Continuum, sheath-limited gyrokinetic simulations of turbulence in NSTX-like SOL

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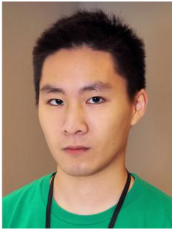
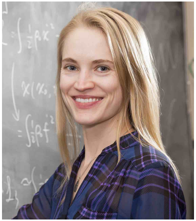
December 5th, 2018

PPPL

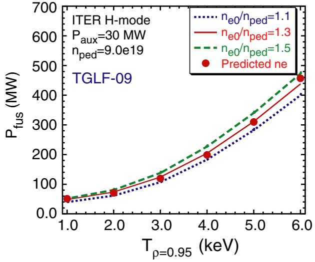
`Gkeyll` Continuum Gyrokinetics Team

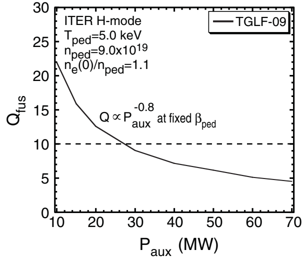
   

The core team: **Eric Shi** (now at LLNL) **Noah Mandell**, **Tess Bernard**, **Mana Francisquez** and **Greg Hammett**. Funded via edge and core SciDACs (C.S Chang, David Hatch).

Boundary-plasma strongly affect fusion performance

- Boundary plasma (edge and SOL) believed to set boundary conditions on the core
- Improved confinement associated with transient suppression of edge turbulence[1]
- ITER projections show fusion performance highly sensitive to the H-mode-pedestal temperature, relatively insensitive to auxiliary power heating ('core profile stiffness')
- Need reliable, fully predictive simulations of the pedestal to quantitatively model the core
- How to increase the pedestal pressure?

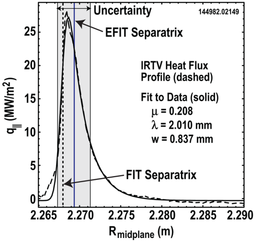




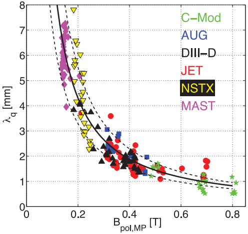
From Kinsey et al., (2011)

SOL power-exhaust problem is potential show-stopper

- Most of power (100 MW on ITER) released in the SOL flows in an extremely narrow channel $\sim$1 mm
- On ITER, need to dissipate most ($\sim$95% (Goldston, 2015)) of this power somehow before it reaches the divertor plates
  - Material limitations $\sim$10 MW m^{-2}, ITER operation can ‘easily’ reach $\sim$30 MW m^{-2}
- If SOL heat-flux width is too narrow, even steady-state power loads can result in material erosion
  - ITER designs have assumed $\lambda_q = 5$ mm, empirical extrapolation[2] of 1 mm ($B_{\mathrm{pol}} \approx 1.2$ T)



DIII-D from Makowski et al. (2012)



Eich et al. (2013)

Approaches for boundary-plasma simulation

- Sophisticated codes for fluid-based modeling of the boundary plasma have been developed.
  - Fluid **transport** codes: Model cross-field transport as diffusion and employ free parameters to match experimental profiles (interpretive use). SOLPS/UEDGE remain the principal tool for ITER boundary-plasma modeling.
  - Fluid **turbulence** codes (fluid and gyrofluid): Qualitatively useful, but cannot fully capture potentially important kinetic effects.

- We need kinetic codes solving 5D $(\boldsymbol{R}, v_{\|}, \mu)$ gyrokinetic equations in the edge and SOL for quantitative prediction
  - First-principles-based approach valid across a wide range of collisionality regimes
  - Parallel variations in T, n, ϕ on order of mean free paths
  - Help improve models and boundary conditions used in much cheaper fluid codes
  - Check empirical extrapolations to ITER

Attempts at gyrokinetic continuum code for boundary

We are not the first ones to attempt this!

- TEMPEST (LLNL, ∼2005–2010) — Finite-difference scheme, performed some axisymmetric studies. Conservation issues?
- G5D (JAEA, ∼2007–present) — Conservative finite-difference scheme, stated goal of open-field-line turbulence appears to have been dropped.
- FEFI (IPP Garching, ∼2009–?) — 4th-order Arakawa scheme. Went directly to electromagnetics. Issues with Alfvén dynamics and sheath-model stability.
- COGENT (LLNL, ∼2008–present) — 4th-order finite volume. Axisymmetric 4D transport simulations in realistic divertor geometry and initial tests in a 5D performed

This is a very hard problem and has required us to overcome many numerical and physics challenges.

Status of Gyrokinetics in Gkeyll

- Pioneering work by Eric Shi[3] led to 5D electrostatic full-F GK simulations of LAPD and NSTX-like helical SOL with sheath BCs
- Over past year, we have been rapidly developing a new version of Gkeyll
  - Moving from nodal to modal DG representation $\rightarrow$ orthonormal basis functions, quadrature-free, computer algebra-generated solver kernels (much easier to generalize to higher dimensionality/polynomial order), $\mathcal{O}(10)$ faster
  - Much simpler user interface, details abstracted away
- Have reproduced many of Shi's results with new version of Gkeyll; **Will discuss Eric's results today and show priliminary simulations with nonlinear EM terms turned on.**

[3] See 2017 thesis; JPP 2017 paper on LAPD; and PoP 2018 paper on Helical SOL

Gyrokinetic Model in Gkeyll

- Gkeyll currently solves the gyrokinetic system in the **long-wavelength (drift-kinetic) limit** for the gyrocenter distribution function $f(\boldsymbol{R}, v_\parallel, \mu, t)$:

$$\frac{\partial \mathcal{J} f}{\partial t} + \nabla \cdot (\mathcal{J} \{\boldsymbol{R}, H\} f) + \frac{\partial}{\partial v_\parallel}(\mathcal{J} \{v_\parallel, H\} f) = \mathcal{J} C[f] + \mathcal{J} S,$$

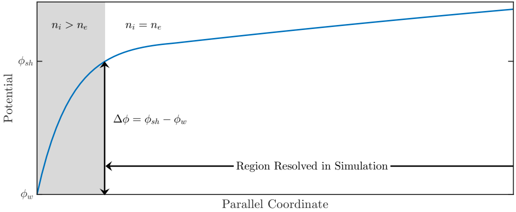
$$-\nabla_\perp \cdot \left(\frac{n_{i0}^g m_i}{q_i B^2} \nabla_\perp \phi \right) = \sigma_g = e \left[n_i^g(\boldsymbol{R}) - n_e(\boldsymbol{R}) \right],$$

$$H = \frac{1}{2} m v_\parallel^2 + \mu B + e\phi,$$

where $\mathcal{J} = B_\parallel^*$, $\{\cdot, \cdot\}$ is a non-canonical Poisson bracket, and $C[f]$ represents a model of collisions.

- Linearized ion polarization density for now (constant n_{i0}^g)

Conducting-Sheath Boundary Conditions



- Need to model effects of non-neutral sheath using BCs
- Get ϕ_{sh} from solving GK Poisson equation, then use $\Delta\phi = \phi_{sh} - \phi_w$ to reflect low-$v_{||}$ electrons entering sheath
  - Kinetic version of sheath BCs used in some fluid models that determine $v_{||,e}$ BC from ϕ (also similar to some gyrofluid sheath BCs)
- Potential self-consistently relaxes to ambipolar-parallel-outflow state
- Allows local currents into and out of the wall
- No BC applied at sheath to ions (free outflow)

Sheath-Model Boundary Conditions for Electrons

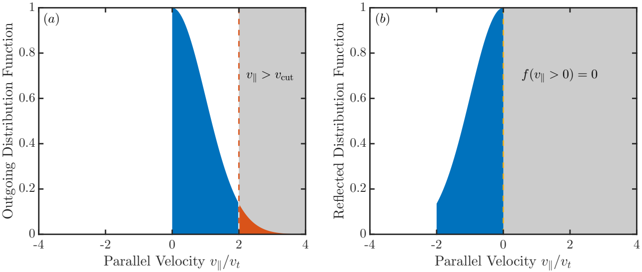
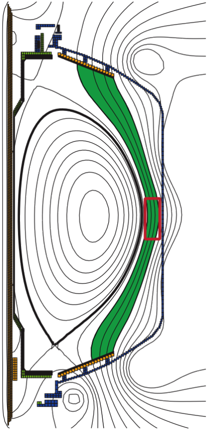


Figure: Illustration of sheath-model boundary condition. (a) Outgoing electrons with $v_\parallel > v_c = \sqrt{2e\Delta\phi/m} = 2$ are lost into the wall, where $\Delta\phi = \phi_{sh} - \phi_w$, ϕ_s is determined from the GK Poisson equation, and $\phi_w = 0$ for a grounded wall. (b) The rest of the outgoing particles $(0 < v_\parallel < v_c)$ are reflected back into the plasma.

Turbulence in NSTX-like helical SOL



- Simple helical model of tokamak SOL
  - Like the green region, but straightened out to vertical flux surfaces
  - Field-aligned simulation domain that follows field lines from bottom divertor plate, around the torus, to the top divertor plate
  - All bad curvature; brings in interchange instability drive
- Parameters taken from NSTX SOL measurements; Real deuterium mass ratio, Lenard-Bernstein collisions
- Conducting sheath boundary conditions at the divertor plates
- Radially-localized source around $x = 1.3$ cm models flux of particles and heat across separatrix from core
- How does the SOL heat-flux width scale in this simplified model? (Eich: $\lambda_q \propto B_p^{-1.19}$)

NSTX-like SOL modeled with helical field lines

Parameter	Value
ρ_{s0}	2.9 mm
ρ_e	0.048 mm
B_{axis}	0.5 T
B_v/B_z	$0.2, 0.3, 0.6$
L_v	2.4 m
L_z	$12, 8, 4$ m
L_x	14.6 cm
L_y	29.1 cm
n_0	7×10^{18} m^{-3}
$T_{i,\text{src}} = T_{e,\text{src}}$	74 eV
$T_{i,\text{sep}}$	40 eV
$T_{e,\text{sep}}$	25 eV
λ_{ee}	0.96 m
λ_{ii}	3.5 m
$c_s/\sqrt{R\lambda_p}$	1.9×10^{5} s^{-1}

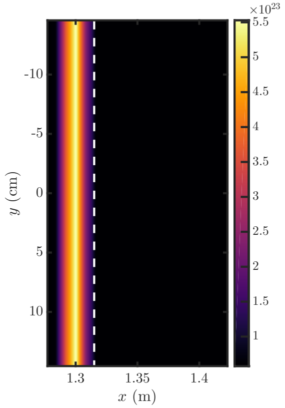


Figure: Midplane particle source for helical-SOL simulations in the perpendicular (x, y) plane.

Blob formation due to bad curvature drive

Figure: Electron density (10^{18} m^{-3}) in $(x, y, z = 0)$ plane vs. time. Starting from the initial condition, radially elongated structures extend out from the source region before flow shear in the source region leaves propagating blobs at large x. The dashed line indicates the separation between the source and SOL regions.

Curvature strongly influences turbulence

Heat-flux profiles narrow with increased B_p

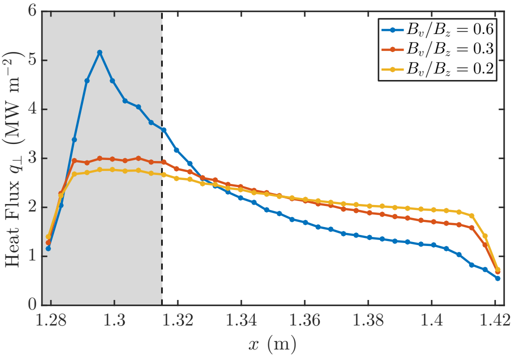


Figure: Time-averaged radial profiles of the total perpendicular heat flux $q_\perp = q_\| \sin\theta = q_\| B_v/B_z$ measured at the sheath entrance for three simulations with different magnetic-field-line pitches. A larger B_v/B_z results in a steeper heat-flux profile, similar to how the SOL heat-flux width scales with B_p in present-day tokamaks.

Particle-flux as function of poloidal field

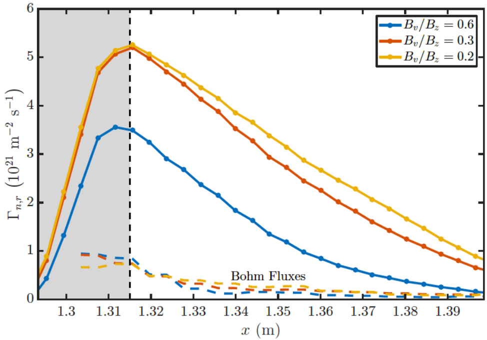


Figure: Comparison of radial $E \times B$ particle flux evaluated at the midplane for three different poloidal fields. Increasing the poloidal field decreases the radial flux, consistent with the heat-flux profiles on the divertor plate. For comparison, Bohm fluxes estimates are shown as dashed lines.

Larger amplitude, more intermittent blobs in SOL

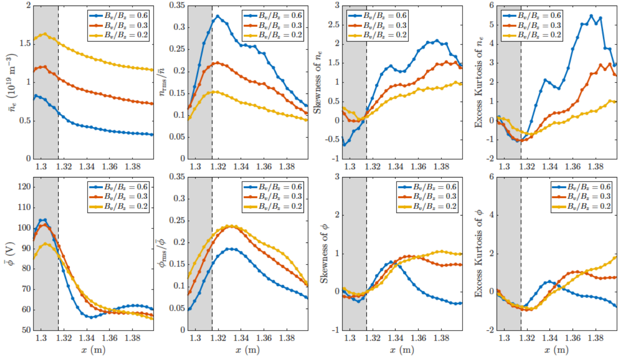


Figure: Comparison of electron-density fluctuations (top row) and electrostatic fluctuations (bottom row) at mid-plane. The density fluctuations (blobs) are larger amplitude and more intermittent than the potential fluctuations which show much smaller skewness and kurtosis.

Ion and electron temperatures are not in equilibrium

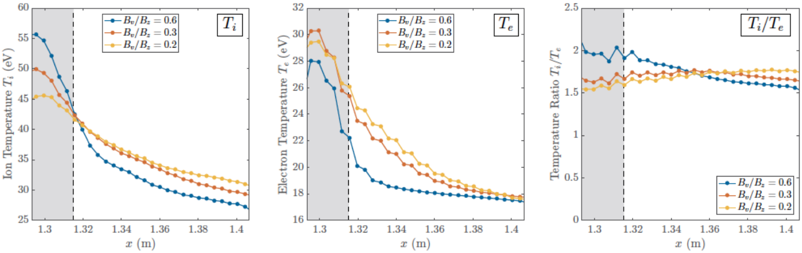


Figure: Radial profiles of steady-state ion (left) and electron (middle) profiles near midplane. Right plot shows ion-to-electron temperature ratio. Although both electrons and ions are sourced at the same temperature, the sheath allows rapid loss of high energy electrons to wall, resulting in lower electron temperatures in the SOL.

Electromagnetic effects important in edge

- Electromagnetic effects are especially important in the edge and SOL, where steep gradients can push the plasma close to the ideal-MHD stability threshold and produce stronger turbulence
- Including electromagnetic fluctuations has proved challenging in PIC codes due to sampling noise, which leads to the well-known Ampère cancellation problem
- Continuum gyrokinetic codes for core turbulence have avoided the Ampère cancellation issue
- As `Gkeyll` uses a continuum formulation, we expect that we can handle electromagnetic effects in the edge and SOL in a stable and efficient manner

Linear Benchmark: Kinetic Alfvén Waves

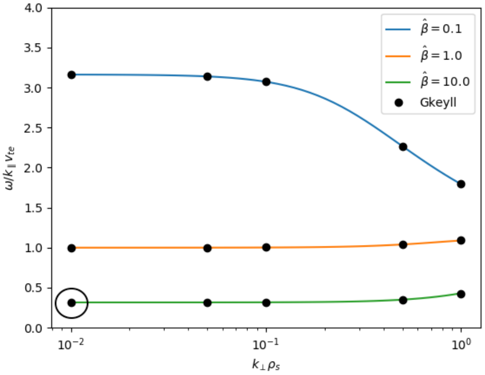


Figure: Alfvén wave dispersion relation computed with `Gkeyll` compared to analytical results.

Linear Benchmark: Kinetic Ballooning Mode

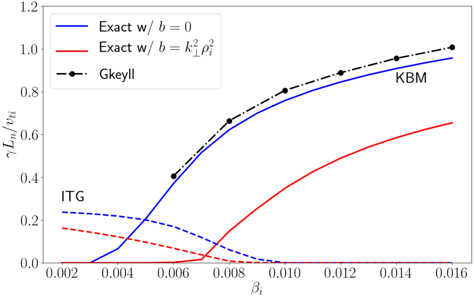
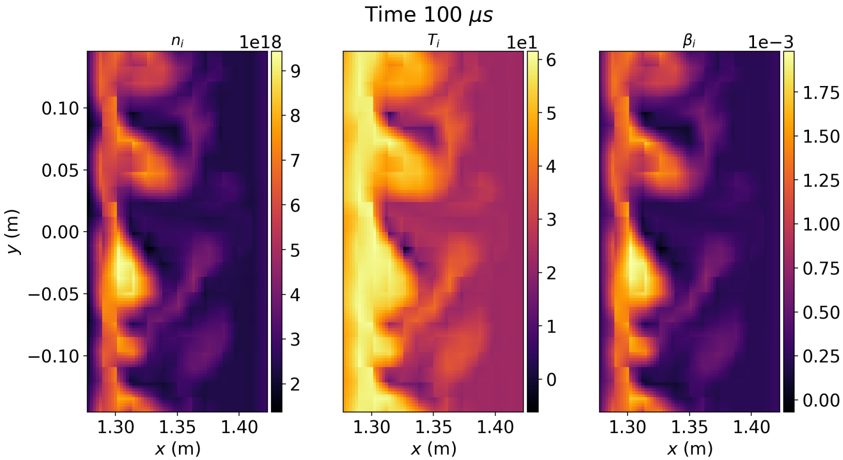


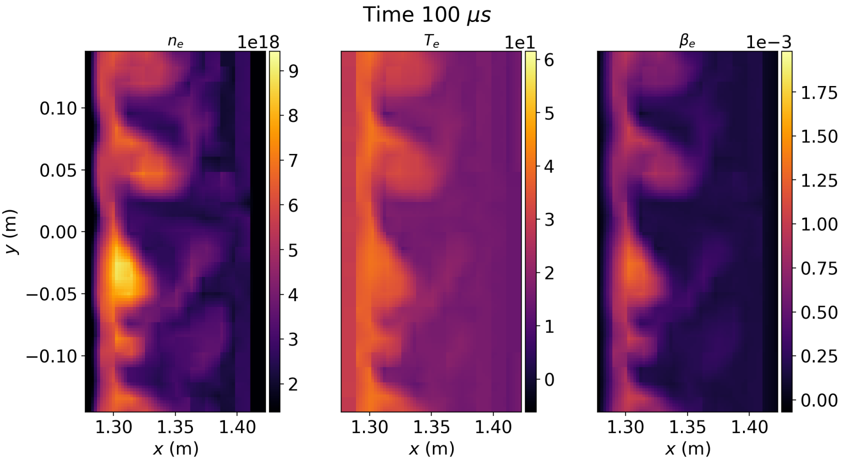
Figure: Kinetic Balloning Mode (KBM) growth rate as function of β_i from `Gkeyll` compared to analytical results.

EMGK turbulence in NSTX-like helical SOL

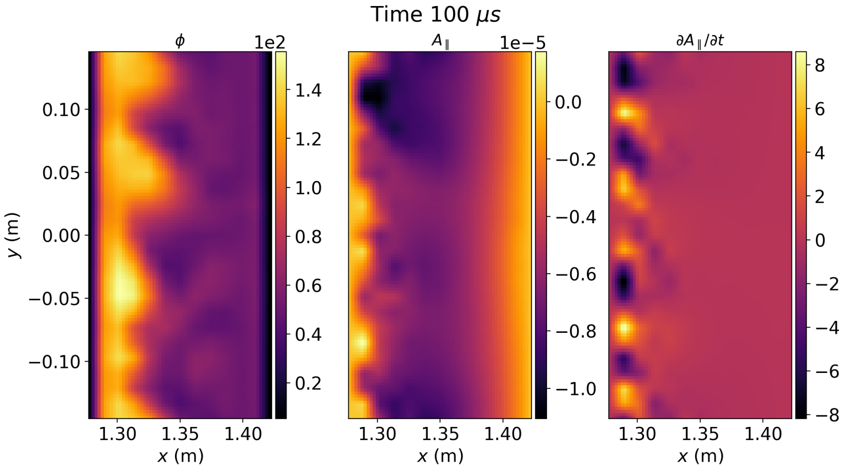
EM turbulence in NSTX-like helical SOL: Ions



EM turbulence in NSTX-like helical SOL: Electrons



EM turbulence in NSTX-like helical SOL: Fields



The Future: Gkeyll

Gkeyll is in a very exciting phase of development at present. Several major physics studies are underway and significant new development is planned. Hiring two new postdocs (one computational, one physics) to work on aspects of GK project.

- Full geometry. Implemented mapped grids; need to extend to multiple blocks to do full tokamak geometry;
- Neutrals via fluids and/or kinetic solvers; recycling and other PMI physics; improved sheath boundary conditions, accounting for field incidence angle
- Compare with NSTX GPI data to extract blob statistics from simulations. Started on this in collaboration with S. Zweben and others.
- Other physics studies on MAST-U super-X divertors; other machines