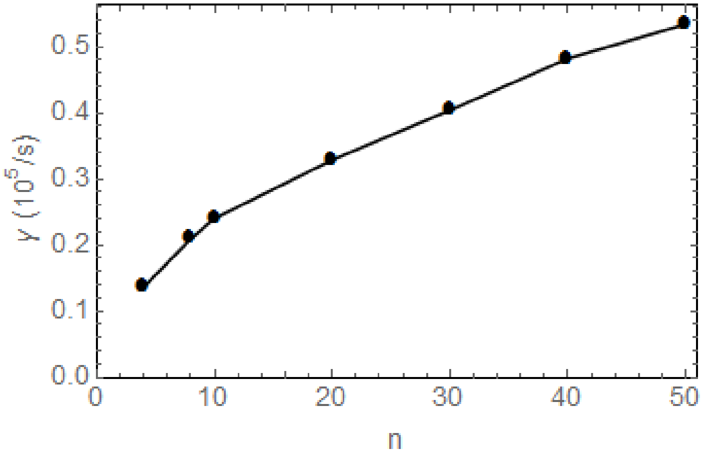
Results

- Parameter scans in torodial mode number:

Inner leg

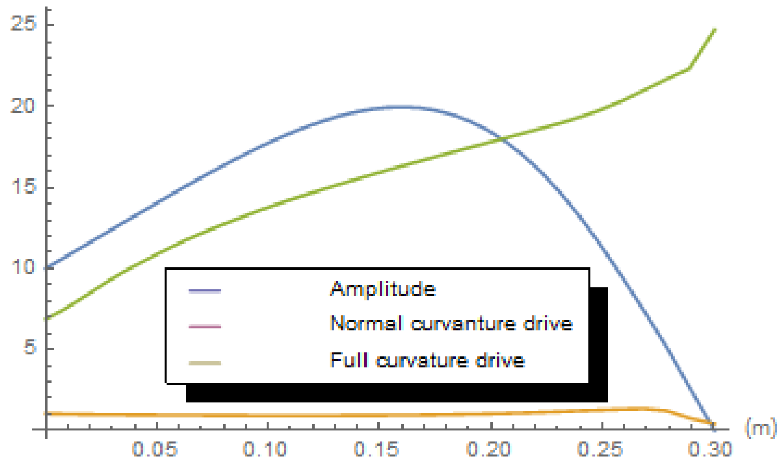


Outer leg

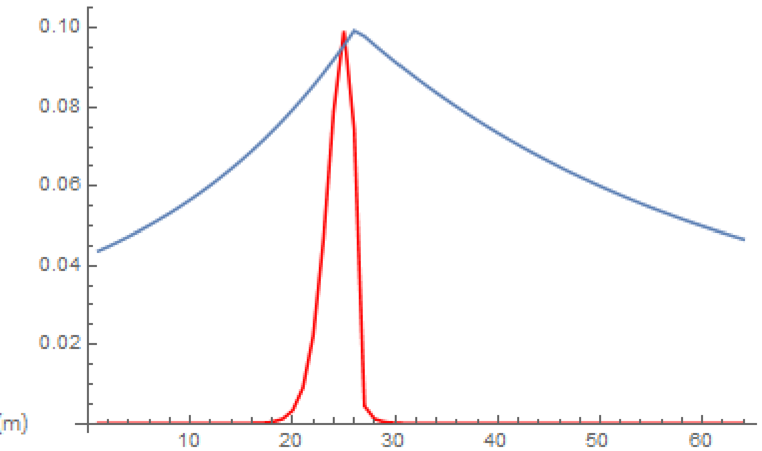


Results

- Mode structure for n=4, inner leg, 3-field model:



Poloidal mode structure vs. drive terms:



Radial mode structure of amplitude temperature

- Confirms mode exists in bad curvature region.
- Normal curvature represents a small fraction of total instability drive.

Future work

- Investigate role of other instability mechanisms.
  – i.e. sheath instability drive.
- Investigate role of other physics effects.
  – i.e. flow shear.
- Investigate alternate domains.
  – i.e. x-point localized grids.

Conclusions

- Linear calculations provide a useful tool for studying properties of filamentary structures.
- Studies of parallel mode structure reveal regular trends.
  - Connection length increases with decreasing collisionality.
- Location of modes can be compared to experiment.
  - Divertor-localized modes observed on inboard and outboard side.
- Studies of structure of divertor-localized modes reveal underlyling physics.