Modeling of chirping toroidal Alfvén eigenmodes in NSTX

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Outline

Motivation

ORBIT simulations

Numerical Equilibrium with Quadriatic q profile

NSTX 205072

NSTX 139048

Conclusion

Motivation

- Complex behavior of Alfvén modes is often observed in tokamak discharges

- Rapid frequency changes referred to as chirps, occur at msec time scales, much shorter than the typical time for changes in the equilibrium.

- Aside from being an interesting test of the capability of numerical simulation, the existence of chirping can significantly modify high energy particle distributions. A spectrum of modes can lead to large scale modification of beam and alpha particle distributions.

- The modeling of chirping is an major tool for describing mode-induced fast ion losses in present tokamaks, as well as in ITER, where chirping cannot be ruled out

ORBIT δf equations

Perturbation $\delta B = \nabla \times \alpha(\psi, \theta, \zeta, t) B$, Potential $\Phi(\psi, \theta, \zeta, t)$

$$\alpha = \sum_{m,n} A_n \alpha_{m,n}(\psi) sin(\Omega_{mn}), \quad \Phi = \sum_{m,n} A_n \Phi_{m,n}(\psi) sin(\Omega_{mn}),$$

$\Omega_{mn} = n\zeta - m\theta - \omega t - \phi_n(t),$ $\vec{E} \cdot \vec{B} = 0$

Canonical Variables and Hamiltonian

$P_\zeta = g\rho_\| - \psi_p, \zeta$ $P_\theta = \psi + \rho_\| I, \theta$ $H = \frac{\rho_\|^2 B^2}{2} + \Phi$

The perturbed distribution δf is represented by a sum over particles

$$\delta f(\psi_p, \theta, \zeta, \rho_\|, t) = \sum_j w\delta(\psi_p - \psi_{p,j}(t))\delta(\theta - \theta_j(t))\delta(\zeta - \zeta_j(t))\delta(\rho_\| - \rho_{\|,j}(t)),$$

with $\rho_\| = v_\|/B$, and the weights are stepped by

$$\frac{dw}{dt} = \frac{w - f/g}{f_0}[\partial_E f_0 \dot{E} + \partial_{P_\zeta} f_0 \dot{P}_\zeta],$$

with g the marker distribution and f the beam distribution.

Time dependence is given by $\Omega_{mn} = n\zeta - m\theta - \omega t - \phi_n(t)$

The mode is stepped in time by

$$\frac{dA_n}{dt} = \frac{-\nu_A^2}{D_n \omega_n A_n} \sum_{j,m} w_n \left[\rho_{\|} B^2 \alpha_{mn}(\psi_p) - \Phi_{mn}(\psi_p) \right] cos(\Omega_{mn}) - \gamma_d A_n,$$

$$\frac{d\phi_n}{dt} = \frac{-\nu_A^2}{D_n \omega_n A_n^2} \sum_{j,m} w_n \left[\rho_{\|} B^2 \alpha_{mn}(\psi_p) - \Phi_{mn}(\psi_p) \right] sin(\Omega_{mn}),$$

$\rho_{\|} = v_{\|}/B$, $D_n = 4\pi^2 \sum_m \int \xi_{mn}^2(\psi_p) d\psi_p$,

Note that if the damping is large, the particle modification of ϕ_n is large compared to the change of A_n, and that $d\phi_n/dt$ gives a frequency modification of the mode.

The time dependence of a mode is given by

$$\Phi(t) = cos(\omega_0 t + \phi(t)) + isin(\omega_0 t + \phi(t)).$$

Analysis is done using the Wigner distribution of quasi-probability

$$W(t, \omega_k) = \int_{-T}^{T} \Phi^*(t+q)\Phi(t-q)e^{-2i\omega_k q}dq,$$

with ω_k a set of frequencies spanning the mode ω

This has advantages over a simple running Fourier analysis.
In particular, if $\Phi = c$ then for large T, $W(t, \omega_k) = \delta(\omega_k)$.

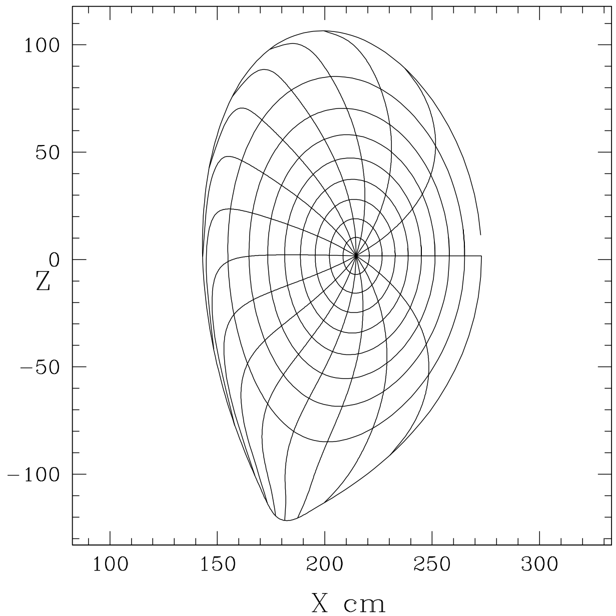
If the mode is a single ha rmonic $\Phi = e^{i\omega t}$ then $W(t, \omega_k) = \delta(\omega_k - \omega)$
Φ is a single chirp, $\Phi = e^{i\omega t^2}$ then $W(t, \omega_k) = \delta(\omega_k - \omega t)$.

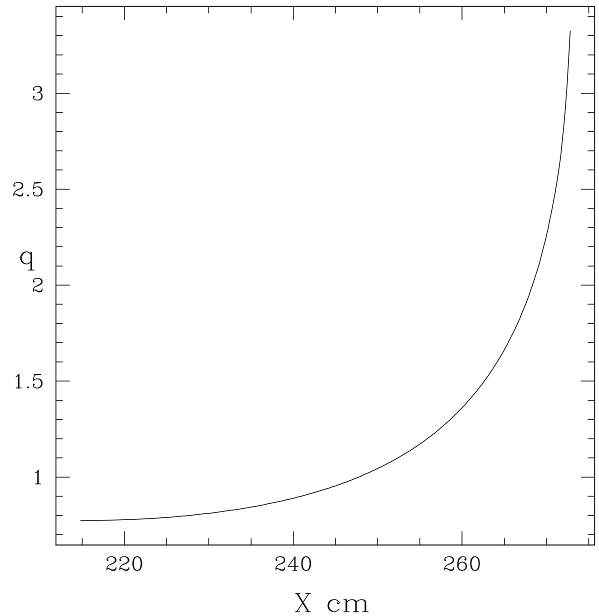
W is second order in Φ, so it gives a stronger resolution of relative amplitudes than a linear Fourier analysis.

Numerical equilibrium with a quadratic q profile

The field on axis was 4.9 kG.

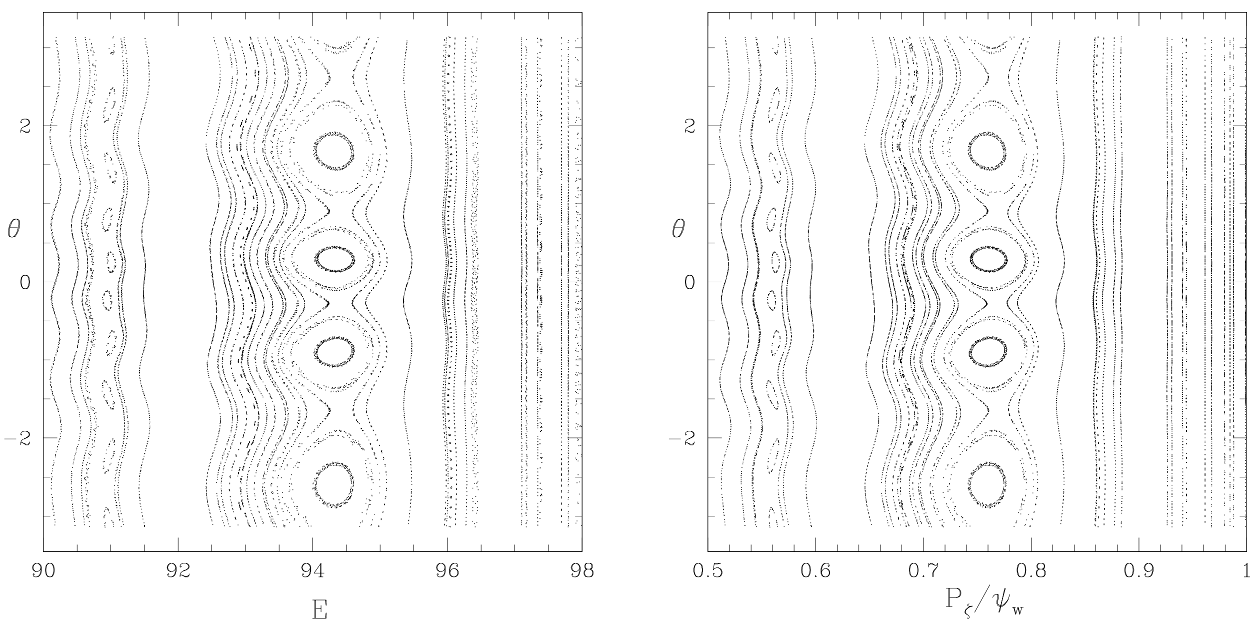
A perturbation consisting of an ideal mode
single harmonic $\xi(\psi_p)$ and $f = 100kHz$
$m = 6$ and $n = 5$, with a simple Gaussian radial profile.



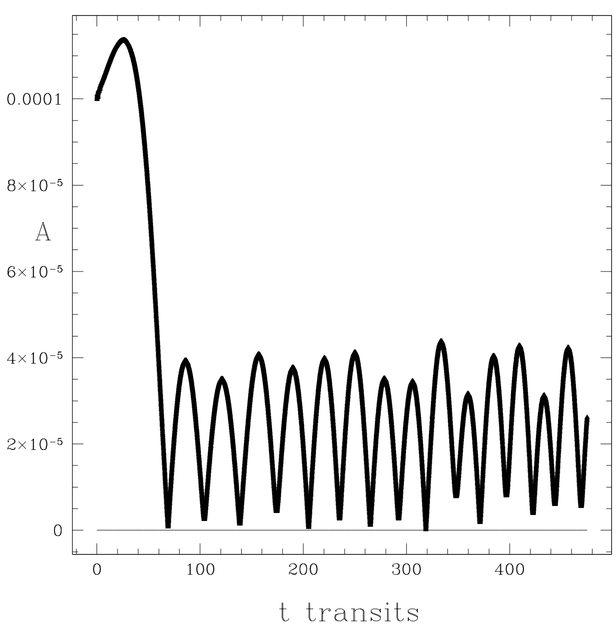


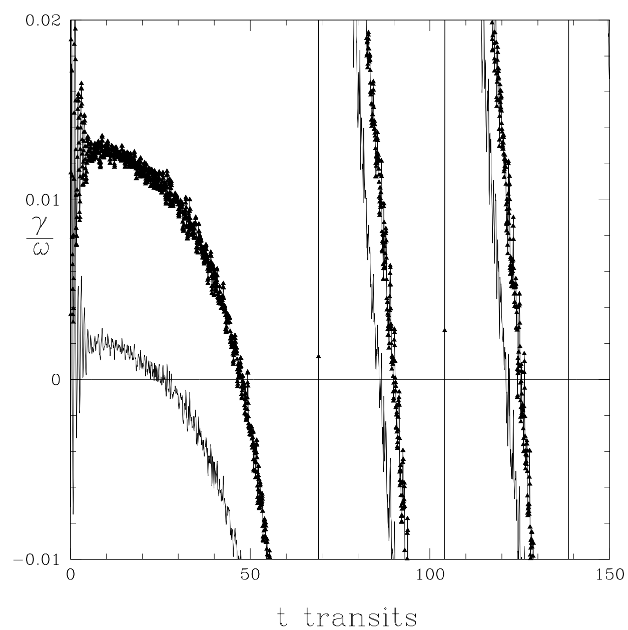
Resonance determined for a deeply passing distribution,
in energy and canonical momentum P_ζ

The distribution used must be broader than the resonance
for the maximum amplitude in the simulation. Here $A = 10^{-3}$.
A is maximum ideal displacement of the mode in terms of R
The resonance has a poloidal structure of four elliptic points.
Large amplitude to show the resonance.

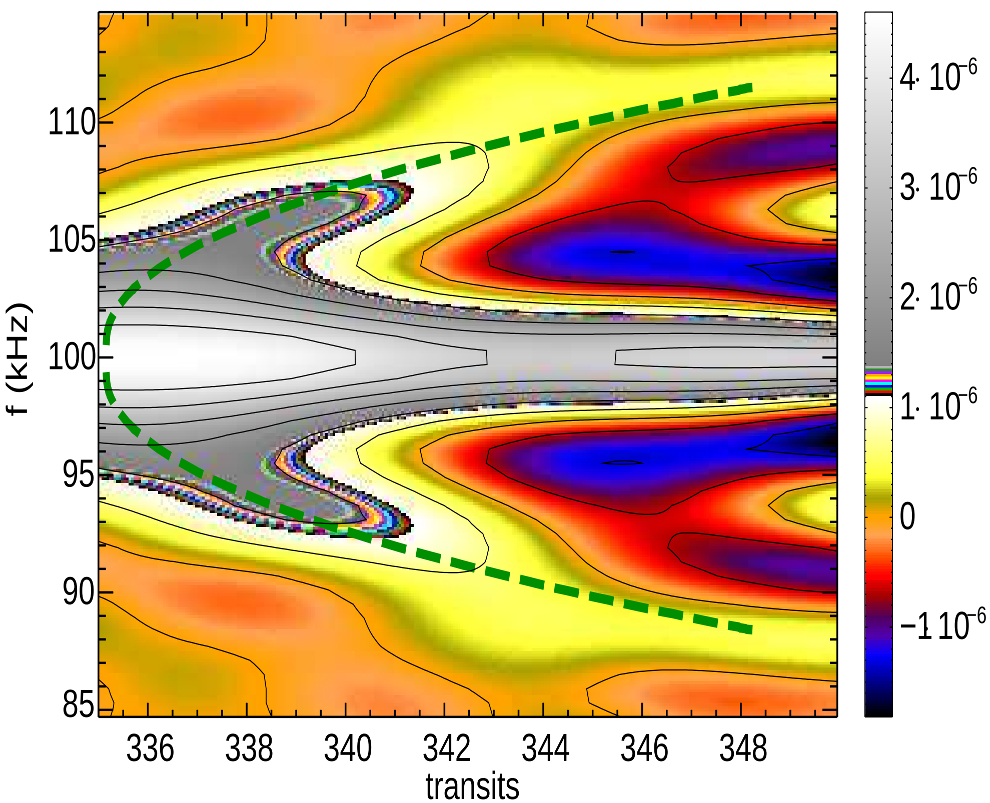


Without collisions the mode grows until it has flattened the distribution function within the resonance, and then decays due to the damping.
The strongly correlated particles in the resonance are then suddenly left without the structure provided by the perturbed field, and as the mode decays they typically leave by forming a clump and a hole giving Fourier sidebands both above and below the mode frequency.
The mode again grows and the sequence is almost periodic.
Initial $A = 10^{-4}$





Chirp



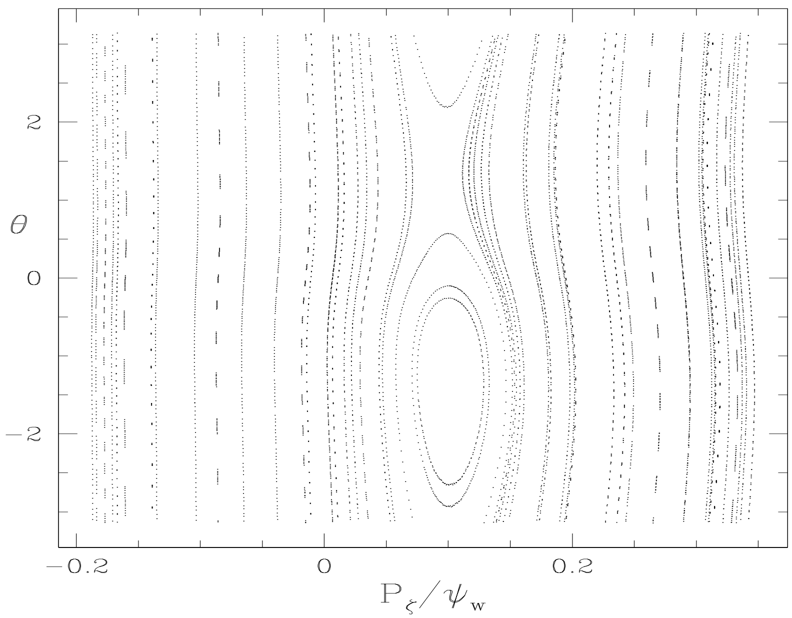
Time evolution of spectrum showing strong chirp, 100 kHz. Green curve = departures from the original eigenmode frequency, from the Berk-Breizman prediction. Phys Lett (1997). Parameters were $\gamma_L/\omega = 0.04$, $(\gamma_L - \gamma_d)/\omega = 0.007$

$$\delta f = \pm \frac{16\sqrt{2}}{\pi^2 3\sqrt{3}} \gamma_L \sqrt{\gamma_d} t.$$

The toroidal transit time is $4.7\mu s$. Time for plot is 0.07 msec

Chirp dynamics

$$\frac{d\phi_n}{dt} = \frac{-\nu_A^2}{D_n\omega_n A_n^2}\sum_{j,m} w_n\left[\rho_{\|}B^2\alpha_{mn}(\psi_p) - \Phi_{mn}(\psi_p)\right] sin(\Omega_{mn}),$$

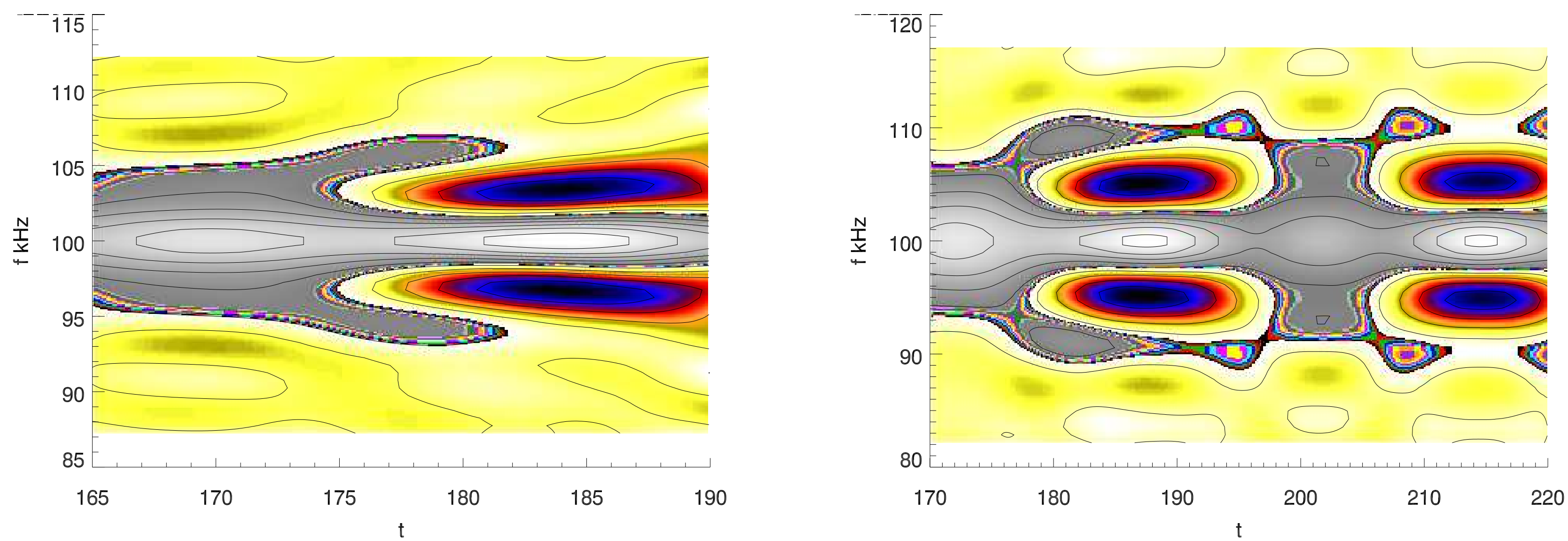


Drop in mode amplitude due to damping, but not due to a change in the drive, leaves particles in the resonance strongly driven but without the guide field of the perturbation. These particles then drive a complicated modification of $d\phi_n/dt$, producing two frequency sidebands associated with a clump and a hole. Modification of f is local in E, P_ζ and not observable.

Examples of chirps.

For most cases the sideband frequencies, after separating from the main frequency as $\sqrt{t}$, track the main frequency for a significant amount of time.

In a few cases as the mode amplitude rebounds from its lowest value the clump and hole are reabsorbed into the main frequency.

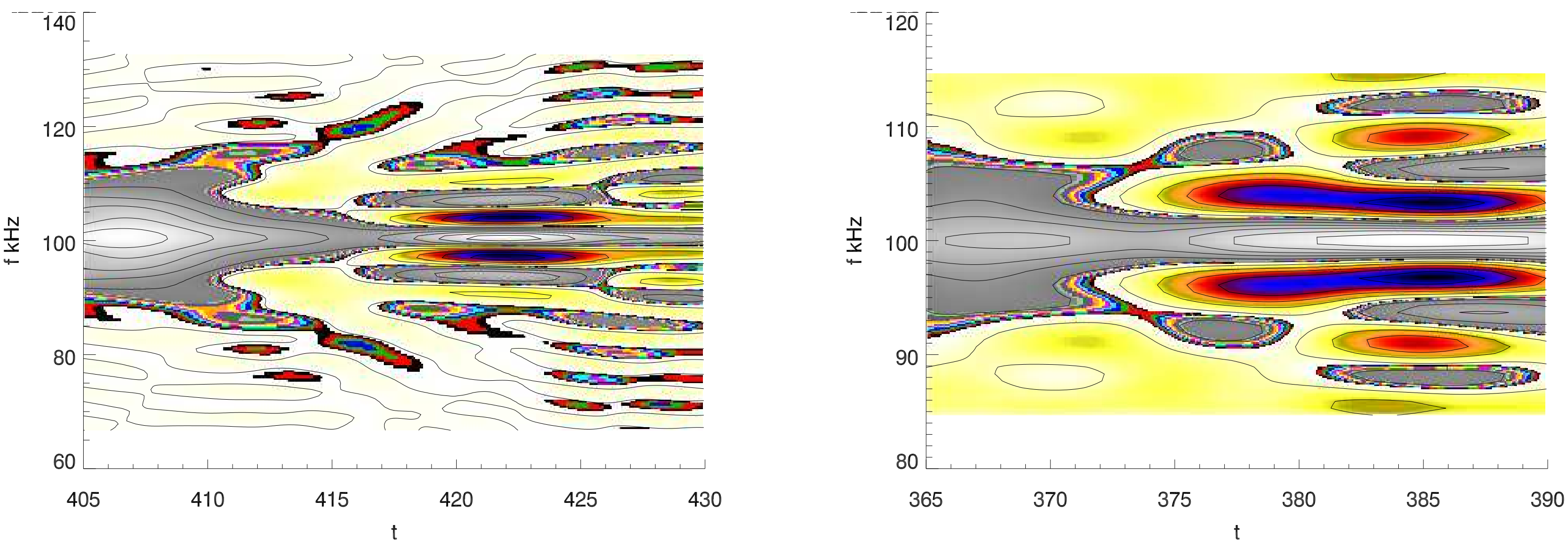


Time evolution of spectrum showing complex chirps

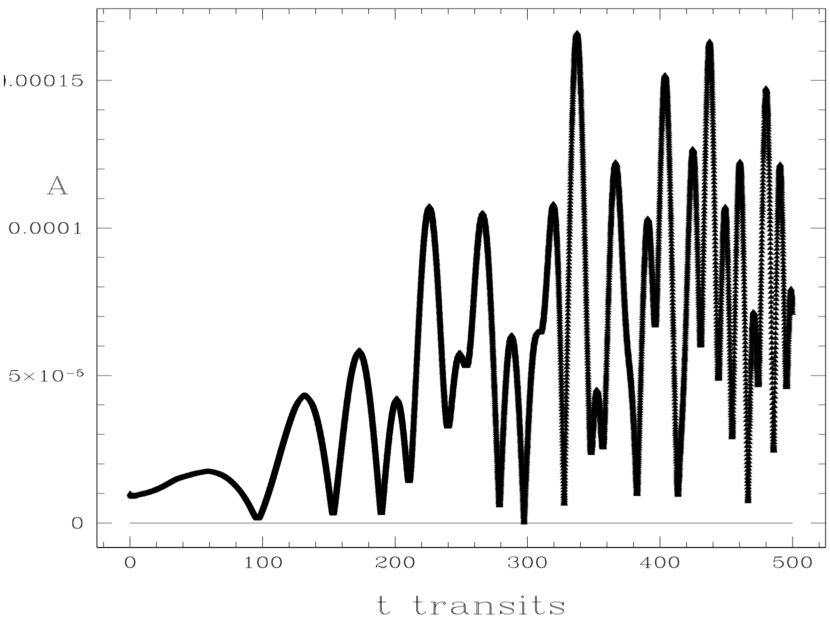
100 kHz, Growth rate and damping were $\gamma_L/\omega = .058$, $\gamma_d/\omega = .053$.

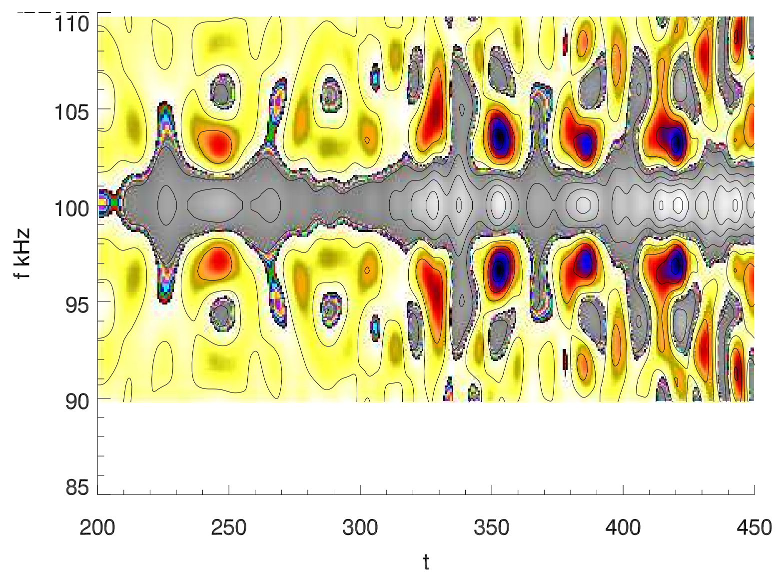
The first case had energy drag of $21/sec$, the second had none

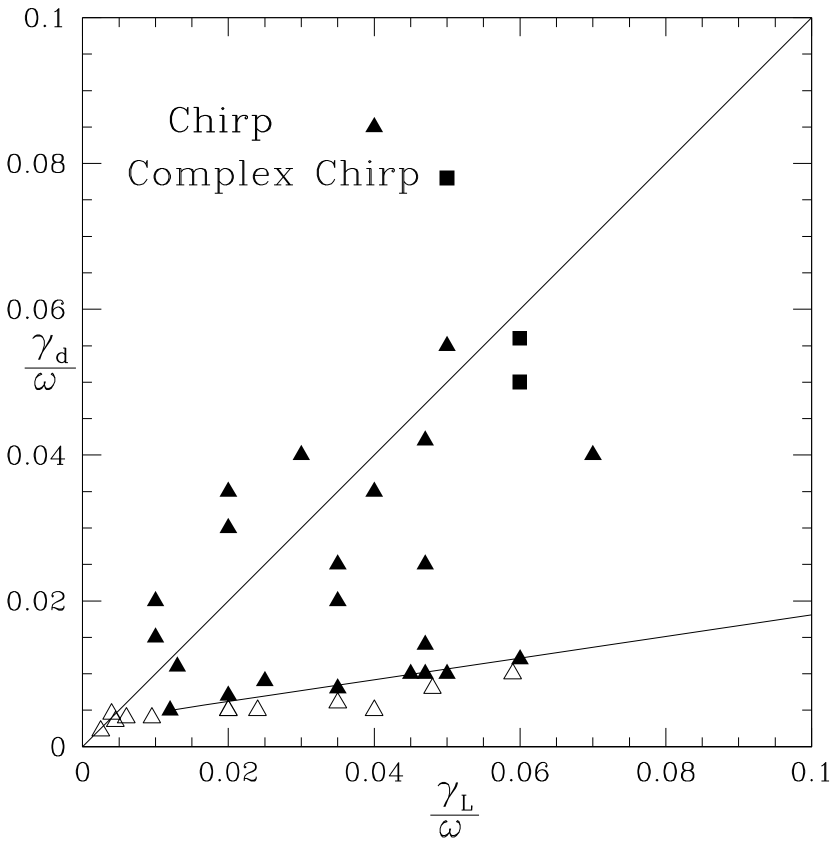
Note secondary chirping and period doubling



Time evolution of mode amplitude and frequency spectrum with multiple chirps, $f = 100$ kHz, Growth rate and damping were $\gamma_L/\omega = .058$, $\gamma_d/\omega = .053$.







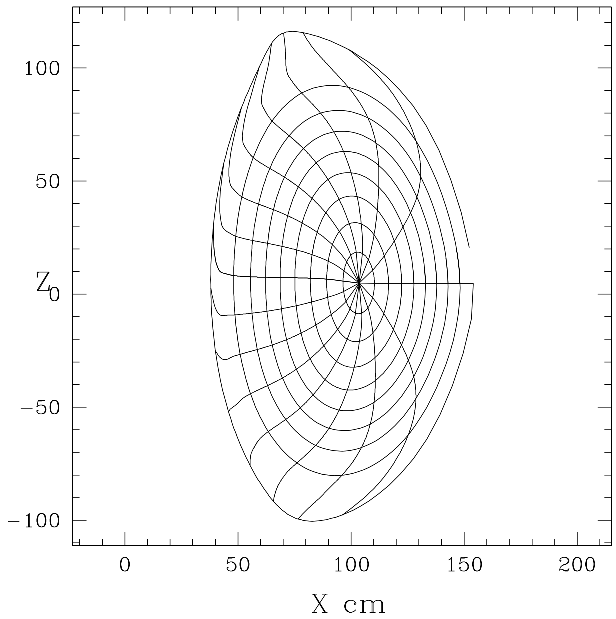
Domain in the space of γ_L/ω and γ_d/ω in which chirping is observed

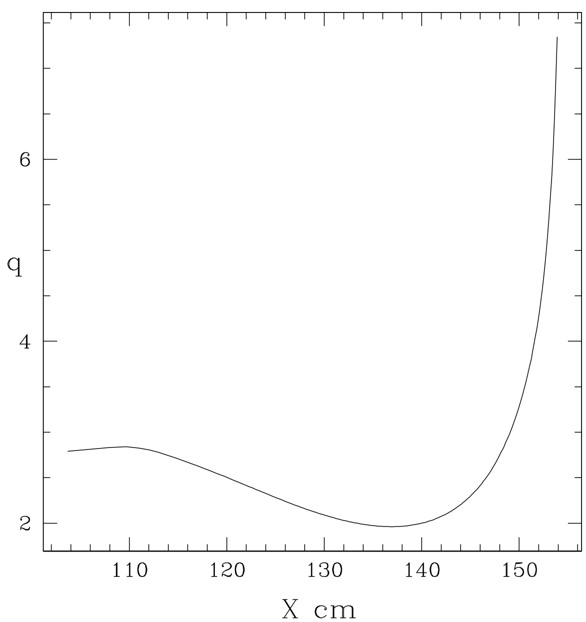
Empty triangles = no chirp

Solid triangles are normal chirps

Solid squares = complex chirping including subsequent period doubling and a profusion of clumps and holes.

There is a threshold for chirping in γ_L/ω of about 0.01, and above this chirping is observed provided approximately that $\gamma_d/\gamma_L > 0.2$.



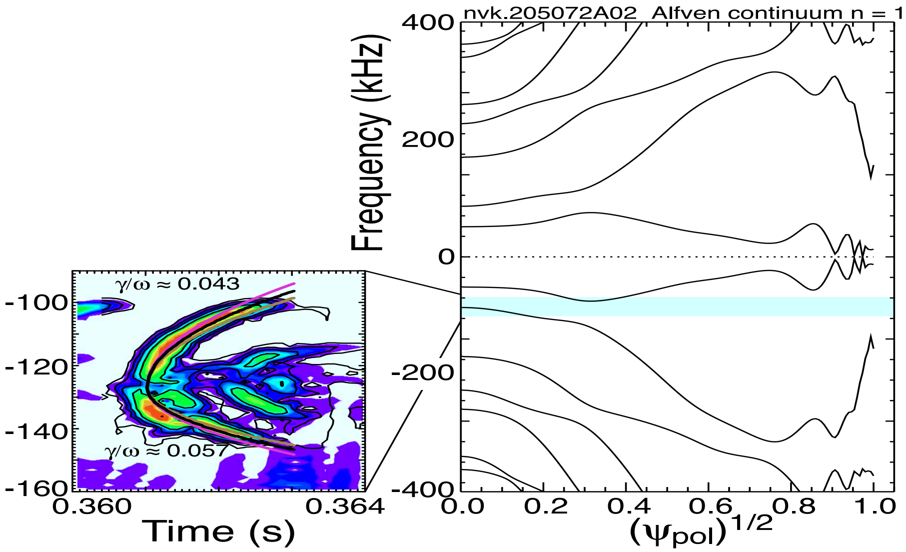


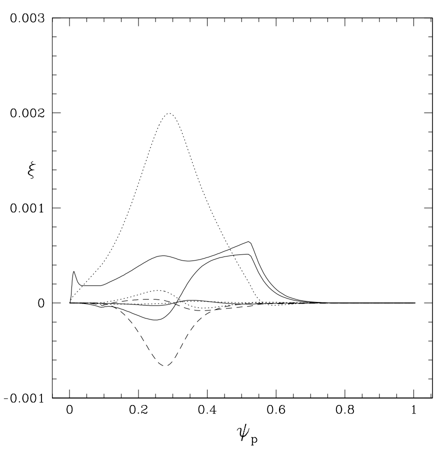
NSTX, shot 205072. With strong reversed shear

Equilibrium given by TRANSP,

Mode structure given by experiment and NOVA

Chirp NSTX 205072





Chirp observed in NSTX, shot 205072.

f = -130 kHz.

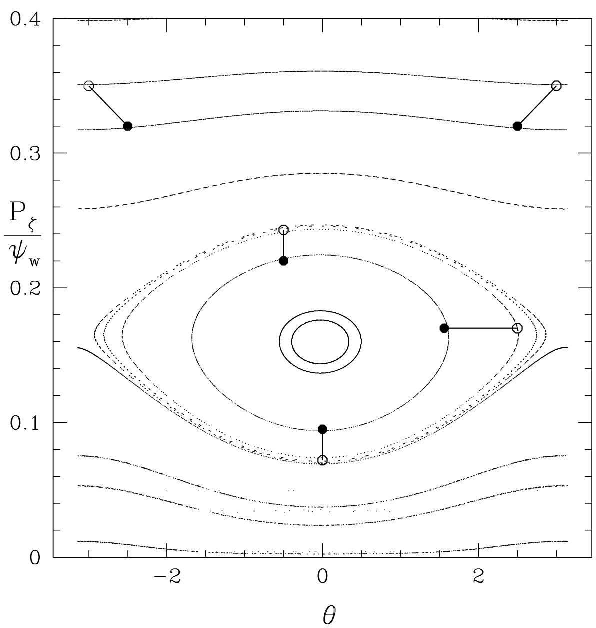
The lines on the left frequency plot show continuum modes, and the mode makes contact with them at two different locations.

Mode is large near axis.

The clump and hole are fit using $\gamma_L/\omega = \gamma_d/\omega$ of 0.043 for the upper branch and 0.057 for the lower branch.

Note the secondary burst

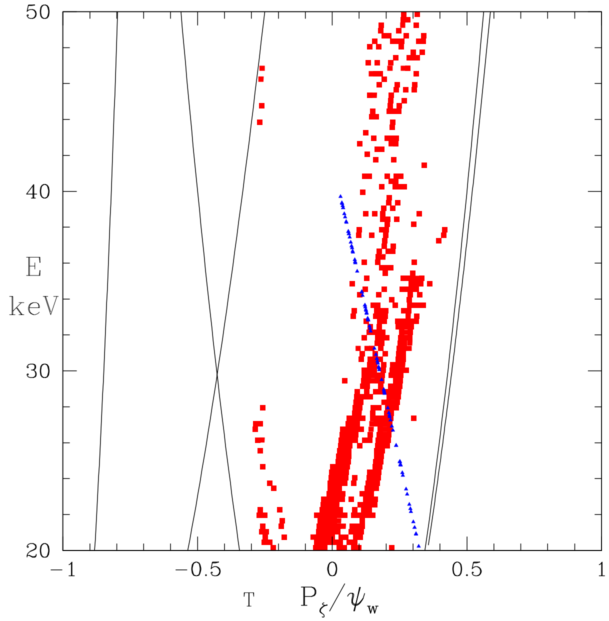
Resonance determination

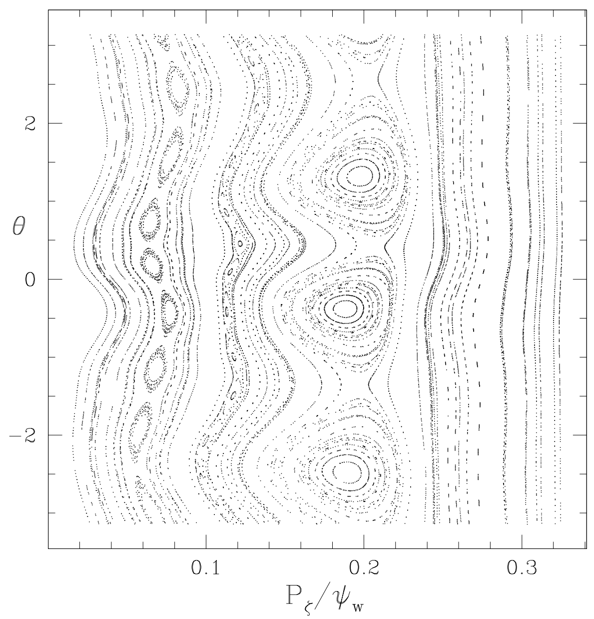


Resonances are found in any system by following pairs of particles and looking for phase vector rotation.

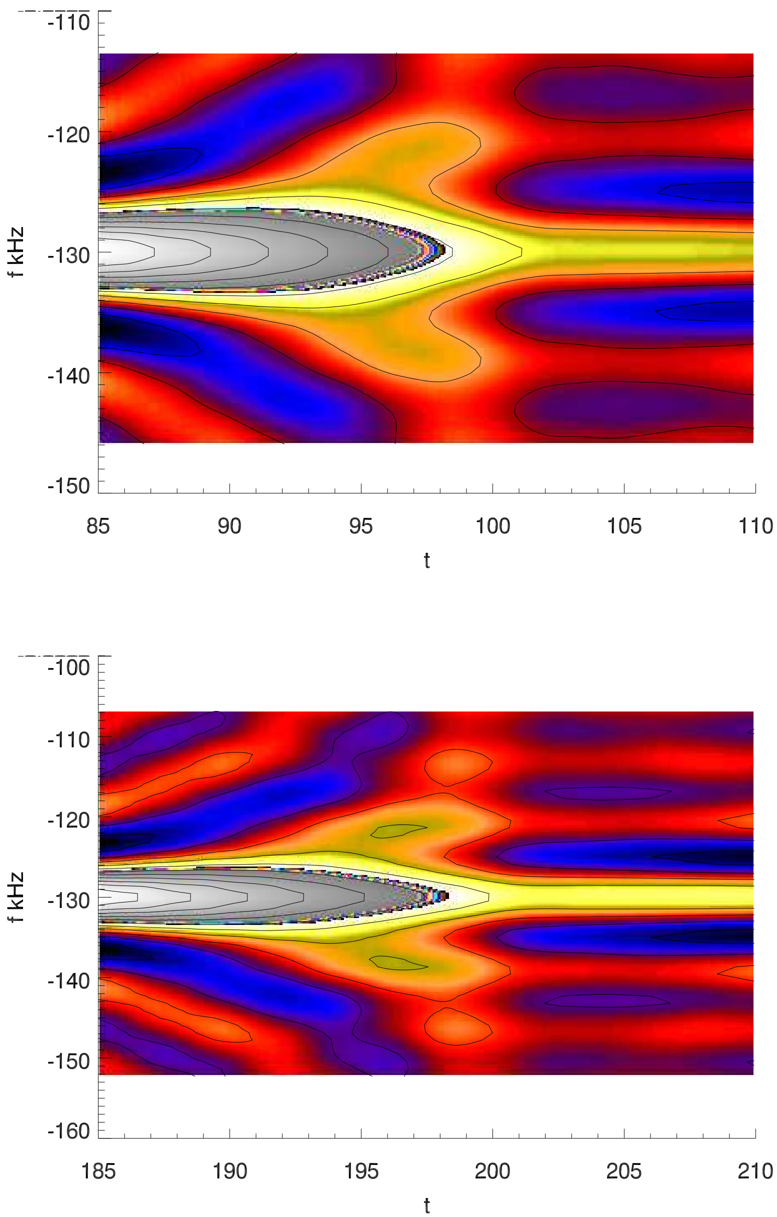
The P_ζ, θ plane showing a single $m = 1$ resonance island, and vectors between nearby points on good KAM surfaces and in the island.

On nearby KAM surfaces the phase vector can rotate by at most π, whereas a phase vector in an island rotates through 2π with a period given by the trapping bounce time.





Resonance, NSTX shot 205072,
Phase vector rotation used to find the domain in E and P_ζ broken KAM surfaces
Poincaré plot along the line $\omega P_\zeta - nE = K$.
The resonance has three poloidal elliptic points
The blue line is $\omega P_\zeta - nE = K$,
Mode given entirely by $m = 1$ harmonic determined by examining Poincaré plots one harmonic at a time

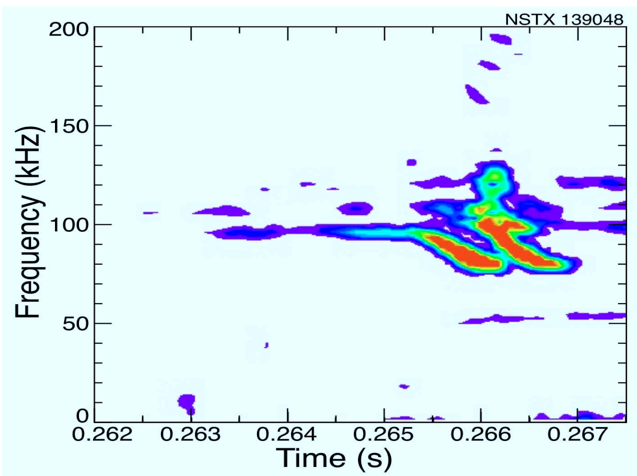


Examples of simulated chirping equilibrium of NSTX shot 205072. Growth, damping $\gamma_l/\omega = .01$, $\gamma_d/\omega = .02$.

The simulation followed the sequence in the experiment.

A mode in steady state was suddenly subjected to a large value of damping, such as would happen if a continuum mode were intersected.

Chirp NSTX 139048



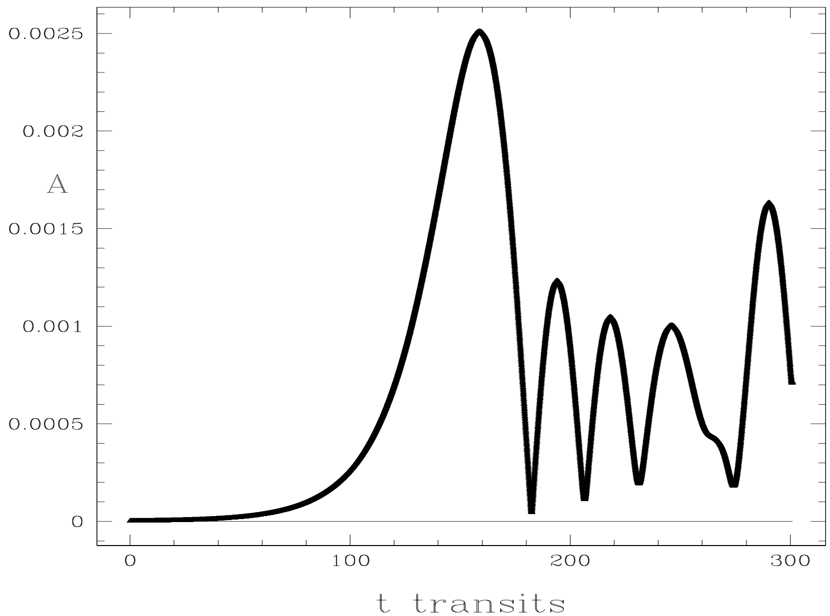
Chirp observed in NSTX, shot 139048.

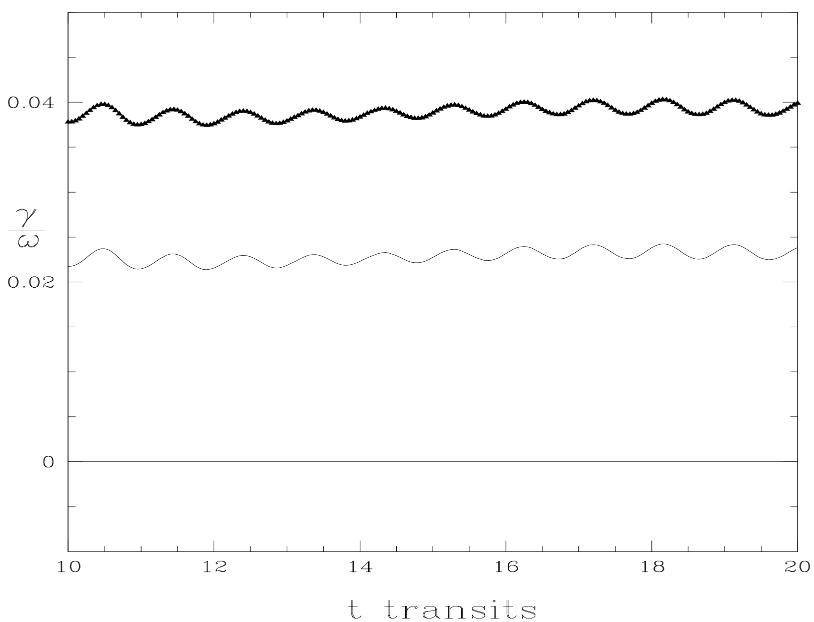
The mode had a frequency of 100 kHz.

Note that the initial part of the chirp is asymmetric, with the frequency chirping only downward, followed immediately by a more symmetric burst.

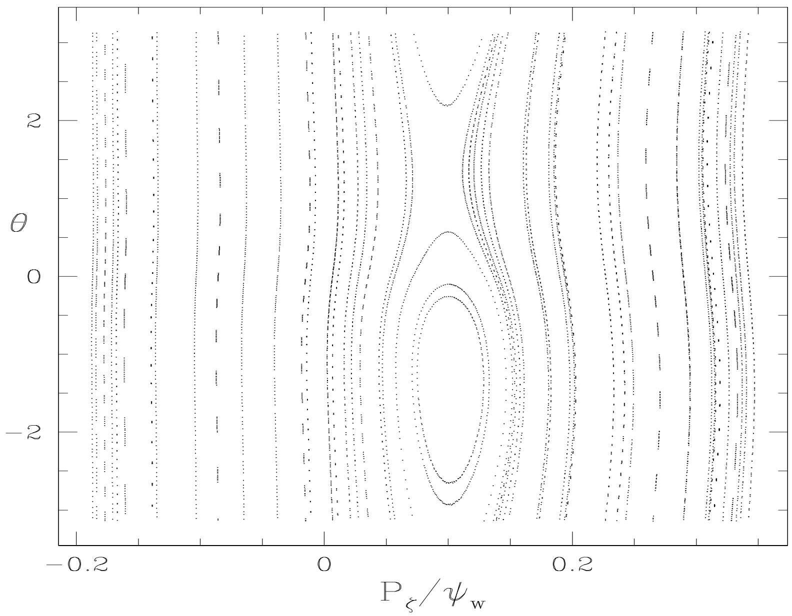
Thus far have not observed any asymmetric chirping,

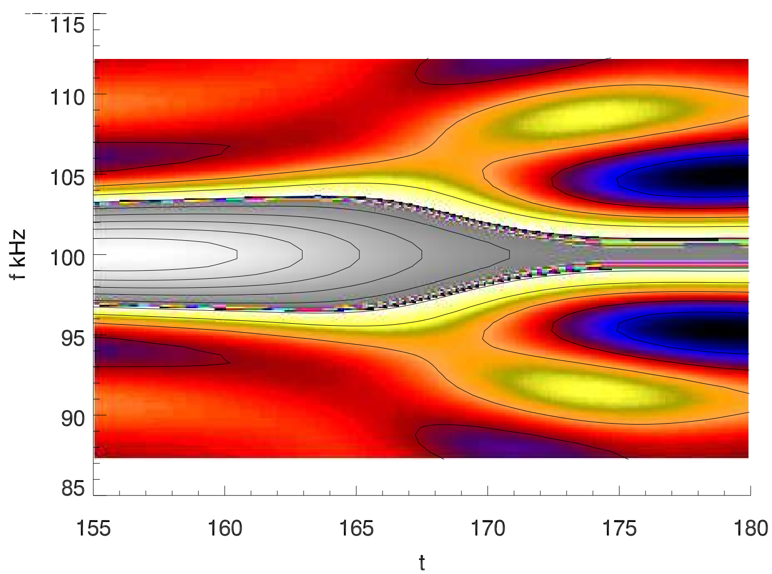
possibly due to energy drag or plasma rotation



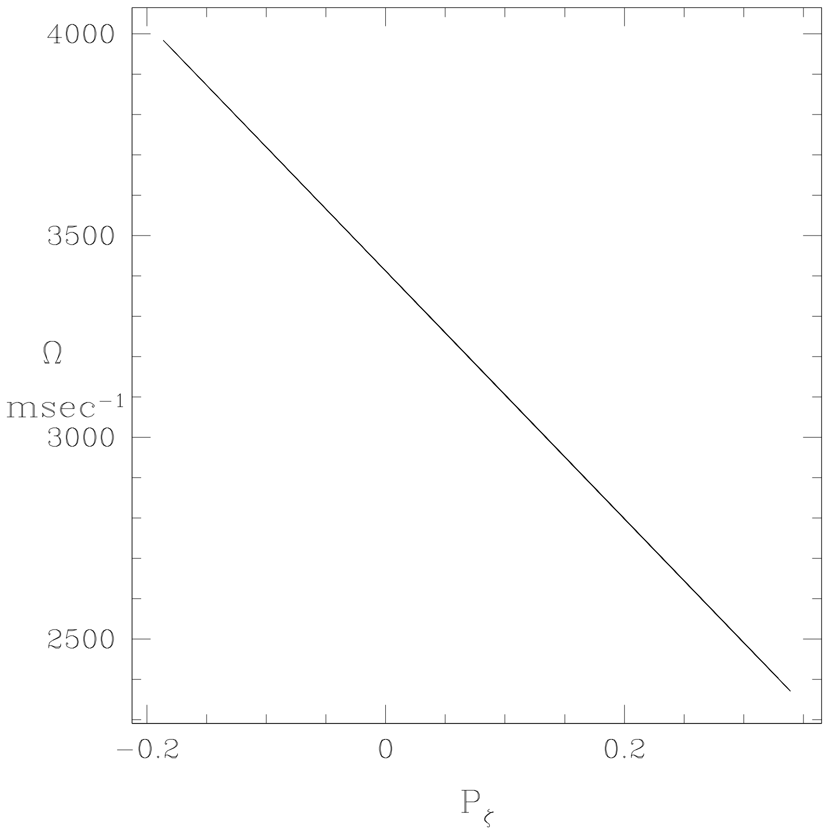


Simulation of chirping in NSTX shot 139048
Frequency of 100 kHz.
Shown is the amplitude and γ_L/ω as well as the value including damping as a function of time.
The damping reduces the growth almost to half its linear value





NSTX shot 139048,
mode resonance in P_ζ
Simulation of a Chirp growth rate and damping as shown above. Growth rate and damping were $\gamma_L/\omega = .04$, $\gamma_d/\omega = .018$.
Unlike shot 205072, the resonance has a poloidal structure of a single elliptic point.



$$\nu_{eff}^3 \simeq 2\nu_\perp R^2 \left[E\frac{B_{pol}^2}{B^2} + \mu B\right] \left(\frac{\partial \Omega}{\partial P_\zeta}\bigg|_{K,\mu}\right)^2 .$$

Numerical determination of $d\Omega/dP_\zeta$.

$\Omega = n\omega_\zeta - p\omega_\theta$ versus P_ζ fixed K,

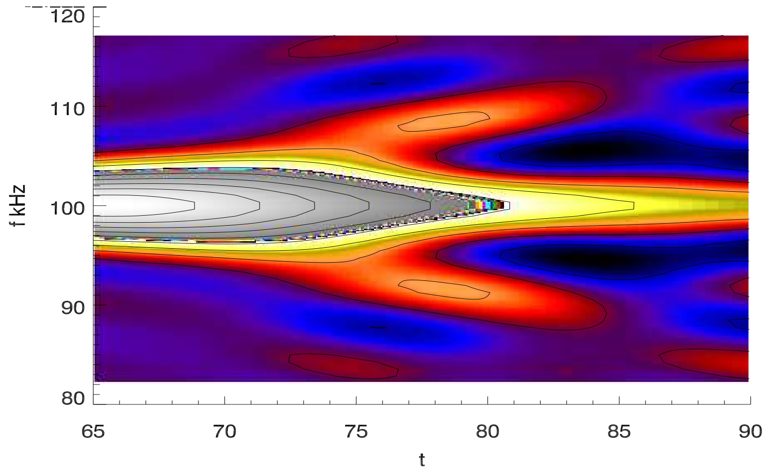
ω_ζ is the mean value of $\Delta\zeta/\Delta t$

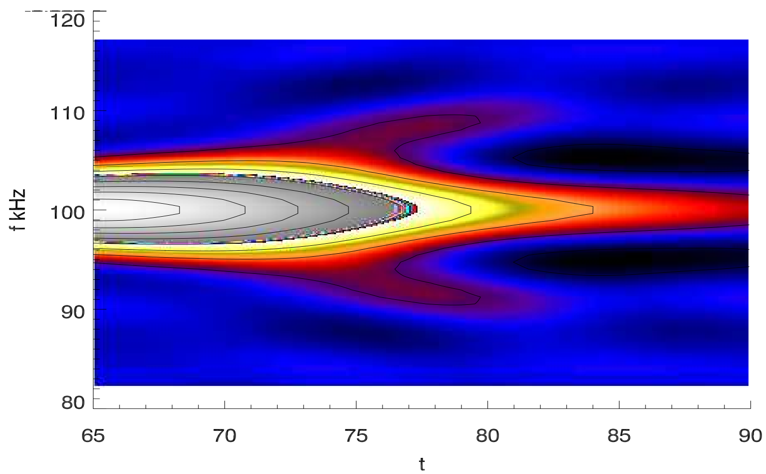
ω_θ is the mean value of $\Delta\theta/\Delta t$

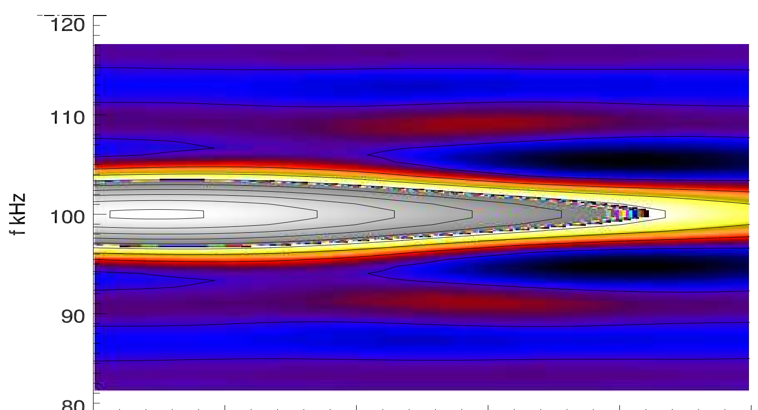
particles with $K = \omega P_\zeta - nE$,

$\mu B = 25$ keV,

$p = 1$ is the number of islands in the resonance.







Collisions kill Chirping

Effect of collisions with energy conserving pitch angle scattering.

A significant collision frequency destroys the coherence of the particles trapped in the resonance and makes chirping impossible.

PLOTS - No collisions,

$\nu = 8.5/sec$

and $\nu = 26/sec$.

Growth rate and damping were $\gamma_L/\omega = .047$, $\gamma_d/\omega = .015$.

Theory predicts $\nu \simeq 10/sec$ to destroy chirping

Conclusion

Mode chirping is observed with the guiding center code ORBIT.
The shape of the sidebands agrees with theory

There must be strong mode damping along with significant drive.
γ_L/ω must be larger than 0.01, $\gamma_d/\gamma_L > 0.2$

The simulations included a numerical equilibrium with a classical q profile quadratic in minor radius and NSTX discharges with strongly reversed shear in the core.

We conjecture that chirping is caused when small changes in the equilibrium make a saturated Alfvén mode come in contact with the continuum and suddenly experience strong damping.

Collisions inhibit chirping by destroying the coherence of the particle distribution in the resonance, and the necessary magnitude of the collisions to do this agrees with theory.