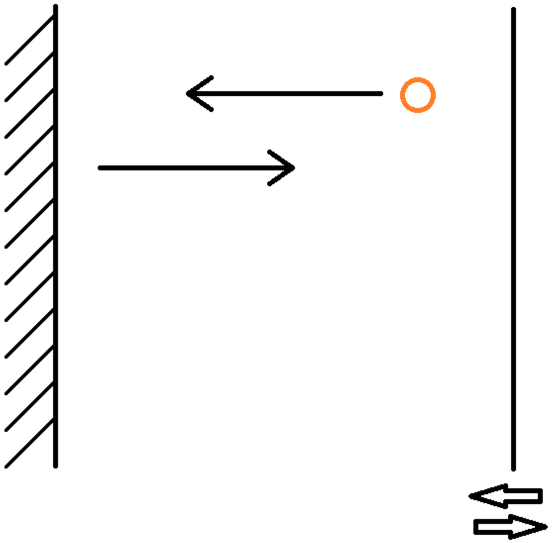
Electron confinement in a plasma-filled magnetic mirror with Fermi-Ulam-like B-field-parallel heating

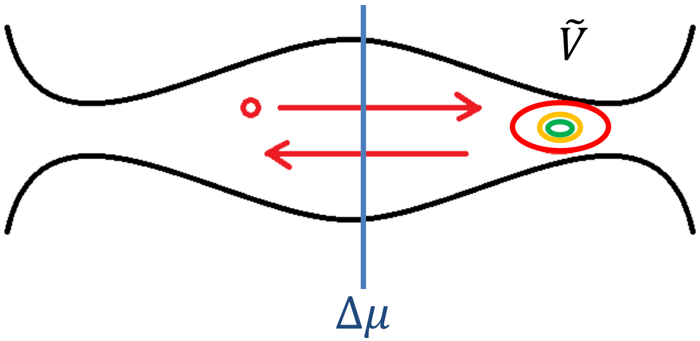
Charles Swanson
2019/09/16

Fermi-Ulam Acceleration in a Magnetic Mirror

- Fermi-Ulam acceleration: to explain cosmic ray energy distribution, particle bounces between moving walls. 1-D process, E or $v_{||}$



- Novel model: particle also experiences non-adiabatic $\Delta\mu$ jumps at mirror midplane. 2-D process, (E and μ) or ($v_{||}$ and $v_{\perp}$)



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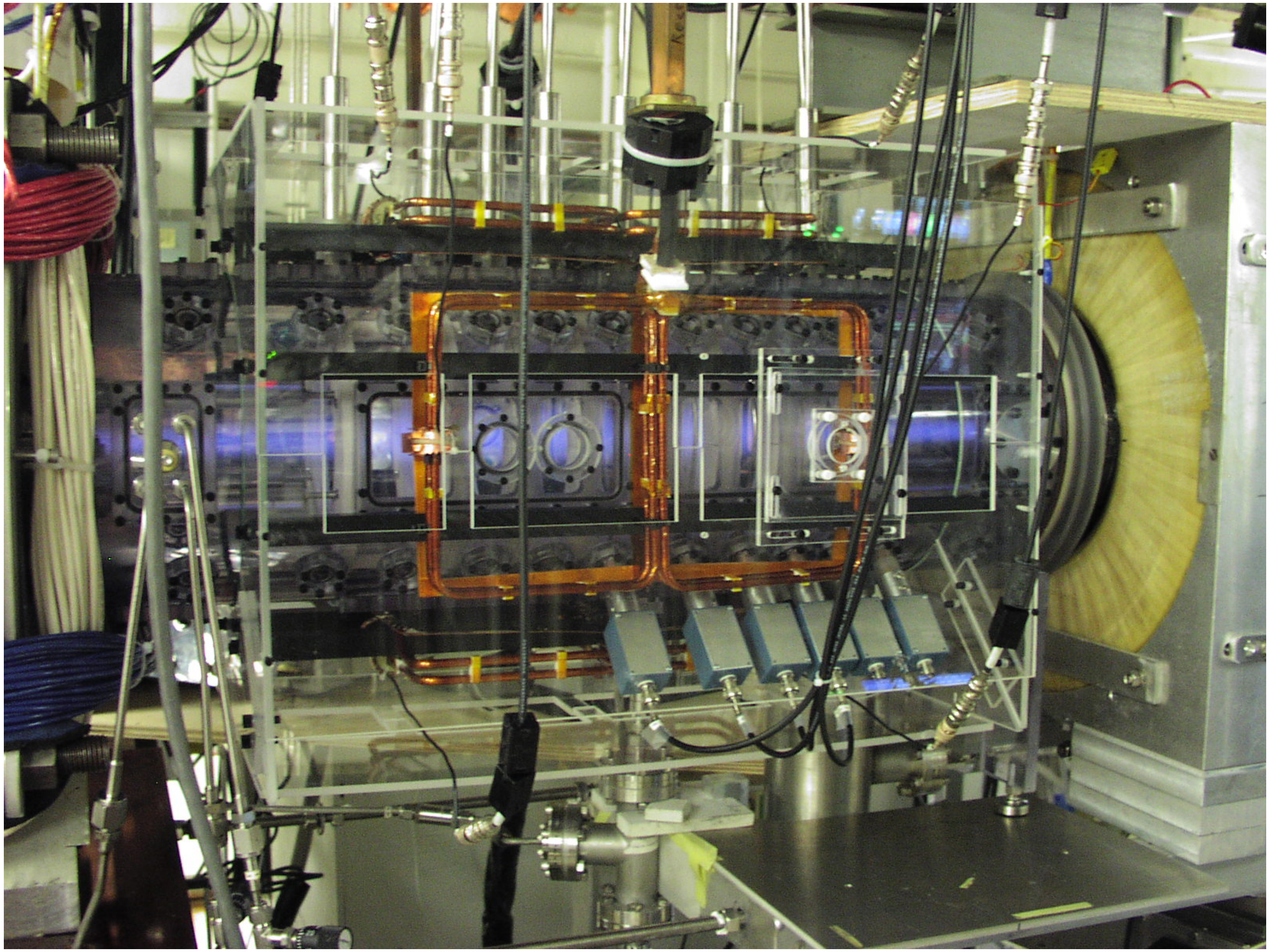
Photograph



88 cm length

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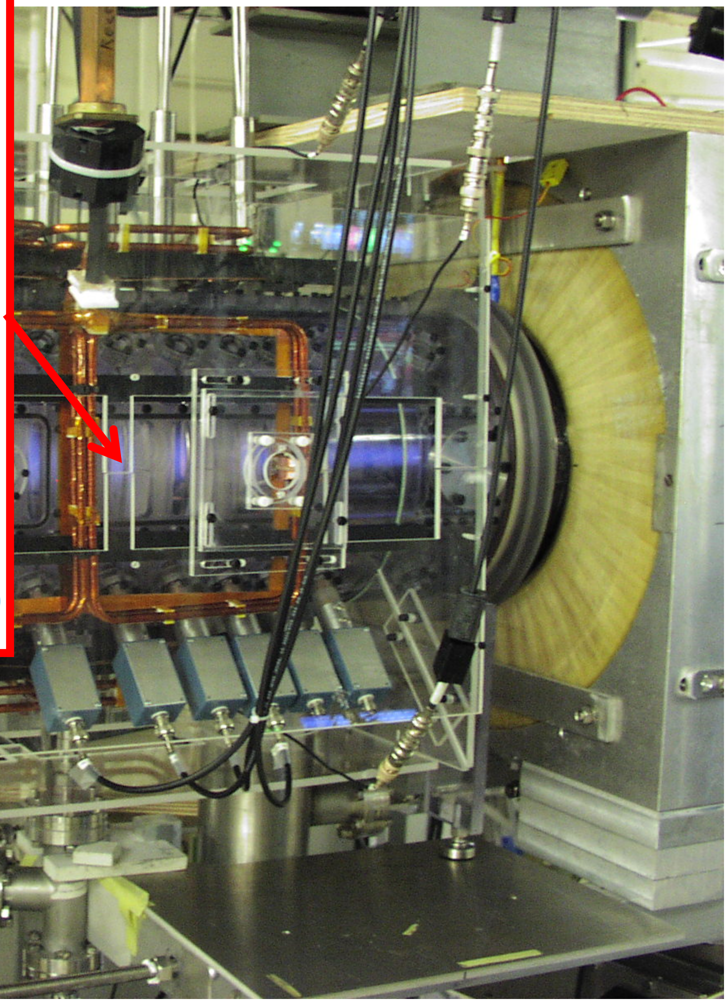

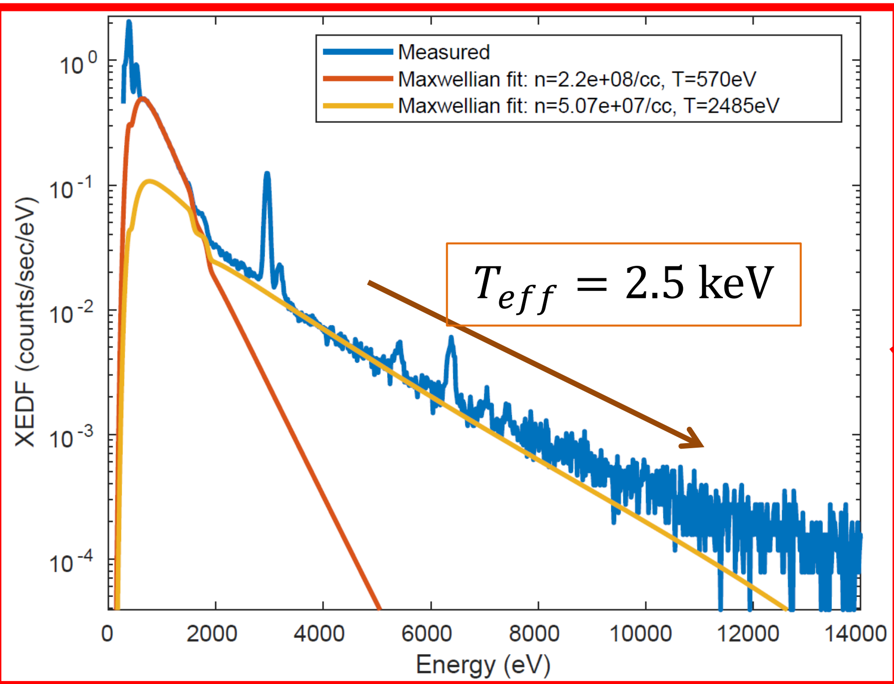

88 cm length

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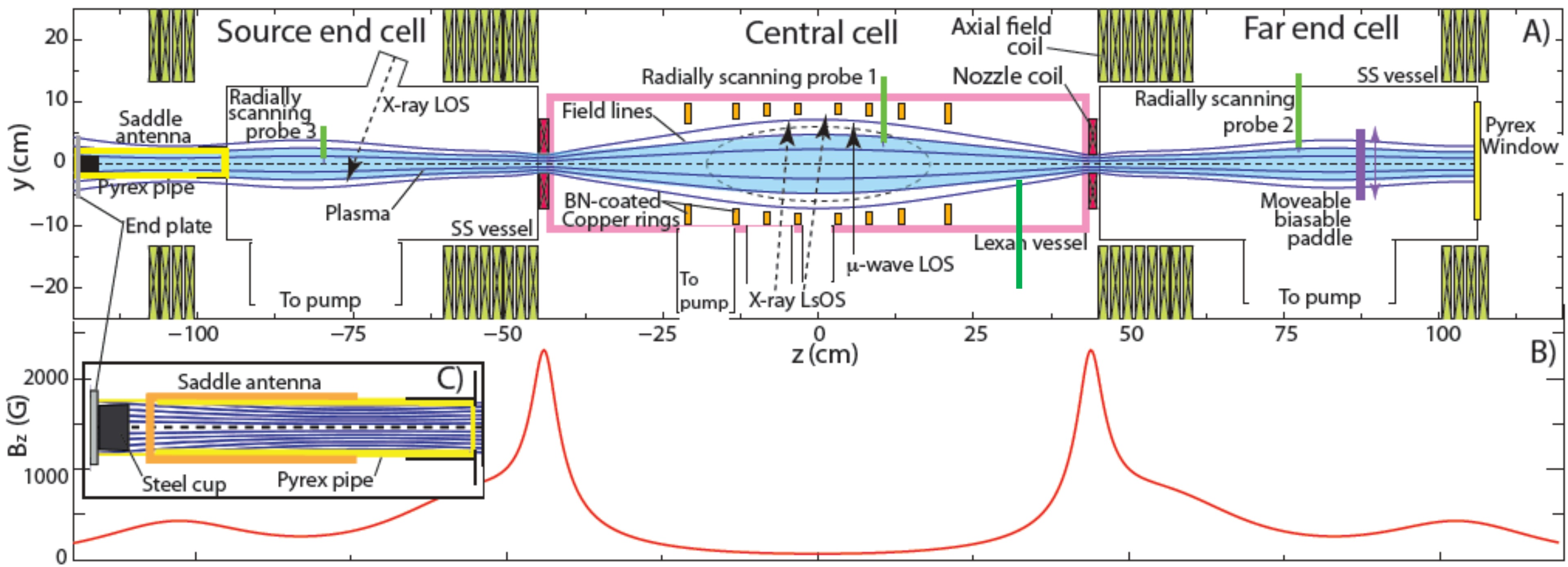
Apparatus

- Apparatus
  - Small (1m) central mirror cell
  - $\mathrm{B_{min}} \approx 200\ \mathrm{G}$
  - $R \approx 10 - 40$
  - 3 cells, all critical
- 25 - 500 W at 14 - 27 MHz
- Bulk plasma at $T_e \approx 5\ \mathrm{eV}$, $n_e \approx 10^{10-11}/\mathrm{cm}^3$
- Probes for 0 – 50 eV
- X-ray for 200 eV – 30 keV



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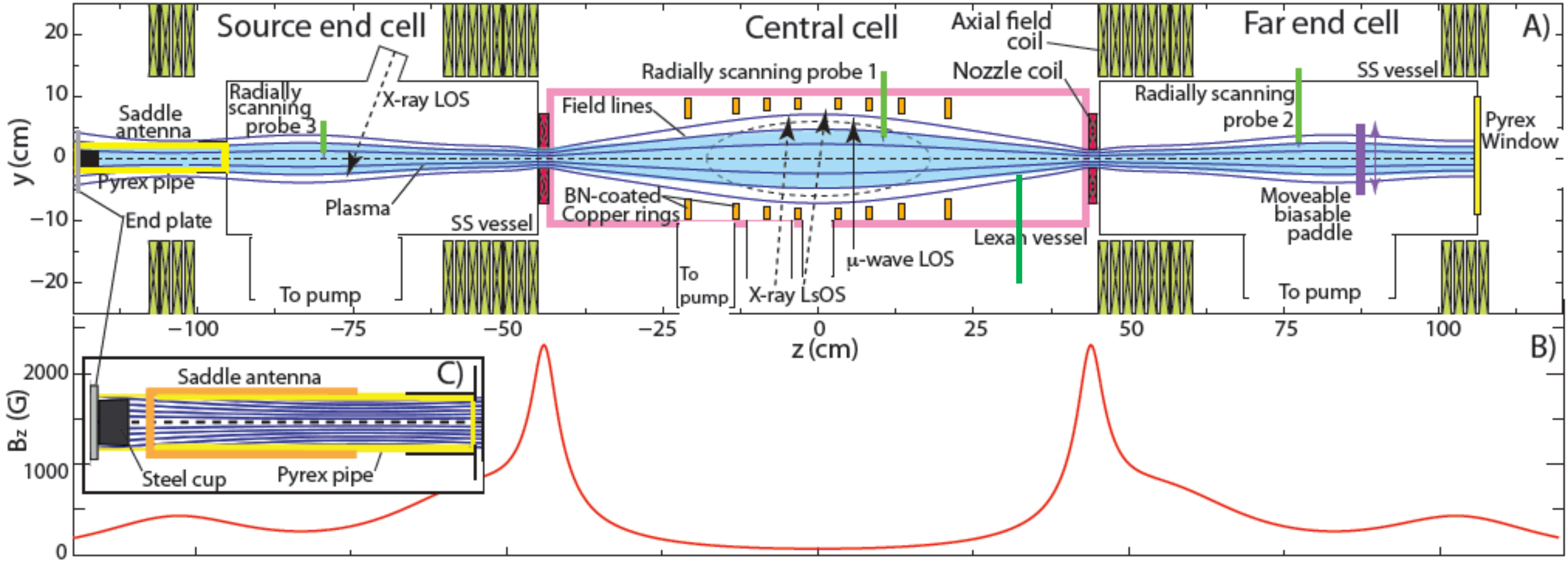

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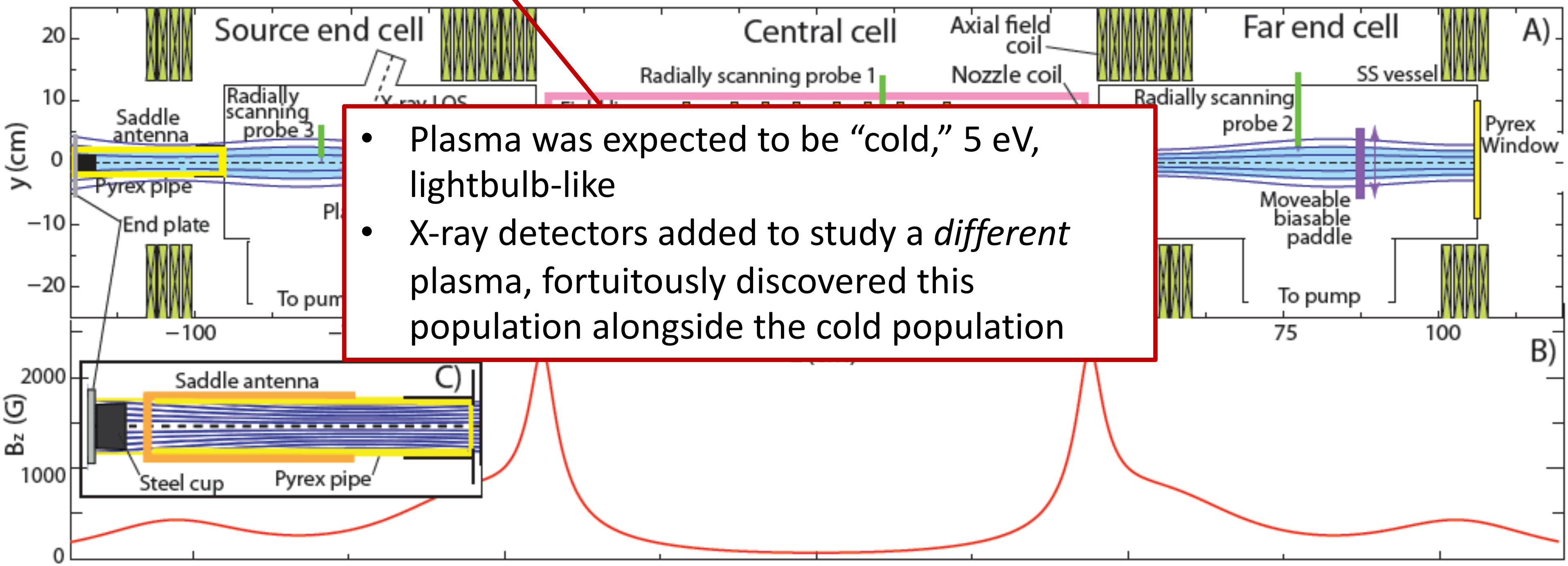

- Plasma was expected to be "cold," 5 eV, lightbulb-like
- X-ray detectors added to study a *different* plasma, fortuitously discovered this population alongside the cold population

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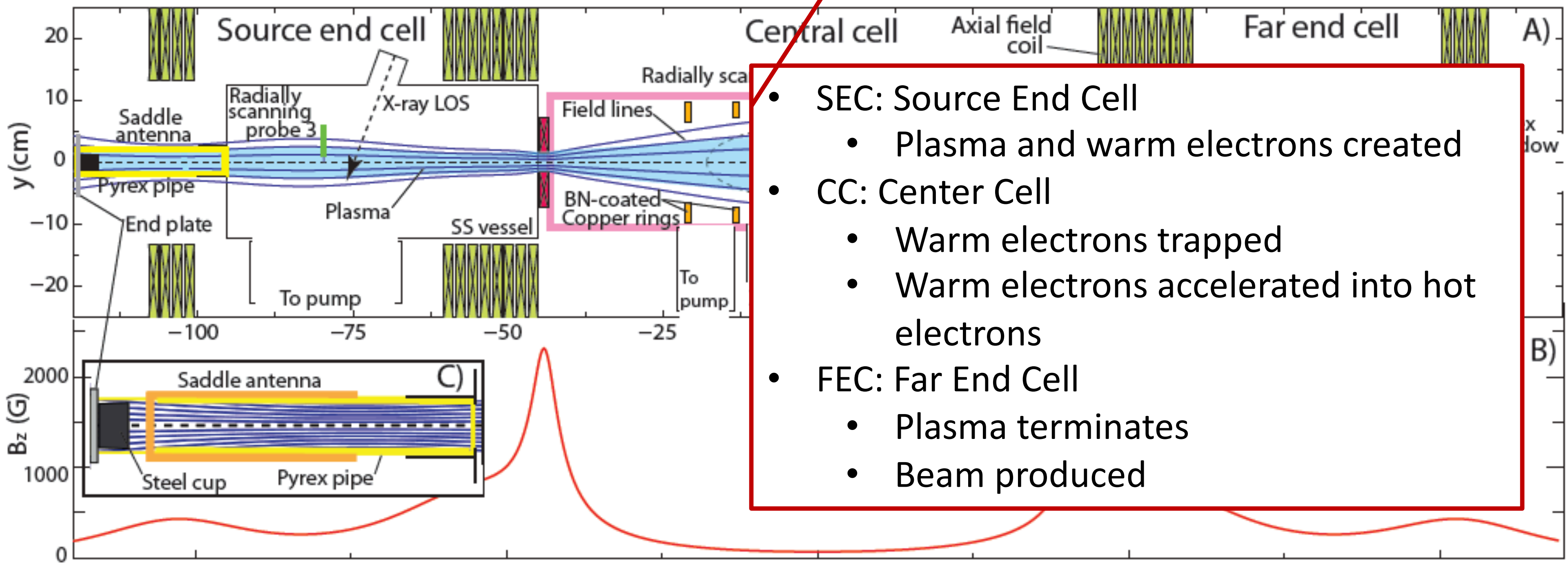


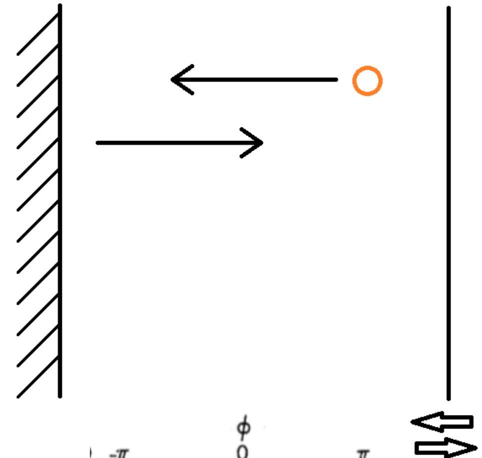
- SEC: Source End Cell
  - Plasma and warm electrons created
- CC: Center Cell
  - Warm electrons trapped
  - Warm electrons accelerated into hot electrons
- FEC: Far End Cell
  - Plasma terminates
  - Beam produced

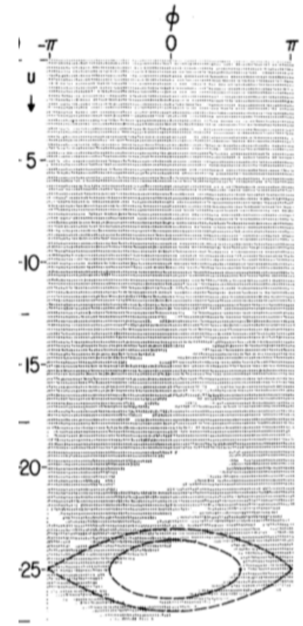
Fermi-Ulam Map

- Fermi-Ulam Map: Ulam (1961)
  - Particle bounces between fixed and oscillating wall. 1-D dynamics!
  - Many variations: Sawtooth wall motion, both walls move, particle returns under gravity, etc.

  - $u_{n+1} = |u_n + \sin\Psi_n|$
  - $\Psi_{n+1} = \Psi_n + 2\pi M/u_{n+1}$
  - u is normalized particle velocity, M/u is normalized transit time, Ψ is phase of oscillating wall

  - Below some critical velocity: particle is free to diffuse through (u, Ψ) space, excepting small islands
  - Above some critical velocity: particle is trapped in quasi-periodic orbits in (u, Ψ) (islands)





Lieberman (1972)

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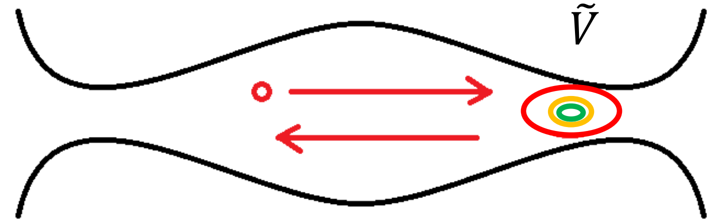
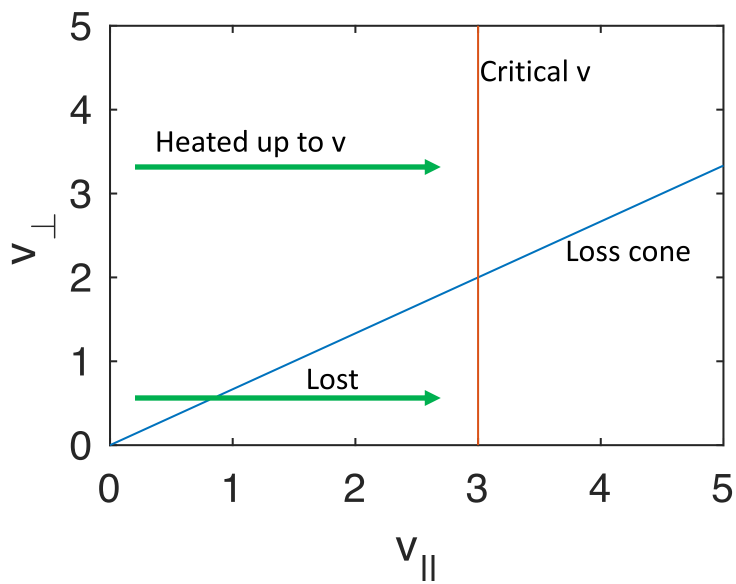
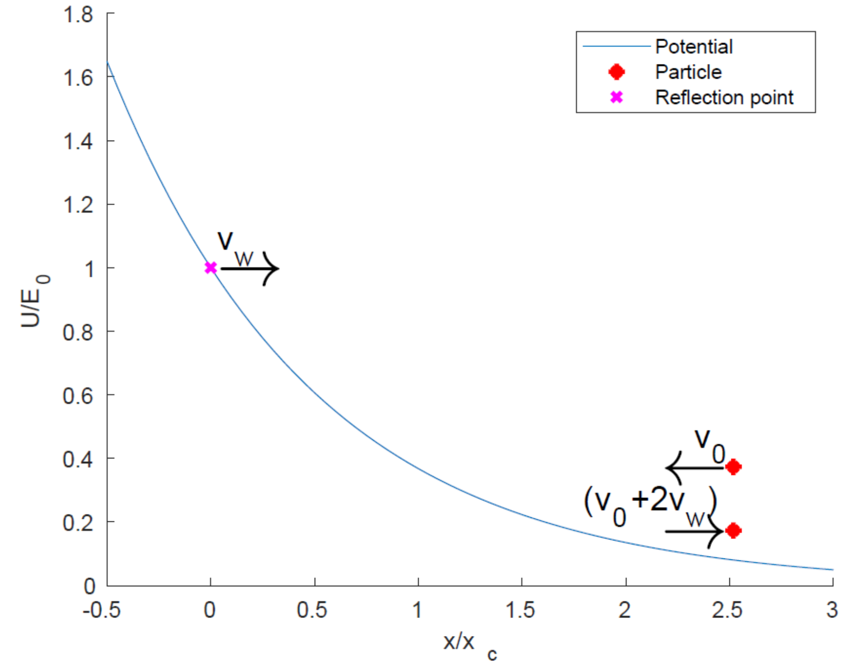
Electrostatic fluctuation in a magnetic mirror

- Localized voltage fluctuation
  - Electrons passing close to the nozzle get increments to $\vec{v}_{||}$ component of velocity, as though bouncing from a moving wall
  - $\vec{v}_{\perp}$ is unchanged
- Particles either de-trapped or prevented from heating



Mirror with localized potential fluctuation





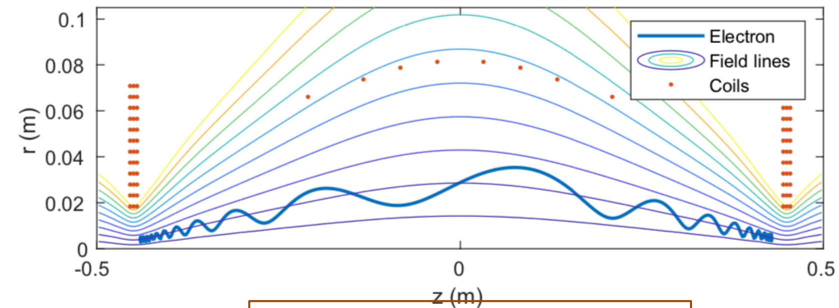
A changing potential acts like a moving wall, reflects particles

Non-adiabaticity of magnetic moment in magnetic mirror

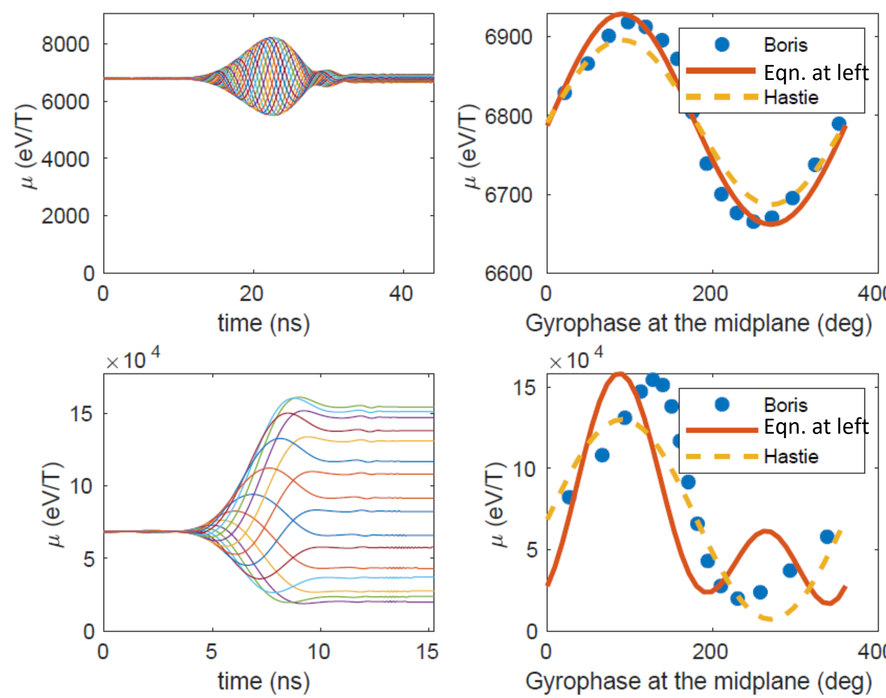
- Single-transit change in μ:
  - Boris algorithm: Numerical integration
  - Hastie, Taylor, Hobbes (1969): Asymptotics
  - Speiser (1970) collisions
  - Novel analysis produces equation:

$$\Delta\mu = \frac{m v_\perp v_\parallel z_0 \sqrt{\pi}}{B_0 R_0} e^{\frac{-z_0^2 \Omega^2}{4\, v_\parallel^2}} \sin\phi_0$$

    - Not produced from asymptotic series; elementary approximations considered.
    - μ conserved around a wire
    - $z_0^{-2} = z_R^{-2} + z_B^{-2}$, *change* in curvature
    - $z_Q^2 = \frac{\partial_{z,z}^2 Q}{2 Q_0}$



5.4 keV electron in PFRC-II



1.5 keV electron (top) and 15 keV electron (bottom) in PFRC-II

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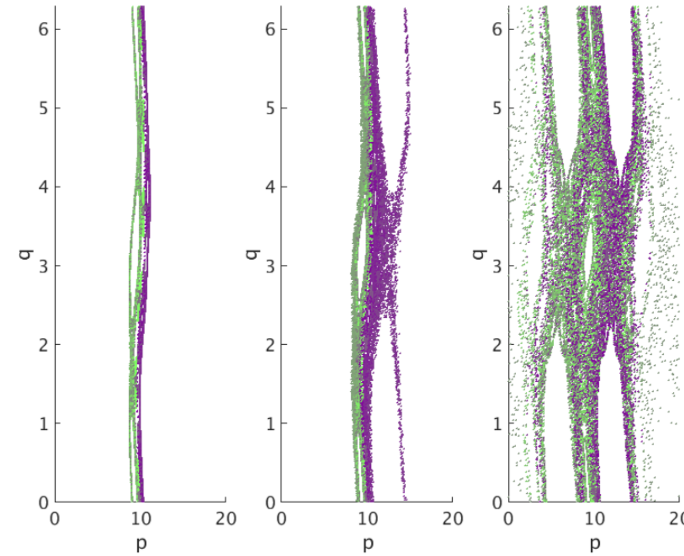
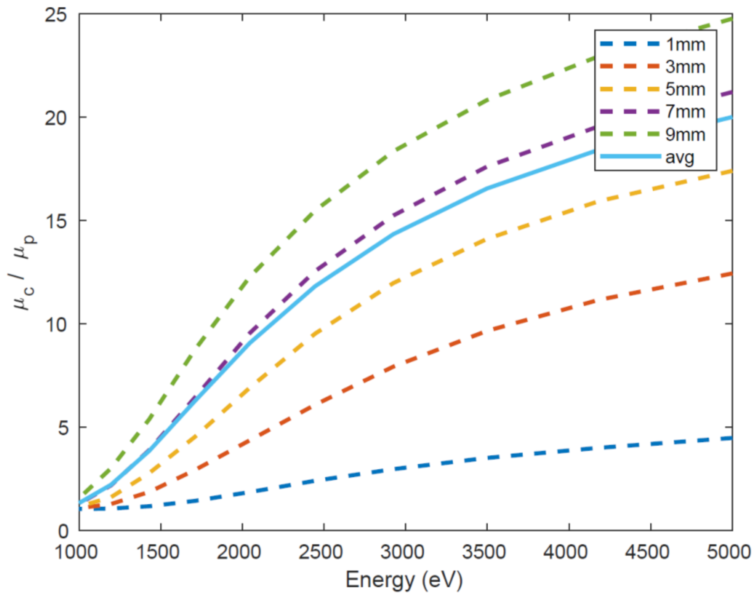
Non-adiabaticity of magnetic moment in magnetic mirror

- Long-time behavior: Chirikov (1978)
  - Equations for μ, ψ reduce to standard map:
    - $p_{n+1} = p_n + K\sin q_n;\ q_{n+1} = q_n + p_{n+1}$
    - $K \equiv -\partial_\mu \Delta\psi \times \delta\mu$
    - $p = \partial_\mu \Delta\psi \times \mu;\ q = \psi + \pi$
    - ψ the gyrophase at the midplane

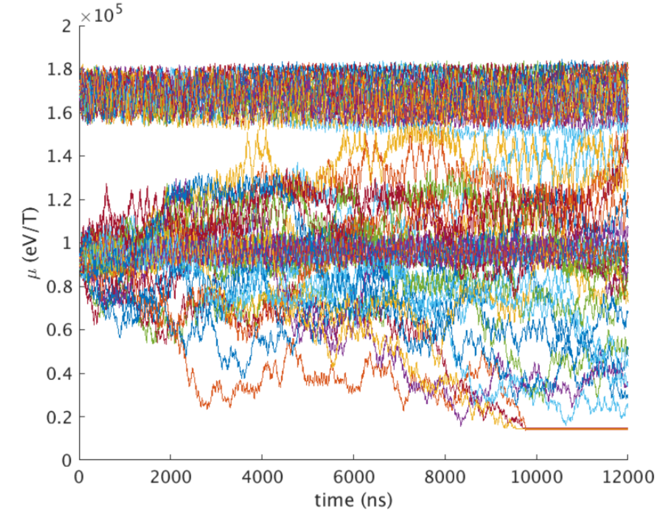


$K = 0.9$ $K = 1.1$ $K = 1.3$

The map at left exhibits a separatrix until $K = 1$



(μ_c, E) for $K = 1$ in PFRC. μ_c is separatrix between diffusive and quasiperiodic.



Boris algorithm confirmation

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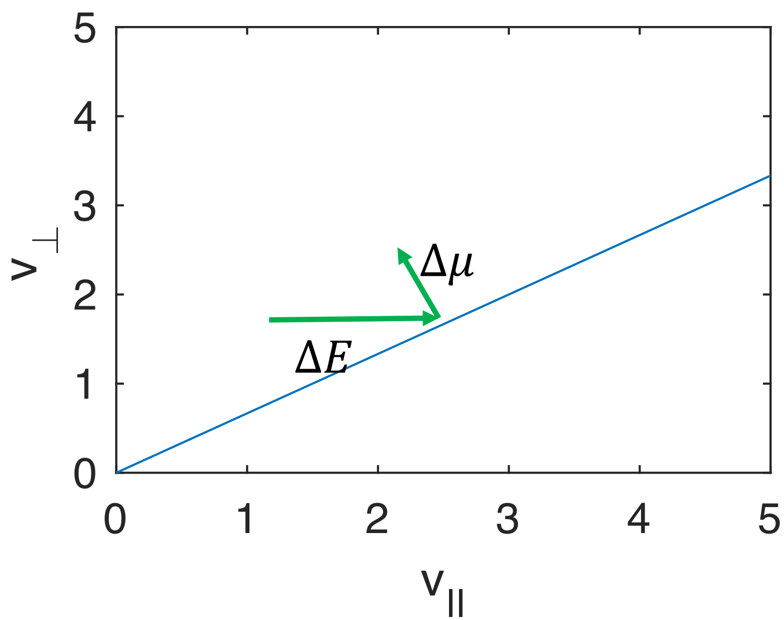
Non-adiabaticity provides trapping mechanism

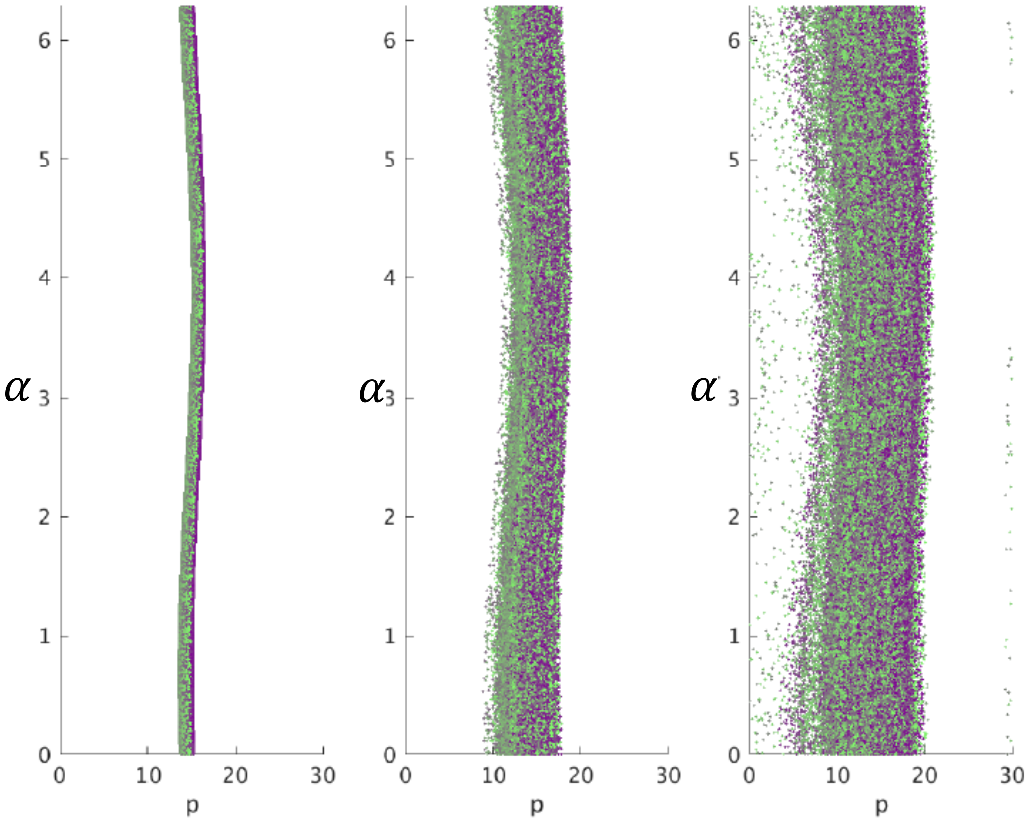
- Trapping by $\Delta\mu$:
  - Fermi acceleration causes ΔE purely in the $\vec{v}_{||}$ direction, toward de-trapping
  - Non-conservation of μ causes diffusion in both directions (mostly $\vec{v}_{\perp}$)
  - Joined diffusion allows arbitrarily large E while remaining trapped



Having diffusion in μ allows Fermi acceleration not to de-trap particles

Non-adiabaticity provides adequate decorrelation

- Decorrelation by $\Delta\mu$:
  - μ, magnetic moment, is *not* conserved for 1+ keV electrons in the PFRC-II.
  - Sign of $\Delta\mu$ determined by *gyro*phase, entirely decorrelated from RF and bounce phase. Essentially random.
    - $p_{n+1} = p_n + K\sin\alpha_n$
    - $\alpha_{n+1} = \alpha_n + p_{n+1} + Rr_{n+1}$
      - $p_n = KE_n/\Delta E$, α_n is the oscillation phase upon particle incidence. $r_n = \Delta\mu_n/\delta\mu$ is random from -1 to 1.
      - $K \equiv \partial_E(\Delta\alpha)\times\Delta E < 0.1$
      - $R \equiv \partial_\mu(\Delta\alpha)\times\delta\mu \approx 0.1$
  - Certainly a ***sufficient*** condition for decorrelation



$K = 0.1, R = 0$ $K = 0.1, R = 0.1$ $K = 0.1, R = 0.2$

Having $R \sim K$ is a sufficient criterion to allow diffusion.
That is, having **the change in RF phase due to μ jumps similar to the change in RF phase due to E jumps.**

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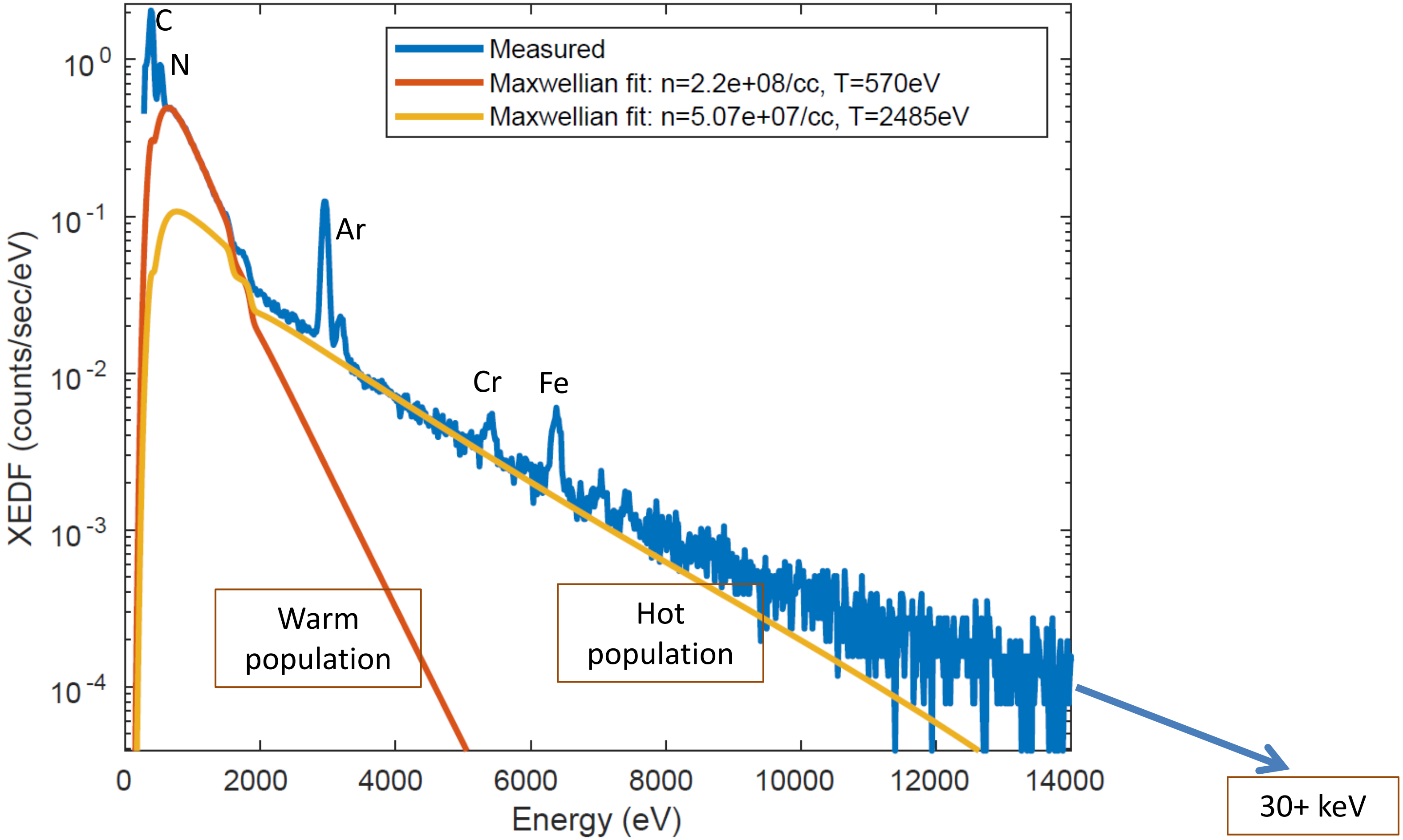
Typical Properties of the Hot Electrons



X-ray measurements reveal that: alongside the **warm population** is a tenuous **hot population**

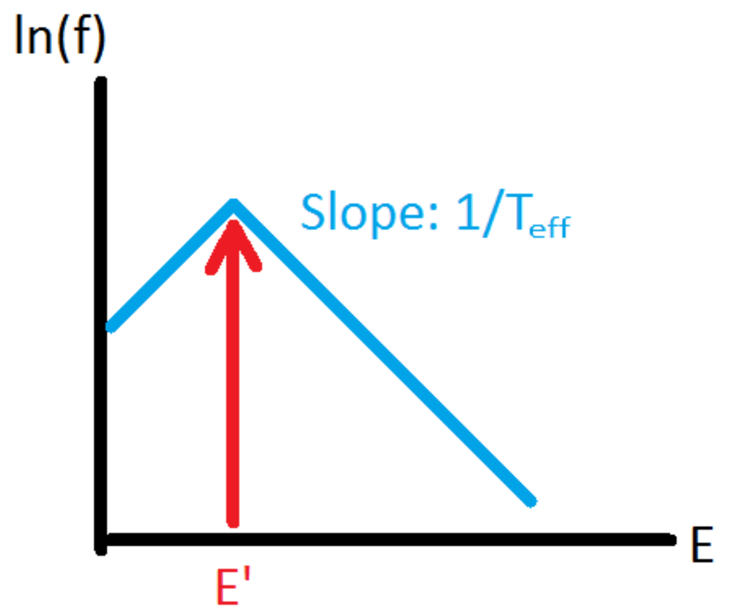
This spectrum fits a Maxwellian distribution with $n_e = 5\times10^7/\mathrm{cm}^3$, $T_{eff} = 2.5$ keV

This spectrum extends measurably beyond 30 keV

Background on particle heating via electrostatic waves

- Decorrelation allows **diffusion** in EEDF
  - $\partial_t f = \partial_E D_E \partial_E f - f/\tau$
  - Green's function for solution: $f \propto e^{\pm E/T_{eff}}, T_{eff} = \Delta E \sqrt{\frac{\tau}{t_t}}$

- f EEDF
- D_E energy diffusivity
- τ loss time
- t_t transit time
- ΔE energy increment



Particles injected at E' are shaped into an exponential function of energy by the action of energy diffusion and loss

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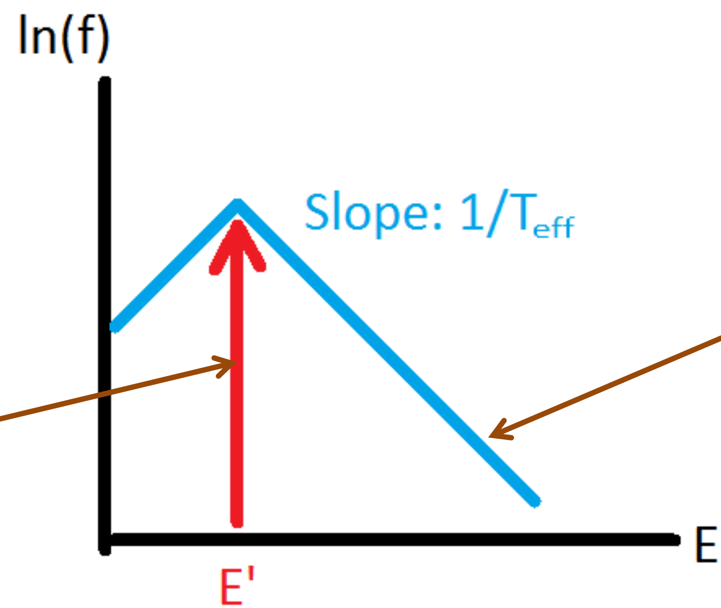
Electrostatic Fluctuation

Confinement

Phase Decorrelation

- f EEDF
- D_E energy diffusivity
- τ loss time
- t_t transit time
- ΔE energy increment



Hot electrons

~ 3 W

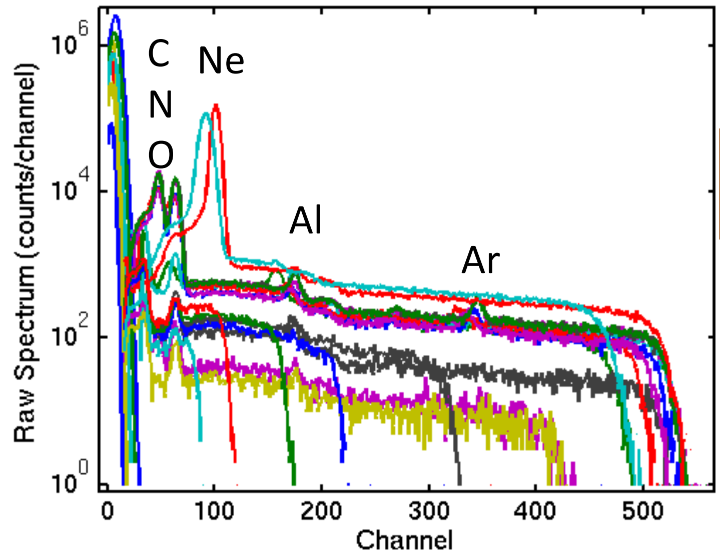
Warm electrons

~ 30 W

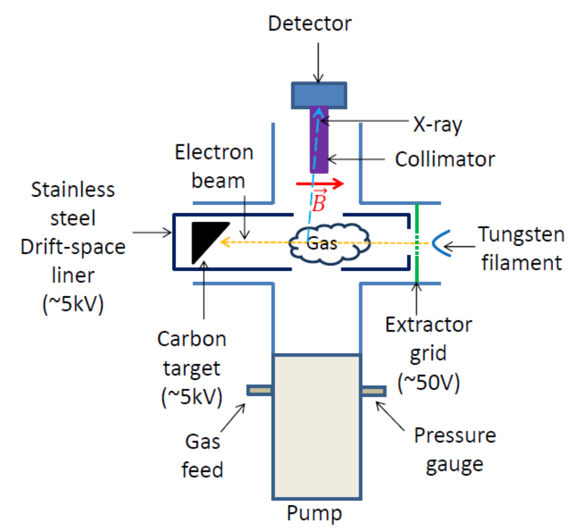
Particles injected at E' are shaped into an exponential function of energy by the action of energy diffusion and loss

Detector Calibration Via Gas-target Bremsstrahlung

- Aside: x-ray measurement (Swanson, 2018)
  - Slope: line-averaged temperature
  - Intensity: line-averaged density
  - 4 detector mounts on PFRC
  - **Calibration, Poisson-regularized inversion**
    - EEDF features from 600 eV to 100 keV
    - Bayesian. Self-consistently includes effects of
      - Window transmission
      - Detector resolution
      - Counting statistics



Energy calibrated



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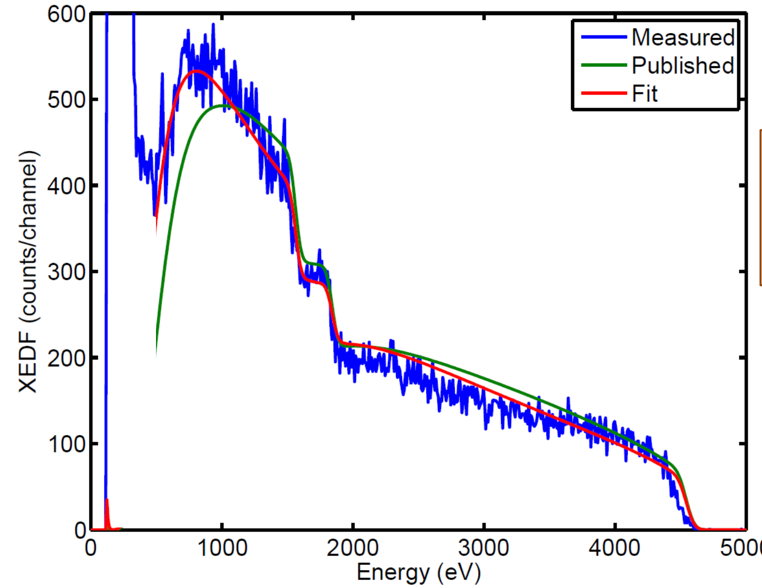
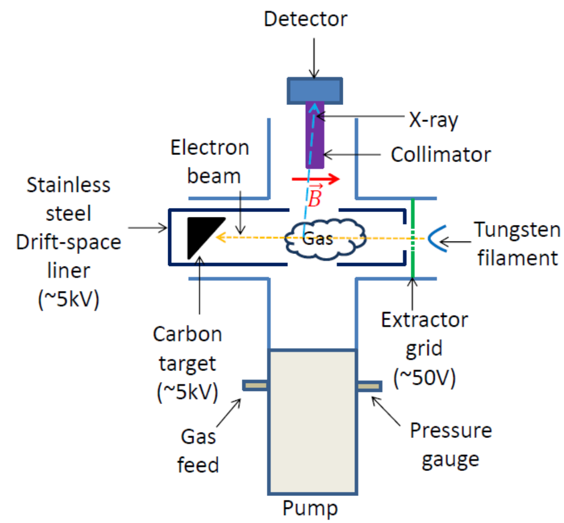
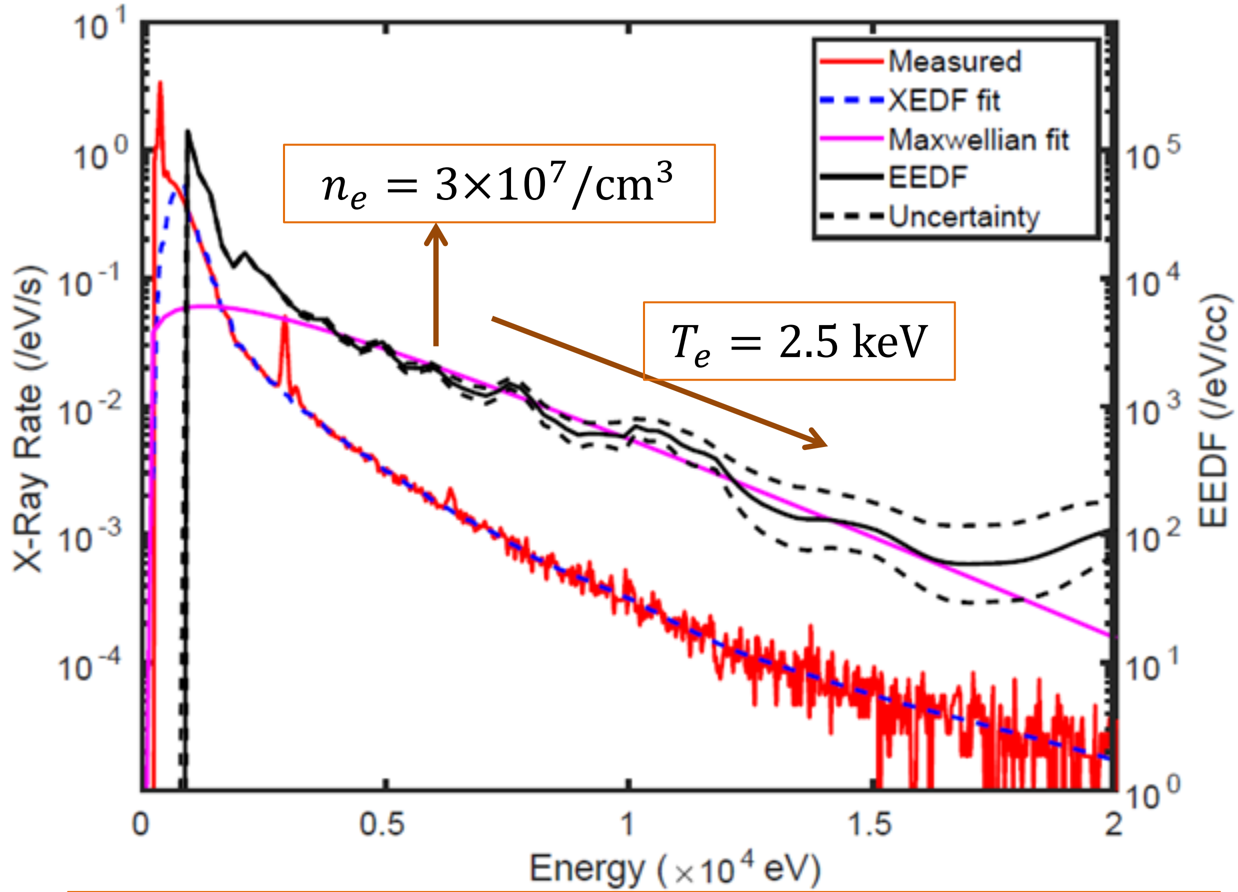


Window transmission calibrated



Typical Properties of the Hot Electrons

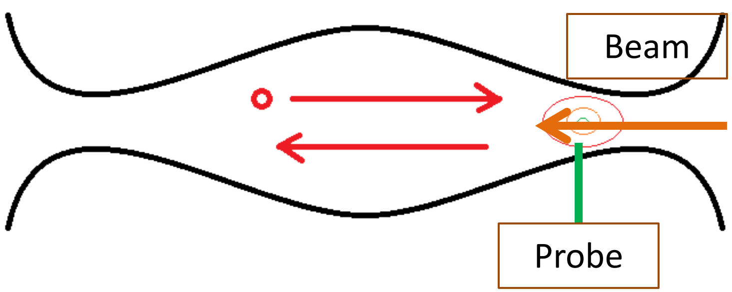
- Measurement of EEDF allows:
  - Non-Maxwellian features to be discovered
  - Energy-dependent quantities (risetime, decay time) to be measured
  - Radial profiles to be measured (Abel transform requires linear quantity)

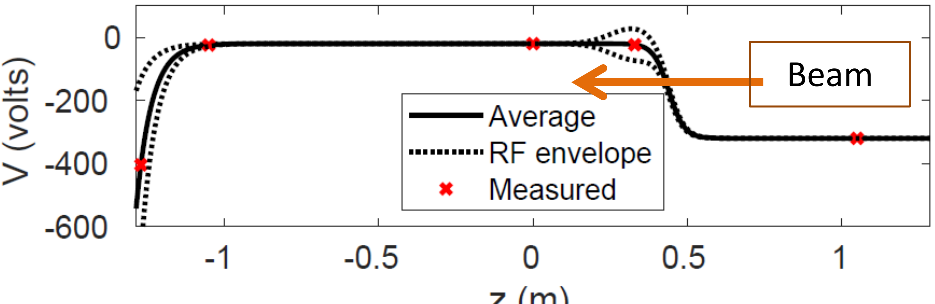


The full EEDF is available for each run condition.
The full EEDF fits a Maxwellian best, and a power law less well. $p = -2.2$

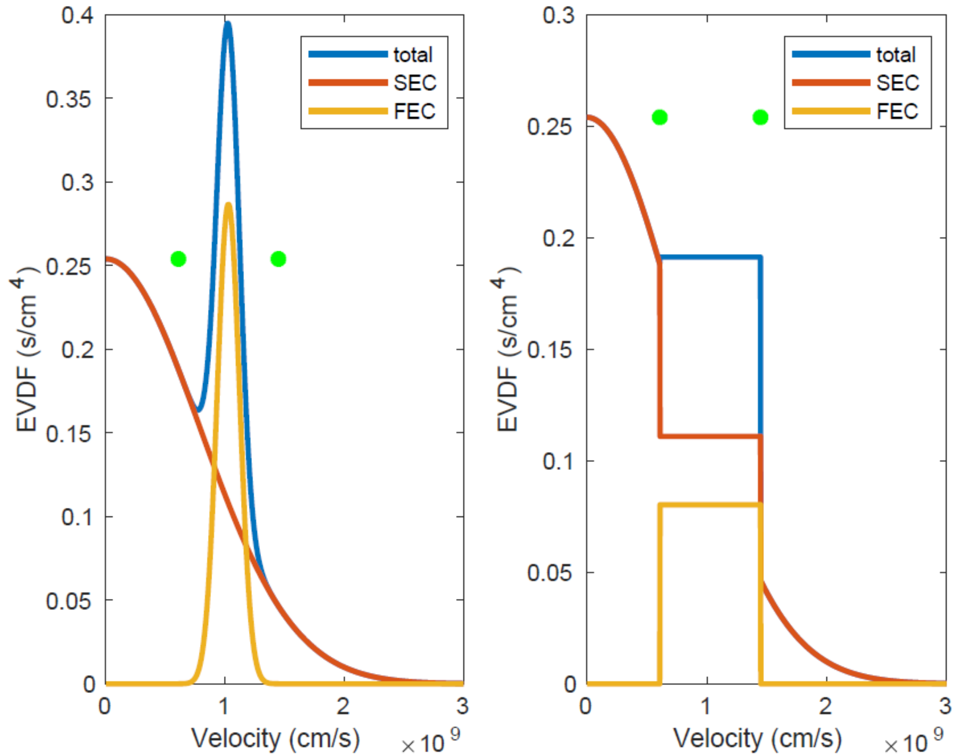
Stronger fluctuation -> Hotter electrons

- Spontaneous potential fluctuation measured via probe
- Measurements are consistent with two-stream instability
- Recall $T_{eff} = \boldsymbol{\Delta E}\sqrt{\frac{\tau}{t_t}}$





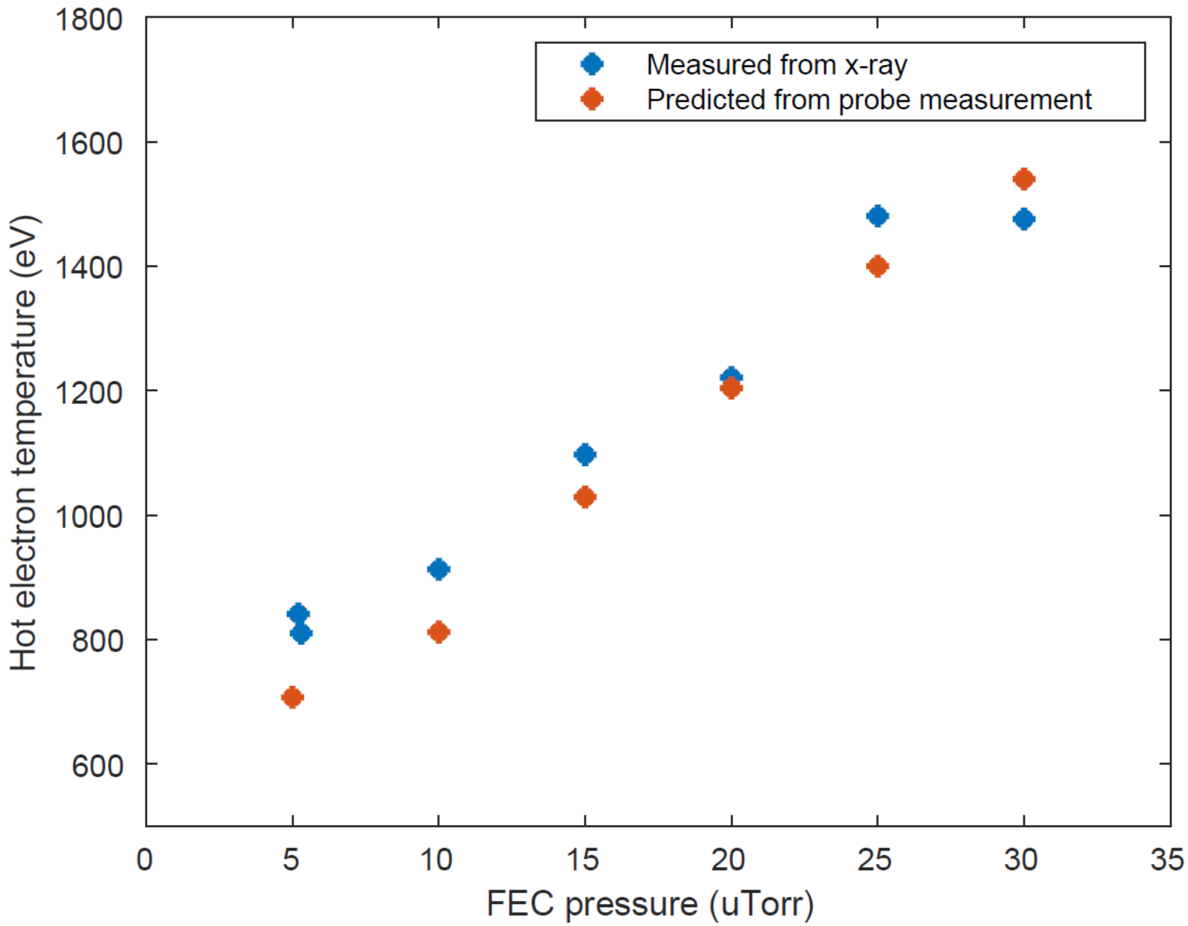
Beam comes from ionization downstream of a potential drop



Expected saturation is ~ 50 V

Stronger fluctuation -> Hotter electrons

- FEC pressure increases *both* electrostatic oscillation *and* x-ray temperature
  - $T_{eff} = \boldsymbol{\Delta E}\sqrt{\frac{\tau}{t_t}}$



Temperature and fluctuation amplitude vary **in the way predicted by the diffusion-loss model**

Stronger fluctuation -> Hotter electrons

- Autocorrelation signal
  - Quantity plotted is $\int dt\ [V(t)\times V(t+t_a)]$, autocorrelation at some time delay
  - White horizontal bands indicate a signal with period t_a
  - Alexeff (1968) measured periodicity changing timescale of 2 – 3 RF cycles, inferred turbulence.

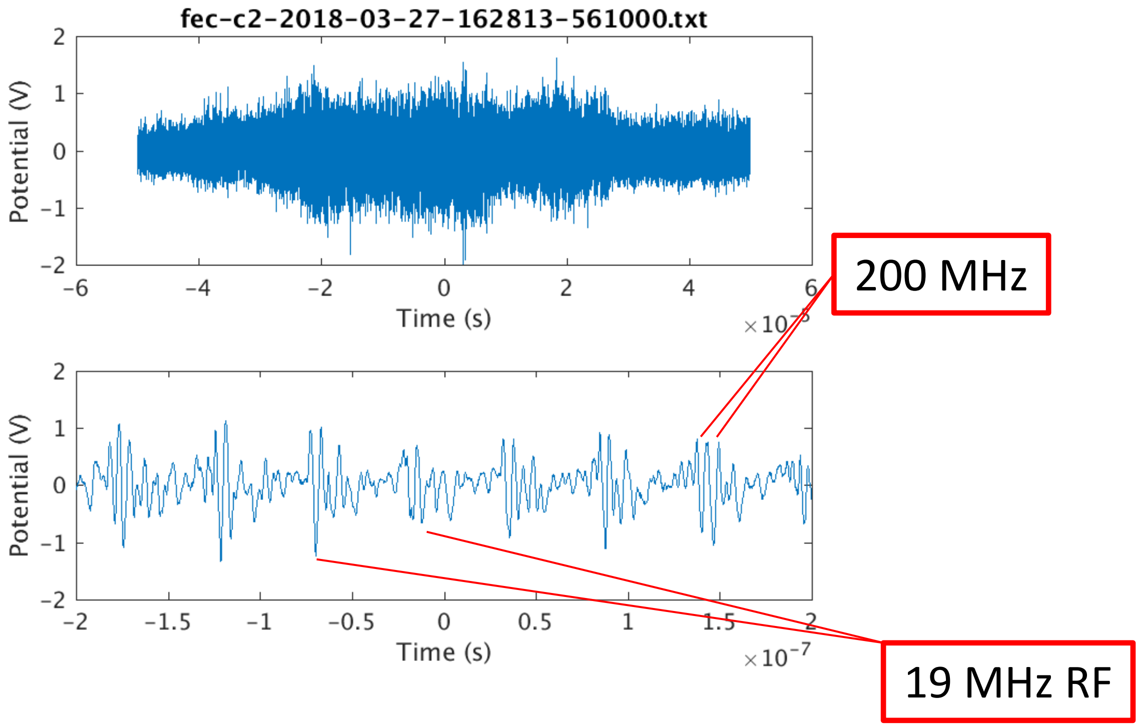
Frequencies:

Bulk f_{pe} ~1 – 3 GHz

Warm f_{pe} ~ 200 MHz

f_{ce} ~ 200 MHz

This **200 MHz signal** is what **increases with increasing FEC pressure** (beam current)



Strong signal near 200 MHz. Changes frequency on a timescale of dozens of μs. **Not turbulent.**

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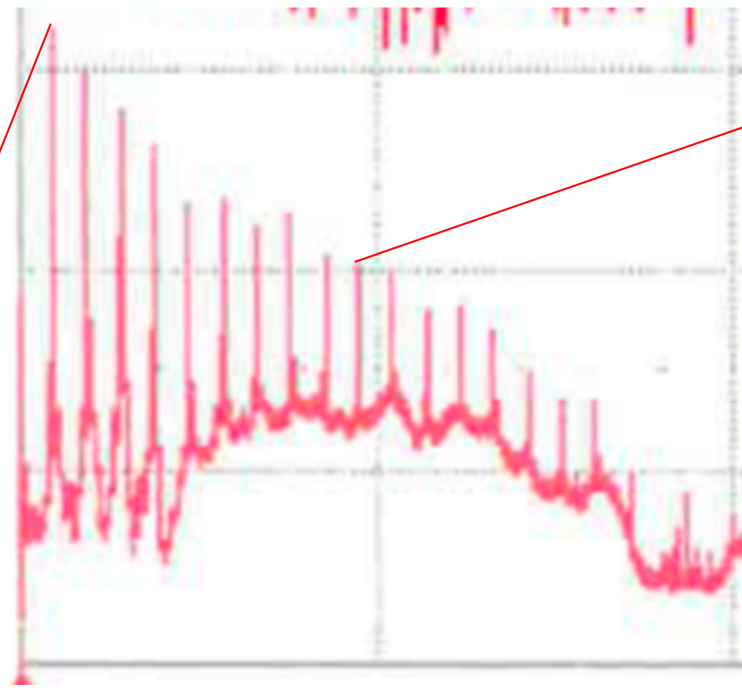

19 MHz RF

200 MHz

Fourier transform, 200 MHz per division. Strong signal near 200 MHz. Changes frequency on a timescale of dozens of μs. **Not turbulent.**

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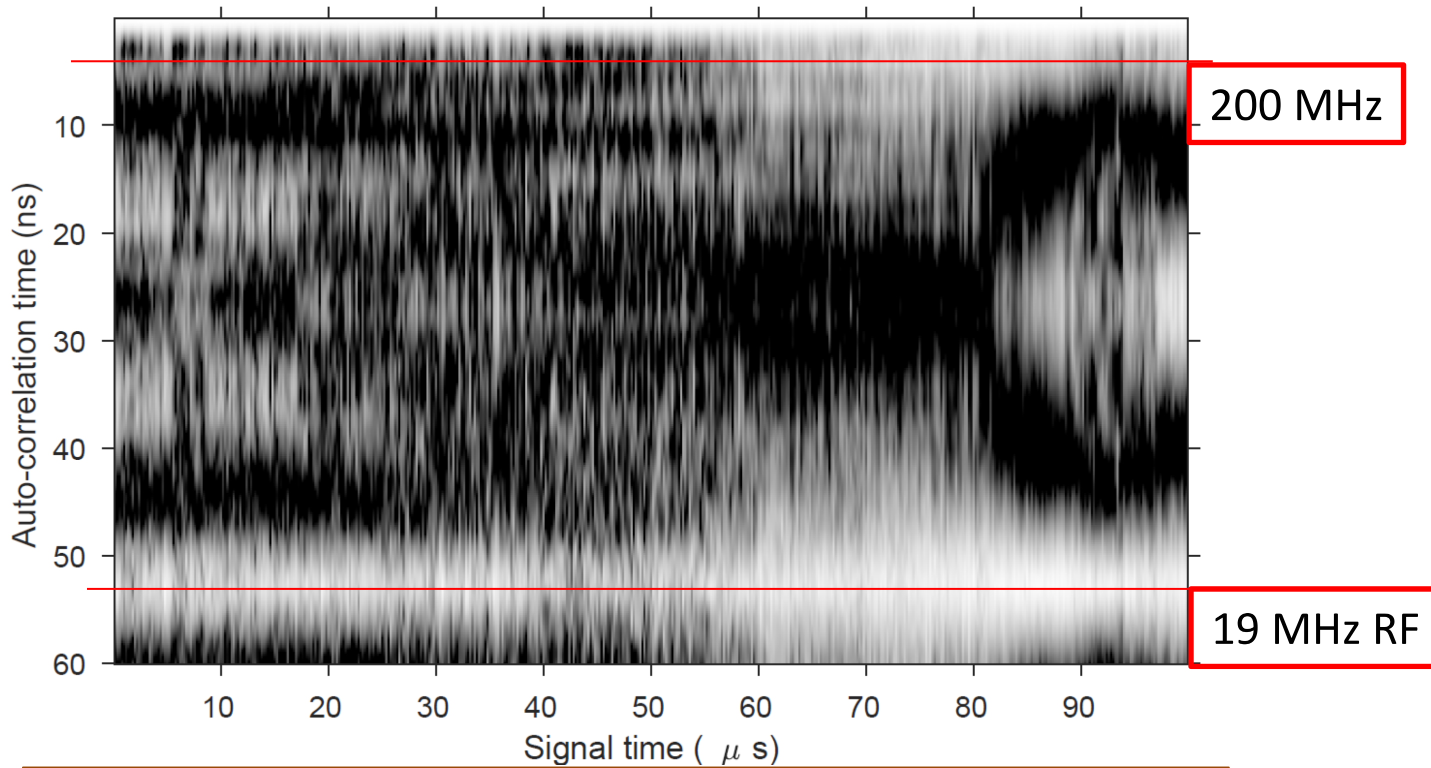

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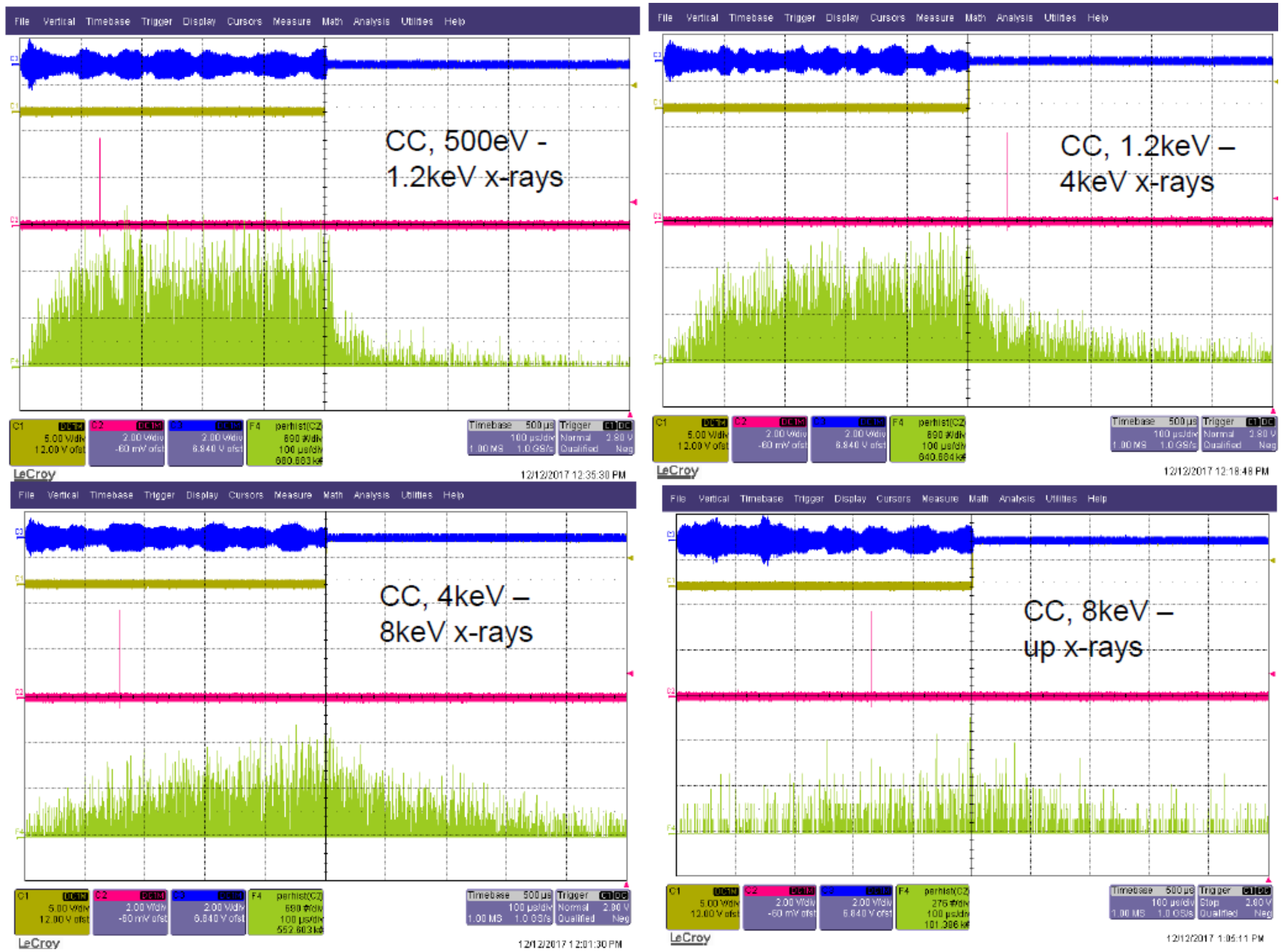
Confinement increases with energy

- Olive: RF on/off
- Blue: RF pickup
- Green: X-ray count histogram
- Takes 100s of μs to *heat up* to high energy also



X-ray detector set to trigger on x-rays of an energy range. Raw oscillographs show that **confinement time is larger for higher energy electrons**.

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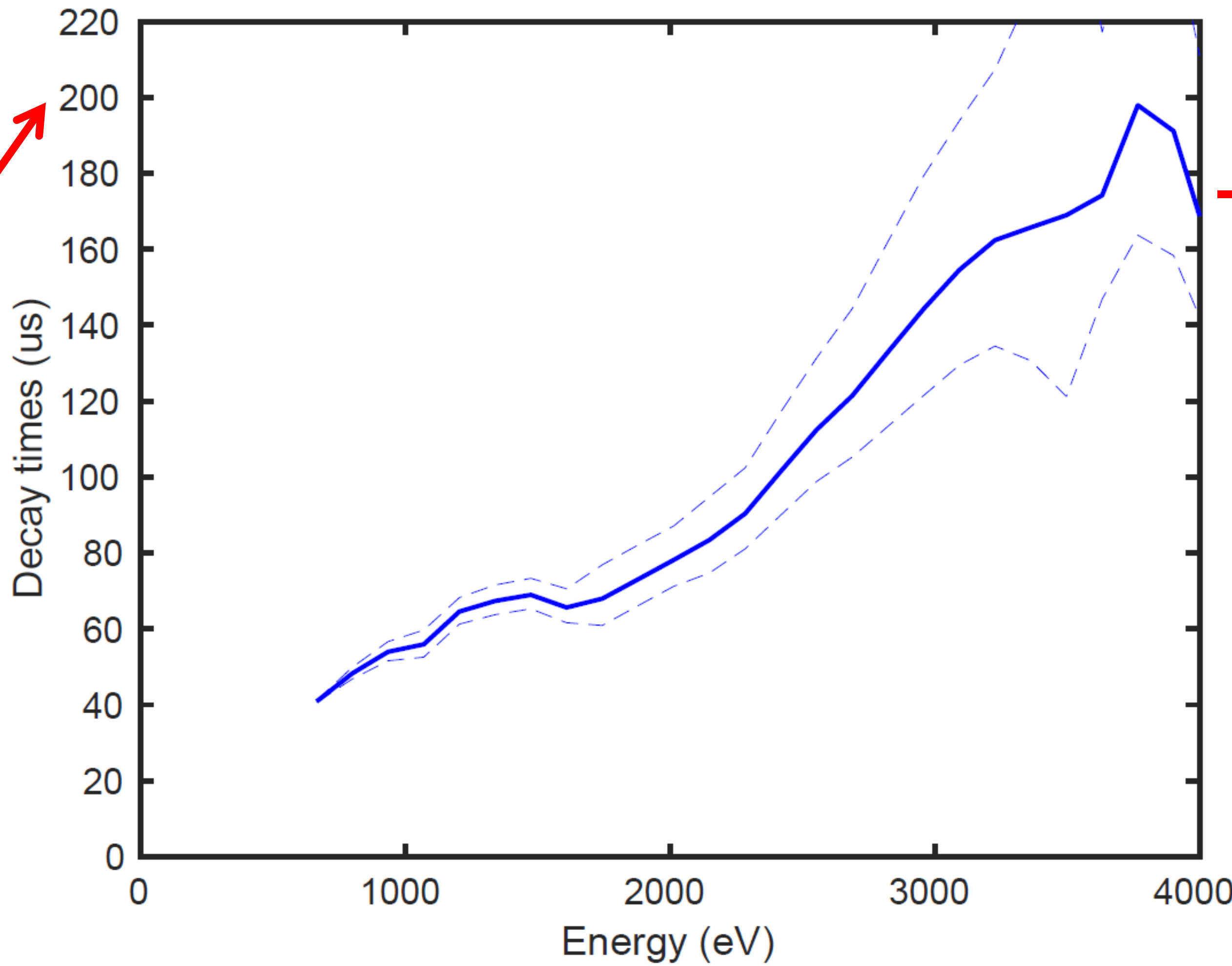
Confinement increases with energy

Hundreds of μs is typical



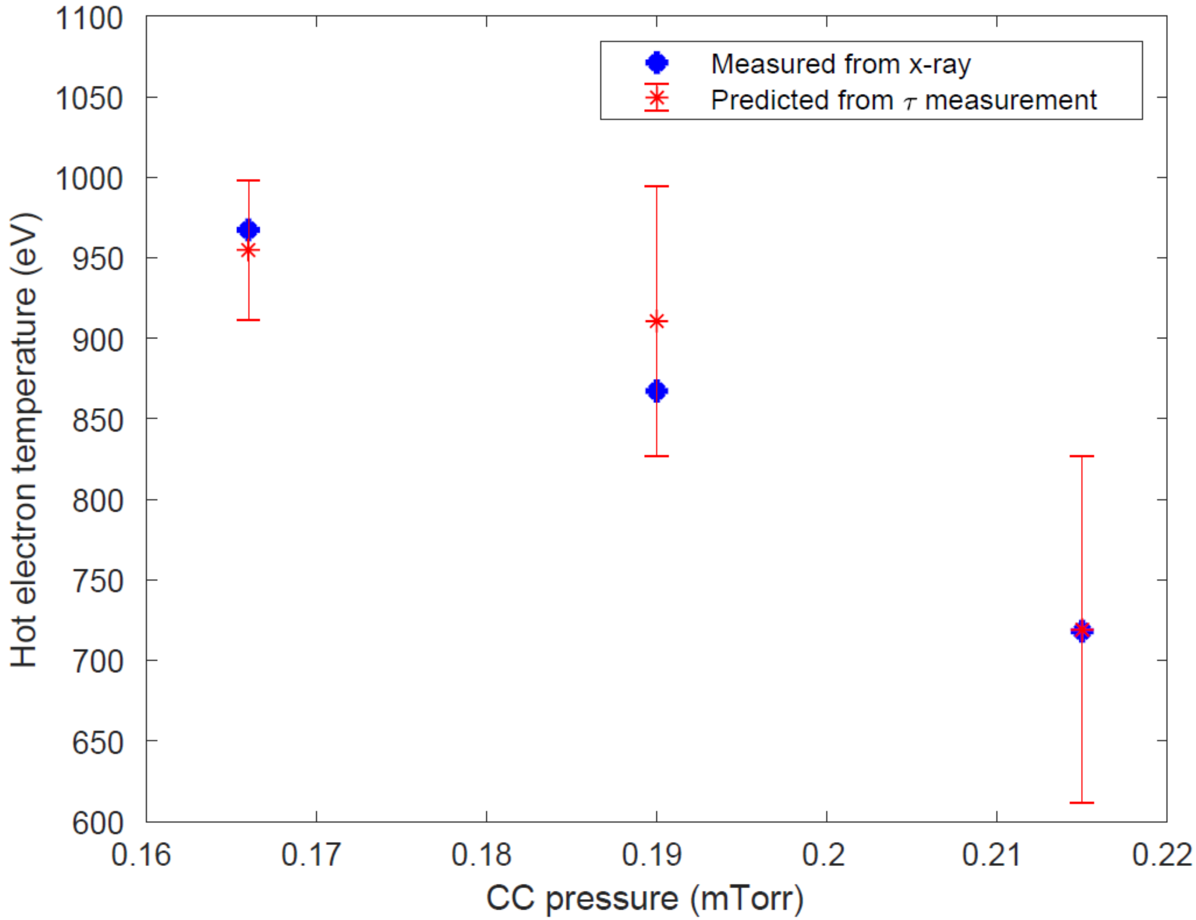
Uncertainty is large

With the help of spectral inversion, we may quantify the e-folding time for each energy. Linear and sub-linear dependences are consistent with data.

Confinement increases with energy

- Increased gas pressure
  - Confinement time measured to decrease
  - Temperature of the hot population measured to decrease in accordance with

$$T_{eff} = \Delta E\sqrt{\frac{\tau}{t_t}}$$



Changing confinement time by changing the gas pressure **has the effect predicted by the diffusion model.**

Conclusions

- One can produce **keV electrons** in a **small, low-power apparatus**
- **Fermi-Ulam acceleration** can create **high energy electrons** in quasiadiabatic conditions
- **μ is mobile**
- Relevance to extant mirror experiments (~10)
  - GOL-3-like multimirrors may be operated in non-turbulent modes (at lower pressure), and effective collisionality can be enhanced by shaping coils to produce $\Delta\mu$
- Relevance to the PFRC-II as an FRC experiment
  - The PFRC-II is designed to study FRCs formed by RMF. We may probe FRC formation in the presence of fast electrons
  - The SOL of the FRC is mirror-like, with high-curvature regions. $\Delta\mu$ will greatly affect SOL pressure by trapping and de-trapping passing electrons.
  - The core of the FRC is also mirror-like, with low-field, high-curvature shell

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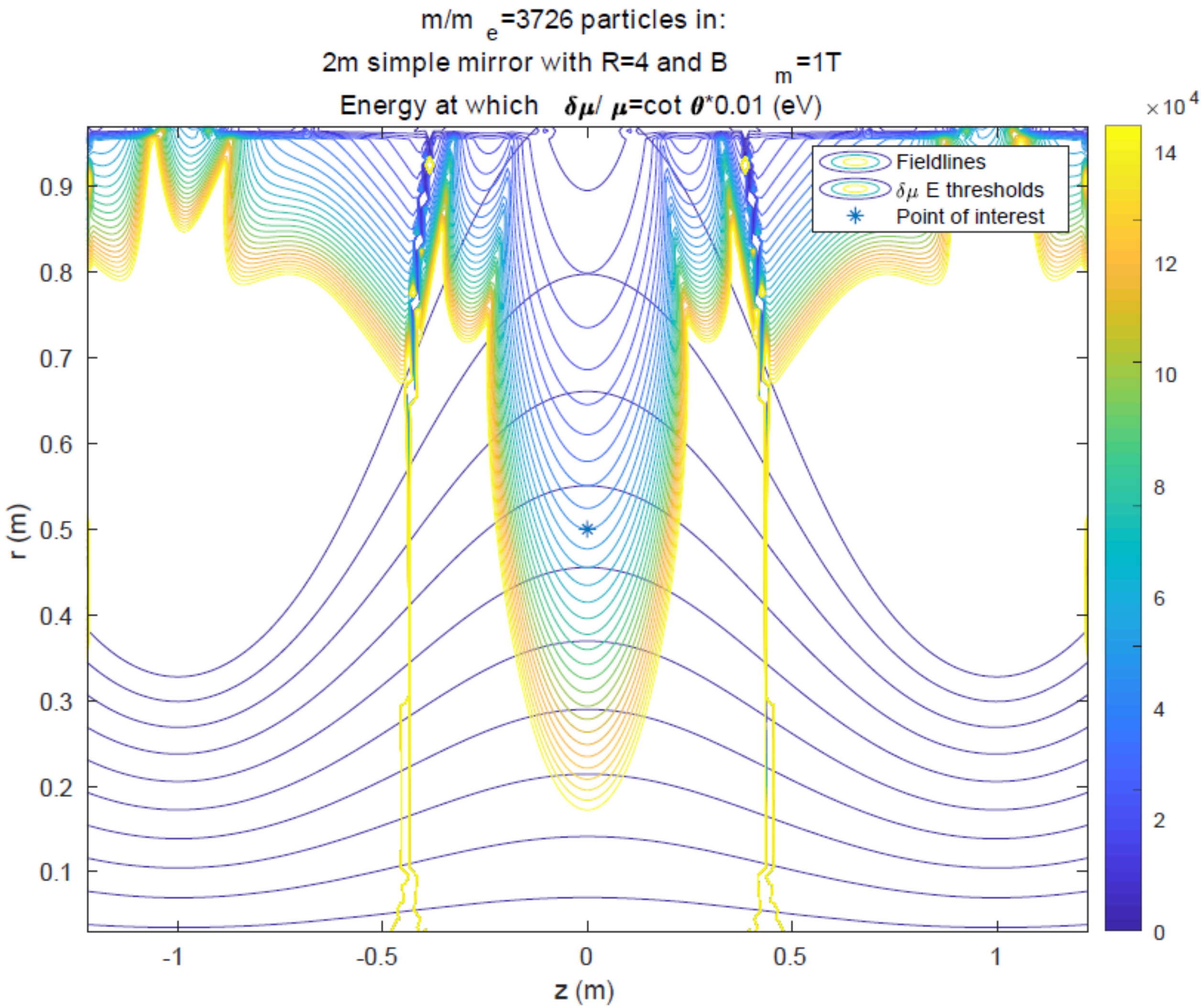
Non-adiabaticity of magnetic moment in magnetic mirror

- Background:
  - μ assumed conserved or nearly so: Alfven (1950), Post (1958), many contemporary and modern reviews [Herskowitz (1990), Post (1987), Sheffield (1994), Simonen (2010)]
    - Popular models can assume constant μ, such as MHD with anisotropic pressure and gyrokinetics.
    - A common "adiabatic parameter" which is assumed to control constancy of μ is $\epsilon = \rho \nabla B / B$
  - $\Delta\mu$ accurately measured or computed: Henrich (1956), Garren (1958), Gibson (1963), Hastie (1969), Birmingham (1984), Dykhne (1960), Cohen (1978), Chirikov (1979)
    - $\Delta\mu$ *actually* seen to change significantly in regions where $\epsilon = \rho \nabla B / B$ is very small.
    - Actual adiabatic dependence includes $v_\parallel$,R_c, R_c'' the radius of curvature and its derivative.
    - Intuitive picture from Delcourt (1994): centrifugal impulse.
  - Long-term behavior of many $\Delta\mu$ compounded over many mirror transits: Chirikov (1978), Tagare (1986), Nakashima (2002), Zelenyi (2013), Chen (1992)
    - One parameter $K \equiv -\partial_\mu(\Delta\psi)\times\delta\mu$ controls whether μ is free to diffuse or is trapped in a small band.

Non-adiabaticity of magnetic moment in magnetic mirror

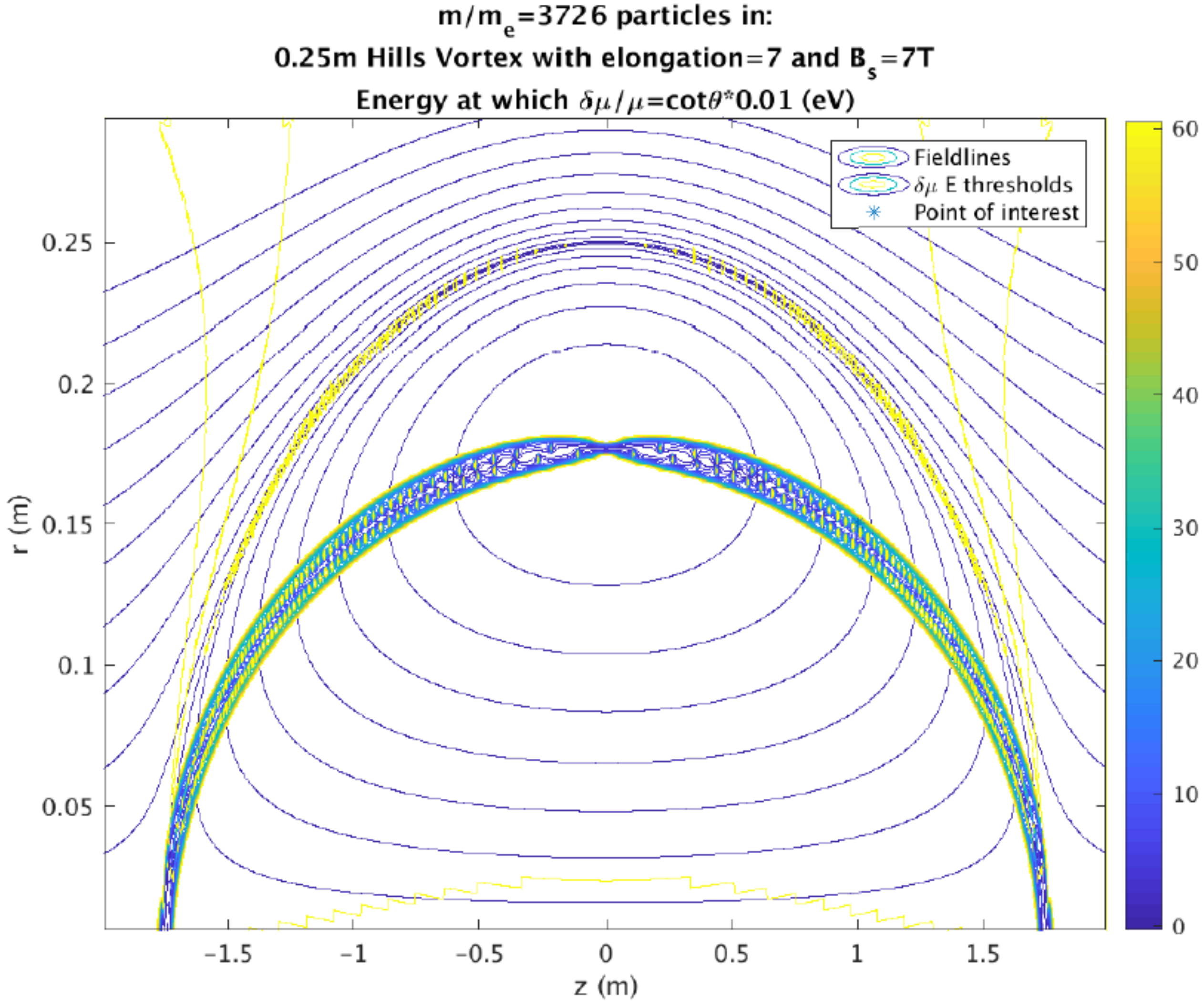
Deuteron in Generic mirror



Plot shows energy at which a particle's μ may change by $1\% \times v_{\parallel}/v_{\perp}$

Non-adiabaticity of magnetic moment in magnetic mirror

Deuteron in large FRC



Plot shows energy at which a particle's μ may change by $1\% \times v_{\parallel}/v_{\perp}$

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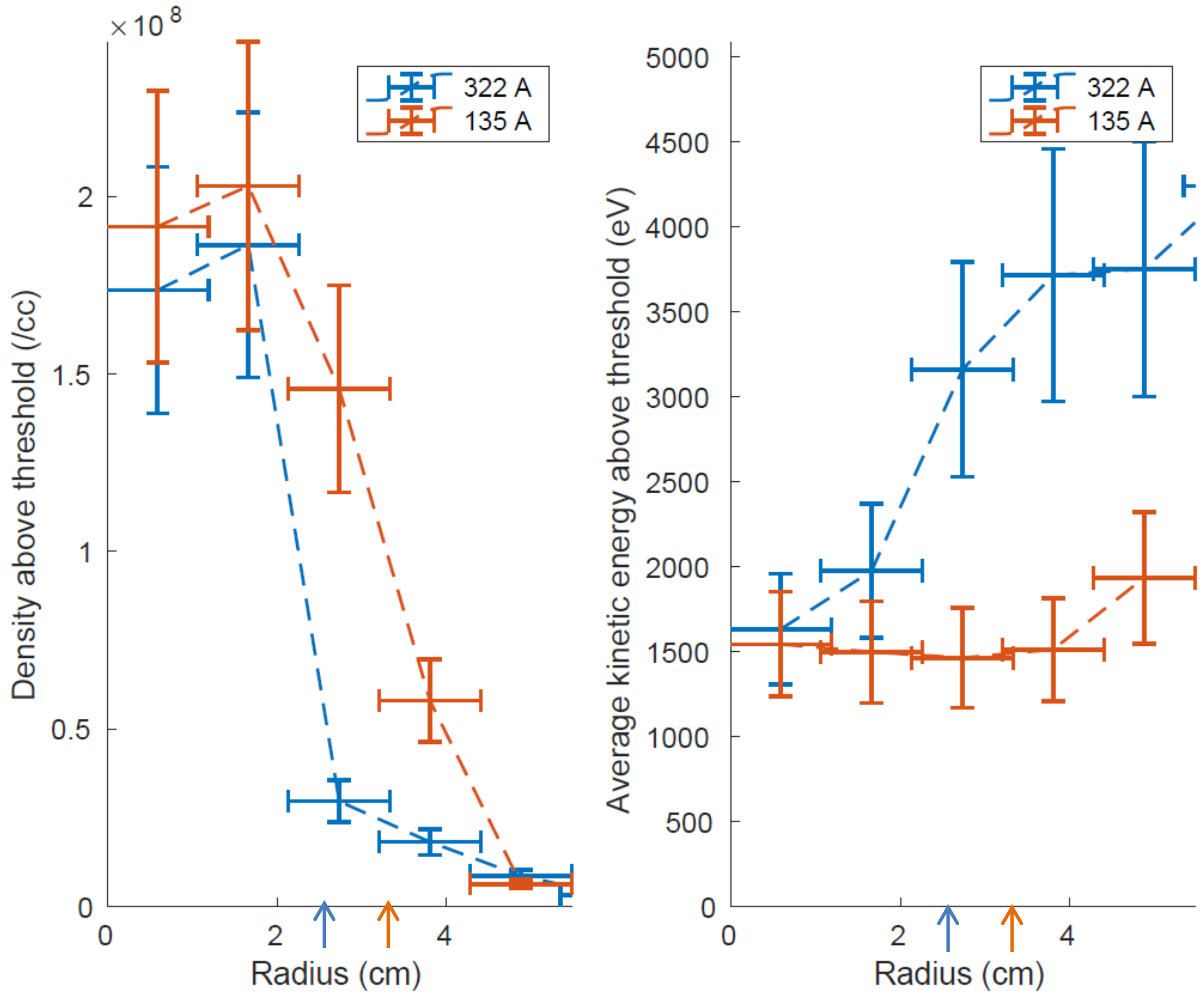
Radial Profile

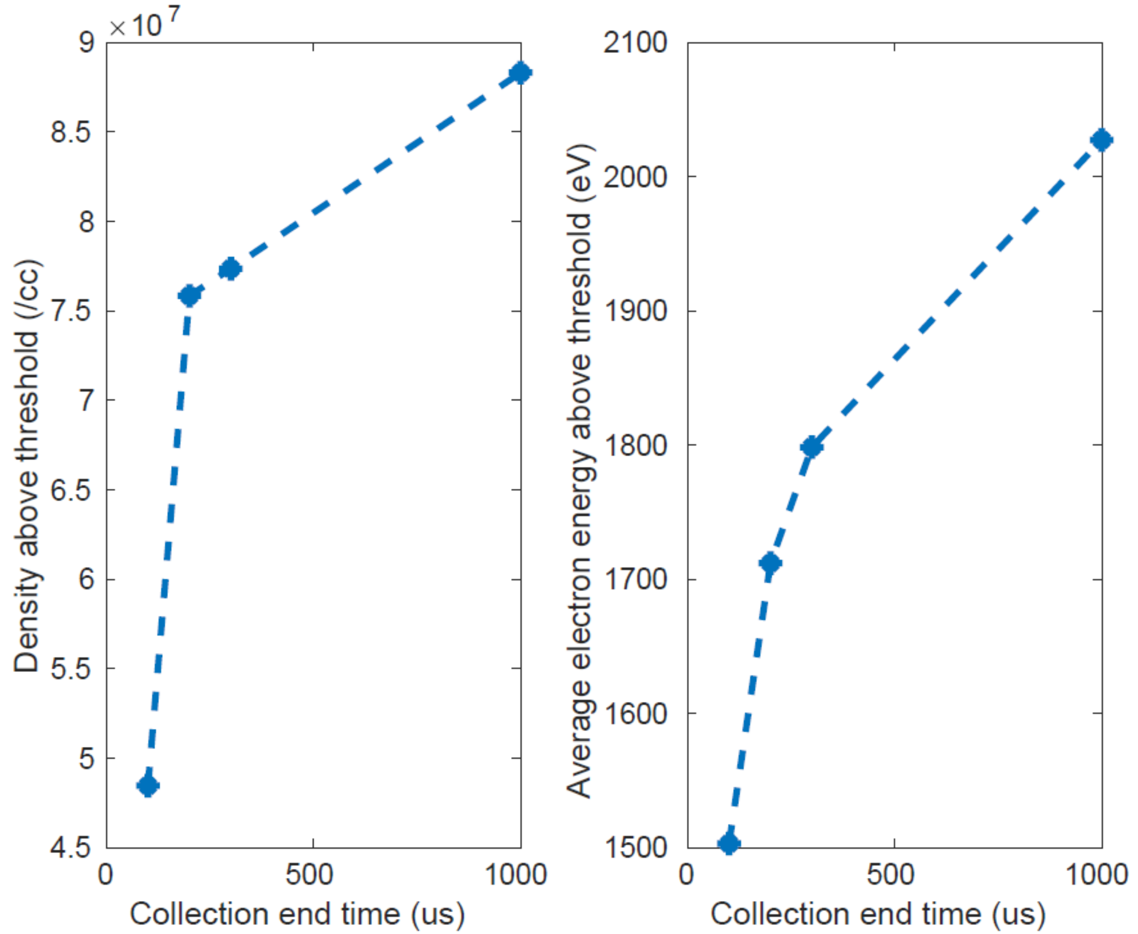
- Profiles as a function of *main* magnetic field
  - "Threshold" energy is 600 eV
  - These are the only non-line-averaged densities in this FPOE
  - B_{min} is 240 G and 100 G, respectively
    - Nozzle-bore-limiting field lines at 2.5 cm and 3.3 cm, respectively.
    - ρ_{max} are 3.1 mm and 7.5 mm, respectively



Density is peaked on-axis or weakly off-axis.
Lower-energy density falls sharply outward of limiting field-line, but higher-energy density does not. **High-energy particles have mobile μ.**

5. Confinement increases with energy

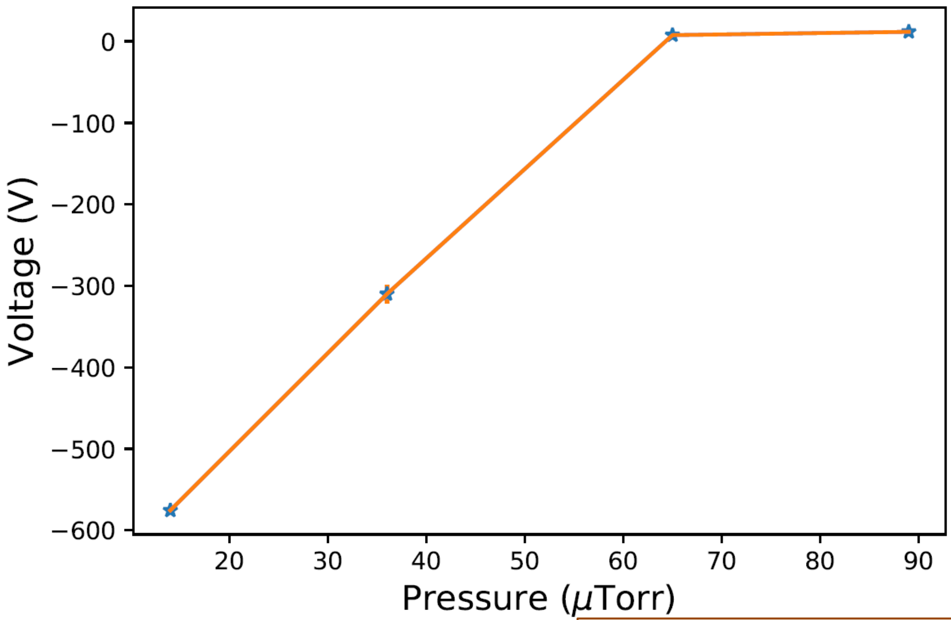
- Rise-time of signal upon application of RF power
  - “Threshold” energy is 600 eV
  - X-rays are collected until a set time after the RF power is turned on

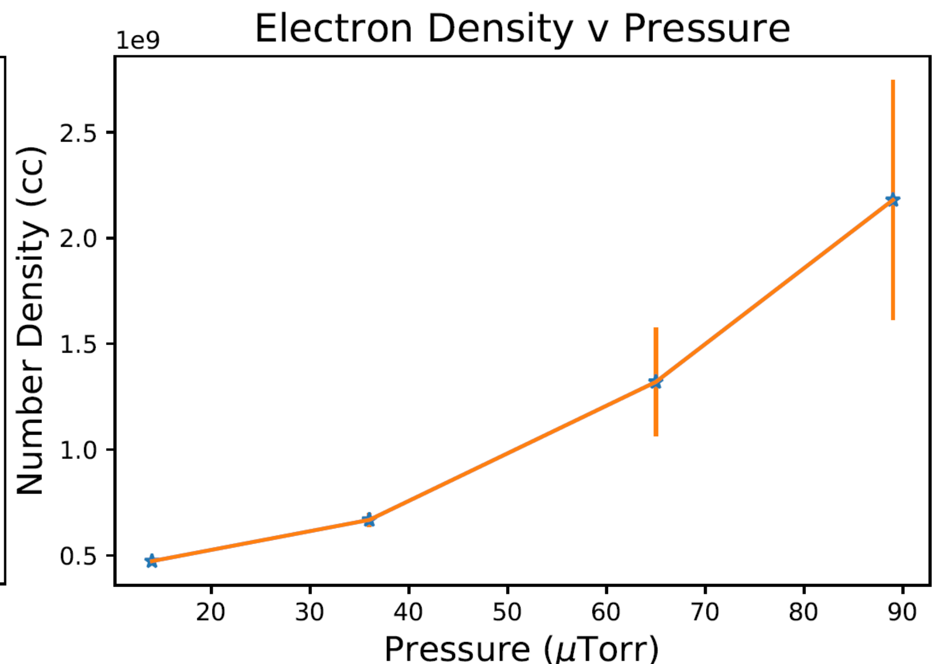


Density and temperature of hot population take **hundreds of μs** to rise. That’s **thousands of mirror transits**.

Fermi acceleration in the PFRC-II creates 3+ keV e⁻

- Conditions in the FEC
  - Negative space potential (-600 V) from SEC fast electrons
  - Ionization in FEC causes 600 eV beam in CC
  - Unstable EVDF (from inverse Landau damping limit *and* Nyquist theorem)
  - Approx. saturation voltage known





Plots showing space potential and density in the FEC vs the gas pressure in the FEC

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Column incident on sapphire window

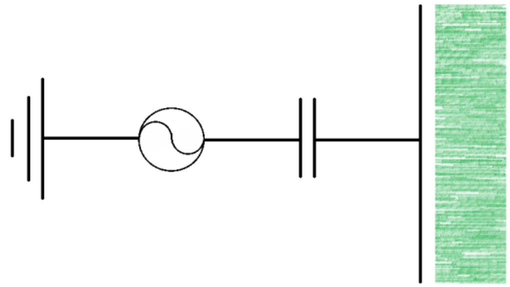
- Hollow profile exhibited
- Explicable via $\Delta\mu$

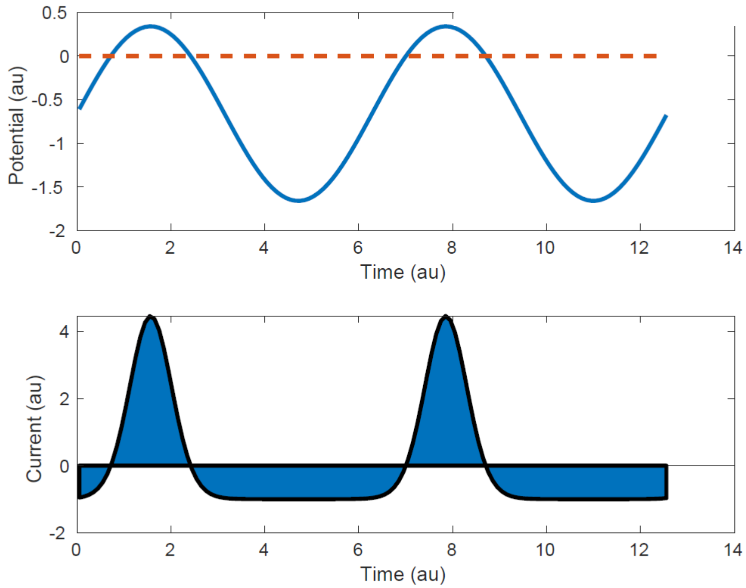
Angle 1: head-on

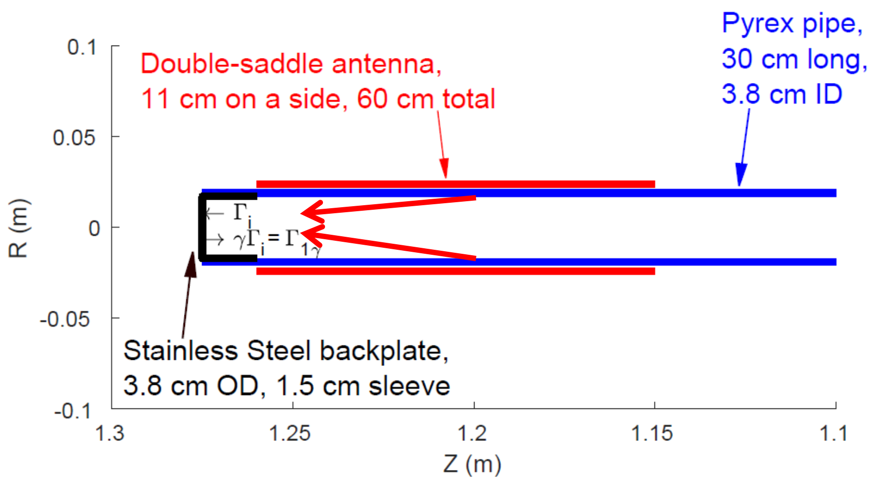
Angle 2: nozzle visible

SEs are created warm by self-bias and SEE

- Self-bias
  - Large electron mobility causes RF electrodes to float at negative potential (Godyak 1990)







Acceleration process couples to extant warm electrons

- Warm electron steady state
- Warm electron average transits

X-ray measurements show 30% *higher* density of fast electrons in the CC, despite what we'd expect from the weaker magnetic field $B_{CC} \approx B_{SEC}/3.5$

Electron density in the CC is anomalously high by a factor of ~5

Acceleration process couples to extant warm electrons

- Warm electron steady state
- Warm electron average transits

X-ray measurements show that **electrons in the CC persist for 300 transits or more** after the cessation of RF power