Real-Time Stability Detection of Multiple MHD Modes (3D MHD Spectroscopy)

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Real-Time Detection of Magnetohydrodynamic (MHD) Instabilities Is Important for Predicting, Controlling and Avoiding Severe Plasma Disruption

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- **Active detection of MHD mode and stability through external 3D coils and 3D plasma response is extensively studied**
  - Single-mode MHD spectroscopy [H. Reimerdes PRL 93 135002 (2004)]
  - Multi-mode plasma response [C. Paz-Soldan PRL 114 105001 (2015)]
  - Multi-mode 3D MHD spectroscopy (**Frequency Domain Method**) [Z.R. Wang NF 59 024001 (2019)]

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- **Real-time monitoring multi-modes stabilities in stable plasma is essential to predict and control plasma disruption**
  - Time Domain Method (TDM): A subspace system identification [P.V. Overschee SPC (1994), M.D. Boyer NF 55 053033 (2015)] process is adopted to develop a linear state-space response model for detecting the stability of each MHD mode

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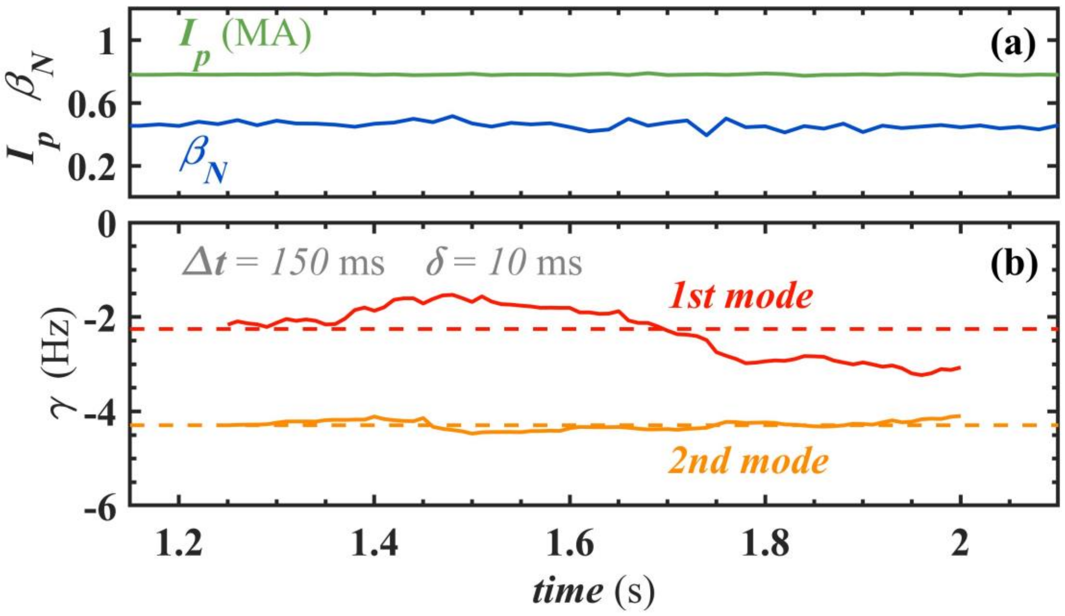

- **In DIII-D experiments, successfully detecting variation of multi-mode instability with time by TDM.**
- **During discharge, two dominant stable modes are observed by short-time fitting.**

- **Real-time monitoring multi-modes stabilities in stable plasma is essential to predict and control plasma disruption**
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Outline

- Method of 3D MHD Spectroscopy
- Detection of Multi-MHD Modes' Stabilities (Post-Processing)
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Time Domain Method Developed by Combining FDM and SSI

Frequency domain method (FDM)

[Z.R. Wang NF 59 024001 (2019)]

Represent dominant eigenmodes

$$P_j(f,\Delta\phi) = \frac{\delta B}{I_{up}} = \sum_{i=1}^{N} \frac{a_i^j + b_i^j e^{-i\Delta\phi}(I_{low}/I_{up})}{i2\pi f - \gamma_i} + c^j + d^j e^{-i\Delta\phi}(I_{low}/I_{up})$$

- Eigenvalue of ith mode is same at any sensor
- Scanning phasing and frequency to observe more eigenmode behaviors

Subspace system identification (SSI) process was adopted to develop a linear state-space response model of the system.

[P.V. Overschee SPC (1994), M.D. Boyer NF 55 053033 (2015)]

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Time domain method (TDM)

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

$$\delta x_{k+1} = \bar{A}\delta x_k + \bar{B}u_{fb,k}$$

$$\delta y_k = C\delta x_k + \mathrm{D}u_{fb,k}$$

Sensor Measurement — System States — Coil Current

Eigenvalue matrix

$$\mathbf{A} = \mathbf{Q\Lambda Q^{-1}}$$

$$\mathbf{\Lambda} = \begin{pmatrix} \gamma_1 & & \\ & \ddots & \\ & & \gamma_N \end{pmatrix}_{N\times N}$$

- **Discretized system matrices need to be extracted by fitting signal**
  - Eigenvalue matrix A should have a connection to the eigenmodes in the FDM
  - **No initial guess required**

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Fitting System Dynamic Model to Extract A B C and D Matrices

$$\delta x_{k+1} = \bar{A}\delta x_k + \bar{B}u_{fb,k}$$

$$\delta y_k = C\delta x_k + \mathrm{D}u_{fb,k}$$

Estimated from U and Y

Mathematical Operation

$$\begin{pmatrix} X_{i+1}^d \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X_i^d \\ U_{i|i} \end{pmatrix}$$

3D Coil signal

$$U = \begin{bmatrix} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+2} & \cdots & u_{2i+j-2} \end{bmatrix}$$

3D magnetic sensor measurement

$$Y = \begin{bmatrix} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & y_{2i+2} & \cdots & y_{2i+j-2} \end{bmatrix}$$

- Extract system state sequence $\boldsymbol{X_i}$ directly from experiment data.
- $\boldsymbol{U_i}$ and $\boldsymbol{Y_i}$ can be constructed by input and output data.
- Use least square to fit the system matrices **A B C D**. Then calculate eigenvalue of matrix **A**.

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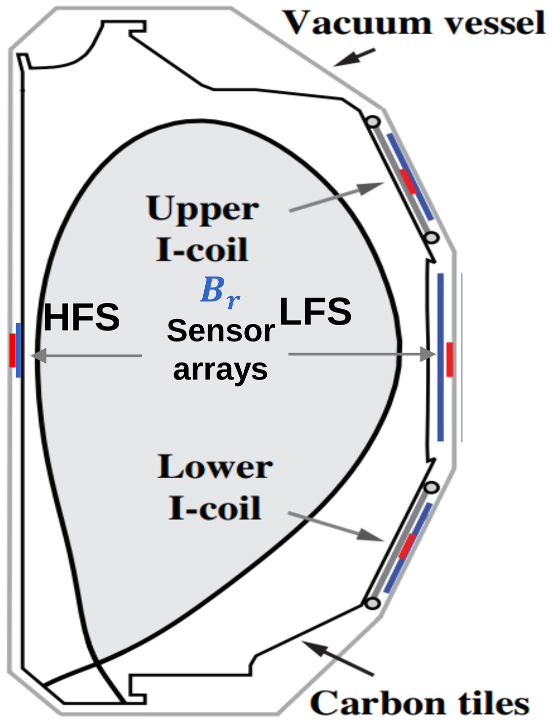
Geometry of Actuators and Sensors in Tokamak Devices

- **Time Domain Method (TDM) is applied in DIII-D**
  – Coil current generated by multi-coils as input
  – Magnetic perturbation picked up by multi-arrays of sensors as output

➢ Two sets of current coils are applied. (Upper I-coil and Lower I-coil)

➢ Two arrays of sensor are applied. (Low field side and High field side)



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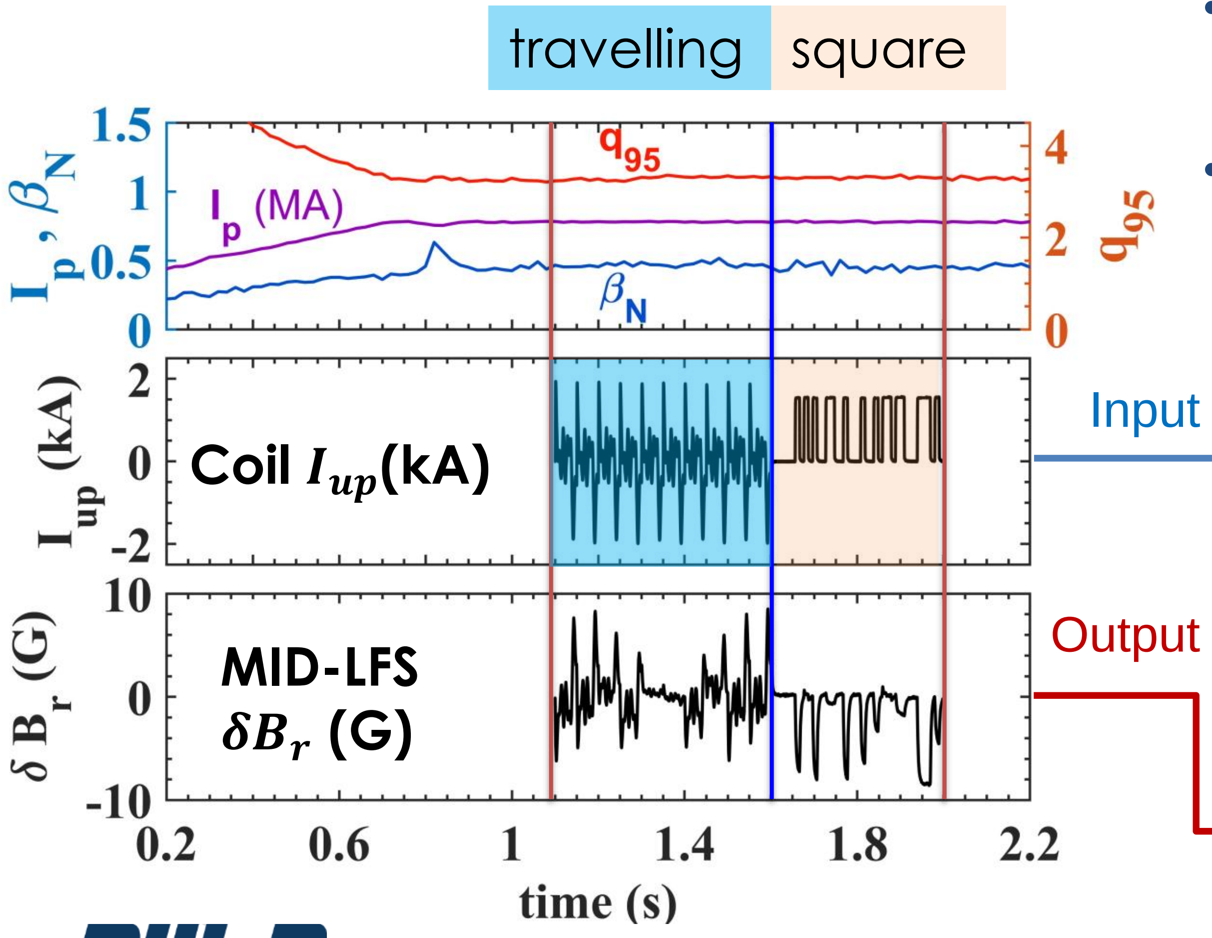
Use Measured n=1 3D Plasma Response and Applied 3D Coil Current to Fit TDM Model

DIII-D Shot No. 178583, $\beta_N \sim 0.5$, $q_{95} \sim 3.3$



- **Keep equilibrium parameters stable with little change**
- **Two waveforms are included**
  - Travelling wave: scan coil phasing and frequency (0 90 180 270 deg) X (±10 ±35 ±60 ±110 Hz)
  - One time interval – one phasing, several frequency with several periods.
  - Square wave: random time interval, same phasing in one interval. (0 45 90 135 180 225 270 315 deg)

$$\delta x_{k+1} = \bar{A}\delta x_k + \bar{B}u_{fb,k}$$
$$\delta y_k = C\delta x_k + \mathrm{D}u_{fb,k}$$

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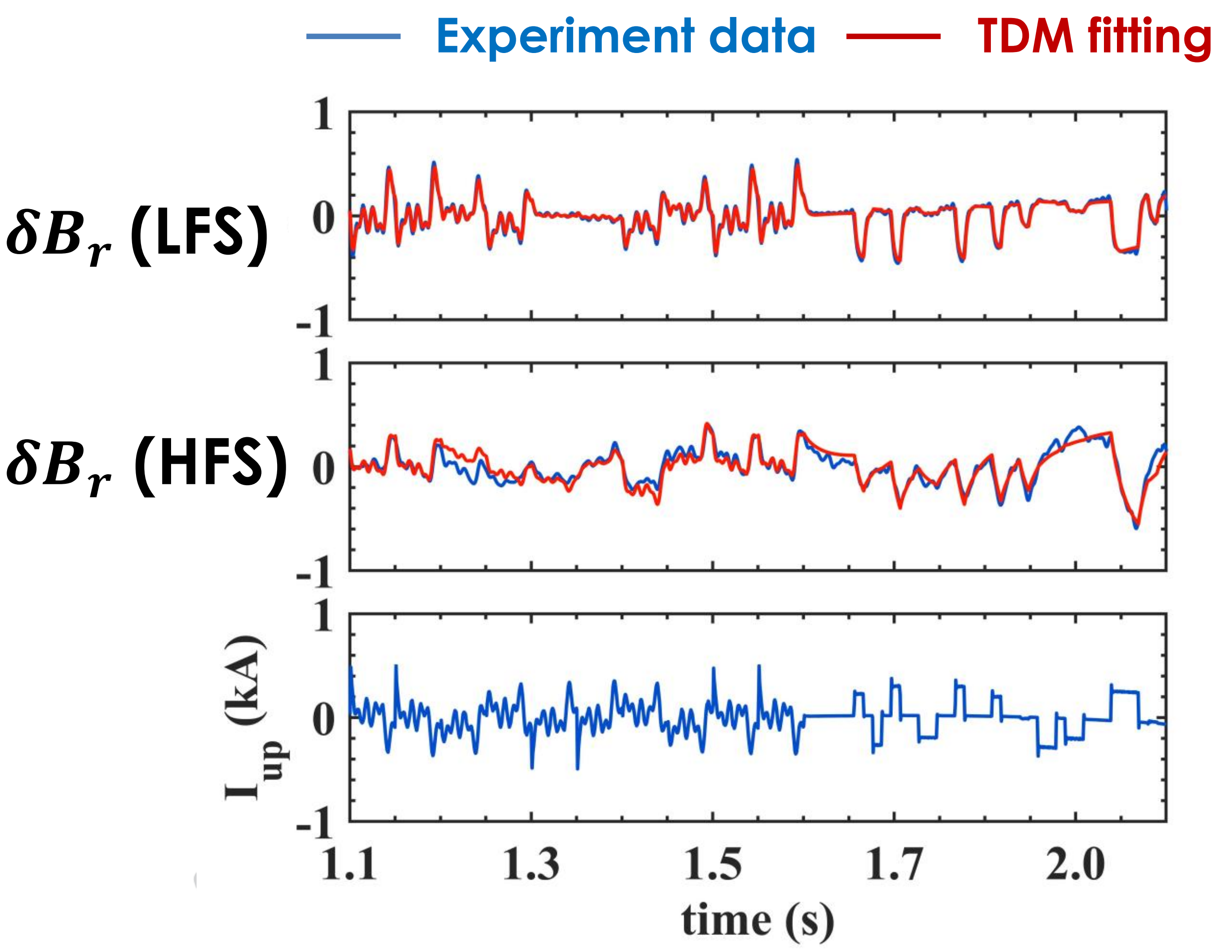
Fitting of TDM Shows a Good Agreement with DIII-D Experiment Data

Time domain method fitting model in DIII-D experiments (Shot No. 178581-178622):



- **Fit signals measured by 3D sensors located at LFS and HFS**
- **Two dominant modes are observed**

γ_1= -62.22-1.17i Hz Least stable

γ_2= -277.13-1.55i Hz Second least

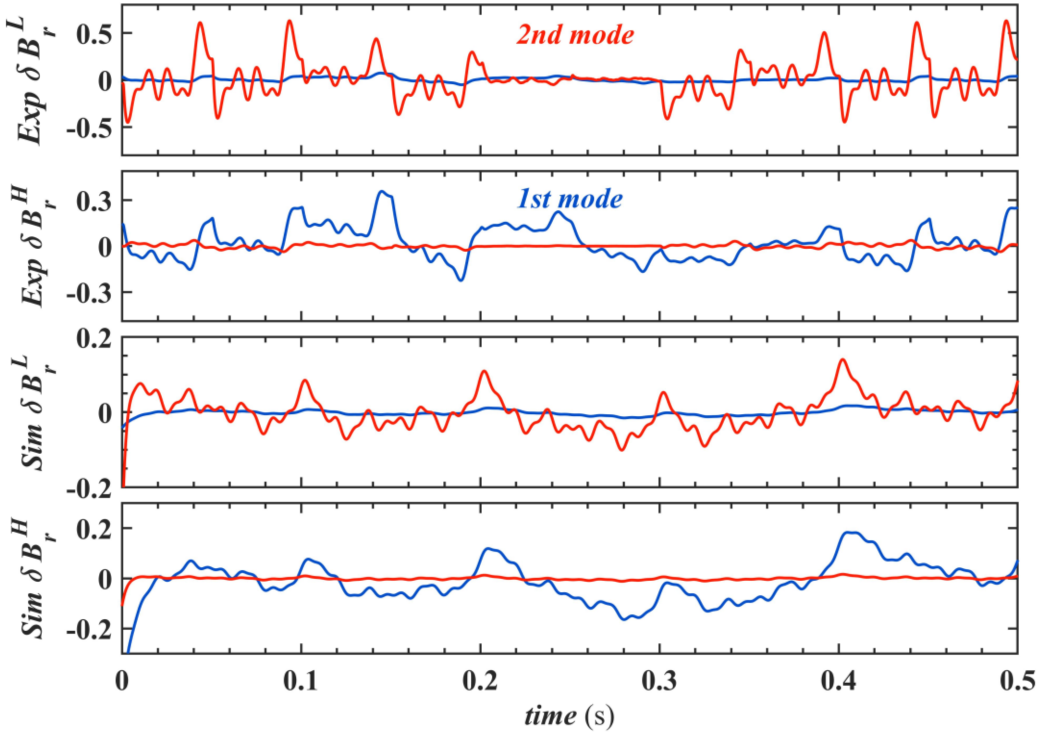
FDM results :

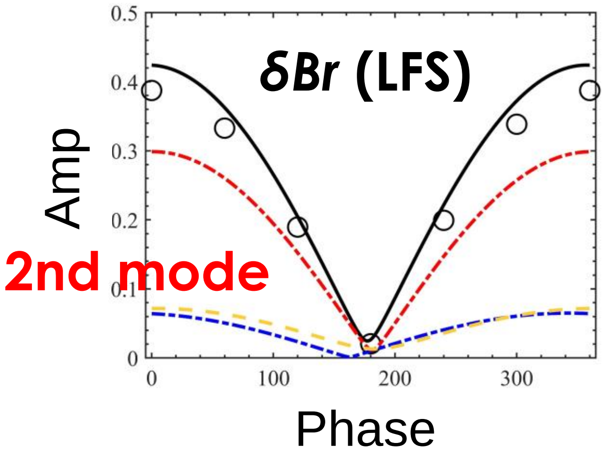
γ_1=-68.57-0.23i Hz γ_2=-274.10+10.1i Hz

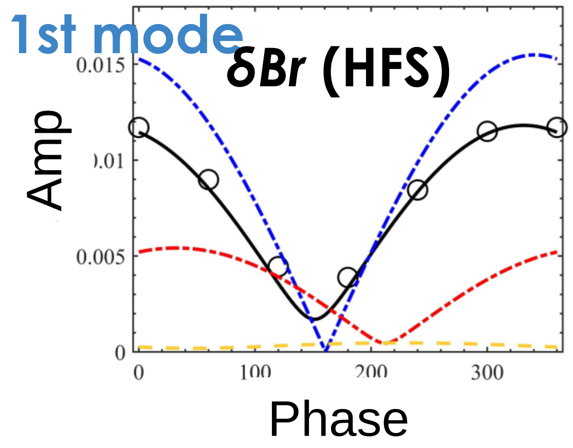
Fitting Simulation Data by MARS Shows Good Agreement btw Two Methods

Running MARS code by adopting experiment equilibrium, simulation data are obtained.

	TDM (Sim)	FDM (Sim)	TDM (Exp)	FDM (Exp)
γ_1	-62.56+0.01i Hz	-64.07+0.37i Hz	-62.22-1.17i Hz	-68.57-0.23i Hz
γ_2	-299.92-0.003i Hz	-265.69+30.65i Hz	-277.13-1.55i Hz	-274.10+10.1i Hz



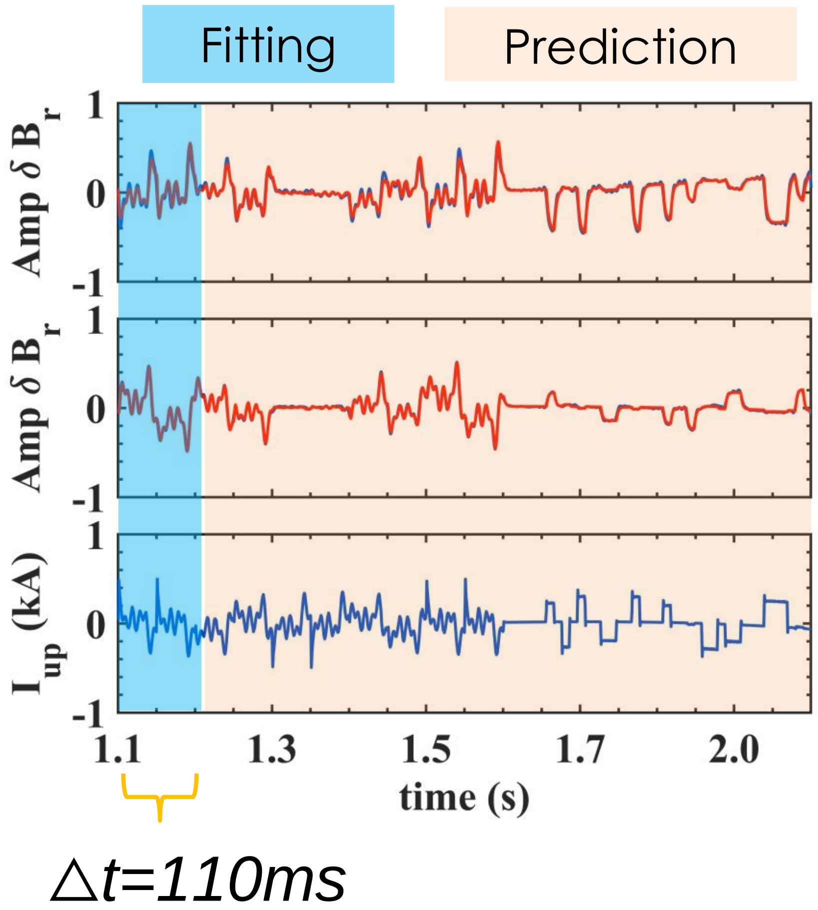


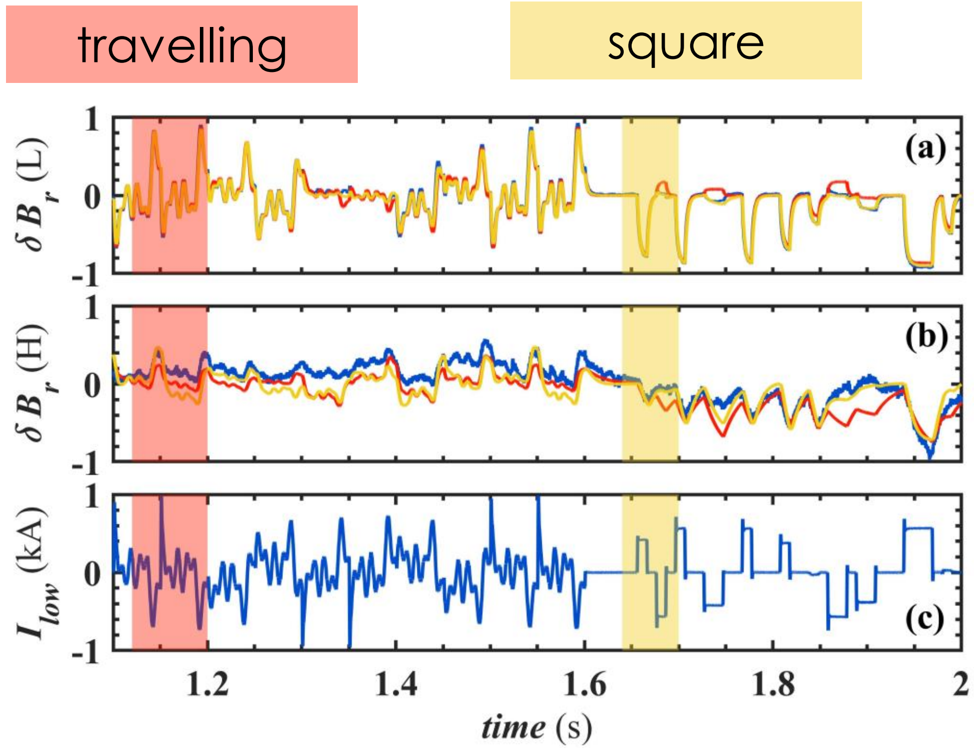


- ➢ Methods show good agreement with each other and with experiment results.
- ➢ Least stable mode dominate at HFS, while second mode dominate at LFS

Predicted Plasma Response Based on a Short Time Fitting Shows a Good Agreement with DIII-D Experiment Data

Fitting model by using short time interval in DIII-D



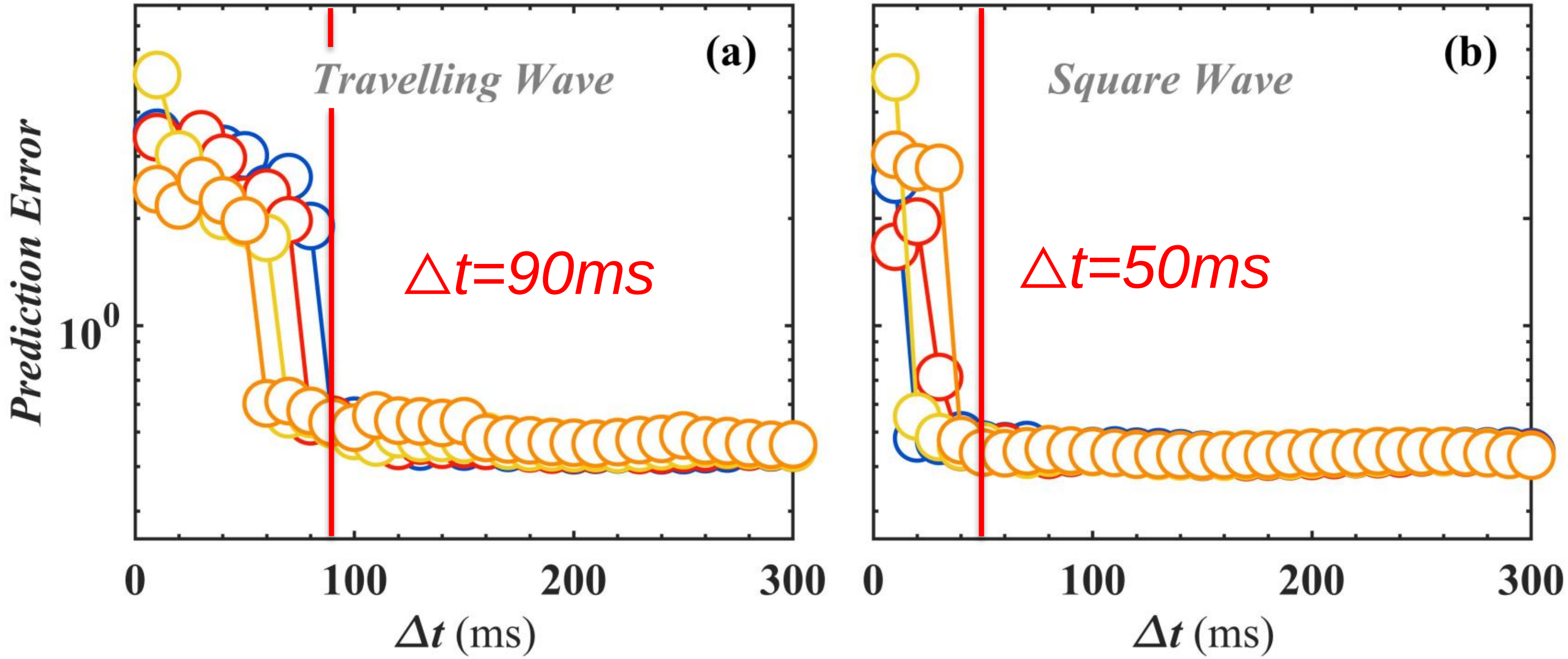


- Long time prediction of plasma response (low error) is successfully achieved by fitting 3D sensor signals in a small time window (Δt=110ms)

Square Wave has shorter fitting window than Travelling Wave

Fitting model by adjusting time window to find shortest window.

$$\text{Prediction Error} = \frac{1}{N}\sum_{i}^{N}\left(y_p(i) - y_o(i)\right)^2$$



Fitting time window

- Shortest window for travelling wave and square wave are 90ms and 50ms, respectively.
- As long as enough information are included, travelling wave and square wave both can be applied to extract a good result.

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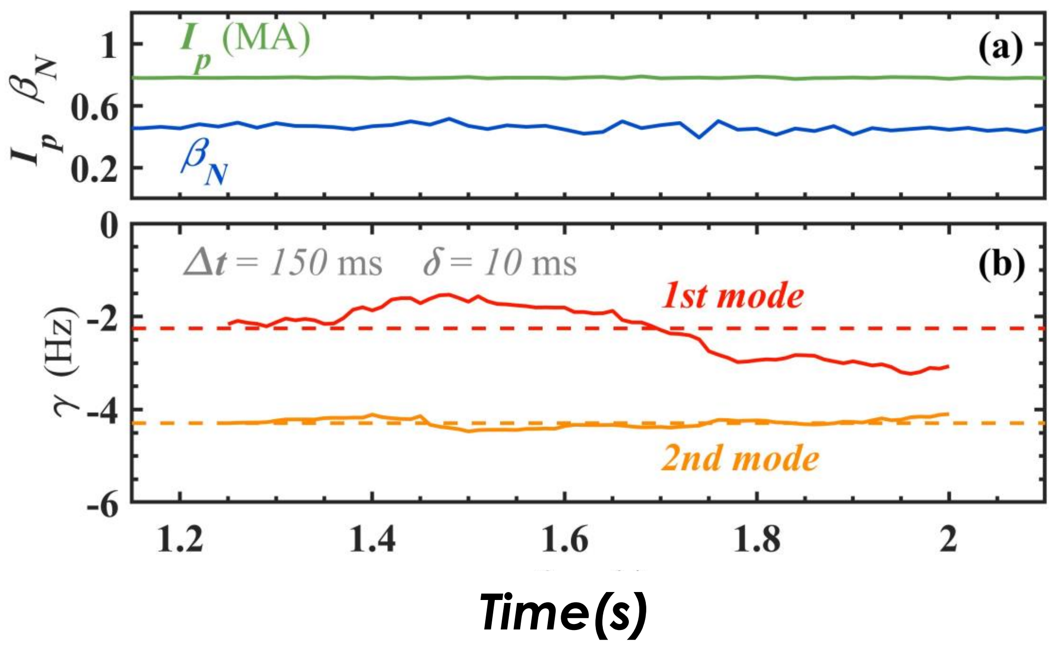
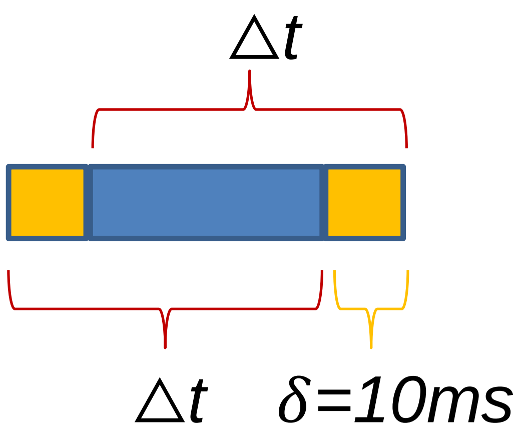
Monitoring Time Evolution of Multi-Mode Instability by TDM in DIII-D Tokamak

Streaming analysis by TDM are tested in DIII-D experiments (Shot No. 178583):

Two dominant modes are observed





- ➢ **Δt : Fitting time window**
- ➢ **δ : Time interval updating eigen value (10ms or faster)**

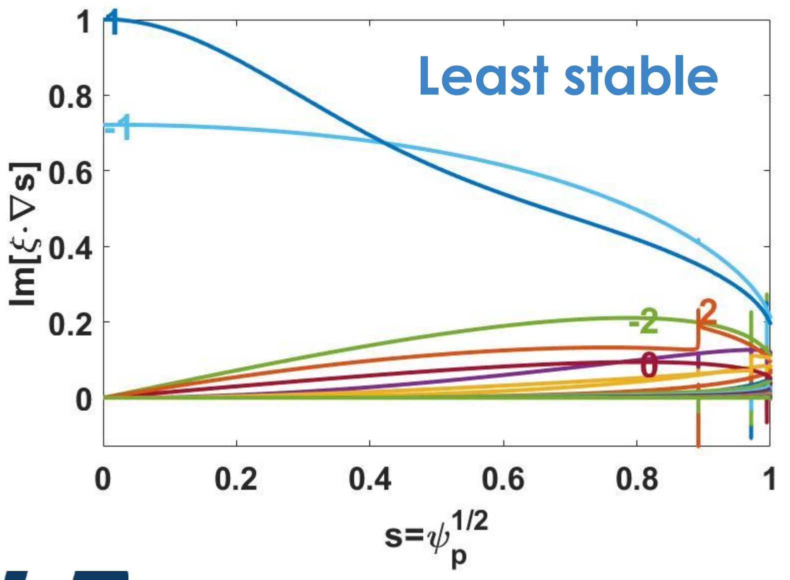
- ➢ Eigenvalue approaching 0 means eigenmode becomes unstable.
- ➢ The 2 dominant modes only fluctuate around a constant value as a result of steady equilibrium.
- ➢ Changing equilibrium parameters may be applied in the future.

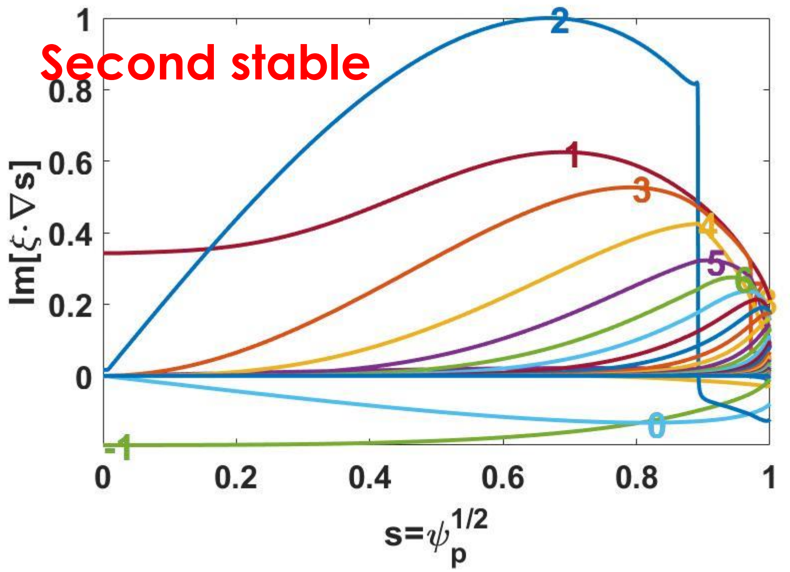
MARS-F Found Two Stable Eigenmodes by Using Experimentally Extracted Eigenvalues as Initial Guess

Two eigenmodes both have global structure. Least stable mode has more core structure, second stable mode has peeling structure with more edge perturbation

	Experiment	MARS-F
γ_1	-62.22-1.17i Hz	-68.91-0.001i Hz
γ_2	-277.13-1.55i Hz	-336.10+0.001i Hz

- Amplification of 2 mode may help ELM suppression
- MARS-F eigenvalue problem is difficult due to Alfven continue spectrum





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Eigenmode Extraction of FDM and TDM can be Obtained from both Experimental and Numerical Data

FDM and TDM are both developed from generalized linear-MHD theory:

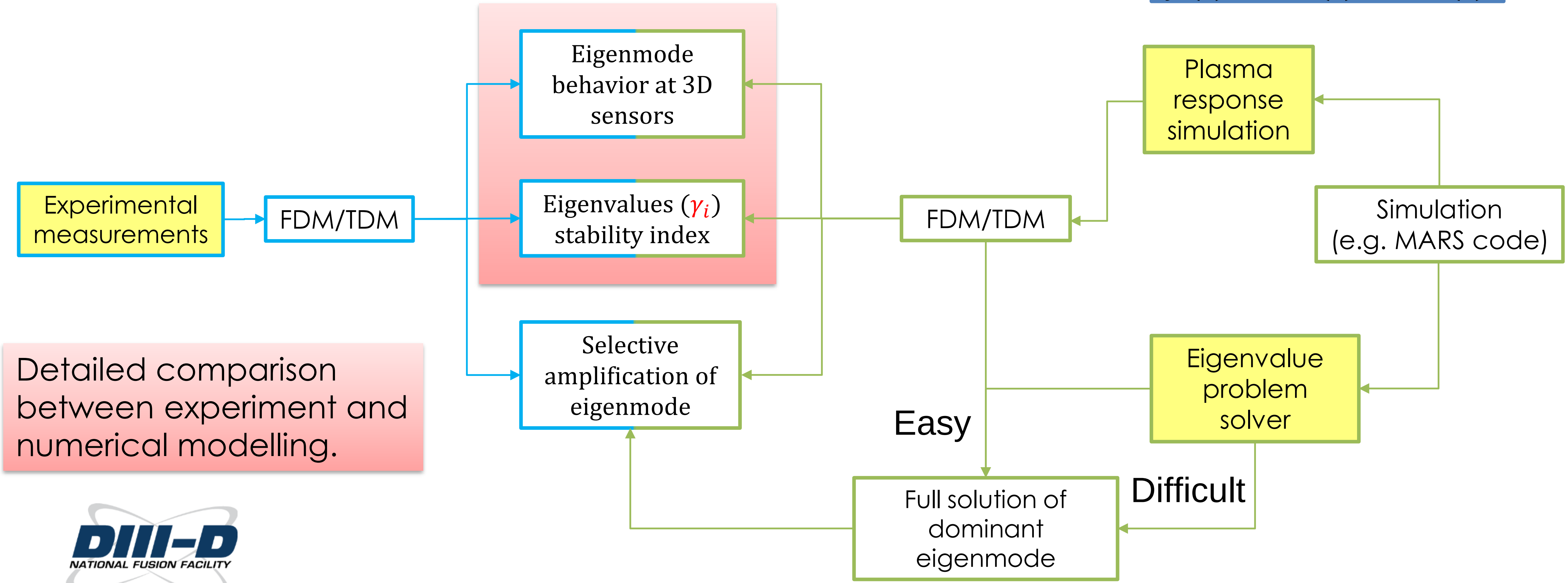
FDM :

$$P_j(\Delta\phi, f) = \frac{\delta B}{I_{up}} = \sum_{i=1}^{N} \frac{a_i^j + b_i^j e^{-i\Delta\phi}(I_{low}/I_{up})}{i2\pi f - \gamma_i} + c^j + d^j e^{-i\Delta\phi}(I_{low}/I_{up})$$

TDM :

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$



Detailed comparison between experiment and numerical modelling.

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Need a Balance between Accuracy and Efficiency

3D Coil signal

$$U = \begin{bmatrix} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+2} & \cdots & u_{2i+j-2} \end{bmatrix}$$

3D magnetic sensor measurement

$$Y = \begin{bmatrix} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & y_{2i+2} & \cdots & y_{2i+j-2} \end{bmatrix}$$

- All the operations are **based on U and Y**.
- TDM parameters decide structure of U and Y, thus the **calculation time**.
- TDM parameters:
  - Time sequence of sample data: Ns.
  - Row number parameter of matrix U and Y: i.
  - Column number parameter of matrix U and Y: j=Ns-2i+1.
  - Number of eigenmodes included: N.

Note: Increasing **i** can improve accuracy but increase the amount of **U** and **Y**. Then lower efficiency.

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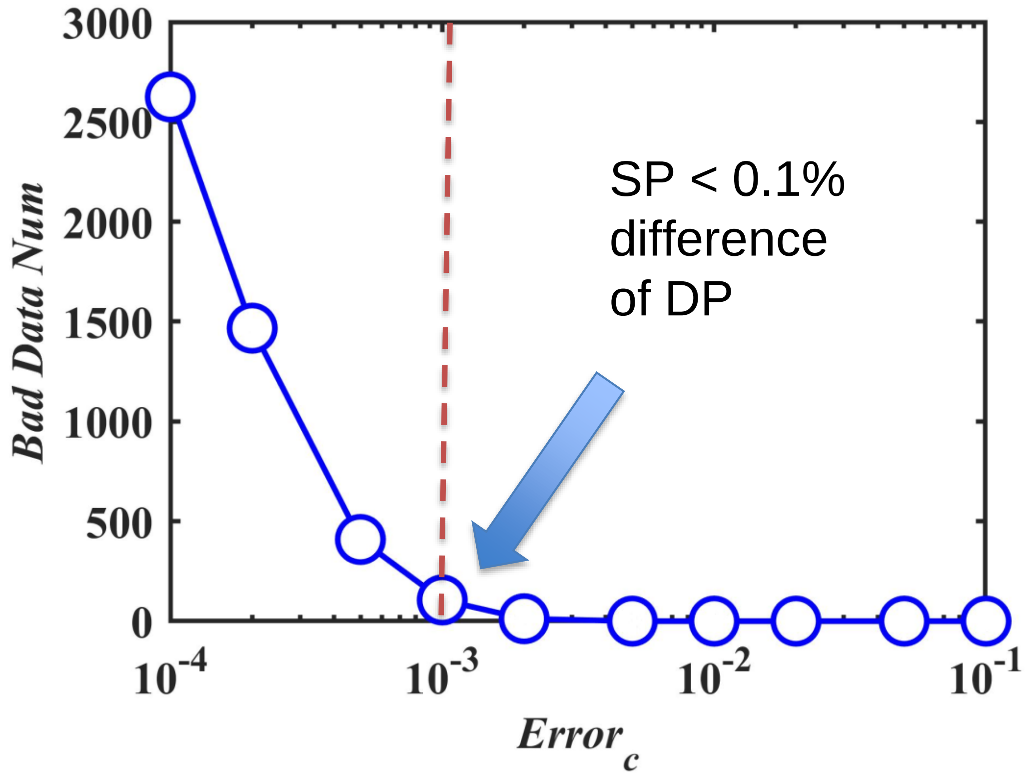
Essential to Develop Single precession Version of RT 3D MHD Spectroscopy

PCS cannot handle double precession (DP), it is necessary to test the difference between single precession (SP) and double precession



$$Error = \frac{|Eig_d - Eig_s|}{|Eig_d|}$$

$$BadNum = \sum_{\substack{i=11\sim20 \\ N=2\sim4 \\ case=40*3}} Error > Error_c$$

- The difference between SP and DP is less than 0.1%.
- SP is sufficient for this method.

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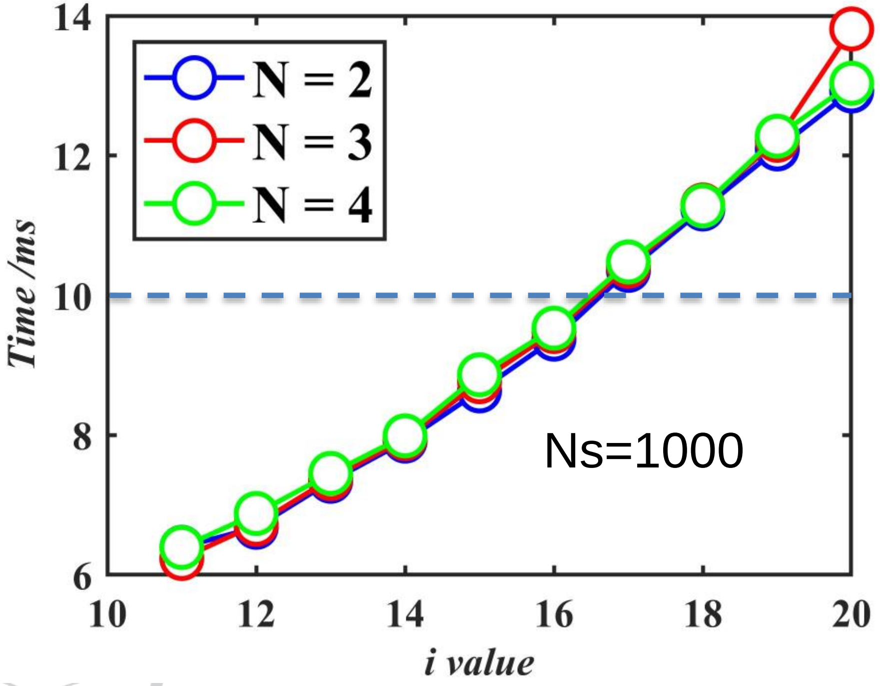
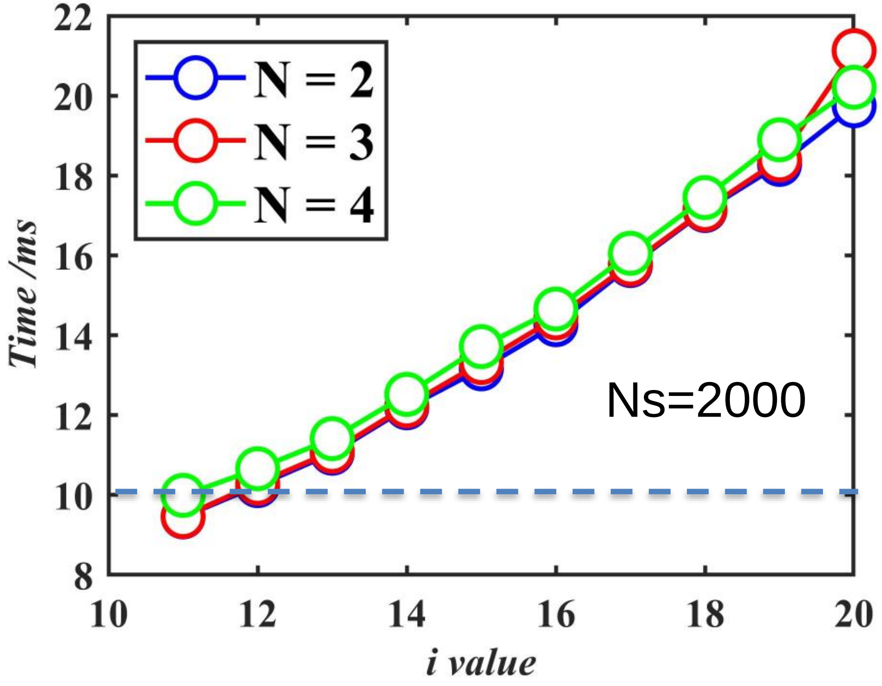
Selection of TDM Parameter Can Insure Calculating Time Less Than 10ms

Different TDM parameters are scanned to test efficiency

- Time sequence of sample data: Ns=1000-2000.
- Row number of U and Y: i=11-20.
- Number of eigenmode included: N=2-4.

$$\begin{pmatrix} X_{i+1}^d \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X_i^d \\ U_{i|i} \end{pmatrix}$$





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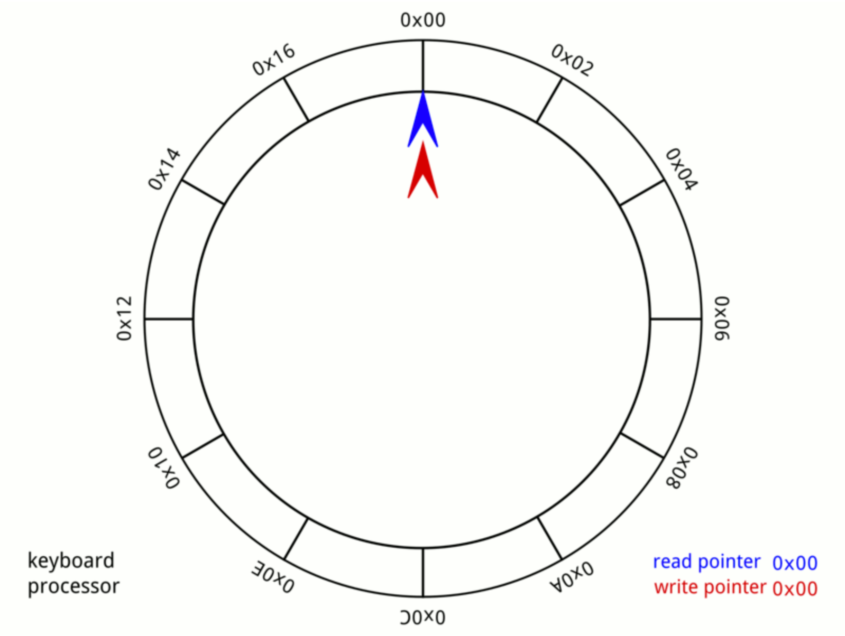
Ring Buffer is Adopted to Buffer Data

The old data will be covered up from top every time when data update



Buffer length should be set up at initial

First time fill up the buffer, then only update 2 data everytime.

0x00	0x02	0x04	0x06	0x08	…	0x14	0x16
1	2	3	4	5	…	15	16

t0 → t1

t1 → t1+ε

0x00	0x02	0x04	0x06	0x08	…	0x14	0x16
17	18	3	4	5	…	15	16

t1 → t2

t2+ ε1 → t2 (t2+ ε) → t2+ ε1

0x00	0x02	0x04	0x06	0x08	…	0x14	0x16
17	18	19	20	5	…	15	16

t2 → t3

t3+ ε2 → t3 (t3+ ε) → t3+ ε2

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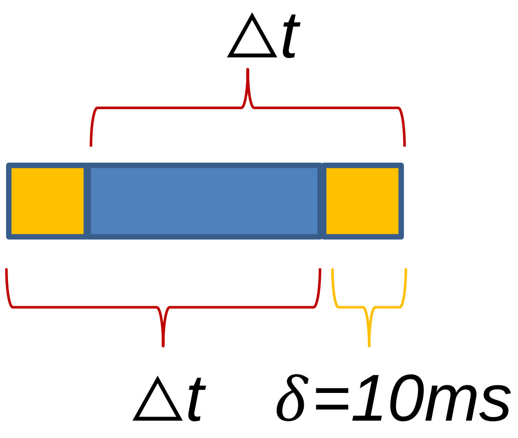
Total Time for Data Passing and Calculating Should Be Less Than Updating Eigenvalue Time Interval ~ 10ms

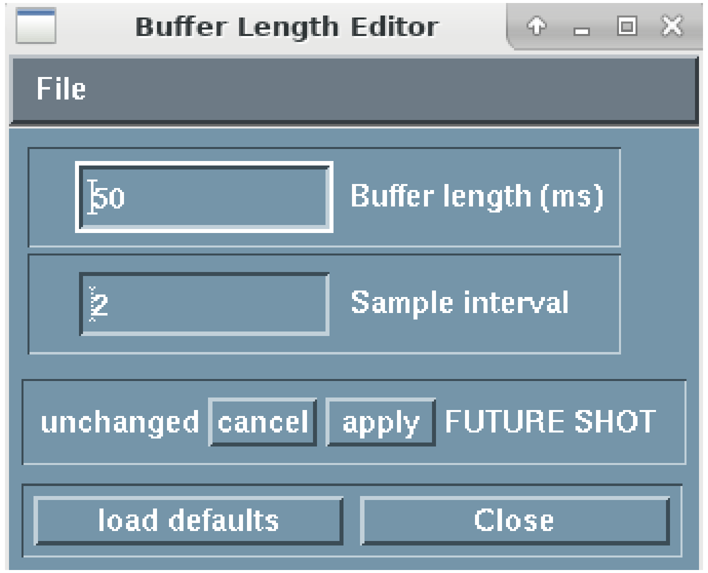
0x00	0x02	0x04	0x06	0x08	…	0x14	0x16
17	18	19	20	5	…	15	16

δ=10ms

Δt



- ➢ **Δt : Fitting time window**
- ➢ **δ : Time interval updating eigen value (10ms or faster)**



- ➢ Sample time in PCS is 5e-5s.
- ➢ 100ms will include 0.1/5e-5=2000 sample data.
- ➢ Skip data to lower amount of sample data.

Real-Time Extraction of n=1 2 3 Component in PCS

Real number signal measured on sensors should be transformed into complex signal before calculation.

$$Ae^{i(n\varphi+\omega t)} = A\cos(n\varphi)\cos(\omega t) - A\sin(n\varphi)\sin(\omega t)$$

$$A\cos(\omega t) = n1\cos_1 * sensor_1 + \cdots + n1\cos_8 * sensor_8$$

$$A\sin(\omega t) = n1\sin_1 * sensor_1 + \cdots + n1\sin_8 * sensor_8$$

$$\omega t = \arctan\frac{A\sin(\omega t)}{A\cos(\omega t)}$$

$$A = \sqrt{A\sin(\omega t)^2 + A\cos(\omega t)^2}$$

- Transformation could be done easily by C-Matrix.
- Constant Matrices are set in PCS for each sensor arrays and each current coils.
- It is convenient to exclude broken sensors by filling out the coefficient row with zeros.

Select matrix

[0] LFS
[1] HFS

	n1 sin	n1 cos	n2 sin	n2 cos	n3 sin	n3 cos
Sensor1	0.22	-0.16	0.96	-0.27	1.01	-0.97
Sensor2	0.18	0.03	0.75	0.11	-0.33	0.39
Sensor3	0.19	0.34	0.29	0.13	-0.68	0.10
Sensor4	0.30	0.02	-0.61	0.86	0.69	-0.70
Sensor5	-0.43	0.44	-0.53	-0.09	0.34	0.50
Sensor6	-0.53	-0.28	0.12	-0.42	-0.48	-0.18
Sensor7	-0.02	-0.28	-0.40	0.52	-0.26	0.46
Sensor8	0.08	-0.11	-0.58	-0.84	-0.29	0.41

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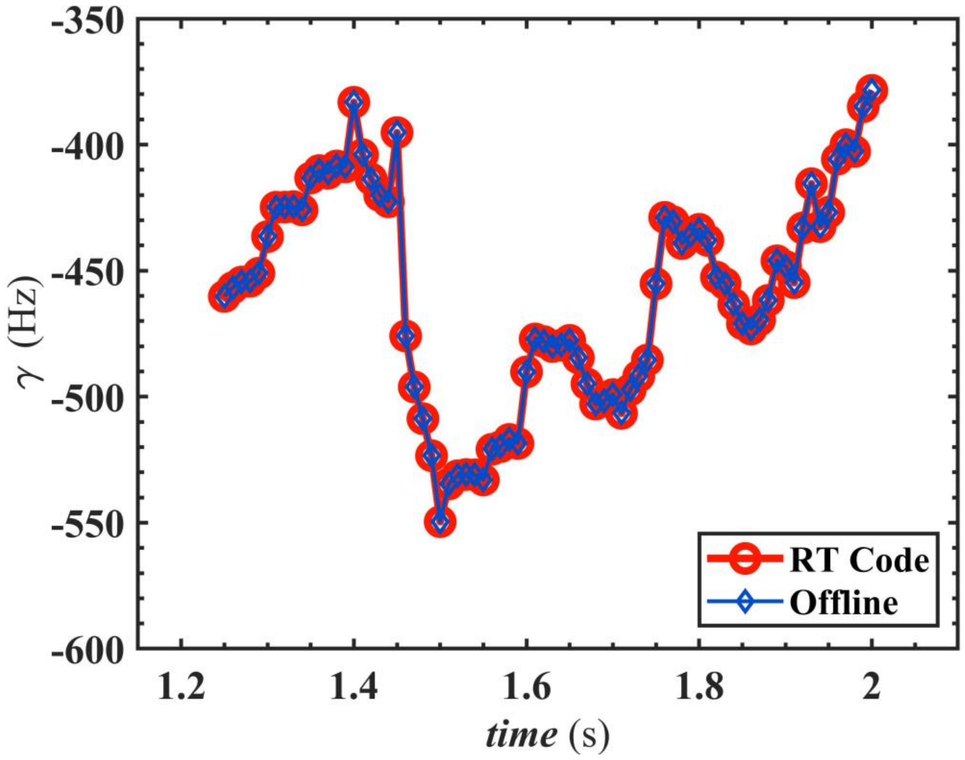
Results of RT Code's Calculation Are Same as Offline Code

Eigenvalue extracted by RT code can be exactly the same as offline code.



- Offline code is written by Matlab. While RT PCS code is written by C.
- There are some limits in PCS such as precession.
- More complicated when operating matrices but more efficient by C than Matlab.

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Summarize Progress of TDM Real-Time Detection of MHD Stabilities Developed

- **Measured multi-modes evolving with time are observed in DIII-D experiments**
- **TDM is applied in DIII-D experiment to validate the efficiency of potential to implement real-time detection**
  - The shortest fitting time window for DIII-D is 50ms
  - The cleaner signal improves the convergence and enable the shorter fitting window
- **TDM has been developed in DIII-D Plasma Control System for real-time detecting of plasma stability**
  - Calculation part is finished and tested well.
  - Data passing part is almost done. (Needs Multiple CPUs Version)
- **The efficiency has been validated to be feasible for real-time detection**
- **The PCS Code will be tested in DIII-D experiment to show the stability change**

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Backup

Calculation Amount Depends on Initial Parameters While Constructing U and Y

- Time sequence of sample data: Ns.
- Row number parameter of matrix U and Y: i.
- Column number parameter of matrix U and Y: j=Ns-2i+1.
- Number of eigenmodes included: N.

$$U = \begin{bmatrix} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+2} & \cdots & u_{2i+j-2} \end{bmatrix}$$

Note:

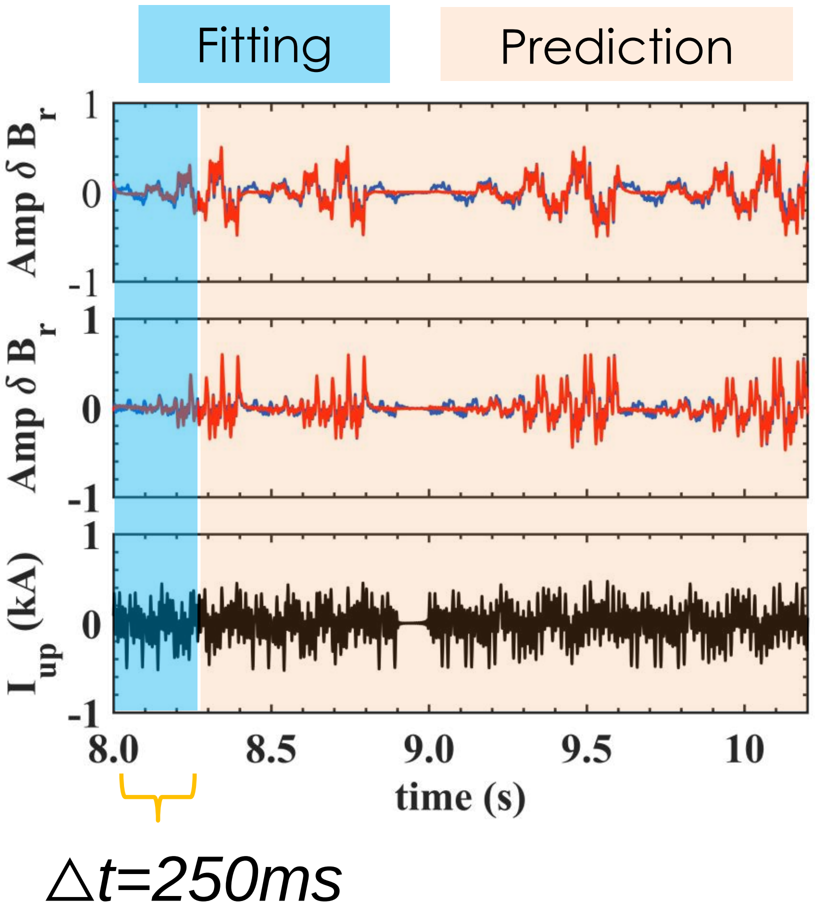
1. At least i>N, j>2i.
2. Increasing i can improve accuracy but increase the amount of U and Y. Then lower efficiency.

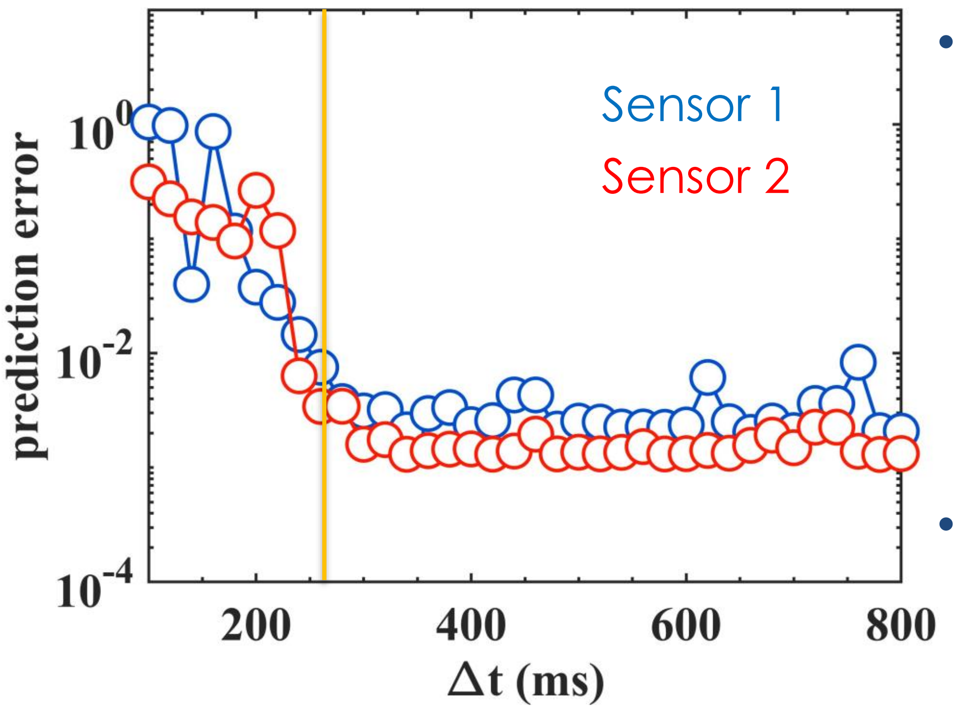
$$Y = \begin{bmatrix} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & y_{2i+2} & \cdots & y_{2i+j-2} \end{bmatrix}$$

Fitting Window for K-STAR Is 250ms to Show a Good Agreement with Experiment Data

Fitting model by using short time interval in K-STAR

$$\text{Prediction Error} = \frac{1}{N}\sum_{i}^{N}(y_p(i) - y_o(i))^2$$





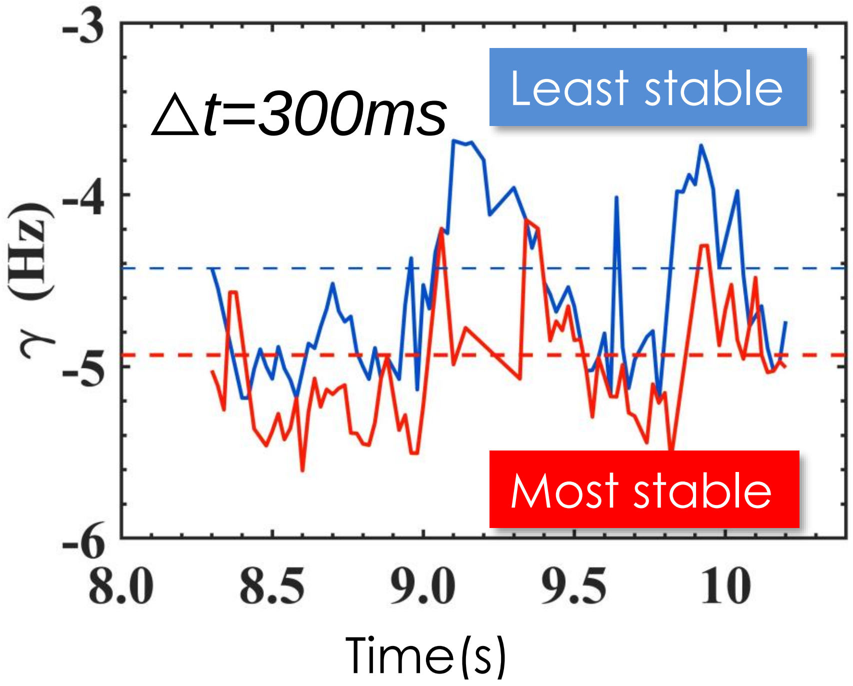
Fitting time window

- Long time prediction of plasma response (low χ^2) is successfully achieved by fitting 3D sensor signals in a small time window (Δt=250ms)
- Travelling wave is better for short time fitting

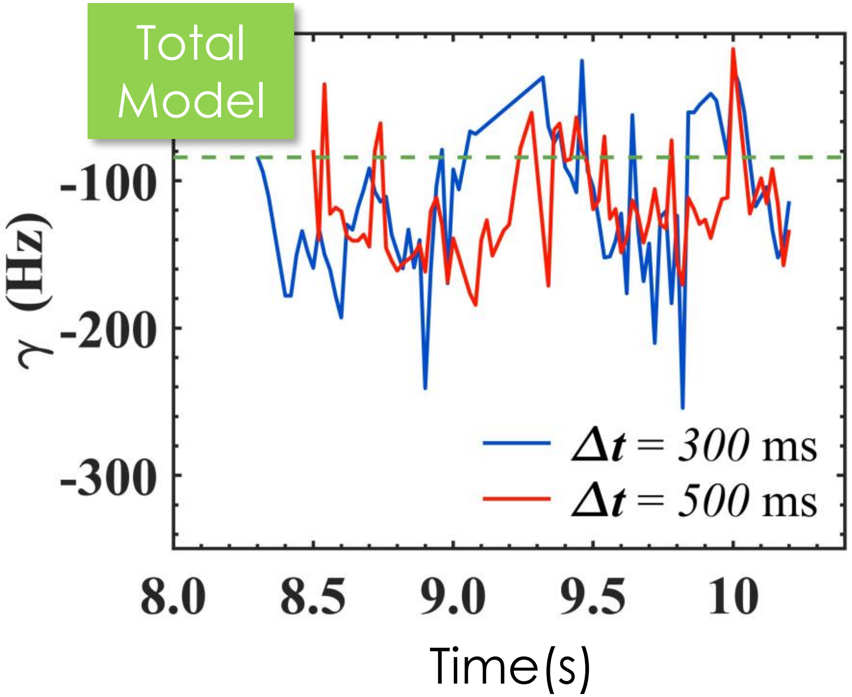
Monitoring Time Evolution of Multi-Mode Instability by TDM in K-STAR Tokamak

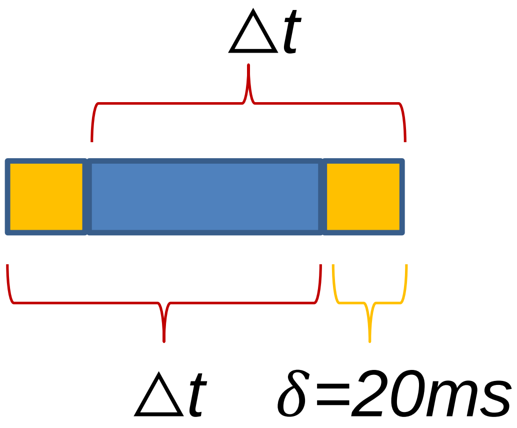
Streaming analysis by TDM are tested in K-STAR experiments (Shot No. 21030):

Two dominant modes are observed



Evolution of least stable mode for different time window





- Δt : fitting time window
- δ : time interval updating eigen value (20ms or faster)