Prediction of the nonlinear character of Alfvénic instabilities and their induced fast ion losses

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in collaboration with

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[4]University of California, Irvine

Work funded by DOE under contracts DE-AC02-09CH11466 and DE-FC02-04ER54698 and through the SciDAC project "*Integrated Simulation of Energetic Particles in Burning Plasmas*"

NSTX-U / Magnetic Fusion Science Meeting - March 16th 2020

Outline

Part I: Prediction of the nonlinear spectral character of Alfvén waves

- Chirping vs quasi-steady frequency responses and the applicability of reduced models
- A criterion for the chirping onset
- Analysis of NSTX and DIII-D data
- Predictions for ITER scenarios

Part II: Reduced quasilinear modeling of fast ion losses

- Development of the broadened quasilinear framework
- Verification and validation exercises
- Integration of the Resonance Broadened Quasilinear (RBQ) code into TRANSP
    - modeling of DIII-D critical gradient experiments

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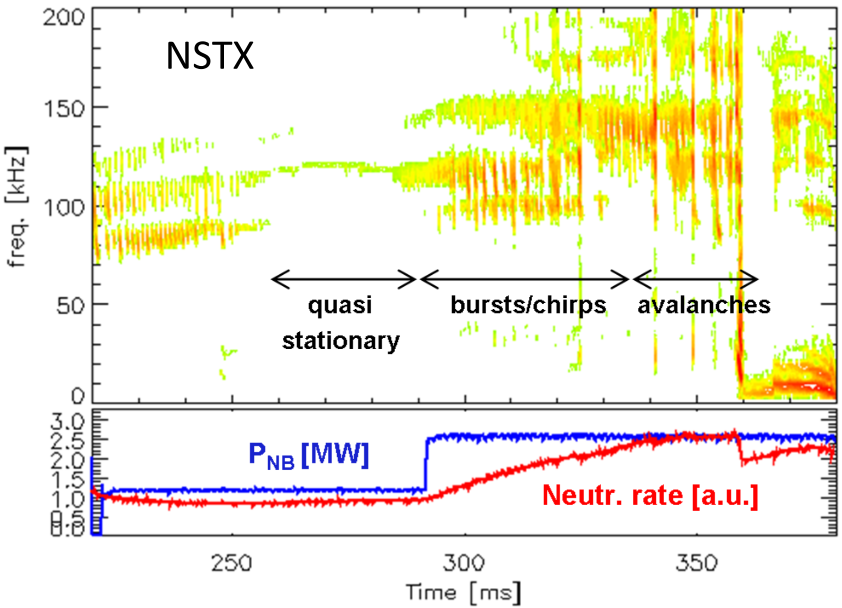
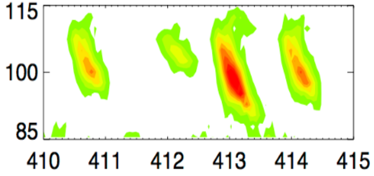
Alfvén waves can exhibit a range of bifurcations upon their interaction with fast ions

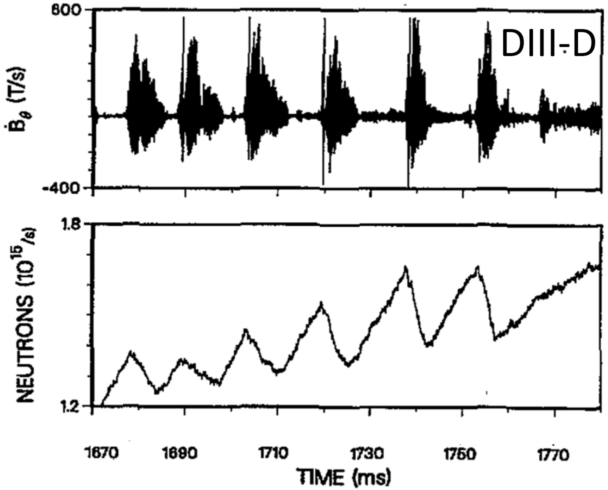


Podestà *et al*, *Nucl. Fusion* **51** 063035 (2011)

Chirping is a gateway to avalanches



Each chirp induces fast ion losses

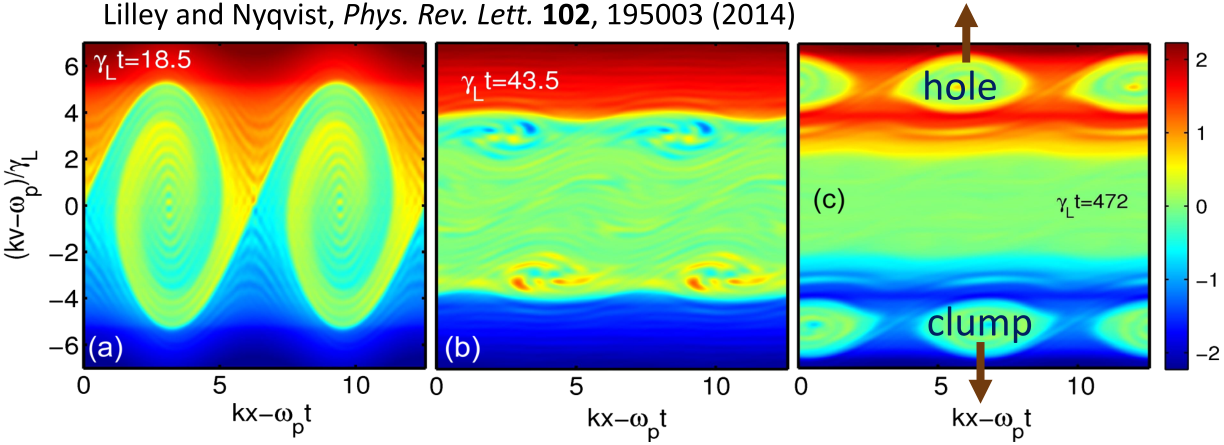


Heidbrink, *Plasma Phys. Control. Fusion* **37** 937 (1995)

Major question: why is chirping common in spherical tokamaks and rare in conventional tokamaks?

Chirping is supported by phase-space holes and clumps

Lilley and Nyqvist, *Phys. Rev. Lett.* **102**, 195003 (2014)



Stochasticity hinders the formation of holes and clumps, preventing chirping

Two typical scenarios for fast ion losses:

Diffusive transport (typical for fixed-frequency modes)

- can be modeled using <u>reduced theories</u>
  - e.g., quasilinear theory

Convective transport (typical for chirping frequency modes)

- Requires capturing full phase-space dynamics
  - <u>expensive nonlinear simulations</u>

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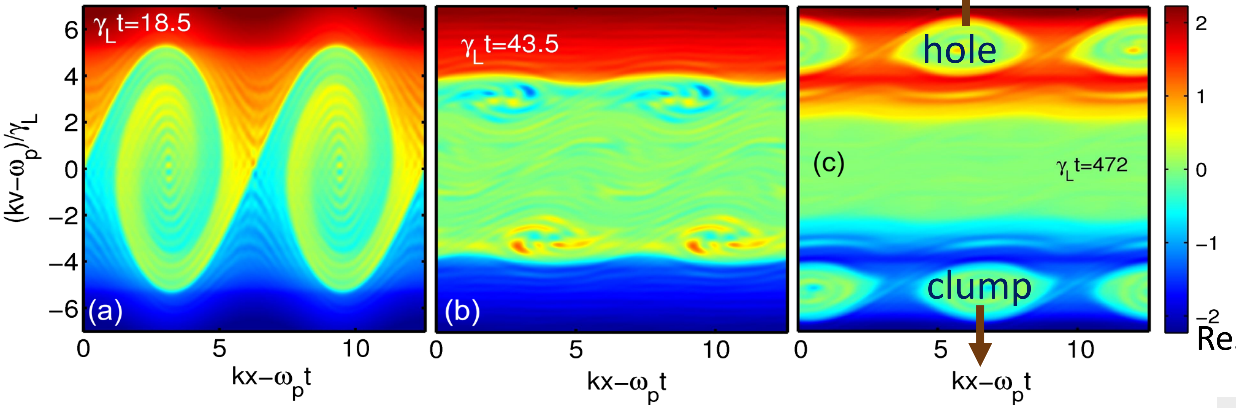

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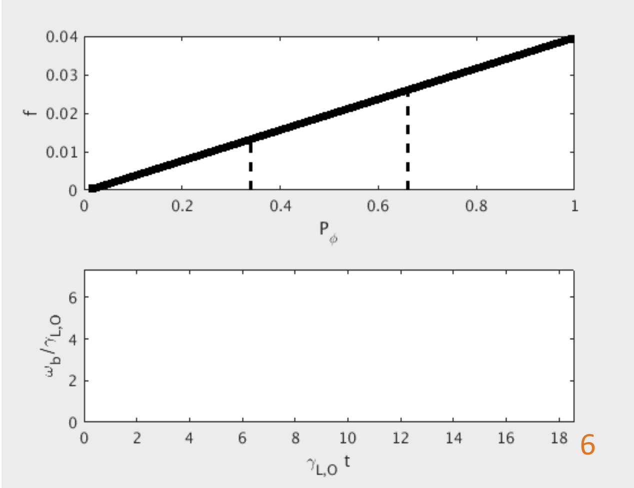
Resonance Broadening Quasilinear (RBQ) code
(later in this talk)



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A criterion for the likelihood of chirping onset in tokamaks

Starting point: the evolution equation for mode amplitude A near marginal stability:

$$\frac{dA(t)}{dt} = (\gamma_L - \gamma_d)\, A(t) - \int_0^{t/2} d\tau \int_0^{t-2\tau} d\tau_1 \tau^2 e^{-\hat{\nu}_{stoch}^3 \tau^2 (2\tau/3+\tau_1) + i\hat{\nu}_{drag}^2 \tau(\tau+\tau_1)} \mathcal{O}\left(A^3\right)$$

Berk, Breizman and Pekker, PRL 1996
Lilley, Breizman and Sharapov, PRL 2009

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stabilizing

destabilizing (makes integral sign flip)

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Blow up of A in a finite time-> system enters a strong nonlinear phase (chirping likely)

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Chirping criterion:

$$Crt = \frac{1}{N}\sum_{j,\sigma_\parallel} \int dP_\varphi \int d\mu \frac{|V_{n,j}|^4}{\omega_\theta \nu^4_{drag}} \left|\frac{\partial \Omega_j}{\partial I}\right| \frac{\partial f}{\partial I} Int\left(\frac{\nu_{stoch}}{\nu_{drag}}\right)$$

- >0: fixed-frequency solution likely
- <0: chirping likely to occur

(nonlinear prediction from linear physics elements->incorporated into the linear NOVA-K code)

Duarte *et al*, *Nucl. Fusion* **57** 054001 (2017)

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The criterion ($Crt \gtrless 0$) predicts that micro-turbulence should be key in determining the likely nonlinear character of a mode, e.g., fixed-frequency or chirping

Duarte *et al*, *Nucl. Fusion* **57** 054001 (2017)

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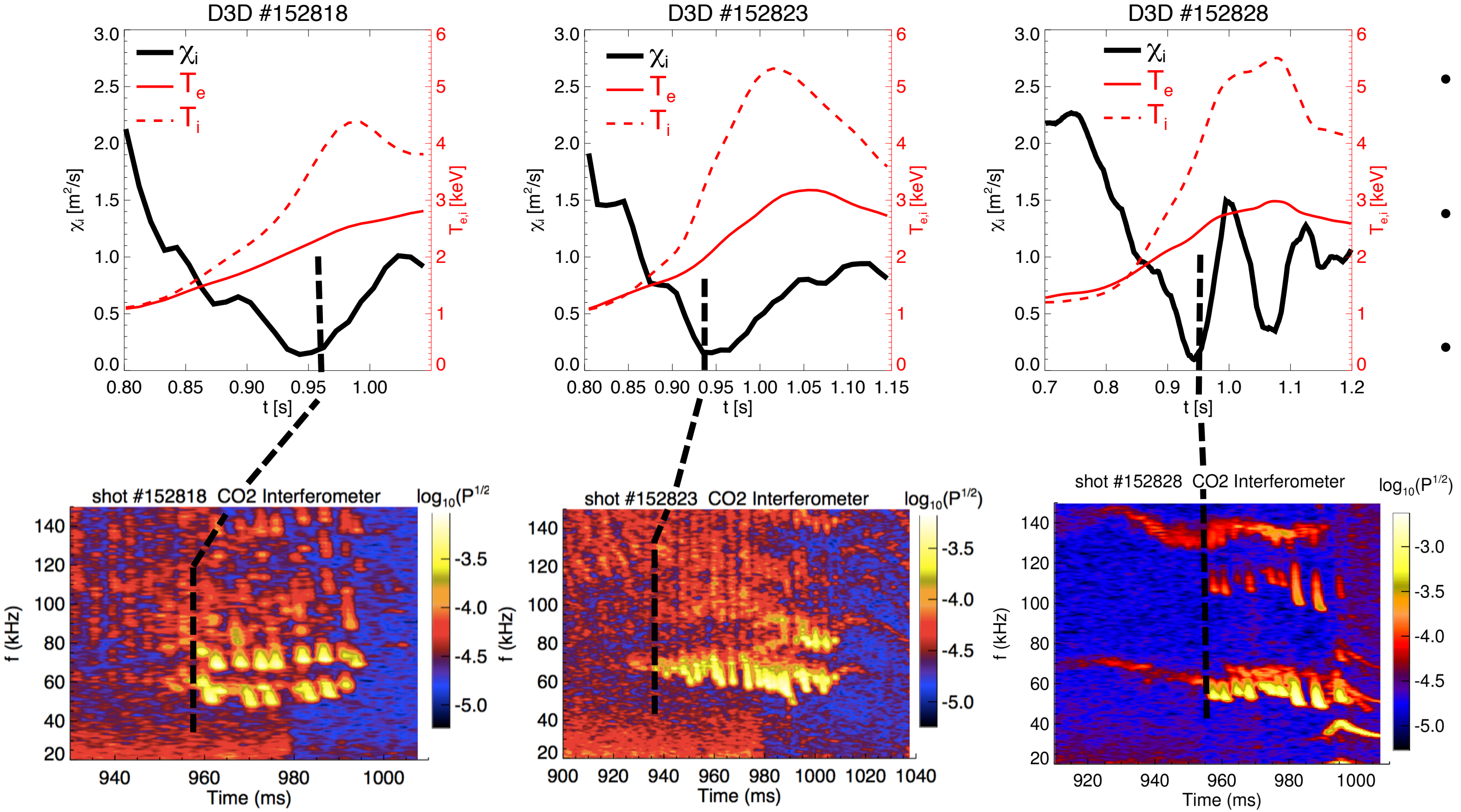
Correlation between chirping onset and a marked reduction of the turbulent activity in DIII-D

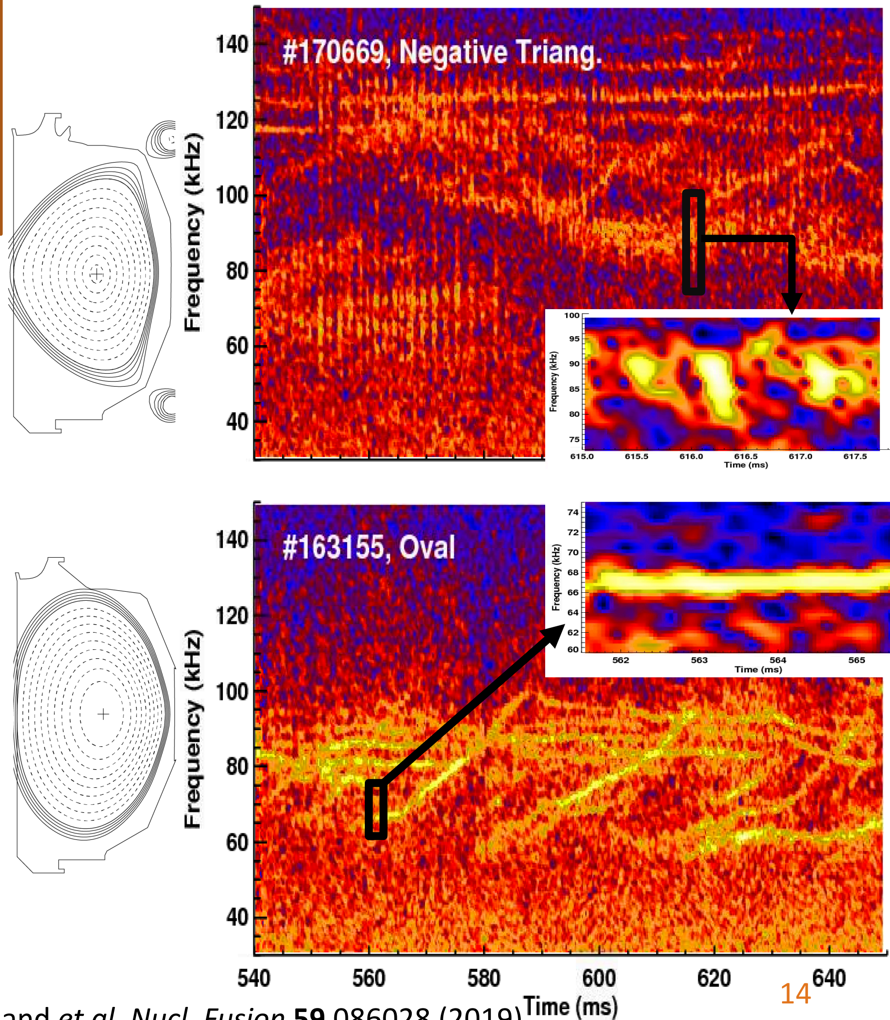


- Diffusivity drop due to L→H mode transition
- Strong rotation shear was observed
- This observation motivated DIII-D experiments to be designed to further test the hypothesis of low turbulence associated with chirping

Duarte *et al*, *Nucl. Fusion* **57** 054001 (2017)

Dedicated experiments showed that chirping is more prevalent in negative triangularity DIII-D shots

- Transport coefficients calculated in TRANSP are 2-3 times lower in negative triangularity, as compared to the the usual positive/oval triangularity



Van Zeeland *et al*, *Nucl. Fusion* **59** 086028 (2019)

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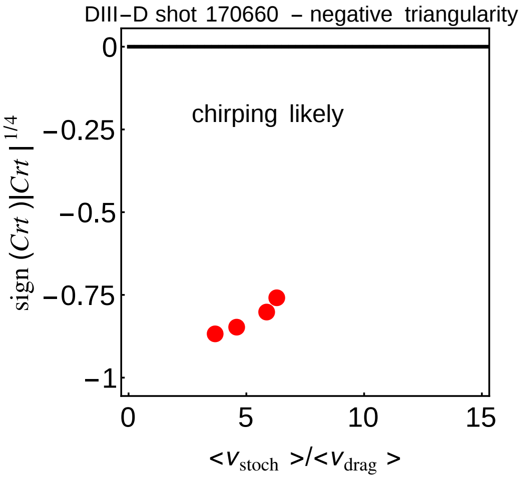
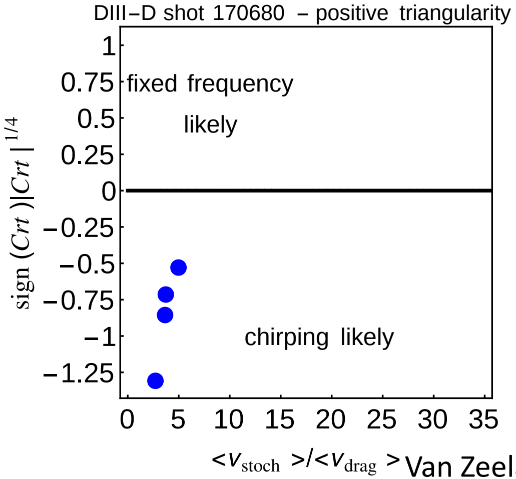
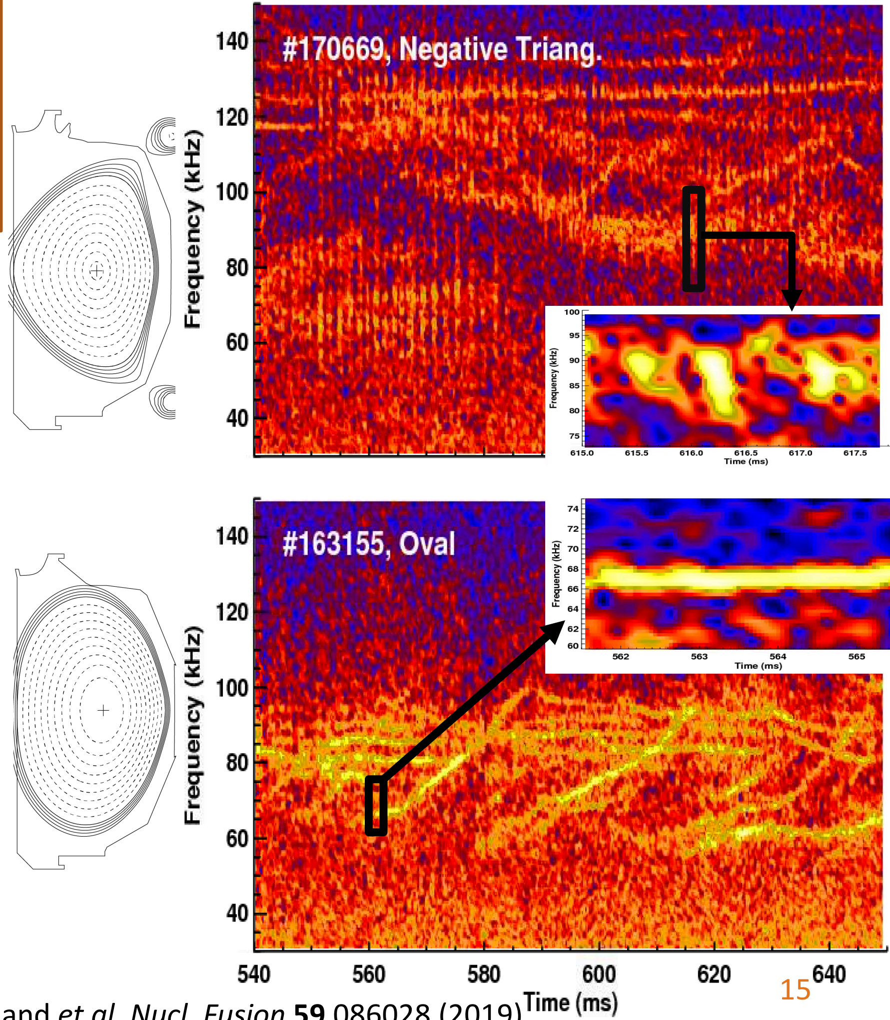
>0: fixed-frequency

<0: chirping







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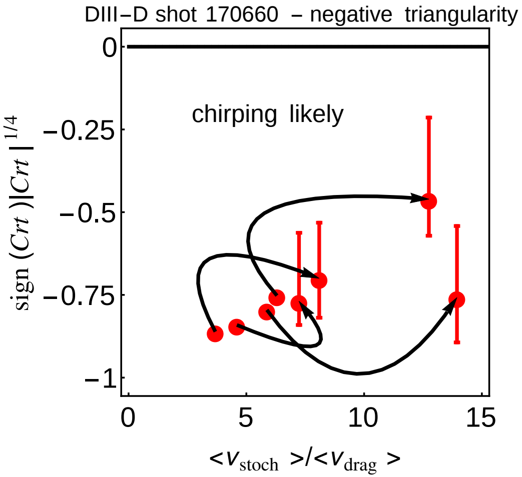
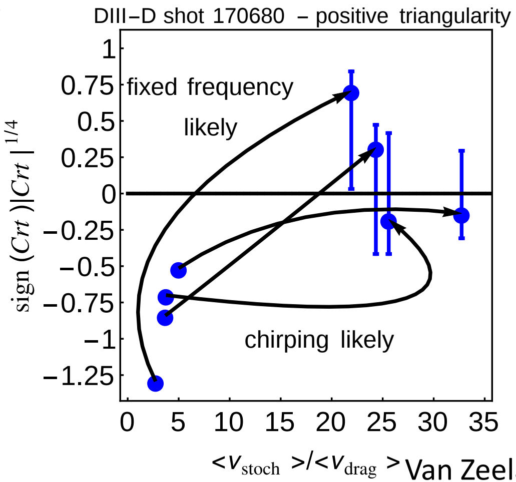
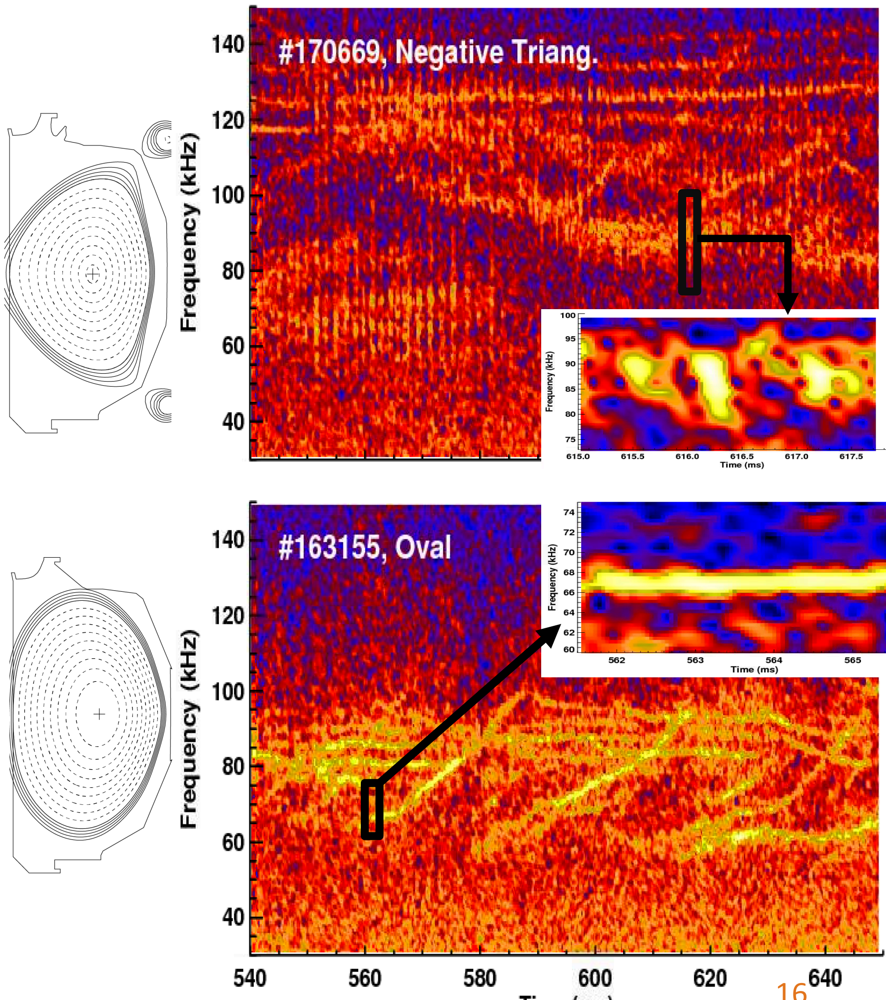
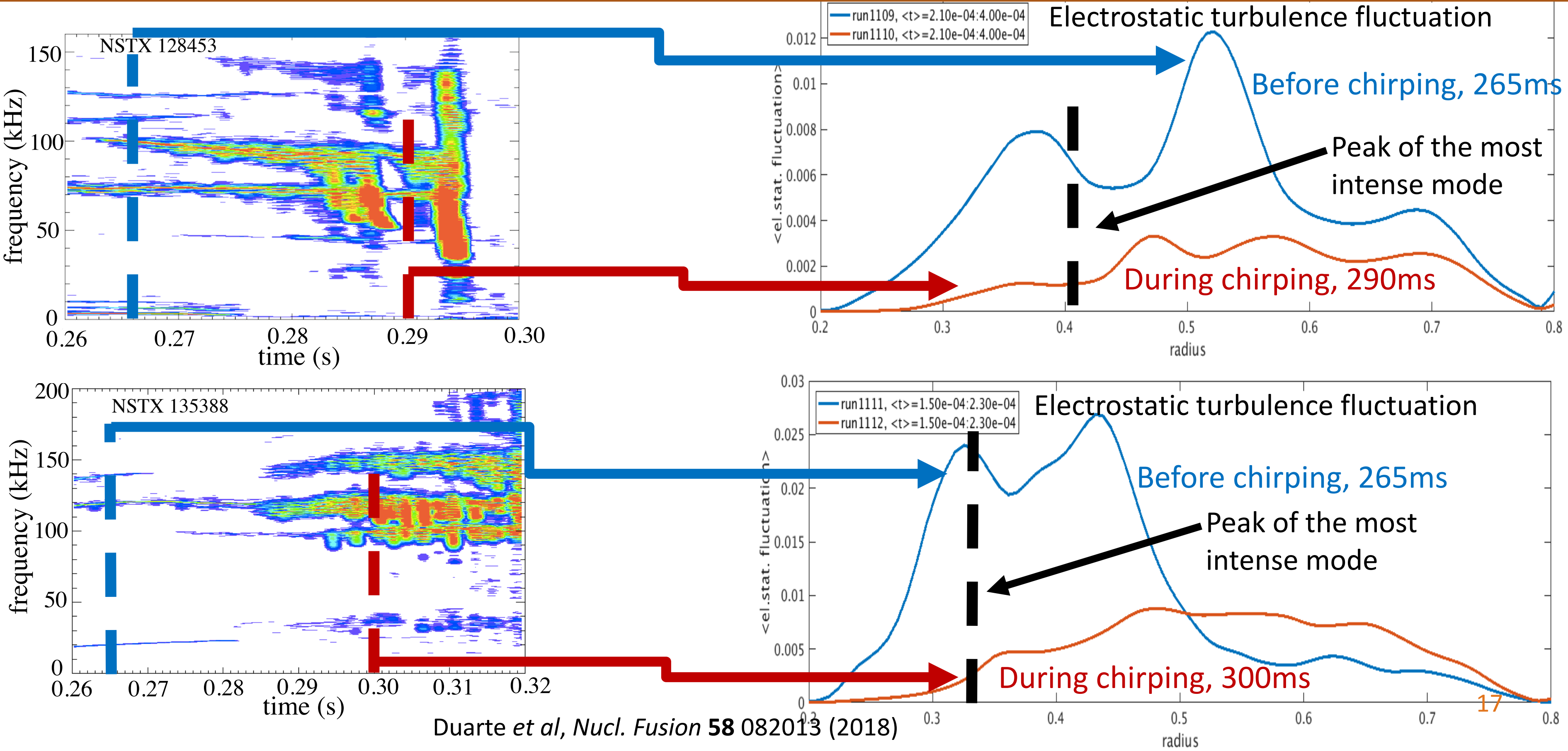






Van Zeeland *et al*, *Nucl. Fusion* **59** 086028 (2019)

GTS global gyrokinetic analyses show turbulence reduction for rare NSTX Alfvénic transitions from fixed-frequency to chirping

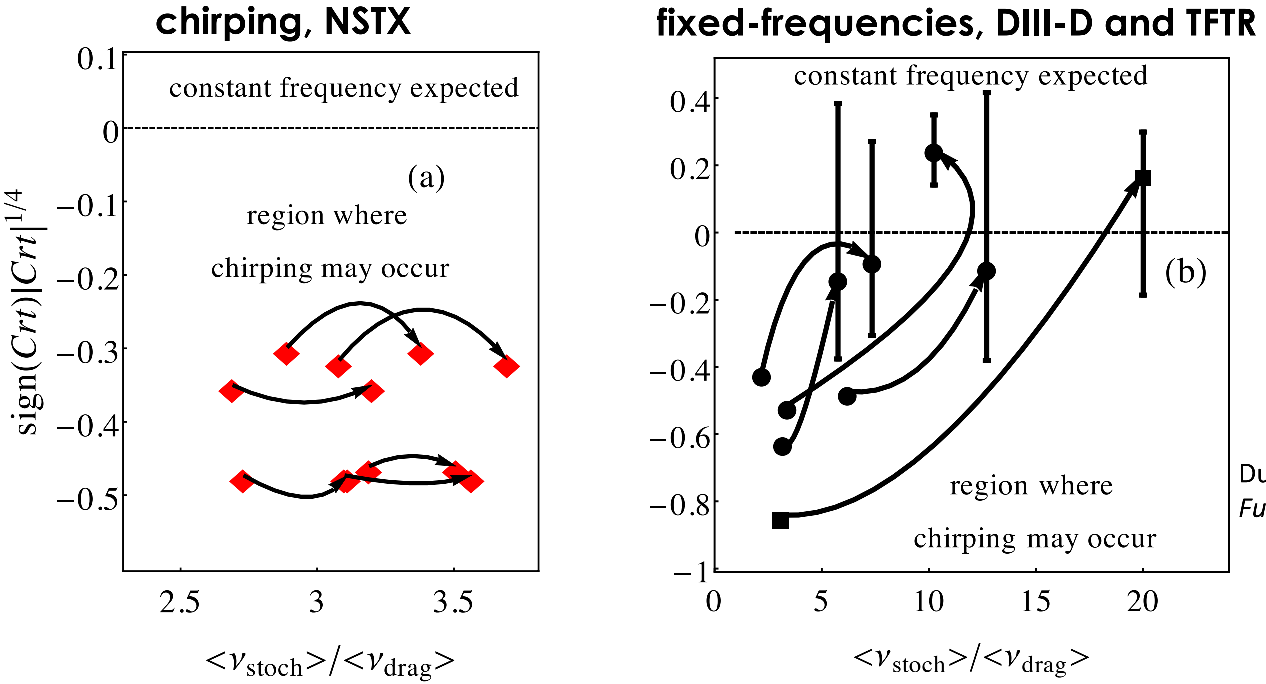


Duarte *et al*, *Nucl. Fusion* **58** 082013 (2018)

Chirping criterion explains different spectral behavior in spherical vs conventional tokamaks

Arrows represent how much turbulent diffusion changes the prediction with respect to collisions alone

Transport in NSTX in mostly neoclassical at the ion scale



Duarte *et al*, *Nucl. Fusion* **57** 054001 (2017)

Chirping is ubiquitous in NSTX but rare in DIII-D, which is consistent with the inferred fast ion micro-turbulent levels

Similar agreement was later found in ASDEX-U [Lauber et al, IAEA inv. talk 2018]

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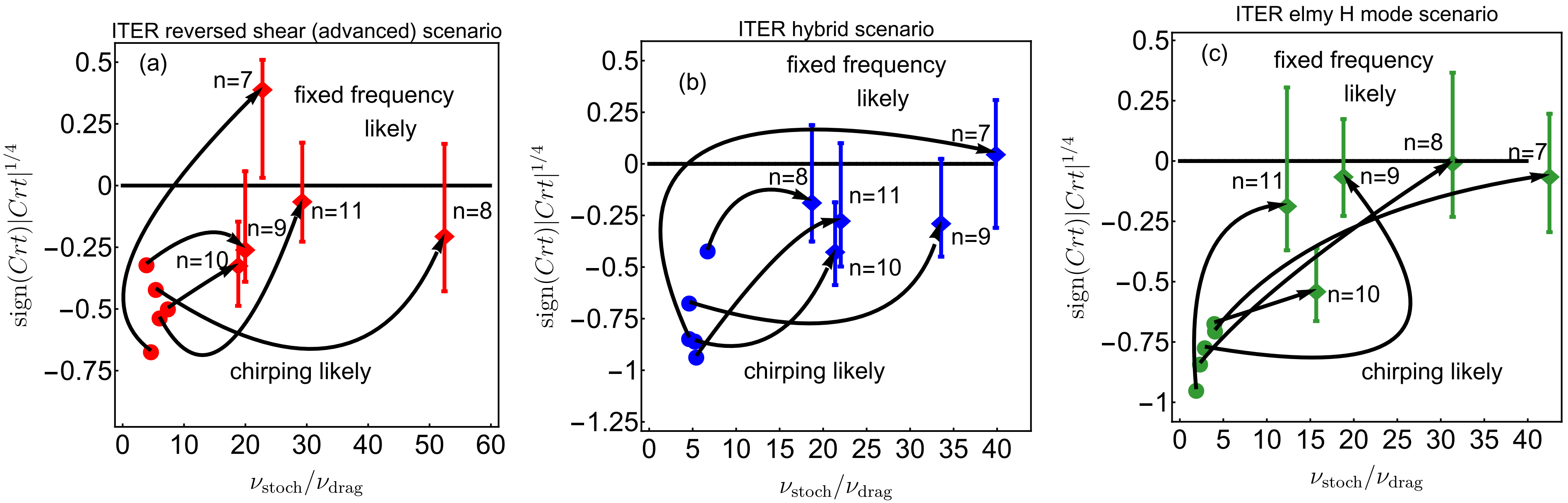
Alfvénic chirping is unlikely but cannot be ruled out in ITER

Predictions for the most unstable (n=7-11) TAEs in ITER are near threshold between fixed frequency and chirping, based on TRANSP analysis, requiring Q>10



Duarte *et al*, *Nucl. Fusion* **58** 082013 (2018)

Validated chirping criterion predicts when fast ion transport can be modeled with reduced models

- Reduced transport models are only applicable when details of phase-space dynamics are unimportant (no chirping modes)

- Nonlinear theory has been used to develop a criterion for when chirping is likely to occur

- Stochasticity destroys phase space coherence necessary for chirping
    - less turbulence favors chirping → confirmed in negative triangularity experiments

- Chirping criterion explains why chirping is ubiquitous in NSTX and rare in DIII-D
    - predicts that chirping is possible but unlikely in ITER

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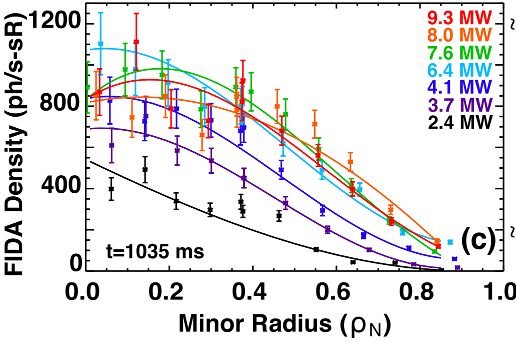
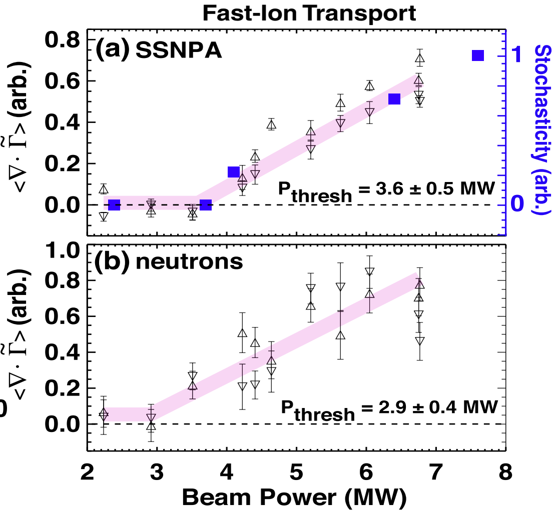
Critical gradient behavior in DIII-D suggests that quasilinear modeling is a viable tool for fast ion relaxation

Critical gradient is often observed in DIII-D

- stiff, resilient fast ion profiles as beam power varies
-stochastic fast ion transport (mediated by overlapping resonances) gives credence in using a quasilinear approach





Collins et al, Phys. Rev. Lett. **116**, 095001 (2016)

- Fully nonlinear modeling of fast ion interaction with Alfvénic modes in a realistic tokamak is numerically expensive

- Reduced (but still realistic) modeling can be exploited if linear mode properties do not change faster than the equilibrium, e. g.,
  - eigenstructure
  - resonance condition

- Simulations need to cope with the simultaneous excitation of multiple unstable Alfvénic instabilities

Resonance-broadened quasilinear (RBQ) diffusion model

-a reduced but yet a realistic framework-

Formulation in action and angle variables $\frac{\partial}{\partial I} = \omega \frac{\partial}{\partial \mathcal{E}} - n \frac{\partial}{\partial P_\varphi}$

- Diffusion equation:

$$\frac{\partial f}{\partial t} = \sum_{k,p,m,m'} \frac{\partial}{\partial I} D_{k,p,m,m'}(I;t) \frac{\partial f}{\partial I} + C[f]$$

Broadened delta

$$D_{k,p,m,m'}(I;t) = \pi A_k^2(t)\, \mathcal{E}^2 \frac{\mathcal{R}_{k,p}(I - I_r)}{\left| \frac{\partial \Omega_{k,p}}{\partial I} \right|} G^*_{m'p} G_{mp}$$

eigenstucture information

- Mode amplitude evolution:

$$\frac{dA_k(t)}{dt} = (\gamma_{L,k} - \gamma_{d,k})\, A_k(t) \qquad \gamma_{L,k} \propto \frac{\partial f}{\partial I}$$

- Broadening is the platform that allows for momentum and energy exchange between particles and waves.
    - Heuristic broadening recipes historically used
    - Physics-based determination of the resonance broadening function was only very recently achieved
- The model is applicable for both single/isolated resonances and also multiple and overlapping ones.

Berk *et al*, *Nucl. Fusion* **35** 1661 (1995); Duarte, PhD thesis (2017); Gorelenkov, Duarte, Podestà and Berk, *Nucl. Fusion* **58** 082016 (2018)

First-principles analytical determination of the collisional resonance broadening

Starting with the kinetic equation:

$$\frac{\partial f}{\partial t} + \Omega \frac{\partial f}{\partial \varphi} + Re\left(\omega_b^2 e^{i\varphi}\right)\frac{\partial f}{\partial \Omega} = C\left[f, F_0\right] \begin{cases} \nu_K \left(F_0 - f\right) \\ \nu_{scatt}^3 \partial^2 \left(f - F_0\right)/\partial \Omega^2 \\ \text{(from collisions, turbulence,...)} \end{cases}$$

Near marginal stability, perturbation theory can be employed to show that a quasilinear transport equation naturally emerges, with the resonance functions given by

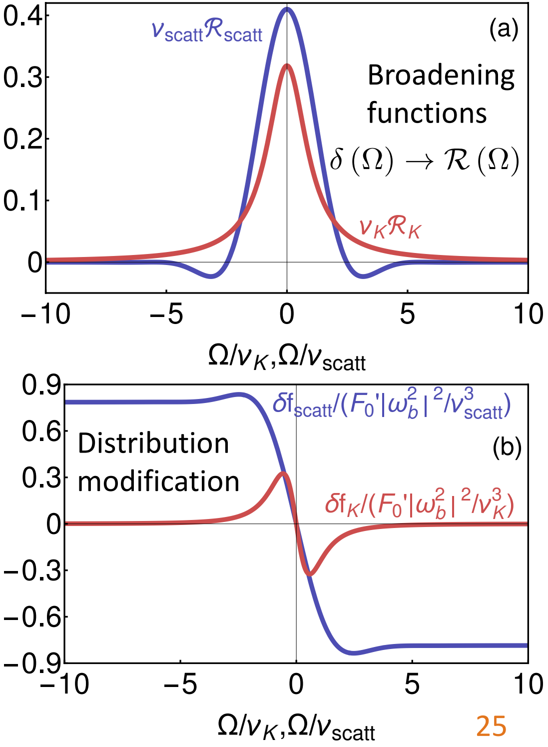
$$\mathcal{R}_K(\Omega) = \frac{1}{\pi \nu_K \left(1 + \Omega^2/\nu_K^2\right)} \qquad \mathcal{R}_{scatt}\left(\Omega\right) = \frac{1}{\pi \nu_{scatt}} \int_0^\infty ds\ \cos\left(\frac{\Omega s}{\nu_{scatt}}\right) e^{-s^3/3}$$

The use of the obtained resonance broadening functions implies that essential features of nonlinear theory, such as the growth rate and the saturation level, are automatically built into a reduced QL theory

Duarte *et al*, *Phys. Plasmas* **26**, 120701 (2019)

Blue curve: pitch-angle scattering

Red curve: Krook collisions



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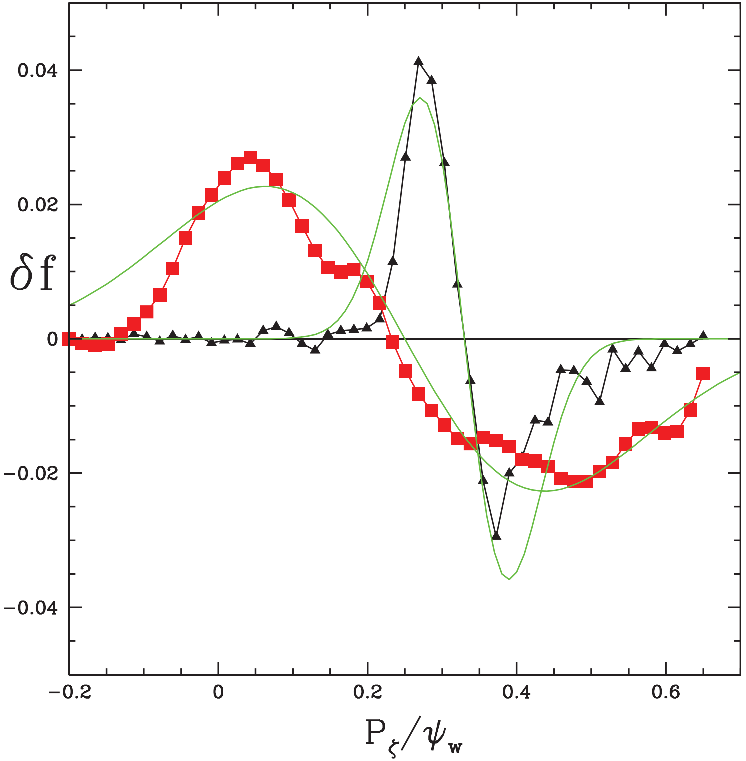
Verification of the analytical predictions against ORBIT simulations of Alfvénic resonances

Modification of the distribution function vs canonical toroidal momentum



Red and black: guiding-center ORBIT simulation results for two different levels of collisionality

Green: analytic fit

White, Duarte *et al*, *Phys. Plasmas* **26**, 032508 (2019)

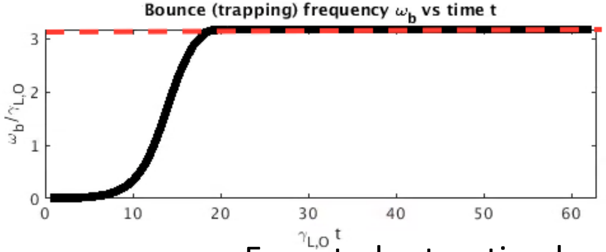
Quasilinear simulations replicate analytical predictions for the mode saturation amplitude from nonlinear theory

Definitions: initial linear growth rate γ_L, mode damping rate γ_d and trapping (bounce) frequency ω_b (proportional to square root of mode amplitude)

Collisionless case

- Undamped case[1]

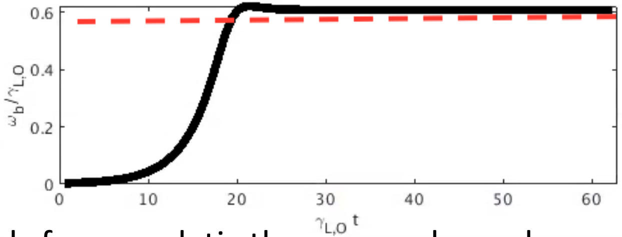
$$\omega_b \cong 3.2\gamma_L$$



Collisional cases

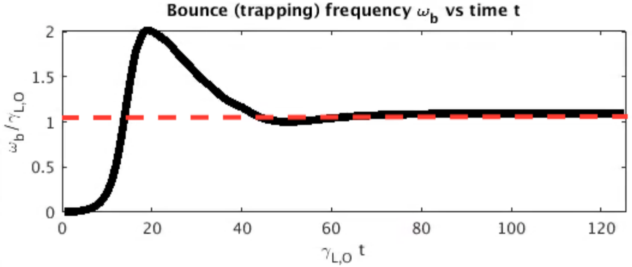
- Close to marginal stability[2]: $\nu_{\rm eff} \gg \omega_b$

$$\omega_b = 1.18\nu_{\rm eff}\left(\frac{\gamma_L-\gamma_d}{\gamma_d}\right)^{1/4}$$



- Far from marginal stability[3]: $\omega_b \gg \nu_{\rm eff}$

$$\omega_b = 1.2\nu_{\rm eff}\left(\frac{\gamma_L-\gamma_d}{\gamma_d}\right)^{1/3}$$



Expected saturation levels from analytic theory are shown by – – –

[1]Fried *et al*, Report No. PPG-93 (UCLA,1972); [2]Berk et al. Plasma Phys. Rep, 23(9), 1997; [3]Berk and Breizman. Phys. Fluids B, 2(9), 1990

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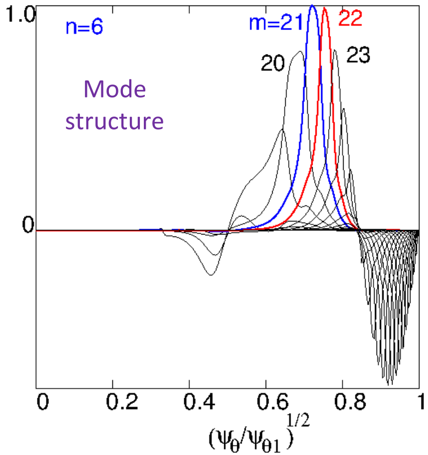
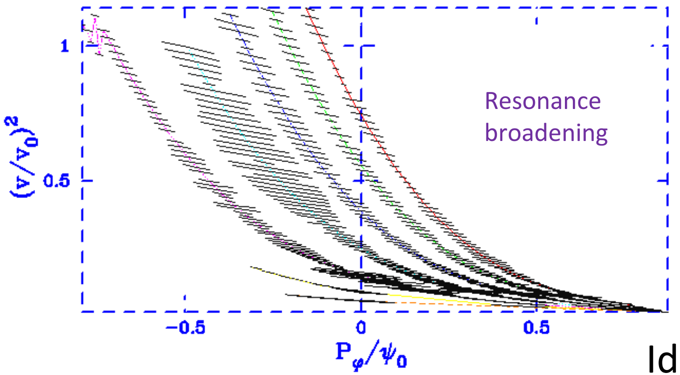
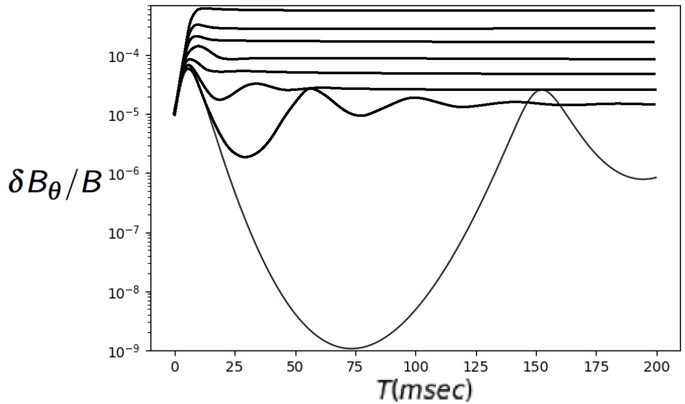
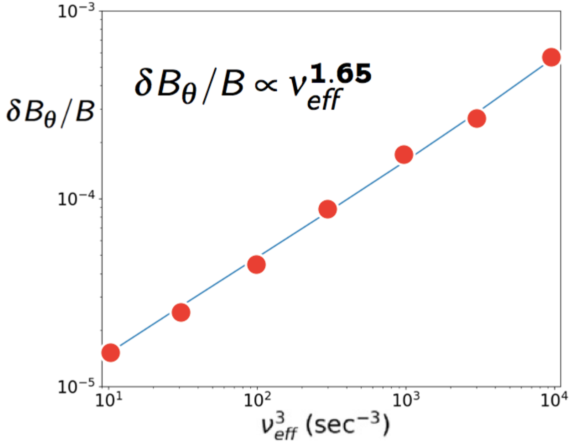
Verification of RBQ runs for the collisional evolution of an Alfvénic wave

DIII-D discharge 153072









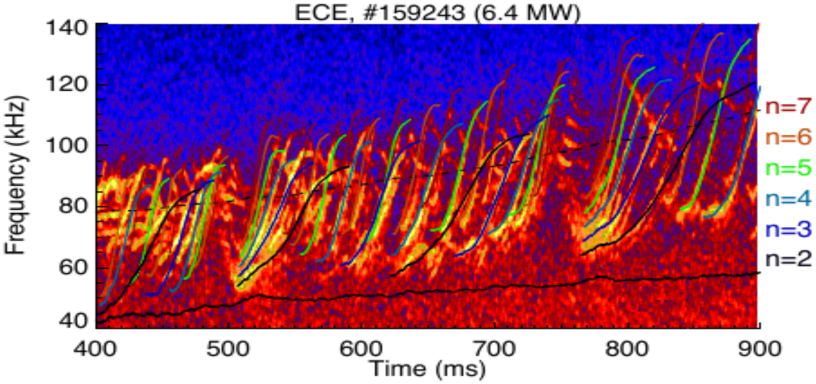
Idealized bump-on-tail scaling (obtained using Dirichlet boundary conditions on both ends of the domain):

$$\delta B_\theta / B = \nu_{eff}^2$$

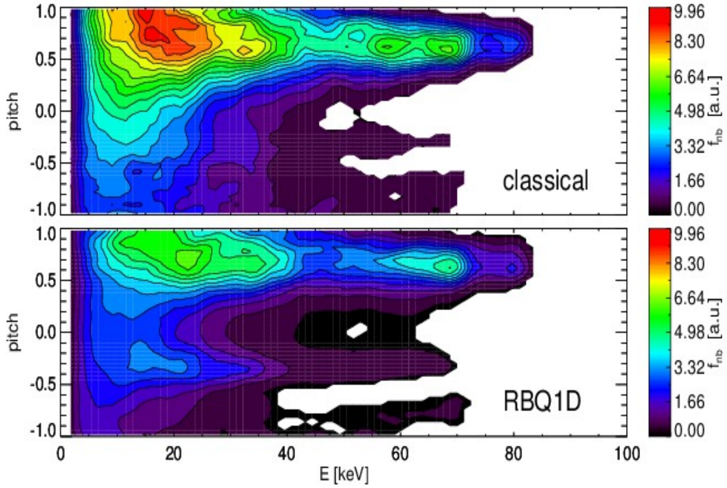
Gorelenkov, Duarte *et al*, *Phys. Plasmas* **26**, 072507 (2019)

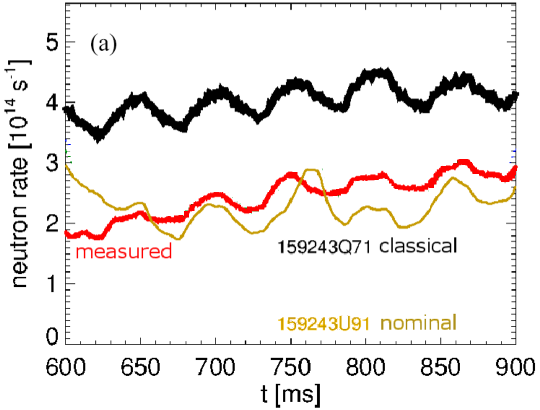
RBQ is interfaced with TRANSP: multi-mode case

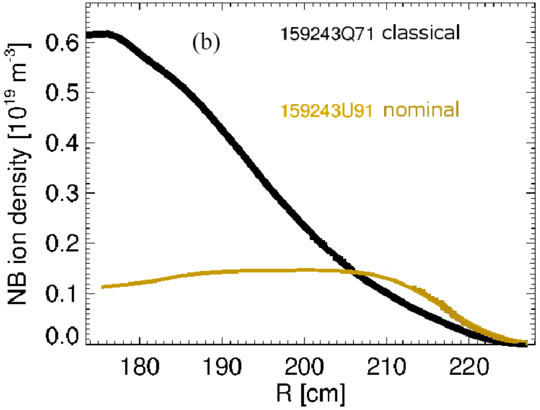
11 unstable Alfvénic modes at 805ms



Distribution at 805ms - DIII-D shot 159243







Gorelenkov, Duarte *et al*, *Phys. Plasmas* **26**, 072507 (2019)

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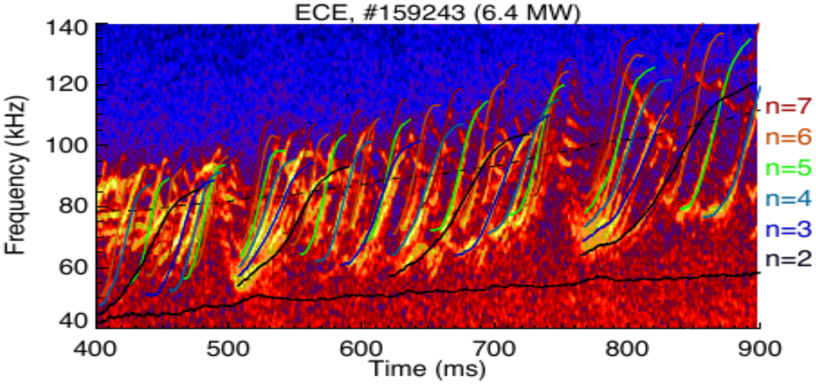

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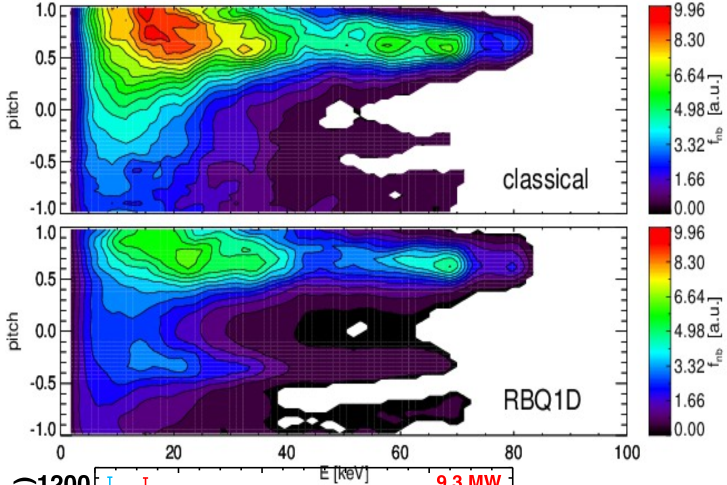
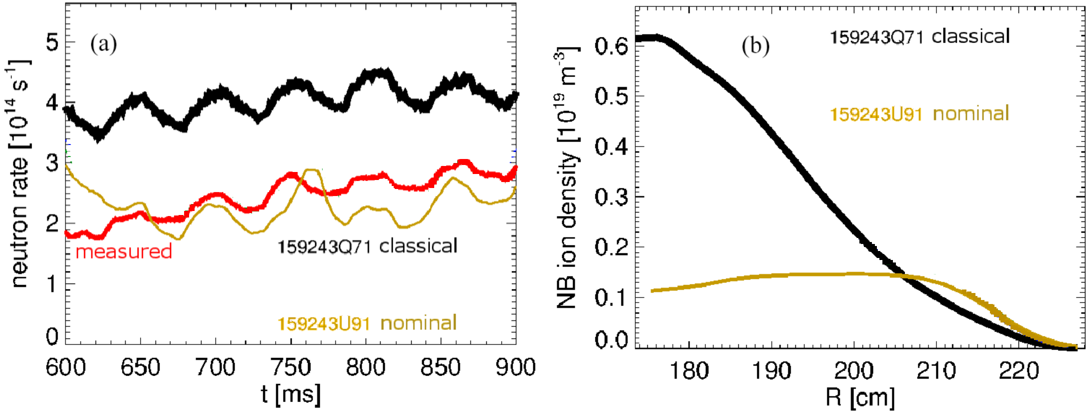
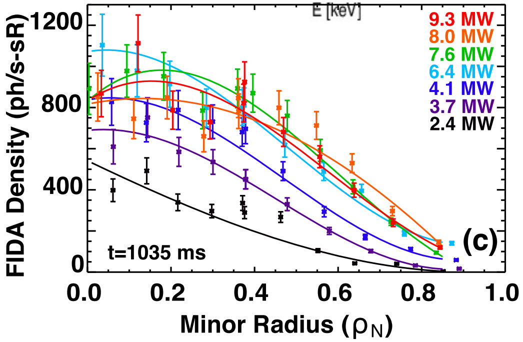






Gorelenkov, Duarte *et al*, *Phys. Plasmas* **26**, 072507 (2019)

Collins et al, Phys. Rev. Lett. **116**, 095001 (2016)

Summary

Prediction of Alfvénic spectral character

- Chirping criterion determines when reduced models, such as quasilinear, can be applied
  - also explains why chirping is more common in spherical tokamaks
- DIII-D negative triangularity experiments confirm prediction that chirping is more prevalent when turbulence is reduced
- The criterion ITER scenarios are predicted to be near boundary between chirping and fixed-frequency behavior

Quasilinear modeling

- The Resonance Broadened Quasilinear (RBQ) model exactly preserves key properties of the full nonlinear system
  - Extensively verified and validated
- Integration in TRANSP enables predictions in realistic scenarios
  - Captures hollow fast ion profiles observed in DIII-D discharges

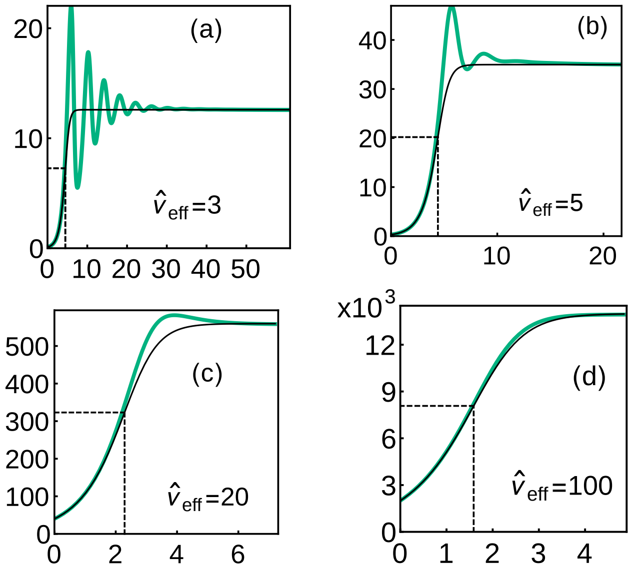
Ongoing study on the evolution of toroidal Alfvén modes in DIII-D, with a changing drive due to beam blips

Amplitude evolution for a single mode near marginal stability:

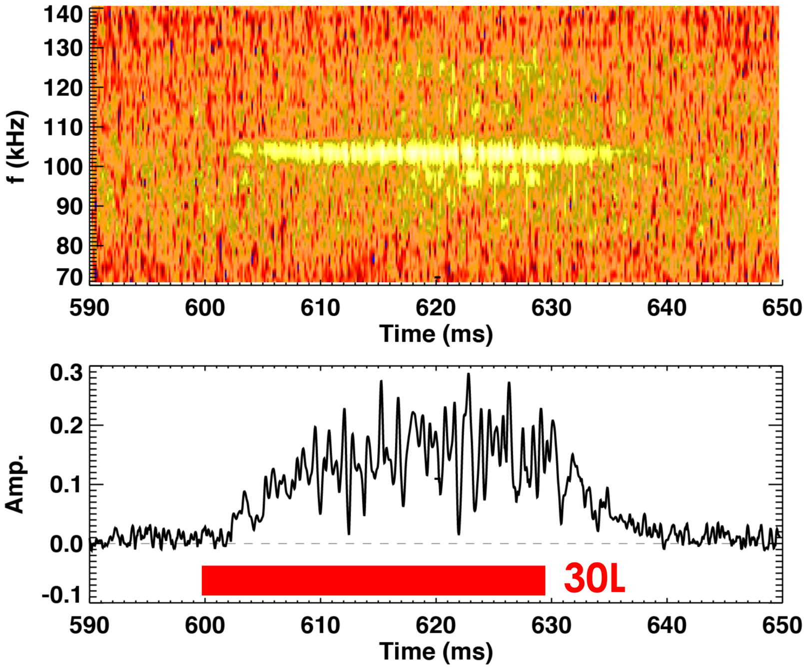
$$A(t) = \frac{A(0)e^t}{\sqrt{1 - gA^2(0)\,(1 - e^{2t})}}$$

Duarte *et al*, *Nucl. Fusion* **59** 044003 (2019)

Amplitude A vs time t for nonlinear simulation (green) and the analytic formula (black)



DIII-D shot 176523



The INPA signal is being used as a proxy for the mode drive

[Duarte and Van Zeeland, in progress]

Thank you

Backup slides

First-principle analytical determination of the collisional resonance broadening – part I

Start with the kinetic equation: $\frac{\partial f}{\partial t}+\Omega\frac{\partial f}{\partial\varphi}+Re\left(\omega_b^2 e^{i\varphi}\right)\frac{\partial f}{\partial\Omega}=C\left[f,F_0\right]$

$\left\{\begin{array}{l}\nu_K\left(F_0-f\right)\\ \nu_{scatt}^3\partial^2\left(f-F_0\right)/\partial\Omega^2\end{array}\right.$
(from collisions, turbulence,…)

Periodicity over the canonical angle allows the distribution to be written as a Fourier series:

$$f\left(\varphi,\Omega,t\right)=F_0\left(\Omega\right)+f_0\left(\Omega,t\right)+\sum_{n=1}^{\infty}\left(f_n\left(\Omega,t\right)e^{in\varphi}+c.c.\right)$$

Near marginal stability, a perturbation theory can be developed in orders of $\omega_b^2/\nu_{K,scatt}^2$ which leads to the ordering $\left|F_0'\right|\gg\left|f_1'^{(1)}\right|\gg\left|f_0'^{(2)}\right|,\left|f_2'^{(2)}\right|$. When memory effects are weak, i.e., $\nu_{K,scatt}/\left(\gamma_{L,0}-\gamma_d\right)\gg 1$,

$$f_1=\frac{\omega_b^2 F_0'}{2\left(i\Omega+\nu_K\right)}\qquad\qquad \frac{\partial f_0}{\partial t}+\frac{1}{2}\left(\omega_b^2\left[f_1'\right]^*+\omega_b^{2*}f_1'\right)=-\nu_K f_0$$

Self-consistent formulation of collisional quasilinear transport theory near threshold

$$\frac{\partial f(\Omega,t)}{\partial t} - \frac{\pi}{2}\frac{\partial}{\partial\Omega}\left[\left|\omega_b^2\right|^2 \mathcal{R}(\Omega)\frac{\partial f(\Omega,t)}{\partial\Omega}\right] = C[f, F_0]$$

$$\gamma_L(t) = \frac{\pi}{4}\int_{-\infty}^{\infty} d\Omega \mathcal{R}\frac{\partial f(\Omega,t)}{\partial\Omega} \qquad d\left|\omega_b^2\right|^2/dt = 2\left(\gamma_L(t) - \gamma_d\right)\left|\omega_b^2\right|^2$$

- A QL theory naturally emerges when considering kinetic theory near threshold when collisions occur at a time scale faster than the phase mixing time scale.

- The QL plasma system automatically replicates the nonlinear growth rate and the wave saturation levels calculated from full kinetic theory near marginality, with a rather complex time-delayed integro-differential equation (Berk, Breizman and Pekker, *Phys. Rev. Lett.* 1996) $|\omega_{b,sat}| = 8^{1/4}\left(1 - \gamma_d/\gamma_{L,0}\right)^{1/4}\nu_K$

$$\frac{d}{dt}\,\omega_B^2 = (\gamma_L - \gamma_d)\omega_B^2(t) - \frac{\gamma_L}{2}\int_{t/2}^{t} dt'\,(t-t')^2\omega_B^2(t')\int_{t-t'}^{t'} dt_1\;\exp[-\nu(2t-t'-t_1)]\,\omega_B^2(t_1)\omega_B^2(t'+t_1-t)$$

Determining the parametric dependencies of the broadening from single mode saturation levels

The broadening is assumed with the parametric form $\Delta\Omega = a\omega_b + b\nu_{eff}$ where the coefficients a and b are determined in order to enforce QL theory to replicate known nonlinear saturation levels:

Limit near marginal stability[3] $\rightarrow$ $b = 3.1$

$$\omega_b = 1.18\nu_{eff}\left(\frac{\gamma_{L0}-\gamma_d}{\gamma_{L0}}\right)^{1/4}$$

Limit far from marginal stability[4] $\rightarrow$ $a = 2.7$

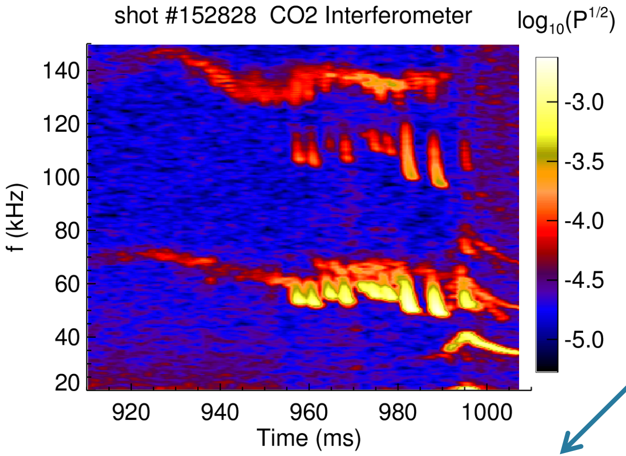
$$\omega_b = 1.2\nu_{eff}\left(\frac{\gamma_{L0}-\gamma_d}{\gamma_d}\right)^{1/3}$$

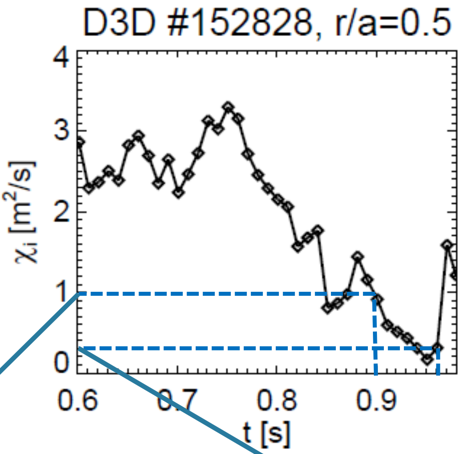
Top-hat, square resonance function heuristically assumed

Resonance-broadened quasilinear formalism can cope with both situations of isolated and overlapping modes

[3]H. L. Berk et al. Plasma Phys. Rep, 23(9), 1997 [4]H. L. Berk and B. N. Breizman. Phys. Fluids B, 2(9), 1990

Characterization of a rarely observed chirping mode in DIII-D



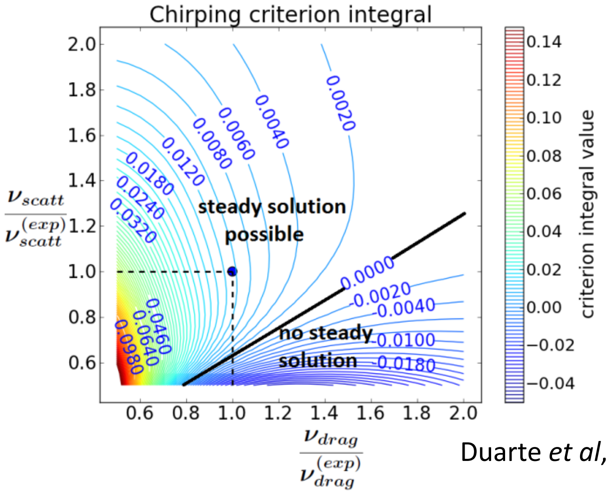


TRANSP run
Diffusivity
vs
time

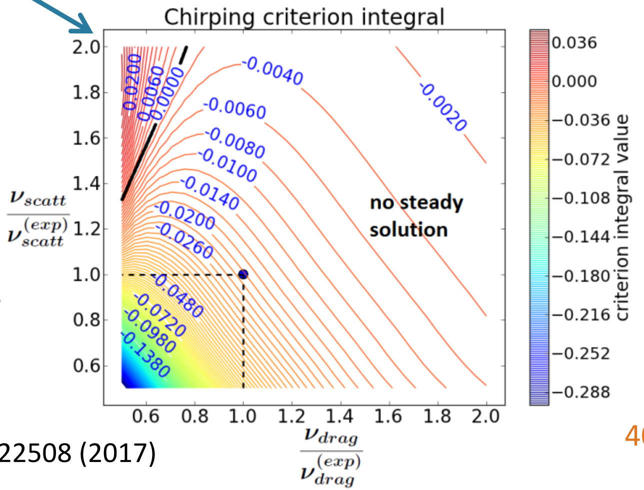
Diffusivity drop due to L→H mode transition

Strong rotation shear was observed

Before chirping, at 900ms: D~1m^2/s



During chirping, at 960ms: D~0.2m^2/s



Duarte *et al*, *Phys. Plasmas* **24**, 122508 (2017)

A criterion for the likelihood of chirping onset in tokamaks

Starting point: lowest-order nonlinear correction to the evolution of mode amplitude A:

$$\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{scatt}^3 \tau^2 (2\tau/3+\tau_1) + i\nu_{drag}^2 \tau(\tau+\tau_1)} A(t-\tau-\tau_1) A^*(t-2\tau-\tau_1)$$

stabilizing

destabilizing (makes integral sign flip)

Berk, Breizman and Pekker, PRL 1996
Lilley, Breizman and Sharapov, PRL 2009

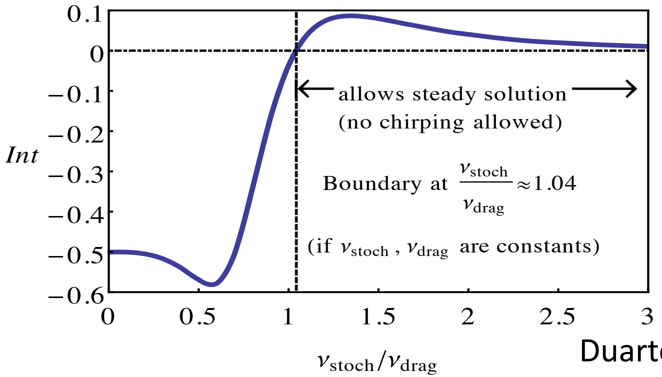
Blow up of A in a finite time-> system enters a strong nonlinear phase (chirping likely)

Chirping criterion:

$$Crt = \frac{1}{N} \sum_{j,\sigma_\parallel} \int \mathrm{d}P_\varphi \int \mathrm{d}\mu \frac{|V_j|^4}{\omega_\theta \nu_{\mathrm{drag}}^4} \left| \frac{\partial \Omega_j}{\partial I} \right| \frac{\partial f}{\partial I} Int$$

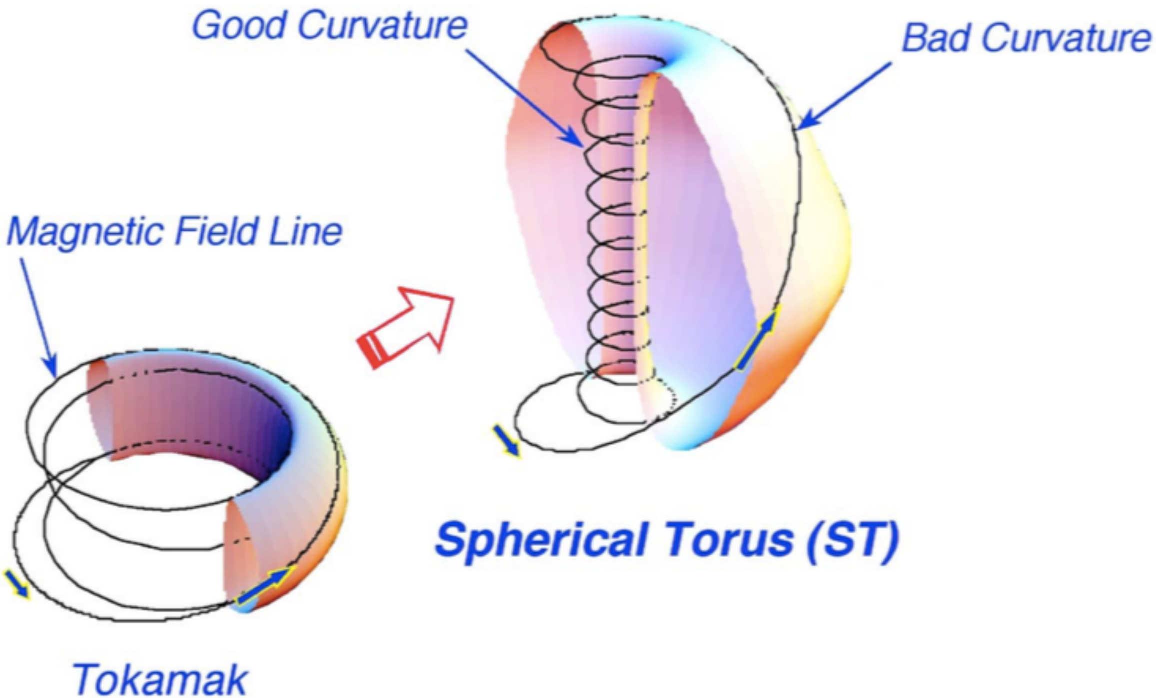
- >0: fixed-frequency solution likely
- <0: chirping likely to occur

(nonlinear prediction from linear physics elements->incorporated into NOVA-K code)



The criterion ($Crt \gtrless 0$) predicts that micro-turbulence should be key in determining the likely nonlinear character of a mode, e.g., fixed-frequency or chirping

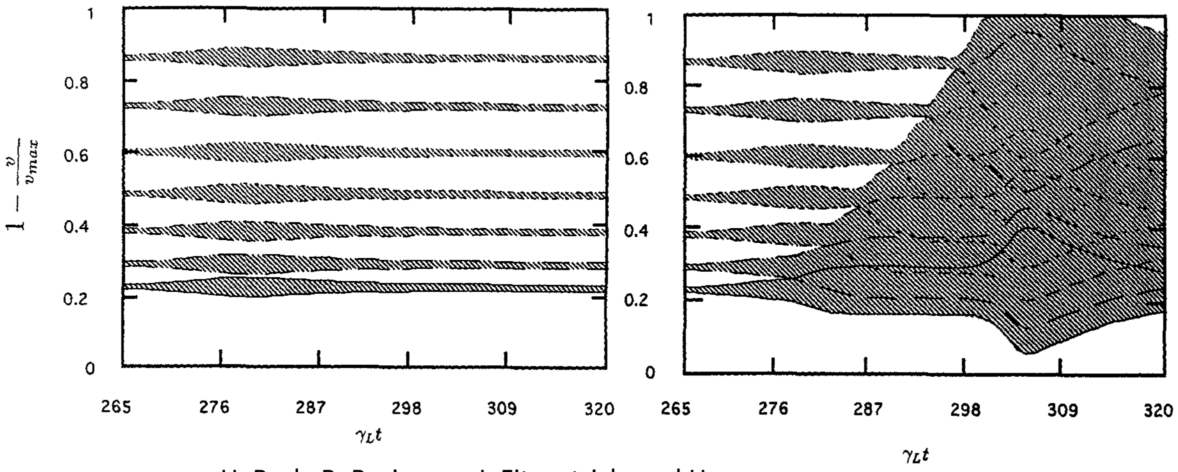
Duarte *et al*, *Nucl. Fusion* **57** 054001 (2017).



Vinícius Duarte, "Prediction of the nonlinear character of Alfvénic instabilities

The overlapping of resonances lead to losses due to global diffusion

- Broadened QL theory is designed to address both regimes of isolated and overlapping resonances
  - the fast ion distribution function relaxes while self-consistently evolving the amplitude of modes



H. Berk, B. Breizman, J. Fitzpatrick, and H. Wong, Nucl. Fusion 35, 1661 (1995).

Correction to the diffusion coefficient: the inclusion of electrostatic microturbulence

- Microturbulence can well exceed pitch-angle scattering at the resonance[1]
- From GTC gyrokinetic simulations for passing particles:[2]

$$D_{EP}\left(E_{EP}\right) \approx D_{th,i}\frac{5T_e}{E_{EP}}$$

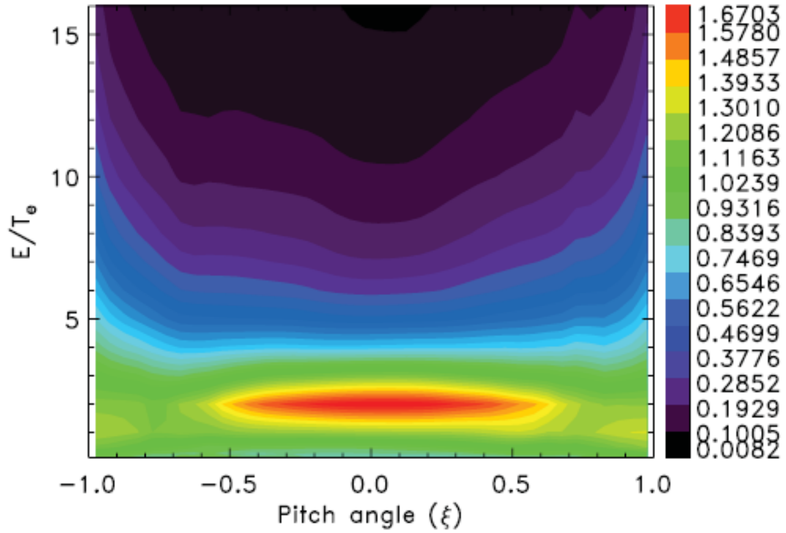
- As pitch-angle scattering, microturbulence acts to destroy phase-space holes and clumps
- Unlike DIII-D and TFTR, transport in NSTX in mostly neoclassical
- Complex interplay between gyroaveraging, field anisotropy and poloidal drift effects leads to non-zero EP diffusivity[3]

[1]Lang and Fu, PoP 2011

[2]Zhang, Lin and Chen, PRL 2008

[3]Estrada-Mila et al, PoP 2005

Ratio of fast ion diffusivity to thermal ion diffusivity[2]

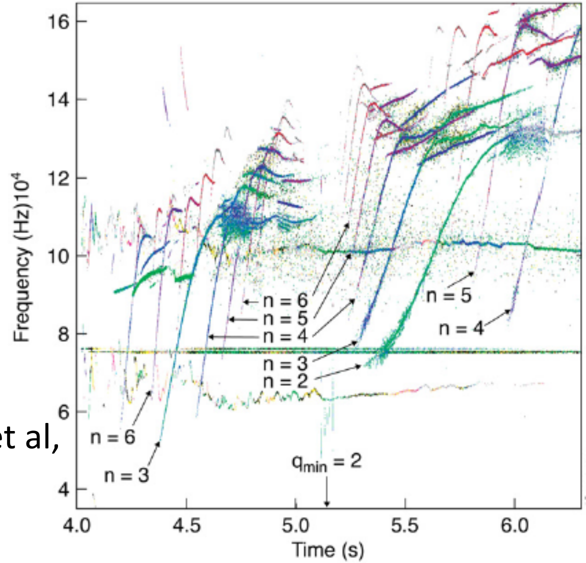


Pueschel et al, NF 2012 gives similar microturbulence levels

Two types of frequency shift observed experimentally

Frequency sweeping

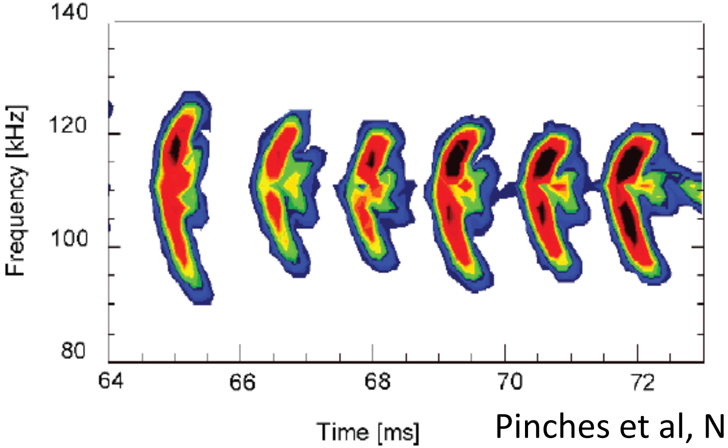
- frequency shift due to time-dependent background equilibrium
- MHD eigenmode
- timescale: ~100ms



Sharapov et al, PoP 2006

Frequency chirping

- frequency shift due to trapped particles
- does not exist without resonant particles
- timescale: ~1ms



Pinches et al, NF 2004

Nonlinear vs Quasilinear approach

- Requires particles to remember their phases from one trapping bounce to another;
- Full kinetic approach necessary;
- Entropy is conserved in the absence of collisions;
- **Convective transport.**

- Requires particles to forget their phase (via collisions, turbulence or mode overlap);
- Assumes that the **modes remain linear** (therefore NOVA is suited) while the distribution function is allowed to slowly evolve nonlinearly in time;
- Entropy increases due to particle memory loss (due to phase averaging).
- **Diffusive transport.**