# Modeling the Scaling Law for Error Field (EF) Penetration by Using TM1

**by**
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**with**
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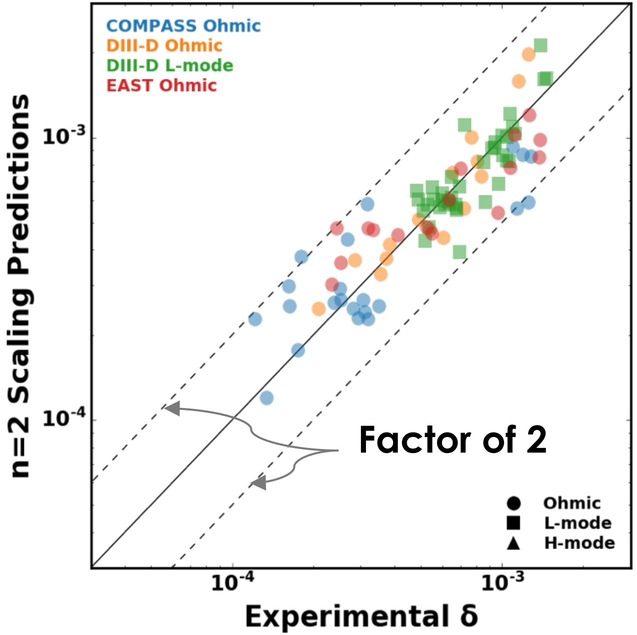
# Simple error field penetration scalings set the tolerances & correction-coil designs for new machines like ITER

## Successes

- Scaling done using simplest possible 0D parameters: $n_e$, $B_T$, $R_0$, $\beta_N/\ell_i$
- Robust perturbation metric, δ, incorporates plasma response

$$b_\mathrm{r} \propto n_e^{0.5} B_\mathrm{t}^{-1.0} R_0^{0.1} (\beta_\mathrm{N}/l_i)^{-0.2} f_0$$



N.C. Logan, et al., Challenges projecting EF correction tolerance 24th MHD workshop, 2019

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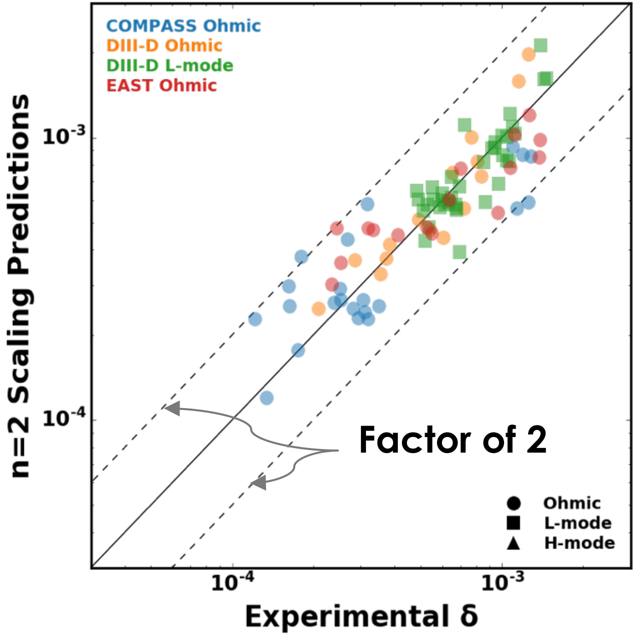
## Challenges

- Profile effects not captured by 0D parameters
- Theoretical prediction isn't enough to address the scaling
- Biases & gaps in database cause, residual EFs

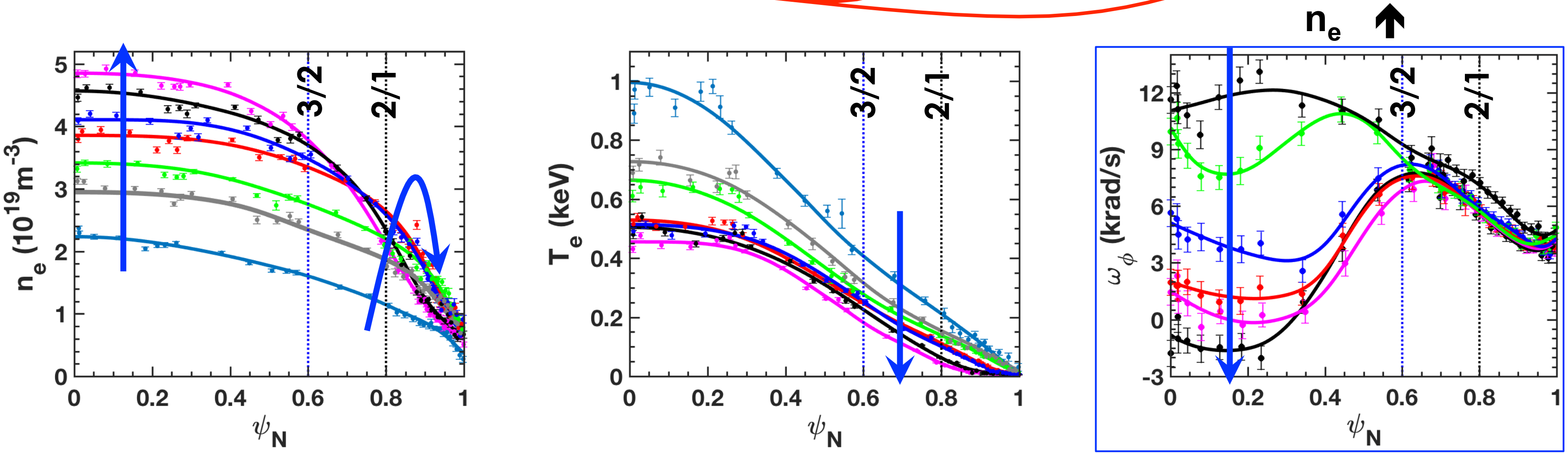


N.C. Logan, et al., Challenges projecting EF correction tolerance 24th MHD workshop, 2019

# EF scaling studies in experiments are more complicated than expected due to the coupling of plasma parameters

$$b_{\mathrm{r}} \propto n_e^{0.5} B_{\mathrm{t}}^{-1.0} R_0^{0.1} (\beta_{\mathrm{N}} / l_i)^{-0.2} f_0$$

[1]



[2]

- **The coupling of plasma parameters makes EF scaling complicated**

[1] N.C. Logan, et al., Challenges projecting EF correction tolerance 24th MHD workshop, 2019
[2] B.A. Grierson, et al., Phys. Plasmas **26**, 042304 (2019)

# Analytical theory is not enough to address the EF scaling

## 1-fluid scaling from Fitzpatrick's theory [1]

$b_r/B_\mathrm{t} \propto n_e^{3/5}\tau_\mathrm{V}^{-3/5}T_e^{3/5}f_0$ *Rutherford* **regime**

$b_r/B_\mathrm{t} \propto n_e^{7/12}\tau_\mathrm{V}^{-7/12}T_e^{5/8}f_0$ *Visco-resistive* **regime**

$b_r/B_\mathrm{t} \propto n_e^{1/2}\tau_\mathrm{V}^{-1/2}T_e^{11/20}f_0^{4/5}$ *Transition* **regime**

$b_r/B_\mathrm{t} \propto n_e^{7/16}\tau_\mathrm{V}^{-7/16}T_e^{9/32}f_0^{5/8}$ *Waelbroeck* **regime**

**No density dependence according to Neo-Alcator scaling [2]**

$\tau_\mathrm{V} \propto \tau_\mathrm{E} \propto n_\mathrm{e}$ ➡ $b_r/B_\mathrm{t} \propto n_e^0$

[1] R. Fitzpatrick, PPOP**5**, 3325 (1998)
[2] W. Pfeiffer and R.E. Waltz, NF **19**, 51 (1979)

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**No density dependence according to Neo-Alcator scaling [2]**

$$\tau_V \propto \tau_E \propto n_e \Rightarrow b_r/B_t \propto n_e^0$$

## Linear 2-fluid scaling [3]

$$b_r/B_t \propto T_e^{1/6}B_t^{-1}R_0^{-7/6}$$ **VR regime**

$$b_r/B_t \propto n_e^{1/4}T_e^{1/8}B_t^{-5/4}R_0^{-1}$$ **SC regime**

## Nonlinear 2-fluid scaling [4]

$$b_r/B_t \propto n_e B_t^{-9/5}R_0^{-1/4}$$ **Polarization regime**

**Inconsistent with observed $n_e^{0.5}$, no rotation scaling**

[1] R. Fitzpatrick, PPOP**5**, 3325 (1998)
[2] W. Pfeiffer and R.E. Waltz, NF **19**, 51 (1979)
[3] A. Cole and R. Fitzpatrick, POP **13**, 032503 (2006)
[4] R. Fitzpatrick, PPCF **54**, 094002 (2012)

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**Motivation:**

- ✓ Examine EF scaling law under single-fluid and two-fluid conditions
- ✓ Examine the scaling difference in core and edge (pedestal) penetration
- ✓ Predict the level of penetration threshold in ITER

[1] R. Fitzpatrick, PPOP**5**, 3325 (1998)
[2] W. Pfeiffer and R.E. Waltz, NF **19**, 51 (1979)
[3] A. Cole and R. Fitzpatrick, POP **13**, 032503 (2006)
[4] R. Fitzpatrick, PPCF **54**, 094002 (2012)

# Outline

- **TM1 modeling of 3/2 EF threshold scaling**
  - → Single-fluid modeling: well reproduced Fitzpatrick's theory
  - → Two-fluid modeling: sensitive scaling on plasma rotation
- **TM1 modeling pedestal-top n=2 RMP threshold scaling**
  - → Two-fluid modeling: similar with core scaling
- **Prediction for ITER**
- **Summary**

# Nonlinear MHD Model TM1 is Used to Simulate EF penetration

- **Cylindrical geometry, circular cross-section**
- **Nonlinear, resistive, two-fluid model [1]:**

$$\frac{d\psi}{dt} = E - \eta j + \boxed{\Omega(\nabla_{\parallel} n_e + \nabla_{\parallel} T_e)}$$

**diamagnetic drift**

$$\frac{du}{dt} = -C_s^2 \nabla_{\parallel} P / n_e + \mu_{\perp} \nabla_{\perp}^2 u$$

$$\rho \frac{d}{dt} \nabla^2 \phi = \vec{e}_{\varphi} \cdot (\nabla \psi \times \nabla j) + \rho \mu \nabla^4 \phi + S_m$$

$$\frac{dn_e}{dt} = \boxed{\frac{\omega_{ce}}{\nu_e} \nabla_{\parallel} j} - \nabla_{\parallel}(n_e u) + \nabla \cdot (D_{\perp} \nabla n_e) + S_n$$

**Ion polarization current**

$$\frac{3}{2} n_e \frac{dT_e}{dt} = \boxed{\frac{\omega_{ce}}{\nu_e} T_e \nabla_{\parallel} j} - T_e n_e \nabla_{\parallel} u + n_e \nabla \cdot (\chi_{\parallel} \nabla_{\parallel} T_e) + n_e \nabla \cdot (\chi_{\perp} \nabla_{\perp} T_e) + S_e$$

  - Able to perform modeling under **single-fluid** or **two-fluid** conditions
  - Two-fluid condition includes **diamagnetic drift effect**, and **ion polarization current** in transport
  - **Long time-dependent modeling** with > 1 s evolution

[1] Q. Yu, et al., POP **10**, 797 (2004); NF **51**, 073030 (2011)
S. Gunter, et al., JCP **209**, 354 (2005)

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$$\boxed{\frac{d\psi}{dt} = E - \eta j} + \cancel{\Omega(\nabla_\parallel n_e + \nabla_\parallel T_e)}$$

**Diamagnetic effect**

$$\frac{du}{dt} = -C_s^2 \nabla_\parallel P / n_e + \mu_\perp \nabla_\perp^2 u$$

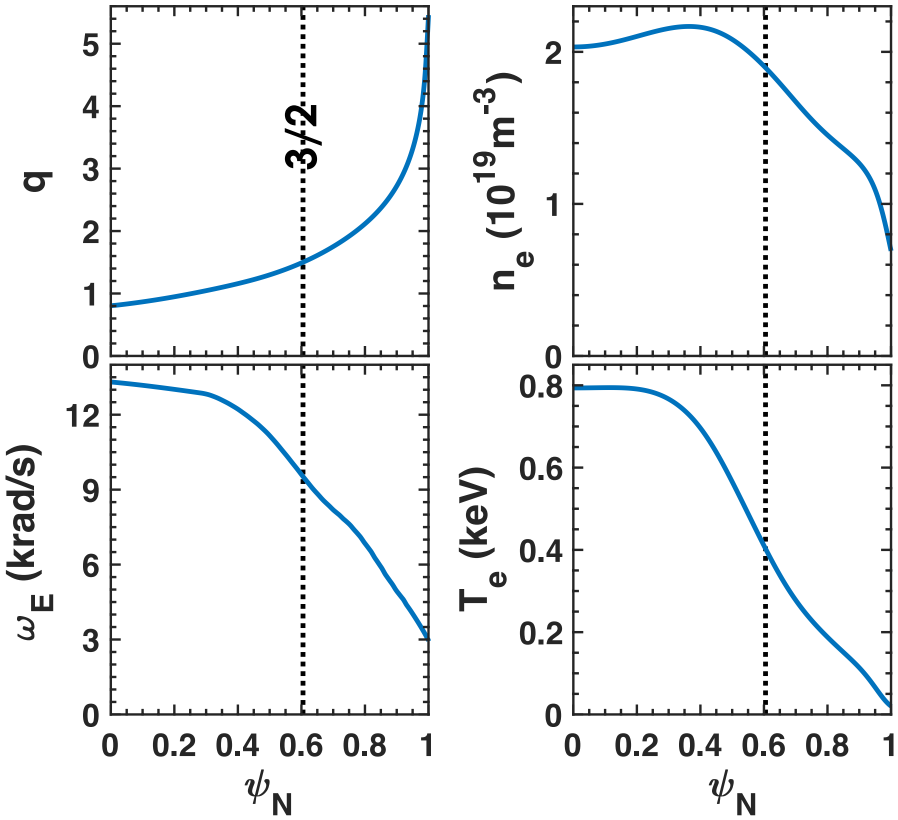
$$\boxed{\rho \frac{d}{dt} \nabla^2 \phi = \vec{e}_\varphi \cdot (\nabla \psi \times \nabla j) + \rho \mu \nabla^4 \phi + S_m}$$

$$\frac{dn_e}{dt} = \frac{\omega_{ce}}{\nu_e} \nabla_\parallel j - \nabla_\parallel (n_e u) + \nabla \cdot (D_\perp \nabla n_e) + S_n$$

$$\frac{3}{2} n_e \frac{dT_e}{dt} = \frac{\omega_{ce}}{\nu_e} T_e \nabla_\parallel j - T_e n_e \nabla_\parallel u + n_e \nabla \cdot (\chi_\parallel \nabla_\parallel T_e) + n_e \nabla \cdot (\chi_\perp \nabla_\perp T_e) + S_e$$

# Input Parameters for TM1 Modeling

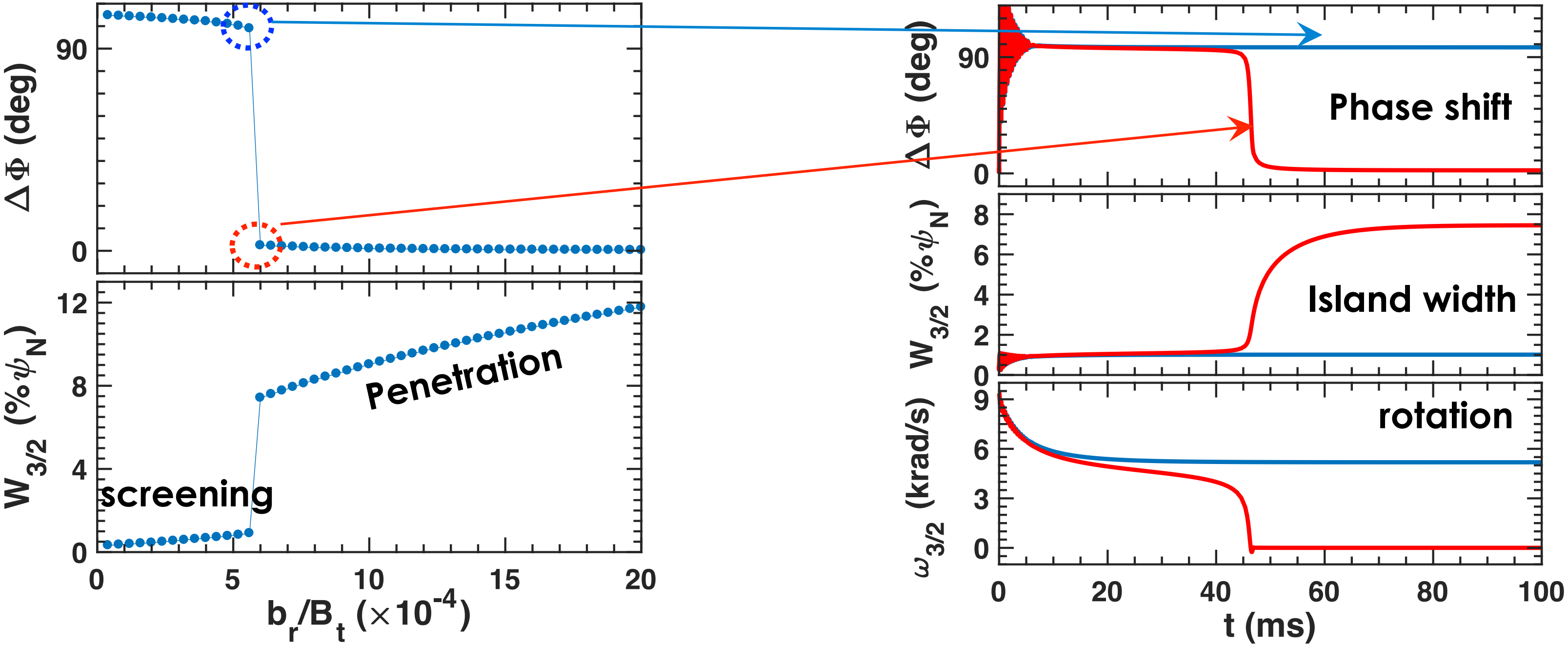
- **Input parameters**
  - Equilibrium and profiles from DIII-D EF L-mode database
  - Transport coefficients derived from measured profiles → $\mu = \chi_\perp = 2D_\perp = 0.5 m^2/s$ are used
  - m/n=3/2 EF



# TM1 modeling scans RMP strength to determine the m/n=3/2 EF penetration threshold

**Scanning RMP strength**

**Detail evolution**

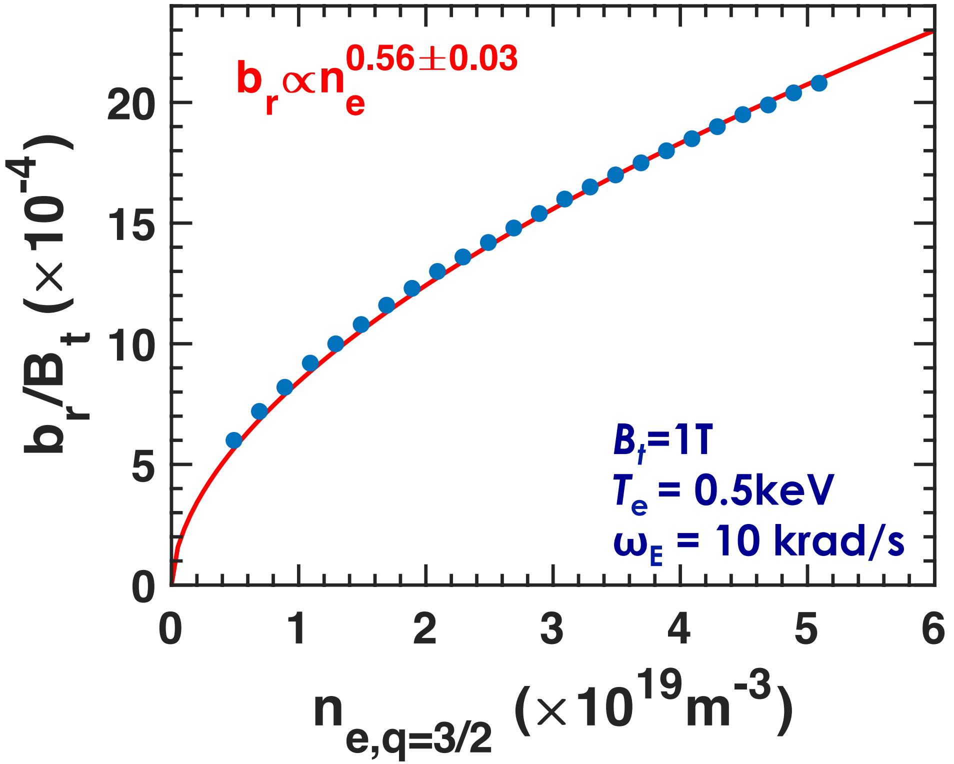


- **EF penetration: phase shift jump, rotation braking, and island growth**

# TM1 single-fluid modeling shows EF threshold positively depends on plasma density

- **EF threshold scales as $n_e^{0.56}$ from TM1 modeling**
  - Higher density
    → higher inertia
    → **stronger EF threshold**

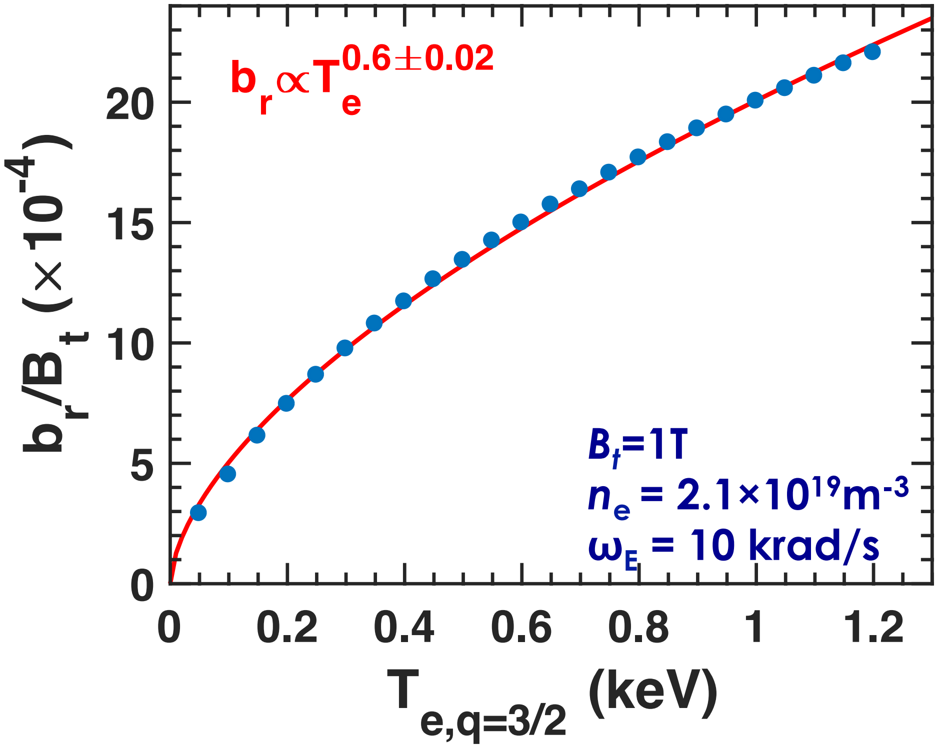
Scaling on density



# TM1 single-fluid modeling shows EF threshold positively depends on electron temperature

- **EF threshold scales as $T_e^{0.6}$ from TM1 modeling**
  - Higher temperature → lower resistivity → stronger screening → **stronger EF threshold**
  - Most experimental scaling ignored temperature effect

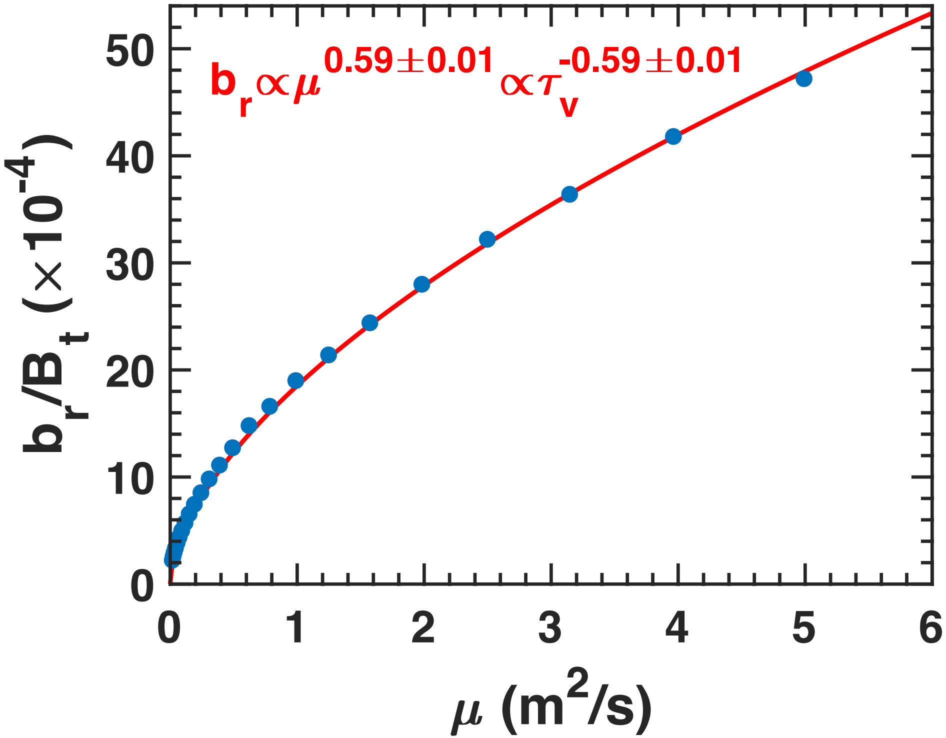
Scaling on temperature



# TM1 single-fluid modeling shows EF threshold positively depends on plasma viscosity

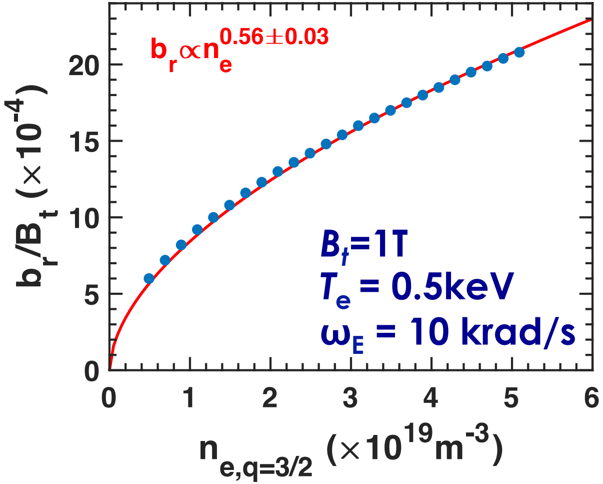
- **EF threshold scales as $\tau_v^{-0.59}$ from TM1 modeling**
  - Stronger viscosity → stronger torque damping → **stronger EF threshold**

Scaling on viscosity

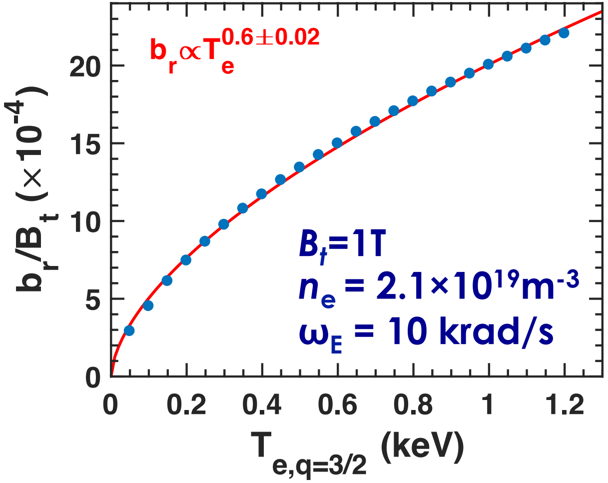


# Scaling laws from TM1 single-fluid modeling agree with Fitzpatrick’s analytical theory

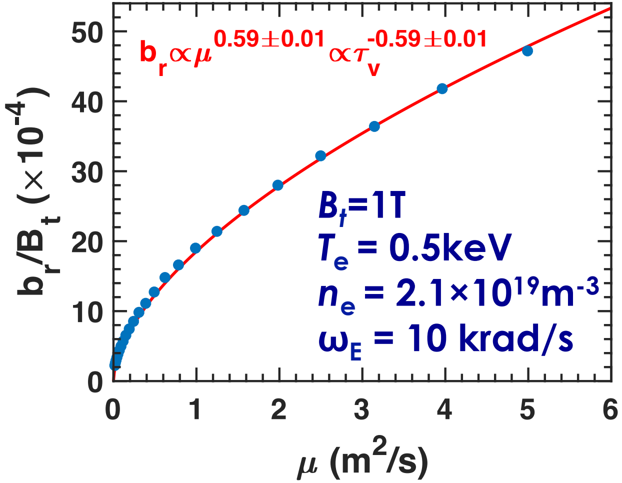
Scaling on density



Scaling on temperature



Scaling on viscosity



**TM1 scaling law:** $b_\mathrm{r} \propto n_\mathrm{e}^{0.56}\, T_\mathrm{e}^{0.6}\, \tau_\mathrm{v}^{-0.59}\, f_0$

**Fitzpatrick’s theory [1]:** $b_\mathrm{r} \propto n_\mathrm{e}^{0.6}\, T_\mathrm{e}^{0.6}\, \tau_\mathrm{v}^{-0.6}\, f_0$ **Rutherford regime**

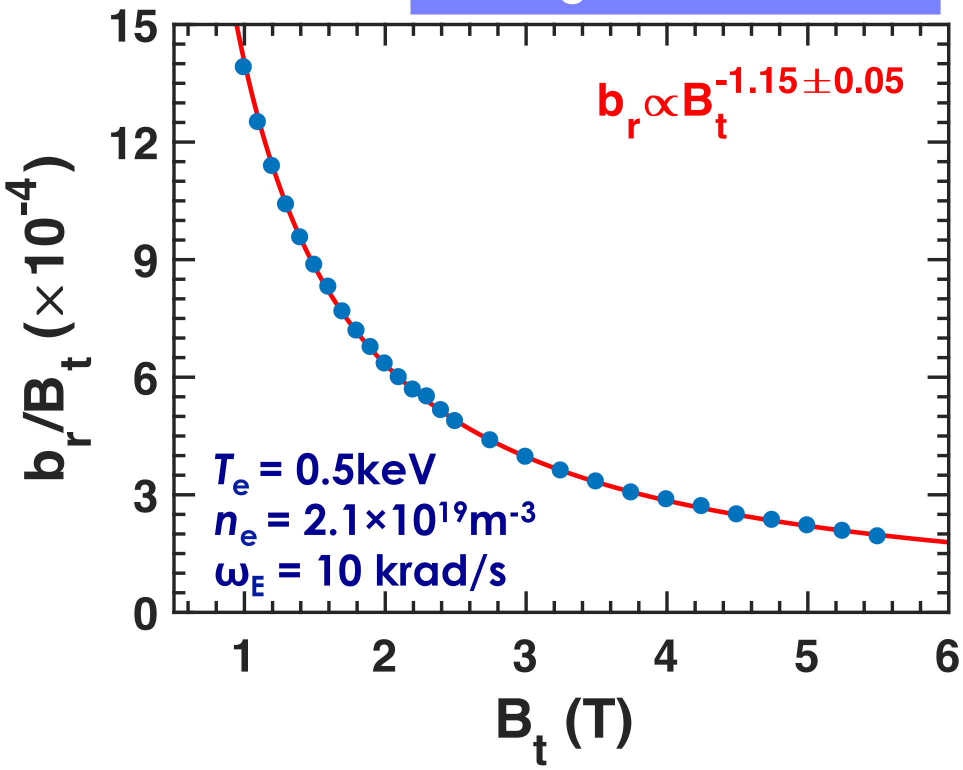
$b_\mathrm{r} \propto n_\mathrm{e}^{7/12}\, T_\mathrm{e}^{5/8}\, \tau_\mathrm{v}^{-7/12}\, f_0$ **Visco-resistive regime**

[1] R. Fitzpatrick, Phys. Plasmas **5**, 3325-3341 (1998)

# TM1 single-fluid modeling shows EF threshold negatively depends on toroidal magnetic field

- **EF threshold scales as $B_t^{-1.15}$ from TM1 modeling**

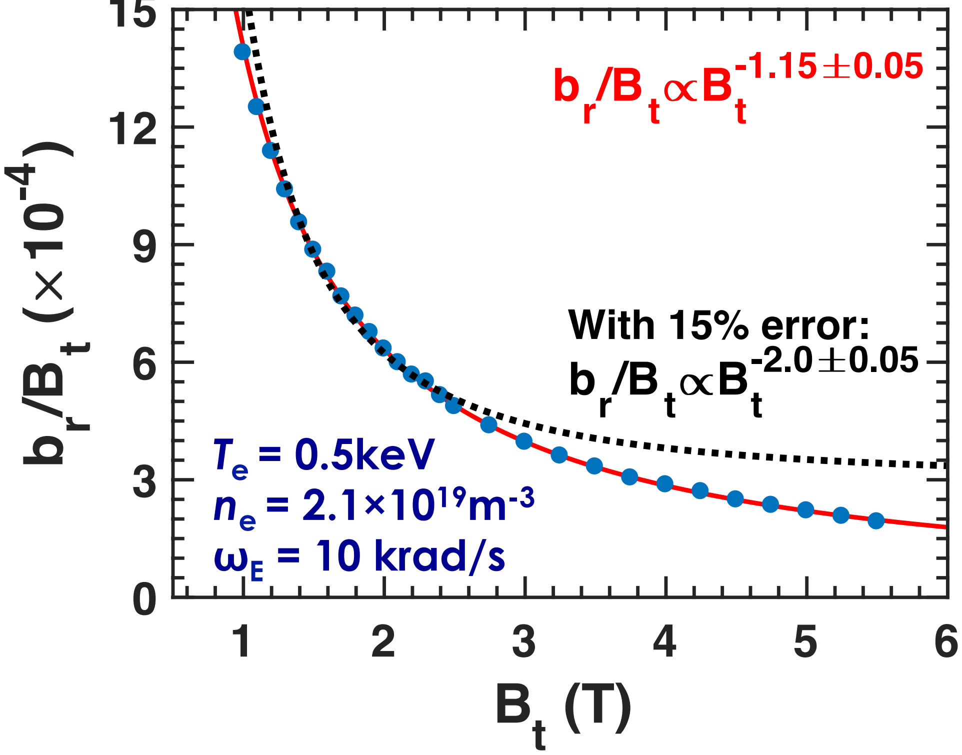
Scaling on toroidal field



# TM1 single-fluid modeling shows EF threshold negatively depends on toroidal magnetic field

- **EF threshold scales as $B_t^{-1.15}$ from TM1 modeling**
- **Hard to obtain the accurate scaling in narrow $B_t$ region**
  - Fast damping in low $B_t$ region
  - **15% error** from residual EF or measurement error can lead to $\mathbf{B_t^{-2}}$

**Scaling on toroidal field**



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- **Prediction for ITER**
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$$\frac{d\psi}{dt} = E - \eta j + \boxed{\Omega(\nabla_\parallel n_e + \nabla_\parallel T_e)}$$

**Diamagnetic effect**

$$\frac{du}{dt} = -C_s^2 \nabla_\parallel P / n_e + \mu_\perp \nabla_\perp^2 u$$

**Momentum,**

$$\rho \frac{d}{dt} \nabla^2 \phi = \vec{e}_\varphi \cdot (\nabla \psi \times \nabla j) + \rho \mu \nabla^4 \phi + S_m$$

**particle,**

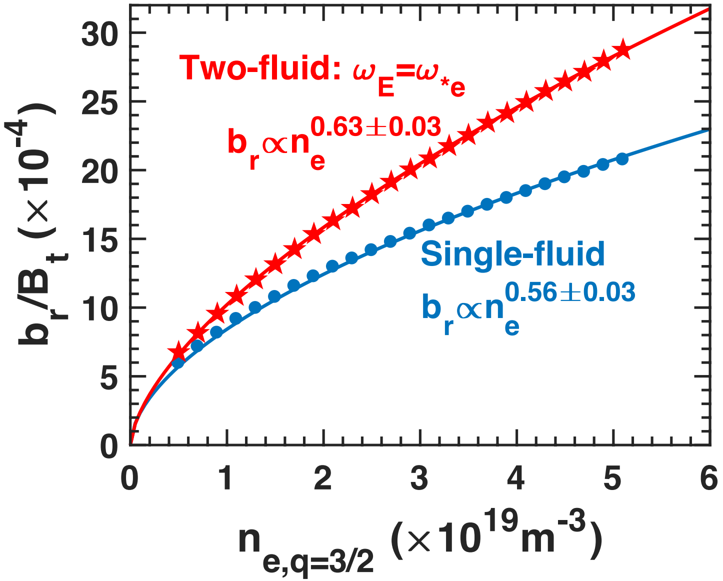
$$\frac{dn_e}{dt} = \frac{\omega_{ce}}{\nu_e} \nabla_\parallel j - \nabla_\parallel (n_e u) + \nabla \cdot (D_\perp \nabla n_e) + S_n$$

**energy transport**

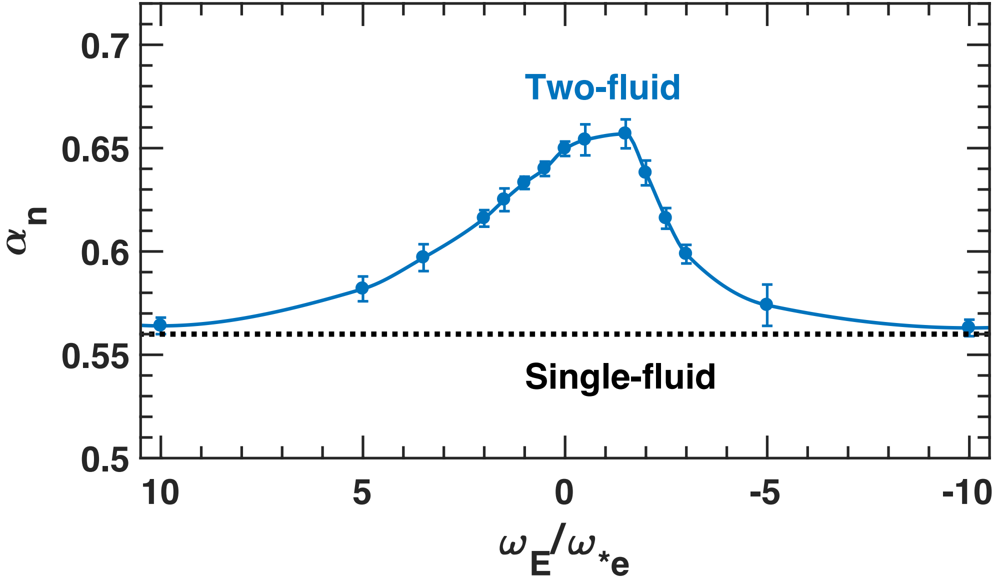
$$\frac{3}{2} n_e \frac{dT_e}{dt} = \frac{\omega_{ce}}{\nu_e} T_e \nabla_\parallel j - T_e n_e \nabla_\parallel u + n_e \nabla \cdot (\chi_\parallel \nabla_\parallel T_e) + n_e \nabla \cdot (\chi_\perp \nabla_\perp T_e) + S_e$$

# TM1 two-fluid modeling shows EF threshold scaling law on density depends on plasma rotation

**Scaling on $n_e$**



**Scaling coefficient on $n_e$ vs. rotation**



- **Two-fluid effect affects $\alpha_n$ when $|\omega_E| \sim |\omega_{*e}|$**
  - Two-fluid effect becomes important when $|\omega_E| \sim |\omega_{*e}|$
  - Two-fluid effect is negligible when $|\omega_E| >> |\omega_{*e}|$

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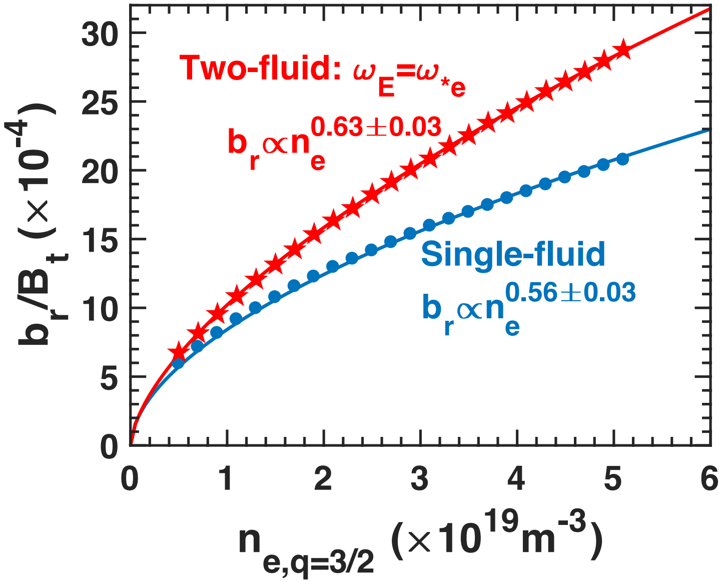
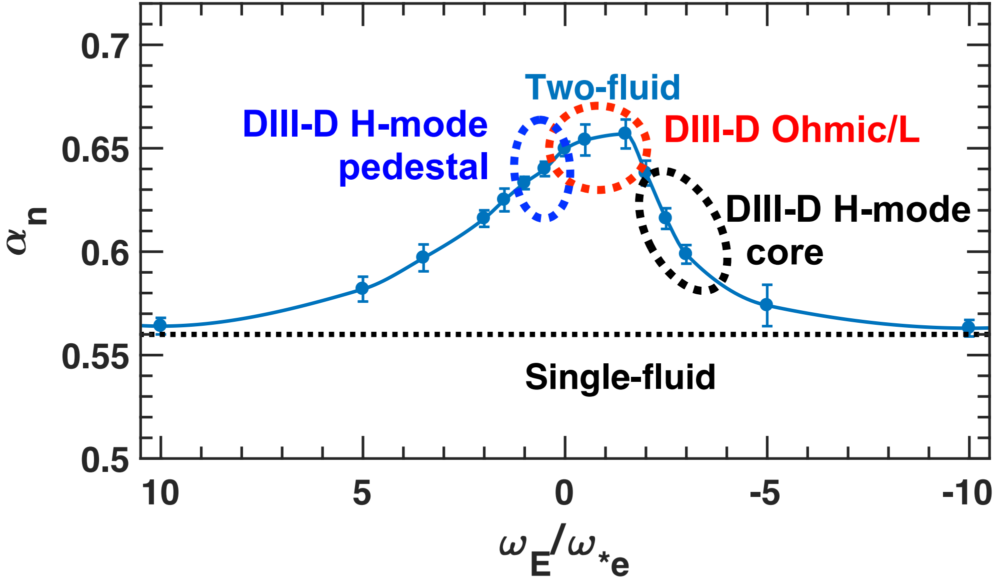
**Scaling on $n_e$**



**Scaling coefficient on $n_e$ vs. rotation**



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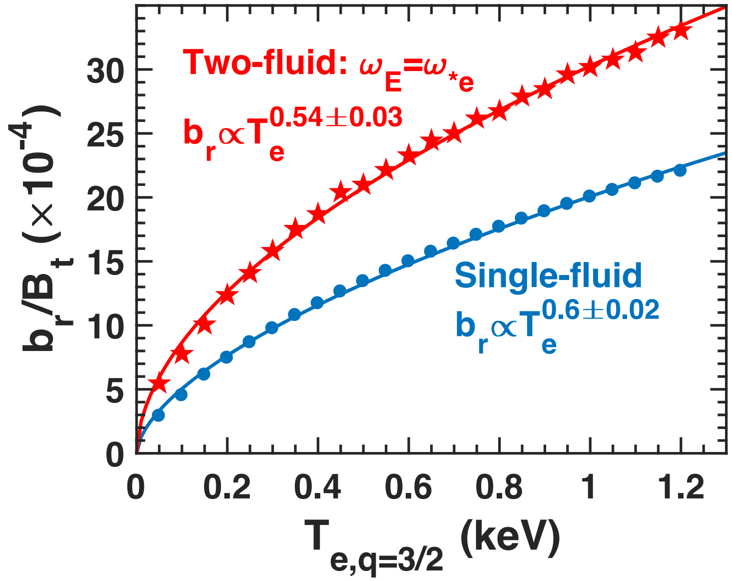
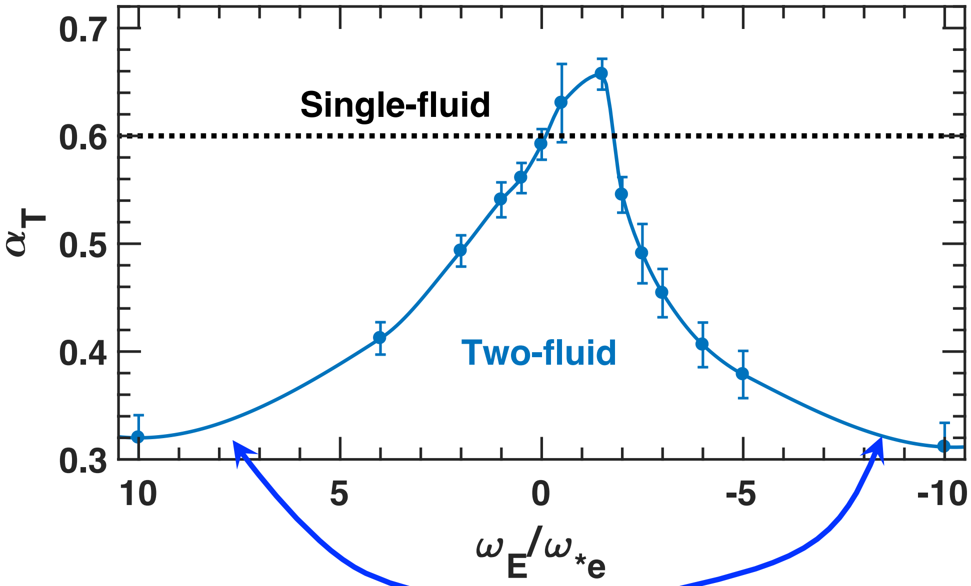
# TM1 two-fluid modeling shows EF threshold scaling law on temperature depends on plasma rotation

**Scaling on $T_e$**



**Scaling coefficient on $T_e$ vs. rotation**

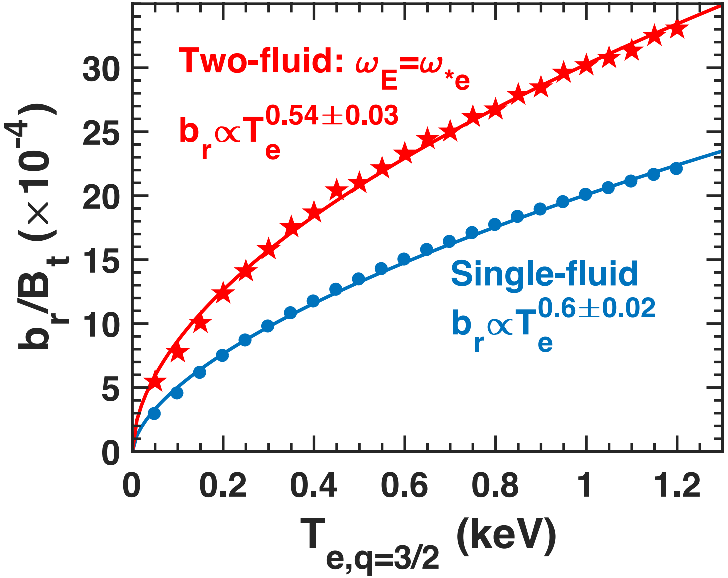


- **Two-fluid effect affects $\alpha_T$ when $|\omega_E| \gg |\omega_{*e}|$**
  - Two-fluid effect becomes important when $|\omega_E| \gg |\omega_{*e}|$
  - Two-fluid effect negligible when $|\omega_E| \sim |\omega_{*e}|$

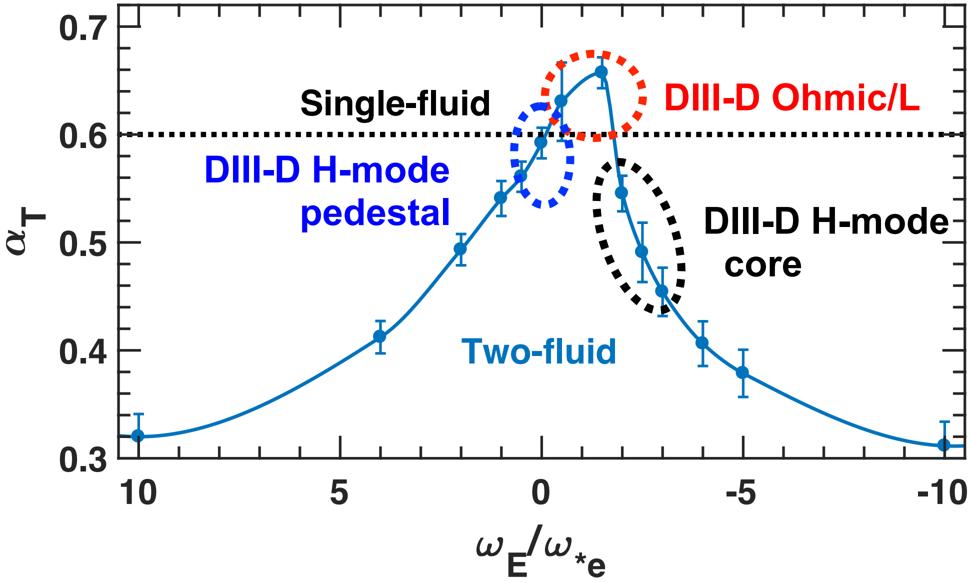
**High sensitivity of parallel transport to resistivity ($T_e$)**

# TM1 two-fluid modeling shows EF threshold scaling law on temperature depends on plasma rotation

**Scaling on $T_e$**



**Scaling coefficient on $T_e$ vs. rotation**



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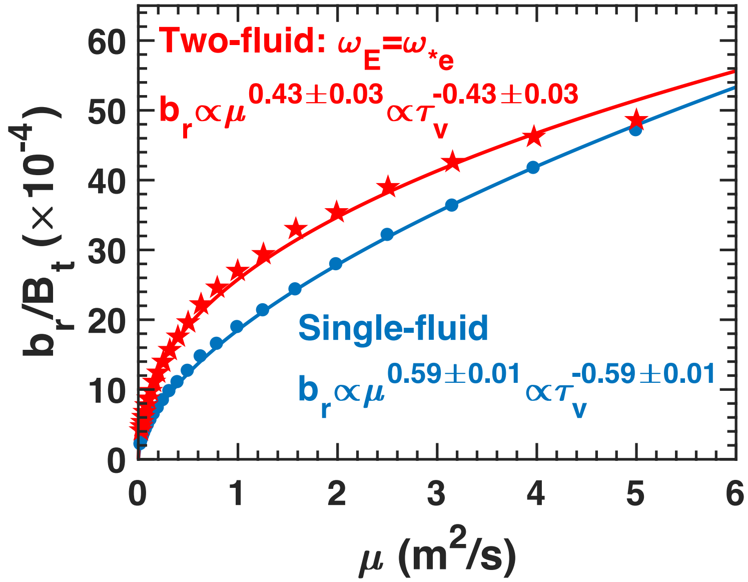
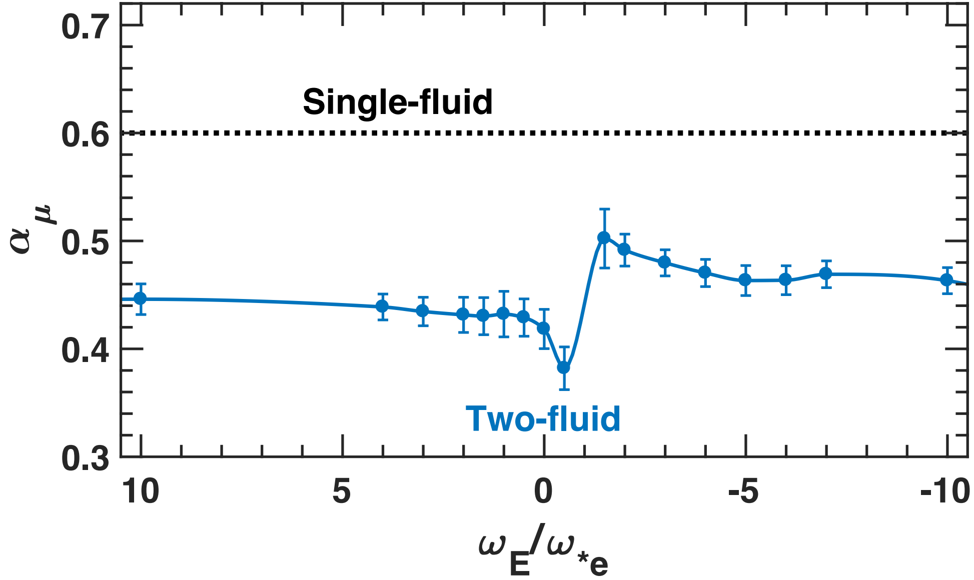
# TM1 two-fluid modeling shows EF threshold scaling law has weaker dependence on visvosity

## Scaling on viscosity



## Scaling coefficient on viscosity vs. rotation

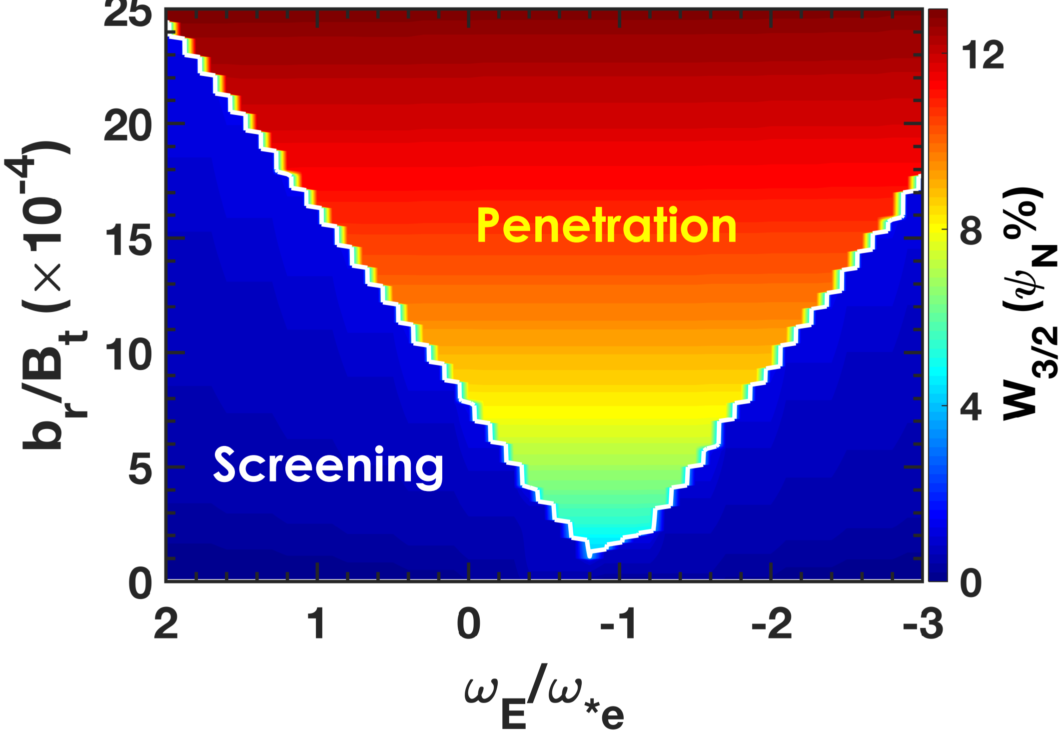


- Two-fluid effect affects $\alpha_v$ and leads to a weaker dependence on viscosity compared to single-fluid → can't cancel the density dependence

# TM1 two-fluid modeling shows linear dependence of EF threshold on plasma rotation

- **3/2 EF penetration threshold linearly depends on plasma rotation**
  - Diamagnetic drift cancels rotation (minimum threshold) when $|\omega_E| \sim |\omega_{*e}|$ → perpendicular flow
  - Almost linear growth of island width vs. RMP strength when $\omega_E+\omega_{*e}=0$

Scaling on rotation



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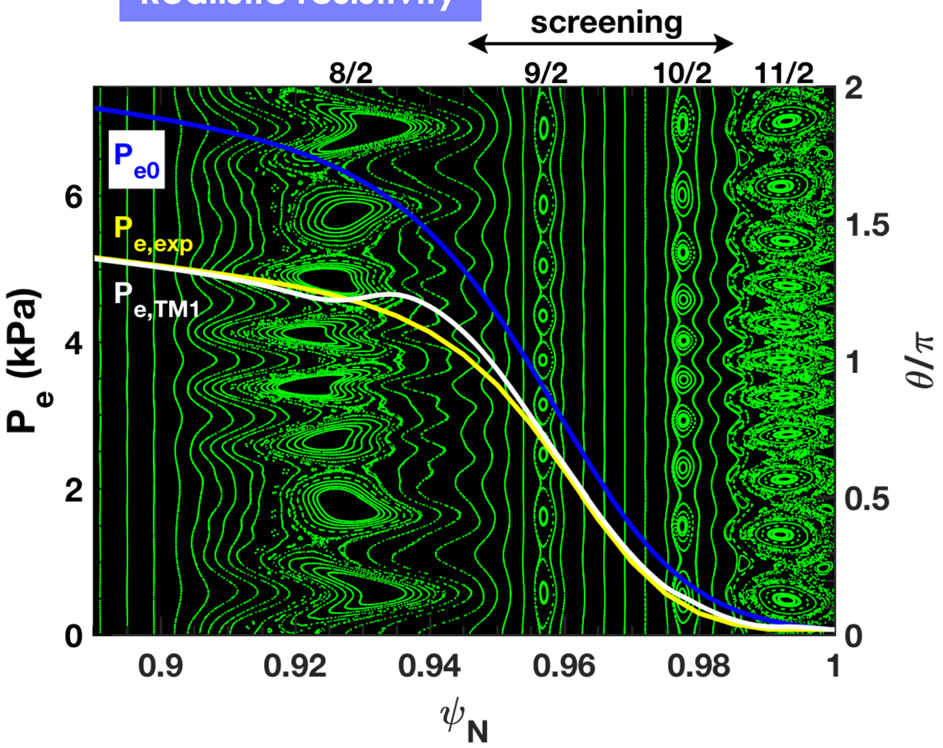
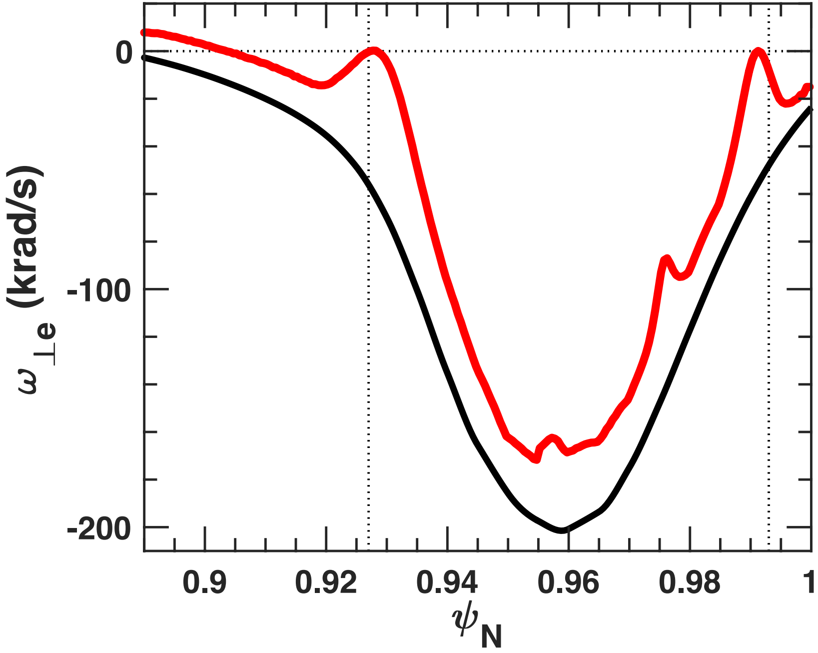
# TM1 two-fluid modeling shows that RMP usually triggers pedestal-top and -foot field penetration in the ELM control experiments

**Realistic resistivity**



**Perpendicular flow frequency becomes zero due to island formation**



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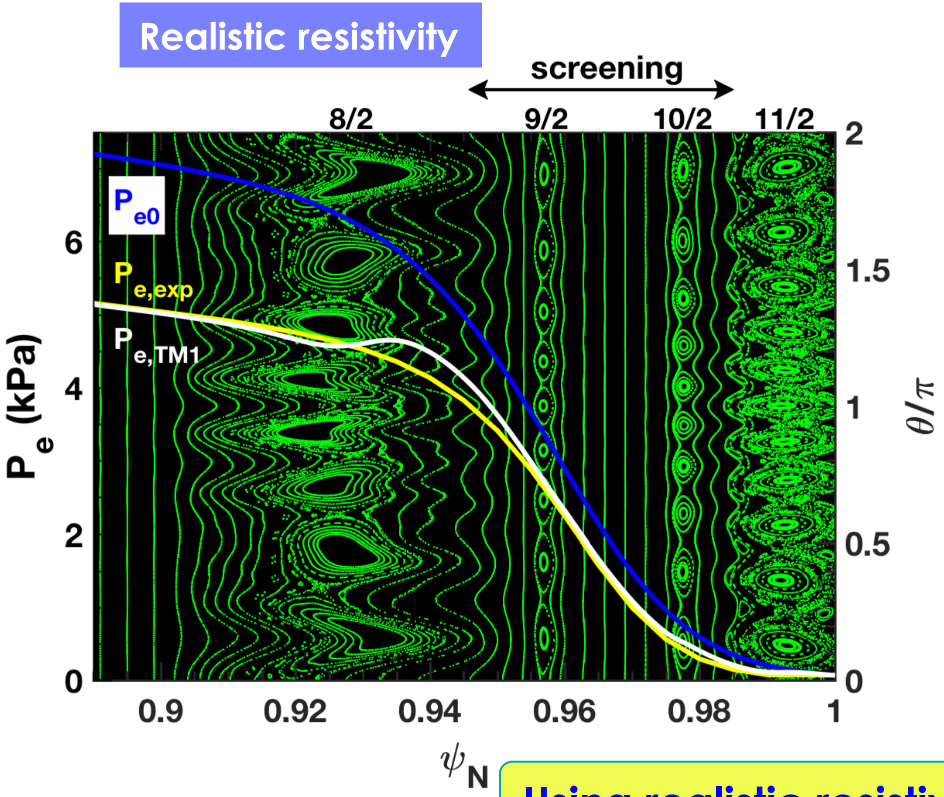
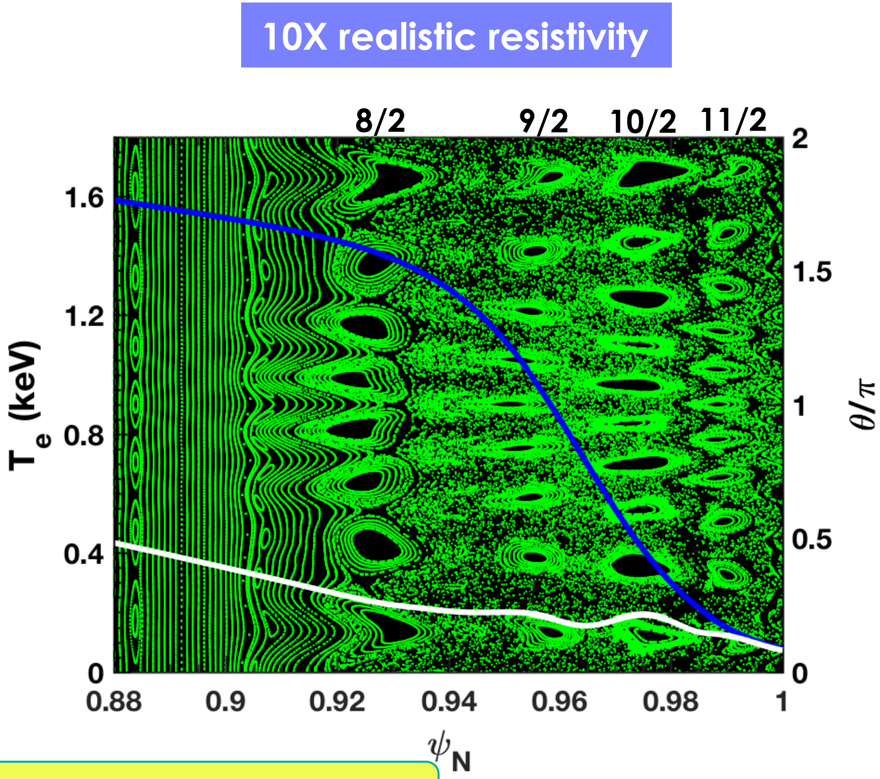




**Using realistic resistivity is important to simulate the plasma response in pedestal**

# TM1 simulated density and rotation dependence for pedestal-top penetration

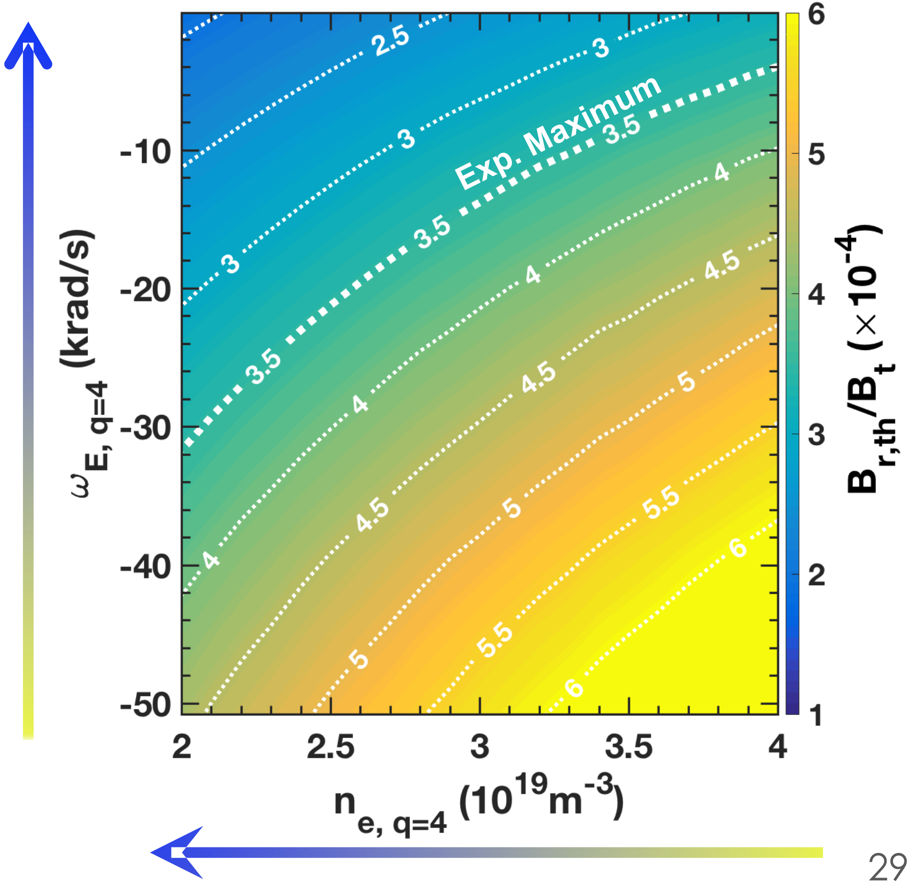
- **2D scaling power law obtained**

$$B_{th}/B_t=3.5\times10^{-2}n_e^{0.7}|\omega_{\perp e}|^{0.94}B_t^{-1}$$

$n_e$ in unit of $10^{19}m^{-3}$, rotation in krad/s, $B_t$ in T

  - **Similar to core EF scaling**

- **Lower $n_e$ and higher rotation makes penetration easier**



# TM1 simulated density and rotation dependence agree with DIII-D experiment

- **2D scaling power law obtained**

$$B_{th}/B_t=3.5\times10^{-2}n_e^{0.7}|\omega_{\perp e}|^{0.94}B_t^{-1}$$

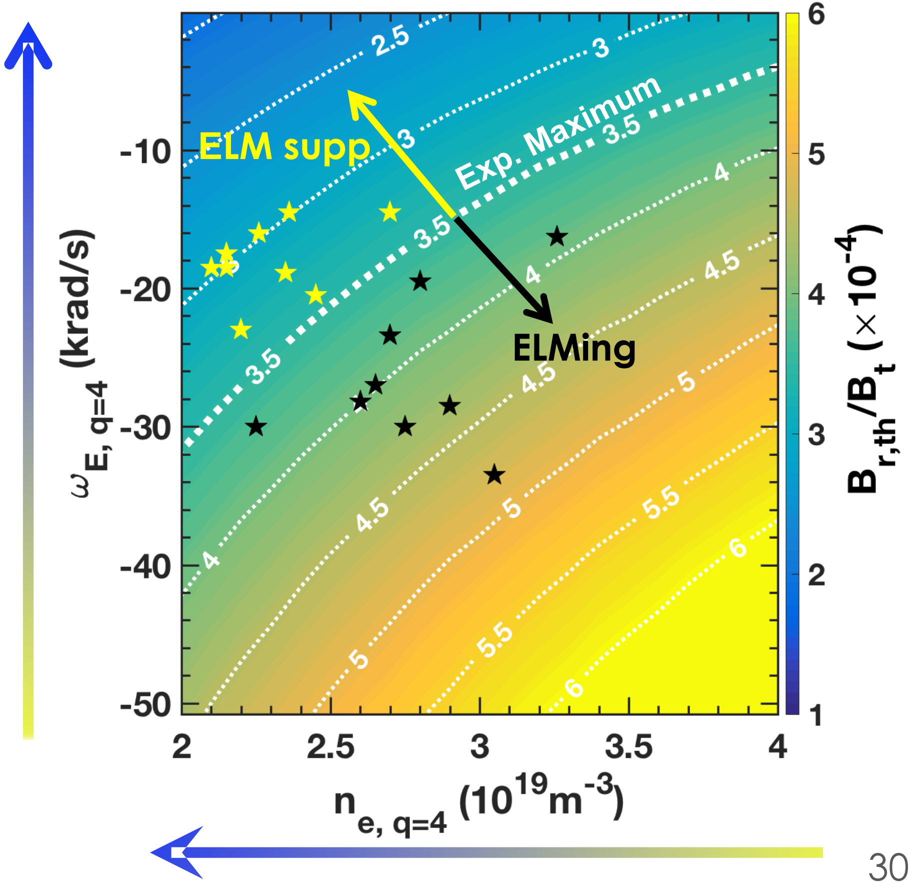
$n_e$ in unit of $10^{19}m^{-3}$, rotation in krad/s, $B_t$ in T

— **Similar to core LM scaling**

- **Lower $n_e$ and higher rotation makes penetration easier**

- **Consistent with experiment [1]**

[1] C. Paz-Soldan, et al., NF **59**, 056012 (2019)

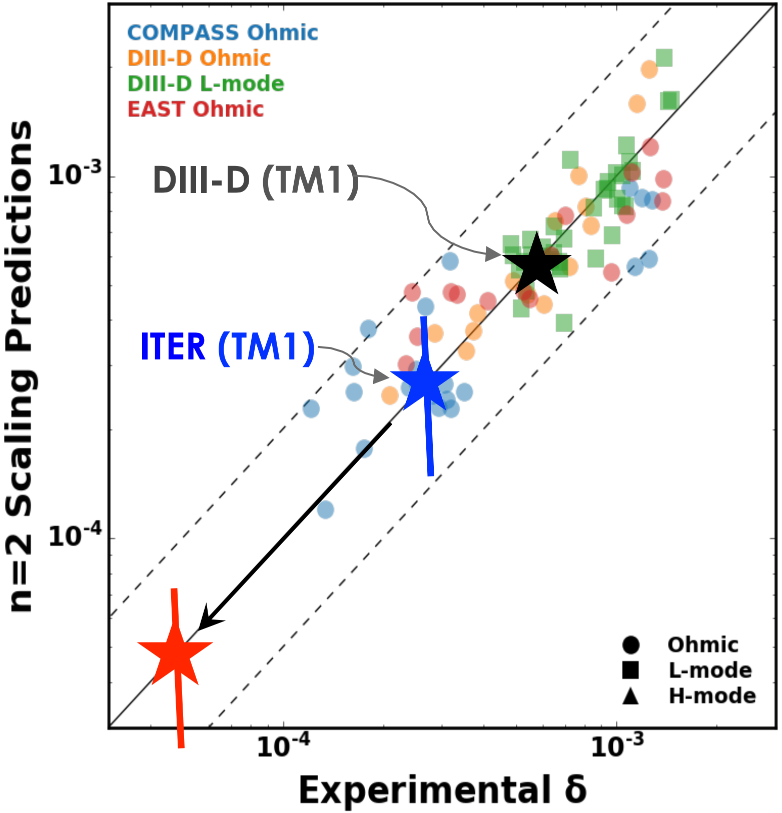


# Outline

- **TM1 modeling of 3/2 EF threshold scaling**
  - → Single-fluid modeling: well reproduced Fitzpatrick's theory
  - → Two-fluid modeling: sensitive scaling on plasma rotation
- **TM1 modeling pedestal-top n=2 RMP threshold scaling**
  - → Two-fluid modeling: similar with core scaling
- **Prediction for ITER**
- **Summary**

# If we make reasonable estimates for ITER profiles, direct TM1 predictions fall within typically observed thresholds

- **TM1 predicts threshold in DIII-D is close to the experiments**
  - $T_e$=0.4keV, $n_e$=2.5×$10^{19}$ $m^{-3}$, $\omega_E$=10 krad/s
- **TM1 predicts a lower threshold for ITER using DIII-D IBS profiles**
  - Scaled to $T_e$=1keV, $n_e$=5×$10^{19}$ $m^{-3}$, $\omega_E$=2-8 krad/s
- **If rotation in ITER is more than one order lower than DIII-D [1], then much lower threshold**

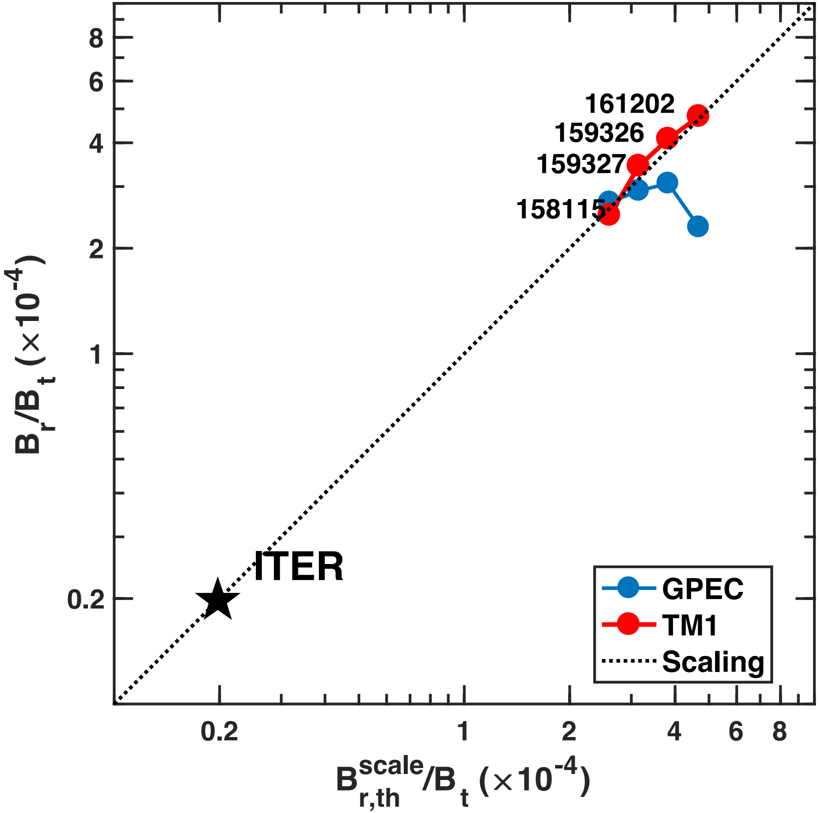


[1] A.M. Garofalo et al Nucl. Fusion **51**, 083018 (2011)

# Pedestal-top penetration threshold in ITER may be also one order lower than DIII-D

| **ITER parameters** |
|---|
| $B_t$ = 5.15 T, $R$ = 6.38 m<br>$a$ = 1.98 m, $q_{95}$ = 3.1 |
| $n_{e,q=3}$ = 9×10$^{19}$ m$^{-3}$<br>$T_{e,q=3}$ = 4 keV |
| $\omega_{E,q=3} \approx \omega_{*e} \approx$ -3 krad/s |

- The penetration threshold for ITER n=3 RMP will be an order lower than DIII-D



# Summary

- **TM1 single-fluid modeling agrees with Fitzpatrick's analytical theory**
- **TM1 two-fluid modeling shows that the scaling coefficients are strongly affected due to two-fluid effect**
- **TM1 predicted penetration in both the core and pedestal of ITER will be weaker than DIII-D due to the slower flow frequency**
- **Next step: compare with DIII-D experiments and perform detail ITER prediction**

# Backup slides

# TM1 single-fluid modeling shows EF threshold scaling law on major radius depends on plasma rotation

- **EF threshold scales as $R_0^{0.78}$ for constant angular frequency**
  - Stronger velocity for larger $R_0$
- **EF threshold scales as $R_0^{-0.18}$ for constant velocity**
  - More like this case [1]

[1] A.M. Garofalo et al Nucl. Fusion **51**, 083018 (2011)

Scaling on major radius